练习12.20

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习题 (12.20). 证明在矩形 $x_0 \le x \le x_1, y_0 \le y \le y_1$ 上,积分

$$\iint \frac{\partial^2 F(x,y)}{\partial x \partial y} dxdy$$

等于

$$F(x_1,y_1) - F(x_0,y_1) - F(x_1,y_0) + F(x_0,y_0).$$

证明. 易得

$$\int \frac{\partial^2 F(x,y)}{\partial x \partial y} dx = \frac{\partial F(x,y)}{\partial y} + C.$$

因此

$$\int_{x_0}^{x_1} \frac{\partial^2 F(x,y)}{\partial x \partial y} dx = \frac{\partial F(x_1,y)}{\partial y} - \frac{\partial F(x_0,y)}{\partial y}.$$

下面我们来看

$$\int \left(\frac{\partial F(x_1,y)}{\partial y} - \frac{\partial F(x_0,y)}{\partial y} \right) dy = F(x_1,y) - F(x_0,y) + D.$$

因此

$$\int_{y_0}^{y_1} \left(\frac{\partial F(x_1,y)}{\partial y} - \frac{\partial F(x_0,y)}{\partial y} \right) dy = [F(x_1,y_1) - F(x_1,y_0)] + [F(x_0,y_0) - F(x_0,y_1)].$$

□