EL9343 Homework 6 Solutions

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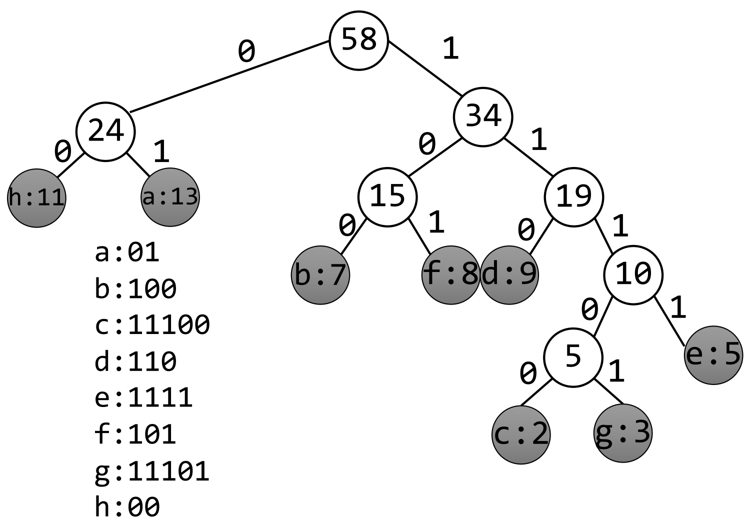
1. Huffman coding



2. (Problem 16-1 in CLRS)

(a) This is known as the cashier's algorithm.

Algorithm 1 cashier's algorithm(x)

$S = \emptyset$
while $x > 0$ **do**
  $c \leftarrow \max\{1, 5, 10, 25\}$ such that $c \le x$
  $x \leftarrow x - c$
  $S \leftarrow S \cup \{c\}$
end while
return S

(i) Denote the number of pennies, nickels, dimes and quarters by P, N, D and Q, respectively. In the optimal solution, we must have $P \le 4$, otherwise we could replace every $5P$ with $1N$. Similarly, we must have $N \le 1$, otherwise replace every $2N$ with $1D$. Also, $D \le 2$, otherwise replace every $3D$ with $1Q$ and $1N$; and also we cannot have $2D$ and $1N$, which is replaceable by $1Q$. The last two constraints can be summarized by $N + D \le 2$.

(ii) Now, let $c_1 = 1, c_2 = 5, c_3 = 10, c_4 = 25$ be the coin denominations. To make $c_k \le x < c_{k+1}$ cents of change, greedy solution takes the largest coin denomination available c_k. If the optimal solution doesn't take c_k, then it needs to reach x using smaller coins. However, this is not possible due to the constraints above.

Examples: For $5 \le x < 10$, maximum we can reach with only pennies is $4P = 4$ (we showed it would be sub-optimal otherwise). For $10 \le x < 25$, maximum we can reach with only pennies and nickels is $1N + 4P = 9$. For $25 \le x$, maximum we can reach with pennies, nickels and dimes is $2D + 4P = 24$.

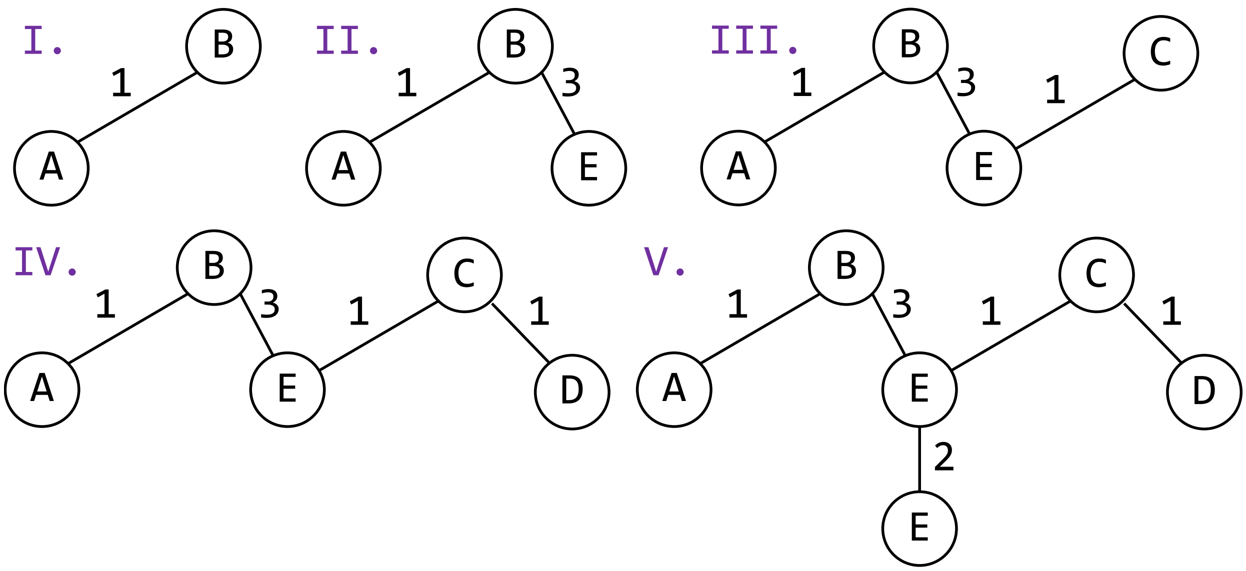
(iii) The problem reduces to making change for $x - c_k$. $\Rightarrow$ this is solved by Algorithm 1.

