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A heuristic approach to long-haul freight transportation with multiple objective functions ☆

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Abstract

This paper studies a long-haul freight transportation problem stimulated by a real-life application, whose underlying vehicle routing problem is a multi-objective one, where travel time and route cost are to be minimized together with the maximization of a transportation mean sharing index, related to the capability of the transportation system of generating economy scale solutions. In terms of constraints, besides vehicle capacity and time windows, transportation jobs have to obey additional constraints related to mandatory nodes (e.g., logistic platform nearest to the origin or the destination) and forbidden nodes (e.g., logistic platforms not compatible with the operations required). Based on the network definition, routes can be multimodal. To solve this problem, we propose a heuristic algorithm that can be applied in the tactical and the operational planning phase, and present the results of an extensive experimentation.

Keywords: Multimodal/intermodal transport; Logistic platform; Multi-objective optimization; Local search

1. Introduction

The improvement and promotion of transport modes alternative to road, both for passenger and freight mobility, is one of the European Union policy priorities in order to develop a sustainable European transport system [1,2]. The modal shift towards more sustainable transport modes, such as rail, short-distance sea shipping and inland waterways, passes through the promotion of multimodality.

Multimodal transport, as defined by the European Conference of Ministers of Transport, is the carriage of goods by at least two different modes of transport, without any handling of the goods themselves in transhipment between the modes [3]. The multimodal transport system is more complex to model and to manage than the unimodal one. Indeed, effective multimodal solutions require both to cope with the customers desiderata in terms of times, costs, reliability and to take into account the shippers operational needs; in other words, transport solutions have to be easy to implement, flexible, reliable, transparent and efficient. In this context, operations research techniques (i.e., optimization models and solution methods) play a crucial role in order to support the specific decisions that have to be made by the different decision

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makers, operating in the multimodal transport system [4–7].

One of the difficulties to face in organizing multimodal transport is to get a wide and deep knowledge about transport offer, such as rail and maritime services routes and schedules, platforms location and facilities, times and fares, and so on. Another crucial point is that multimodality widens the set of the possible solutions and, very often, it is difficult to define an optimum respecting the numerous objectives (cost, time, reliability, etc.); multi-objective optimization techniques are then a very suitable approach for the full exploitation of all the available alternatives.

In this work, we consider a decision maker represented by a logistic operator (LO) that has to manage a set of freight shipments from their origins to their destinations, respecting a number of constraints related to the route and the freight type. To route such shipments, the LO uses both his own transportation means and services from other transport operators (TOs). Some of the latter are timetable services with prefixed starting time and arrival time. The decision maker has to find the routes that optimize certain objective functions, and, for the reasons aforementioned, he would like to encourage multimodal routes. In our problem, when road mode is used, we have full truckload shipments because of the large delivery quantities. We recall that a less than truckload shipment is delivered with other shipments and, in general, is not directly processed to a destination as in full than truckload transportation. For a comprehensive study on less than truckload and full truckload transportation the reader is referred to [5].

On the basis of these considerations, in this paper, referring to a real-life application, we study how to support the LO in all the phases of the decision process for multimodal transport organization. What we propose is able to provide a global answer at the following two different planning levels [5]:

- *services network design* (e.g., tactical phase), in which multimodal operators aim at defining (and then starting up) the best set of transport services and logistics that serves customers' generic requests;
- *transportation programming* (e.g., operative phase), when the operator decides how to satisfy specific customer requests.

The remainder of the paper is organized as follows. Section 2 is devoted to the description of the problem. Section 3 contains the literature review. Section 4 describes the proposed solution approach. In Section 5, the validity of the proposed algorithm is evaluated by considering a real case study, and, for completeness, randomly generated networks. The paper closes with some final remarks.

2. Problem description

In the classical vehicle routing problem, a fleet of capacitated vehicles has to be routed over a network to serve customer demands located in the nodes of a directed weighted graph. There can be one or more than one depot from which routes start and finish within a certain time limit (e.g., defined by the duration of the driver shift). In general, in this scenario, time windows are provided and it is required that each customer must be served within its time window. The basic vehicle routing problem with time windows (VRPTW) has been widely investigated in the literature; for some recent review on the topic the reader is referred to, e.g., [8].

A branch of the VRPTW literature (mainly related to on-demand transportation [9,10,14], hazardous material transportation [11,12], and advanced traffic management systems [13]), focuses on transportation problems where the network is designed to allow alternative routes to serve the demand of an origin–destination pair. In these scenarios, the alternative routes are defined by Pareto optimal arcs that model the chance to travel between two nodes with, for instance, a faster but more expensive way and a slower but cheaper way.

