LAB - Extra DDPM Training Objective Proof

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Start from $\rightarrow$ $E_q\left[-\log \dfrac{p_\theta(x_{0:T})}{q(x_{1:T}|x_0)}\right]$

Note:

$$q(x_t|x_{t-1},x_0) = \frac{q(x_{t-1}|x_t,x_0)\,q(x_t|x_0)}{q(x_{t-1}|x_0)}$$

$$\Rightarrow E_q\left[-\log \frac{p(x_T)\prod_{t=1}^{T} p_\theta(x_{t-1}|x_t)}{\prod_{t=1}^{T} q(x_t|x_{t-1})}\right]$$

$$\Rightarrow E_q\left[-\log \frac{p(x_T)\,p_\theta(x_0|x_1)\prod_{t=2}^{T} p_\theta(x_{t-1}|x_t)}{q(x_1|x_0)\prod_{t=2}^{T} q(x_t|x_{t-1},x_0)}\right]$$

$$\Rightarrow E_q\left[-\log \frac{p(x_T)\,p_\theta(x_0|x_1)}{q(x_1|x_0)} - \log \prod_{t=2}^{T} \frac{p_\theta(x_{t-1}|x_t)}{q(x_t|x_{t-1},x_0)}\right]$$

$$\Rightarrow E_q\left[-\log \frac{p(x_T)\,p_\theta(x_0|x_1)}{q(x_1|x_0)} - \log \prod_{t=2}^{T} \frac{p_\theta(x_{t-1}|x_t)}{q(x_t|x_{t-1},x_0)}\right]$$

$$\Rightarrow E_q\left[-\log \frac{p(x_T)\,p_\theta(x_0|x_1)}{q(x_1|x_0)} - \log \prod_{t=2}^{T} \frac{p_\theta(x_{t-1}|x_t)}{q(x_{t-1}|x_t,x_0)} \cdot \frac{q(x_{t-1}|x_0)}{q(x_t|x_0)}\right]$$

$$\Rightarrow E_q\left[-\log \frac{p(x_T)\,p_\theta(x_0|x_1)}{\cancel{q(x_1|x_0)}} - \log \frac{\cancel{q(x_1|x_0)}}{q(x_T|x_0)} - \log \prod_{t=2}^{T} \frac{p_\theta(x_{t-1}|x_t)}{q(x_{t-1}|x_t,x_0)}\right]$$

$$\Rightarrow E_q\left[-\log \frac{p(x_T)\,p_\theta(x_0|x_1)}{q(x_T|x_0)} - \sum_{t=2}^{T} \log \frac{p_\theta(x_{t-1}|x_t)}{q(x_{t-1}|x_t,x_0)}\right]$$

$$\Rightarrow E_q\left[-\log p_\theta(x_0|x_1) - \underline{\log \frac{p(x_T)}{q(x_T|x_0)}} - \underline{\sum_{t=2}^{T} \log \frac{p_\theta(x_{t-1}|x_t)}{q(x_{t-1}|x_t,x_0)}}\right]$$

$$\Rightarrow E_q\left[-\log p_\theta(x_0|x_1) + D_{KL}(q(x_T|x_0)\,\|\,p(x_T)) + \sum_{t=2}^{T} D_{KL}(q(x_{t-1}|x_t,x_0)\,\|\,p_\theta(x_{t-1}|x_t))\right]$$

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