EL9343 Homework 4 Solutions

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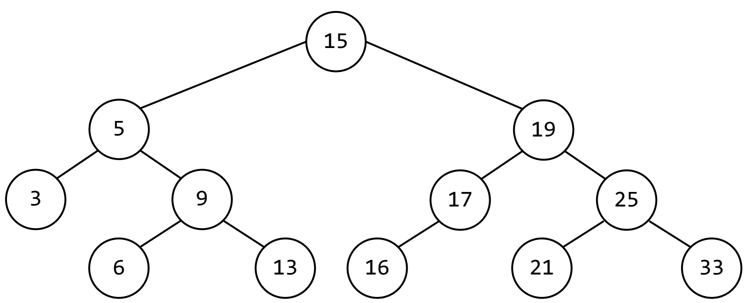
1. (Exercise 12.2-1 in CLRS) Options (c) and (e) are not possible. For (c), we took a left path from 911, but we reach $912 > 911$. For (e), we took a right path from 347, but we reach $299 < 347$.

2. (Exercise 12.2-5 in CLRS) Suppose node x has successor s, which has a left child s'. Since s must be in the right sub-tree of x (prove it with reasoning similar to this one) and therefore so is s', we have $s \geq s' \geq x$. But if s is the successor of x, it must be the smallest in x's right sub-tree, meaning s' cannot be smaller, which is a contradiction. The proof for the predecessor is similar.

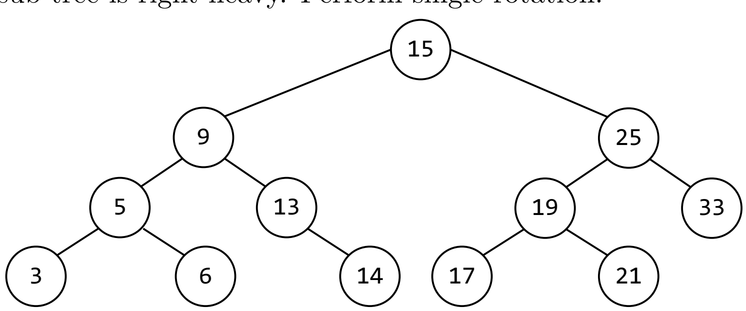
3. (Exercise 12.3-3 in CLRS) If the array is already sorted, creating a BST by calling TREE-INSERT results in a linear tree with height $n-1$ and takes $\Theta(n^2)$ steps. On the other hand, if the BST happens to be balanced, then it takes $\Theta(n \log n)$ steps. In-order tree walk takes $\Theta(n)$ steps.

4. Outside cases

(a) Insertion in the left sub-tree of the left child of 25: Insert $15 < x < 19$, say, 16. Left sub-tree is left-heavy. Perform single rotation.

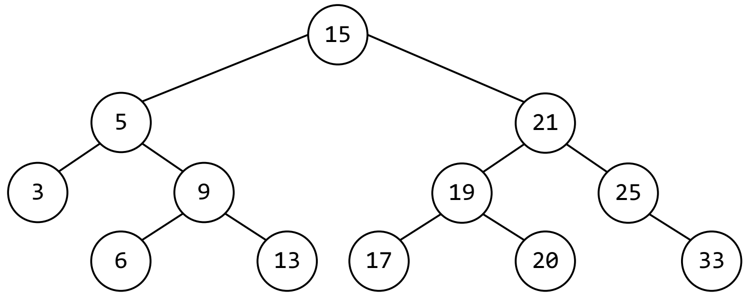


(b) Insertion in the right sub-tree of the right child of 5: Insert $9 < x < 15$, say, 14. Right sub-tree is right-heavy. Perform single rotation.