Our problem is defined along these lines. A LO has to manage a set of commodities that have to be moved from their origins to their destinations on a transportation network. The LO considered in this paper operates mainly in Italy in the distribution logistic sector with particular focus on road and rail transportation. It possesses more than 1500 trains (with more than 30,000 wagons) and manages 15 logistic platforms. Many are the types of freight that it handles: from cloths to food and beverage to building materials.

Each transportation job is associated with given pickup and delivery time windows, meaning that there is a ready time for each freight pickup operation with a related maximum on board loading time, as well as a minimum and a maximum time to reach the destination and unload products, respectively.

The transportation network considered in this paper contains alternative routes; in fact, each arc can be traversed by different transportation means, thus generating a multimodal multigraph (that represents a more general setting than that in the current literature dealing with vehicle routing with alternative paths, see, e.g., [14]). This means that there are arcs linking the same couple of nodes with the same transportation mode, but with different fares and traveling times. Also, there are arcs with the same endpoints associated with different modes, overall producing the problem of determining the set of optimal trips from the origin to the destination in the multigraph, choosing the best path composition. However, a hard constraint should be kept into account: mode changes cannot be made arbitrarily at each node (as done in many papers on multimodal networks, see e.g., [15–17]), but only at predefined nodes, the *logistic platforms*, where freight can be unloaded e.g., from a road mode (trucks) and loaded onto a rail mode (trains). At logistic platforms, changing between two road modes or two rail modes is also allowed; indeed, different road TOs and rail transport operators (ROs) operate on the network, and some portions of the latter are served only by a (small) subset of the available TOs or ROs, even by a single operator. This sheds light on the difficulty of managing the network also in a single-mode implementation; in fact, in general, a path from an origin to a destination made by e.g., road mode only, can be infeasible, and one needs to carefully evaluate the existence of a single TO operating on it, or more than one TO with logistic platforms properly located in the route to allow each intermodal change.

We remark that, in the paper, the terms "mean" and "mode" are used in general with different meanings. Consider for instance the road mode; there can be different transportation means associated with this mode, each one with, e.g., its own cost and capacity.

In the transportation problem we study, there are also additional constraints associated with each commodity: indeed, there could be mandatory nodes (e.g., the logistic platform nearest to the origin or to the destination) and forbidden nodes (e.g., the logistic platforms not agreeable with the operations required), meaning that a route may be obliged to contain and/or not to contain certain subsets of nodes. Note that forbidden nodes are often required since each logistic platform is capable of handling a certain set of operations that in general are not compatible with the requirements of all the transportation jobs. Therefore, for certain demands a platform could be inadequate, and, then, it is marked as forbidden.

With respect to the objective function, we cope with a multi-objective scenario where the traveling time and operative cost minimization is sought together with the maximization of a transportation mean sharing index to improve capacity utilization. Multi-objective optimization is quite natural if the network is multimodal: indeed, there could be solutions that minimize the number of vehicles with long length route, and other solutions that minimize the route lengths with a large number of vehicles. If one considers that minimizing the number of vehicles directly affects the vehicle costs and the labor costs, while minimizing route length is directly related to fuel and time costs, one clearly realizes that prioritizing objectives to deal with a single objective approach is very difficult.

In order to model our transportation problem, we start defining a transportation network $G=(V, E)$ structured as follows. We are given a set of nodes V and a set of arcs E. In Fig. 1, an example of a small network with 14 nodes and 19 arcs is depicted.

Each arc is associated with a mode; modes can be *road*, *rail*, *maritime* or *adduction*. The mode adduction is introduced to model logistic platforms. Indeed, we have four kinds of nodes: *road nodes*, *rail nodes*, *maritime nodes* and *logistic platform nodes*. In the example depicted in Fig. 1, nodes $1, 2, 7, 8, 13$ are road nodes, nodes $3, 4, 10, 11, 12, 14$ are rail nodes, and nodes $5, 6, 9$ are adduction nodes. Without loss of generality, maritime nodes are not taken into account.

Since a logistic platform allows a mode change, if, for instance, we are in a road node (e.g., node 8 in Fig. 1) and wish to change to a rail node (e.g., node 4), we need to traverse two adduction arcs (i.e., $(8, 9)$ and $(9, 4)$), one conducting from the road node to the platform (i.e., $(8, 9)$) and the other (i.e., $(9, 4)$) leading from the platform to the rail node. We note that, for implementation purposes, the network has two different adduction nodes for each logistic platform, e.g., see nodes 9 and 6 that are used to model the path from node 8 to 4 and from 4 to 8, respectively.