(b) Similar to the proof above

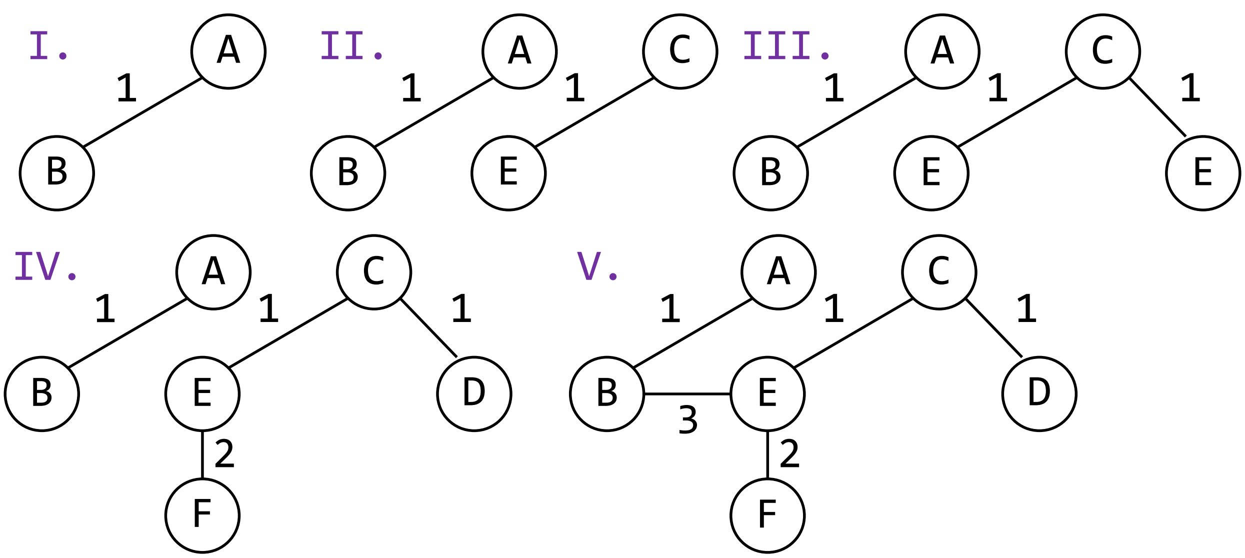
(c) Make 14 cents of change w/ coins of $\{1, 5, 7, 10\}$
Greedy: one 10, four 1s
Optimal: two 7s

(d) Dynamic programming

3. Prim's algorithm:

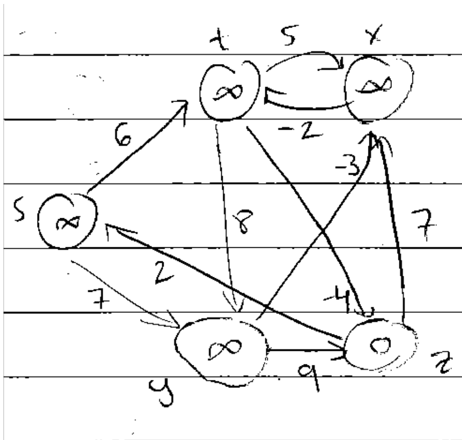


Kruskal's algorithm:

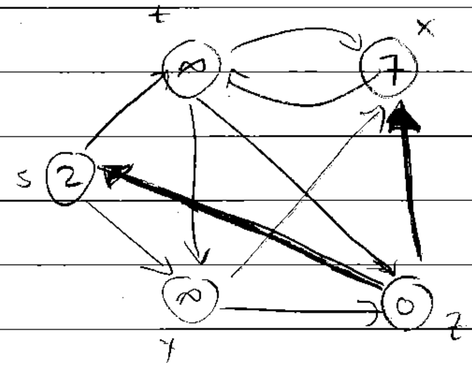


4. (Problem 23-1 in CLRS)

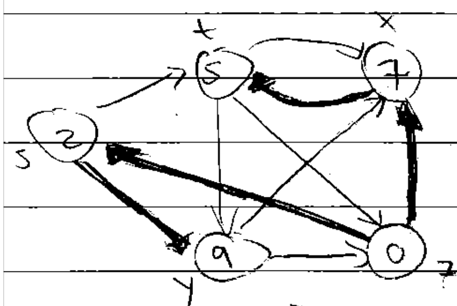
5. (Exercise 24.1-1 in CLRS)



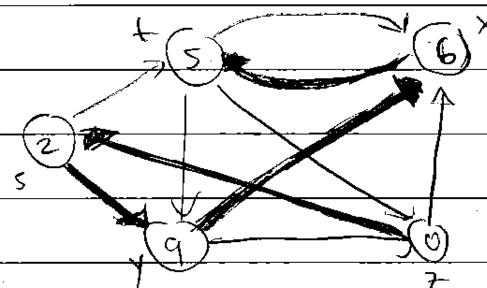
(a)