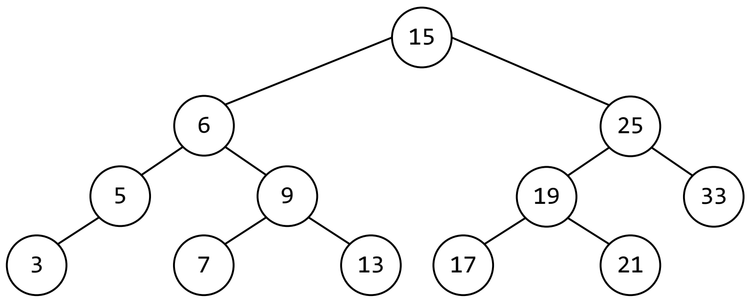


Inside cases

(a) Insertion in the right sub-tree of the left child of 25: Insert $19 < x < 25$, say, 20. Left sub-tree is right-heavy. Perform double rotation.



(b) Insertion in the left sub-tree of the right child of 5: Insert $5 < x < 9$, say, 7. Right sub-tree is left-heavy. Perform double rotation.

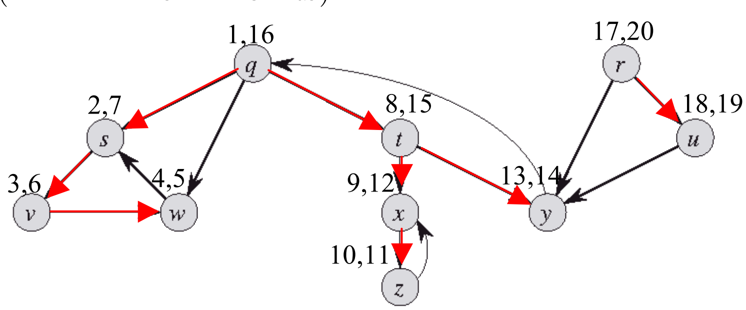


5. (Exercise 22.1-6 in CLRS) Check $A[i,j]$ for $i \neq j$, starting from $[1,2]$. If $A[i,j] = 0$, then j cannot be the universal sink. If $A[i,j] = 1$, then i cannot be the universal sink. Replace the eliminated vertex with the smallest vertex that has not been examined and repeat until only one candidate vertex i remains ($V - 1$ steps). Finally check if vertex i is the universal sink by examining its corresponding row and column in $2V - 1$ steps. $\Rightarrow \Theta(V)$

6. (Exercise 22.2-5 in CLRS) We know that BFS finds a shortest path from the source to any other vertex. Since the minimum distance is independent of the order of the adjacency list, $u.d$ would be the same.

On Figure 22.3, assume x precedes t and u precedes y in the corresponding adjacency lists. Then BFT would include the (x,u) edge instead of (t,u).

7. (Exercise 22.3-2 in CLRS)



Tree edges: $(q,s),(s,v),(v,w),(q,t),(t,x),(x,z),(t,y),(r,u)$
Back edges: $(w,s),(z,x),(y,q)$
Forward edges: (q,w)
Cross edges: $(r,y),(u,y)$

8. (Exercise 22.3-10 in CLRS) For the edge (u,v), if v is

(a) white, (u,v) is tree edge.

(b) gray, (u,v) is back edge.

(c) black and u discovered earlier, (u,v) is forward edge.

(d) black and v discovered earlier, (u, v) is cross edge.

9. (Problem 22-1 in CLRS) BFS on undirected graph:

(a) Since BFS first explores all white neighbors of a vertex, there cannot be any forward or back edges, all such edges would be in the tree.

(b) For each ordered tree edge (u, v), v is added through u, and while adding, we set $v.d = u.d + 1$.

(c) Let u be enqueued before v. v is then closer to the tail. From Lemma 22.3, we know that the difference between the distances of the tail and the head of the queue can at most be one, with tail's distance being greater. So we must have either $v.d = u.d$ or $v.d = u.d + 1$.

BFS on directed graph:

(a) Similar to above.

(b) Similar to above.

(c) Holds for all edges.

(d) Since v was visited before u, we have $v.d \leq u.d$. Also, $v.d \geq 0$ for any v.