Each adduction arc is associated with a coefficient α that represents the time per unit of freight needed to unload freight from the current transportation mean and to load it to a new one. Besides, there is also a (fixed) cost to be paid when a platform is used (this cost is referred to the operations to be executed at the platform).

Each arc associated with a road, rail or maritime mode, instead, is characterized by the following three parameters:

- length, i.e., the distance, in km, between the arc nodes;
- traveling time, i.e., the time, in minutes, to traverse the arc;
- compatibility, e.g., what kinds of trains (and corresponding ROs) or trucks (and corresponding TOs) can serve the arc.

Each transportation service is associated with a variable and a fixed cost. There are two types of variable costs, i.e., $\mathrm{cost}_{\mathrm{vl}}$ and $\mathrm{cost}_{\mathrm{vq}}$, the former being the (variable) unitary length cost and the latter being the

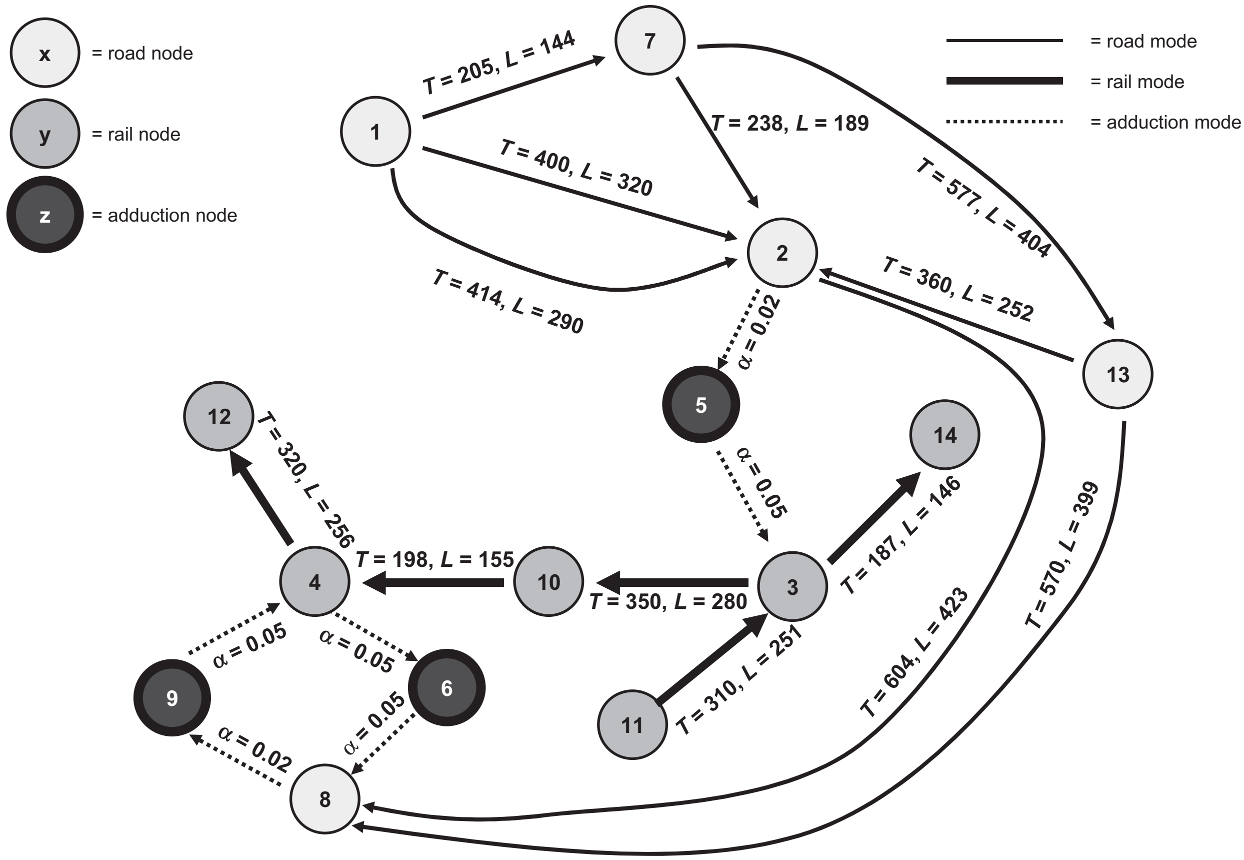

Fig. 1. Example of the multimodal network. L is the arc length in km, T is the time in minutes to traverse the arc, α is the time needed to process a unit of freight in a logistic platform.