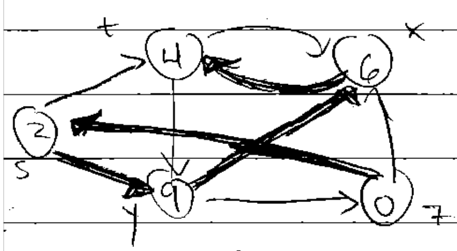
(b) 1



(c) 2



(d) 3



(e) 4

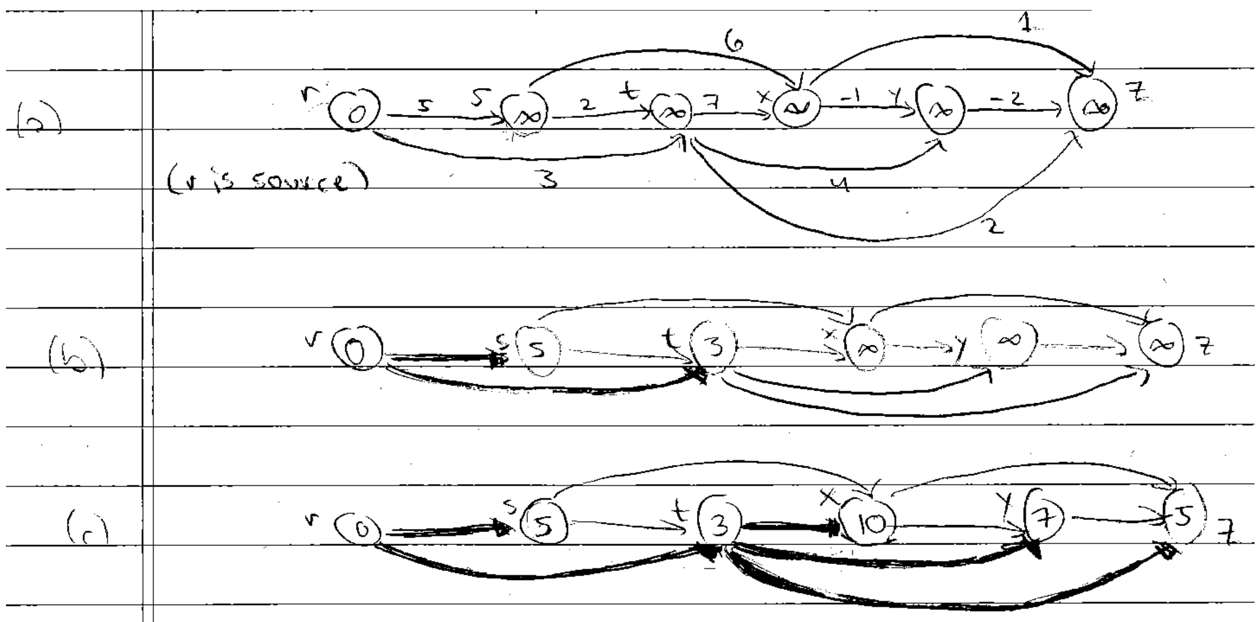
(no more changes after this step)

	s	t	x	y	z
d	2	4	6	9	0
π	z	x	y	s	nil

for $w(z,x)=4$ and s as source:

	s	t	x	y	z
d	0	0	2	7	-2
π	nil	x	z	s	t

6. (Exercise 24.2-1 in CLRS)



(no more changes on future iterations)

	r	s	t	x	y	z
d	0	5	3	10	7	5
π	nil	r	r	t	t	t

7. (Exercise 24.3-1 in CLRS)

Set S	d: s	t	x	y	z	π: s	t	x	y	z	iteration
z	∞	∞	∞	∞	0	–	–	–	–	–	↓
z, s	3	∞	7	∞	0	z	–	z	–	–	
z, s, t	3	6	7	8	0	z	s	z	s	–	
z, s, t, x	↓ (no changes)					↓ (no changes)					
z, s, t, x, y											

8. (Problem 24.3-6 in CLRS) Maximizing the probability along paths is equivalent to maximizing the log-likelihood along paths, which itself is equivalent to minimizing its negative. So define $w(u,v) = -\log r(u,v)$ and run Dijkstra's algorithm.

9. (Exercise 25.2-1 in CLRS)

$$D^{(0)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & \infty & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & 3 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{bmatrix} \quad D^{(1)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{bmatrix}$$

$$D^{(2)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix} \quad D^{(3)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix}$$

$$D^{(4)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix} \quad D^{(5)} = \begin{bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix}$$

$$D^{(6)} = \begin{bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ -5 & -3 & 0 & -1 & -6 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix}$$

10. (Exercise 25.2-2 in CLRS)

Let $w_{i,j} = 1$ if (i, j) is an edge, 0 otherwise. Then run SLOW-ALL-PAIRS-SHORTEST-PATHS, modifying line 7 in EXTEND-SHORTEST-PATHS to $l'_{i,j} = l'_{i,j}$ OR (l_{ik} AND w_{kj}).