Table 1
Example of a cost_{vq} function associated with a transportation mean

Truck_ID	
*	
3.42	20
1.82	70
1.14	145
1.07	250
1.02	$+\infty$

(variable) unitary quantity cost. These are represented as piecewise constant non-decreasing in length and quantity functions: i.e., the longer the portion of a path to be traversed with the same vehicle, the lower the unitary cost per km; similarly, the greater the quantity to be moved, the lower the unitary cost per freight unit (pallet or ton). Each TO has its own fares and thus the choice of the route is strictly affected by this. An example of how the cost function is defined is reported in Table 1. The first row reports the vehicle ID (in this case it is a truck with 35 ton capacity); the second row contains the possible fixed cost (∗ means that no fixed cost is due); the other rows define the corresponding cost_{vq} function. In particular, from 0 to 20 pallets of freight there is a unitary cost of 3.42 Euros, from 21 to 70 pallets the unitary cost is 1.82, from 71 to 145 pallets the unitary cost decreases to 1.14, from 146 to 250 it holds 1.07, and from 251 to the full capacity of the truck it is 1.02.

A road mode has both fixed and variable costs. Rail and maritime modes are associated only with a fixed cost denoted with $\text{cost}_{\text{f}}^{(\text{ra})}$ and $\text{cost}_{\text{f}}^{(\text{ma})}$, respectively, related to purchasing the schedule of a certain service, since their origins, destinations, intermediate stops and capacity are fixed. Moreover, rail and maritime services have a limited capacity, while road services are assumed to have unlimited capacity, since we assume that an arbitrarily large number of trucks can be used.

In order to give an insight on the topology of the network, assume, referring to Fig. 1, that the origin of a certain shipment is 1 and the destination is 12, and that the road mode was used to travel from 1 to 8. Since 4 is a rail node, it will be associated with a timetable defining arrival and departure times for this service. Therefore, to exploit the subpath from 4 to 12 (i.e., the destination), one has to reach 4 early enough, with respect to the train departure time, to process the loading operation; obviously, in case of frequency based trains, there is always a next train to catch regardless of the arrival time at the platform (node 9 in our example) and thus the goal is to limit the waiting time that could be detrimental in terms of arrival time to the destination. Furthermore, note that there are two arcs between nodes 1 and 2: one has a shorter traversing time but a longer length, the other has a longer traversing time and a shorter length. Moreover, note that, even if going from 1 to 2 using path 1–7–2 gives both a longer time and length, it can be associated with a TO offering a cheaper cost_{vl} and/or cost_{vq}. Consequently, in this case, traveling along the path 1–7–2 results more convenient than following the arc $(1, 2)$.

The main objective of the LO is to encourage multimodal paths; practically, the LO tends to dislike unimodal road routes, since a rail mode offers a larger capacity than e.g., a singe truck, and, therefore, allows different commodities to be shared in the same train, with a consequent decrease of the unitary operative costs; for instance, a train has a capacity about 20 times larger than a big truck. In order to achieve this objective, the LO would like to find which trains and corresponding schedules should be purchased from the railway operator. Constraints for LO can be penalty for later delivery, being the penalty a function linearly dependent on the extra time needed to the freight delivery.

When a mode change is operated at a logistic platform, some constrains arise; in fact:

- each platform has a limited capacity, i.e., a maximum number of products can be processed each day;
- daily opening and closure hours are given, and freights cannot be handled outside these time intervals.

Besides the network, we are given a set D of demands to be served; each demand $d \in D$ is associated with a pair of origin–destination nodes, and with a pickup time window PTW_d at the origin node and a delivery time window DTW_d at the destination. Note that each shipment is associated also with a frequency in the planning horizon; indeed, if for instance one considers a two week planning horizon, certain origin–destination demands are to be served once or twice a week. Moreover, let M_d and F_d be the set of mandatory and forbidden nodes, respectively, associated with each demand $d \in D$.

The overall cost C_d of a route associated with demand d is thus given by the following:

$$C_d = \sum_{\text{Platforms}} \text{CP}_d + \sum_{\text{Road}} \text{CRo}_d + \sum_{\text{Rail}} \text{CRa}_d + \sum_{\text{Maritime}} \text{CM}_d, \tag{1}$$

where CP_d represents the cost of the platforms used, CRo_d the cost of the road modes used, CRa_d the cost of the rail modes used, and CM_d the cost of the maritime modes used.

The total traveling time T_d for demand d is instead given by

$$T_d = \sum_{\text{Platforms}} \text{TP}_d + \sum_{\text{Road}} \text{TRo}_d + \sum_{\text{Rail}} \text{TRa}_d + \sum_{\text{Maritime}} \text{TM}_d, \tag{2}$$

where TP_d is the time computed by means of coefficient α related to the platforms used, and TRo_d, TRa_d, and TM_d are the traveling times associated with the road modes, rail modes and maritime modes used, respectively.

For this problem, we consider a multi-objective scenario in which time and cost should be minimized together with the maximization of a third objective function that is the transportation mean sharing index (1 − solution_cost/sum_route_costs). Note that the fraction numerator *solution_cost* is the cost associated with all the routes found for the demand set under consideration: this takes into account transportation mean sharing occurrences where transportation cost can decrease significantly. The fraction denominator *sum_route_costs*, instead, is the sum of the costs associated with all routes, $\sum_{d \in D} C_d$, considered mutually independent, i.e., without taking into account transportation mean sharing. In order to give an example, assume to have two routes using the same transportation mean, and let d_1 and d_2 be the amount of freight associated, respectively, with them. If we sum the two route costs independently, on the one hand, we sum twice the fixed cost associated with that transportation mean that, instead, has to be considered only once; on the other hand, we take the risk to have a greater cost_{vq} since the amount $d_1 + d_2$ can have a lower unitary cost than that associated with d_1 and d_2 separately. Therefore, the higher the transportation mean sharing, the higher the difference between *sum_route_costs* and *solution_cost*, and the higher the sharing index so defined.

Table 2
Constraints and objectives in the transportation problem studied

Constraints	Vehicle capacity
	Pickup and delivery time windows
	Mandatory nodes
	Forbidded nodes
	Freight-transportation mean compatibility
	Network arc-transportation mean compatibility
Objectives	Route time minimization
	Route cost minimization
	Transport-sharing index maximization

In Table 2, we summarize the set of constraints and objective functions considered in our problem.

3. State of the art

For the long-haul transportation problem, we refer the reader to the survey of Crainic [5], and the book by Ghiani et al. [22]. Referring to the multi-objective vehicle routing, authors in the literature have investigated the problem with either two or three objective functions, and with different optimization techniques. The minimization of the number of routes and of the travel costs has been considered in, e.g., [18] where the authors proposed a genetic algorithm approach to cope with this problem. Gambardella et al. [19] studied the problem with the minimization of the number of vehicles and the total costs with a hierarchical approach, in which they designed two ant colonies each one dedicated to the optimization of an objective function. Murata and Itai [20] considered the minimization of the number of vehicles and of the maximum routing time among the vehicles, using evolutionary multi-criterion optimization algorithms. Liu et al. [21] studied the problem with three objective functions, i.e., the total distance traveled by the vehicles, the balance workloads and the balance delivery times among the dispatch vehicles. They transformed the starting multi-objective program in a goal programming one, and proposed a heuristic based on one-point movement, two-point exchange, and intra-route one-exchange local searches.

In [13], Seo and Choi presented a genetic algorithm based search technique to find alternative paths between origin–destination pairs. The method can provide multiple alternatives which are nearly optimal, and is able to reduce similarities among the paths.

4. The proposed algorithm

For the problem presented in the previous section, we propose an algorithm based on local search, implemented to work both in the *tactical* and the *operational* phase.

Table 3
Functionalities of the proposed algorithm

Step 1	All non-dominated paths computation
Step 2	Path removal
Step 3	Minimizing road service cost
Step 4	Minimizing rail and maritime service time and cost

The two implementations work in cascade since the output of the tactical phase is used to build the input scenario for the operational phase.

4.1. Tactical phase

The tactical phase algorithm works in two steps: the first one denoted as *path search* (see Steps 1 and 2 in Table 3) and the second one as *transportation means assignment* (see Steps 3 and 4 in Table 3).

In searching for "good" paths, the LO is mainly interested in three path types, say pt_1, pt_2, and pt_3:

- pt_1 is a unimodal road path;
- pt_2 and pt_3 are computed so as to generate multimodal routes, forcing the inclusion of logistic platforms. This is done by adding, in the set of mandatory nodes of a certain transportation job, a logistic platform node which is not contained in the corresponding forbidden set, when the algorithm is not able to find a multimodal route with the given constraints. From the implementation viewpoint, the algorithm starts adding a dummy mandatory platform located as close as possible (in terms of time) to the origin. The rationale behind this choice is to find a multimodal path as soon as possible from the origin encouraging rail and maritime modes. If the imposed logistic platform generates an empty solution set, i.e., no feasible path to the destination exists, the algorithm removes the current dummy mandatory node inserted and iteratively moves it towards the terminal node until a feasible multimodal path is found. In case of a negative answer, the algorithm returns only the path type pt_1. Multimodal paths are classified as pt_2 if the platform is close to the origin, and pt_3 if the platform is close to the destination. Note that, even if paths pt_3 could be considered unusual, they are those whose origins are located in logistic platforms; therefore, the paths can start with, e.g., a rail mode and the platform at the end of the route is needed to allow freight to reach the destination by means of road mode.

In what follows, we present in detail the operations executed during the path search phase. The latter is achieved by means of a procedure whose functionalities are described in the following.

A first step (see Step 1 in Table 3) is related to the computation of all non-dominated paths by means of Martins' algorithm (see [23–25]). It is a multiple objective extension of Dijkstra's algorithm (see the appendix for a pseudo-code). It acts by executing a dominance check in place of the evaluation of the minimum. This modification is valid since Bellman's principle of optimality [26] can be applied to the multiple objective path case: for the networks being considered, any subpath of an efficient path is an efficient subpath.

Similarly to Dijkstra's algorithm, in Martins' algorithm, at each iteration, there are permanent and temporary labels associated with nodes to be set. The difference lies in the fact that in Dijsktra's algorithm each node is either permanent or temporary, while in Martins' algorithm each node is associated with both a set of temporary and a set of permanent labels, each one representing a path from the origin. Martins' algorithm selects the lexicographically smallest label from all the sets of temporary labels, converts it to a permanent label, and updates all the temporary labels of its successors as performed by Dijsktra's algorithm.

The algorithm terminates when we run out of temporary labels. Each permanent label corresponds to a unique efficient path. Martins' algorithm guarantees that, on the one hand, Bellman's principle of optimality holds and, on the other hand, that all the efficient paths from one node to all the other nodes of a network are computed. In our problem, Martins' algorithm is used to compute Pareto optimal paths with respect to both time and cost objectives. A slight modification is defined on the original algorithm version, since each arc in a path should be feasible with respect to the freight to be shipped from the origin to the destination.

From the resulting efficient path set, we delete (see Step 2 on Table 3) those paths which do not respect operators compatibility, i.e., those paths in which an operator change is executed without using a logistic platform. For instance, assume that a path has a subpath from i to j to k with (i, j) and (j, k) associated with different TOs; in this case the path can be deleted since it is not viable. An example can be that of assuming that in Fig. 1 arcs $(1, 7)$, $(7, 13)$ and $(13, 8)$, defining a route from node 1 to 8, are served by a certain TO, and arcs $(1, 2)$ and $(2, 8)$, also defining a route from 1 to 8, are traversed by a different TO. In this case, the path 1–7–2–8 is not viable since there is a change of TO made without using any logistic platform.

Once candidate paths have been computed, they are all stored and then transportation means can be assigned. In this phase, we have to check the existence of feasible time windows for those services associated with a prefixed timetable. This is a crucial point, since if a timetable service has a departure time that is much greater than the time at which the freight arrives at the logistic platform node, then we will have a large waiting time. On the contrary, if the freight arrives at the logistic platform after the prescribed starting time then it is no longer possible to continue the route on such a path. Anyway, this latter occurrence generates the chance to consider a possible new service to be opened and considered in the operational phase; indeed, the tactical phase acts as a network design phase, where additional timetable services could be defined and possibly used in the operational phase.

The transportation means assignment phase is performed by the following two procedures, i.e., *minimizing road service cost* (`MRSC`) and *minimizing rail and maritime service time and cost* (`MRMST`) (see Steps 3 and 4 in Table 3). In the following, we describe these two subroutines and will refer to a *sequence* as a unimodal path inside a route.

`MRSC` starts by running a greedy routine that, from the path set of a certain shipment computed by Steps 1 and 2, tries to assign to each sequence the same transportation mean/operator. This implies that, in this phase, constraints related to (i) logistic platforms capability, (ii) timetables of rail and maritime services, and (iii) shipment delivery time are neglected.

Settled the initial assignment, a local search optimization (see Table 4) is performed for the road mode *only*; in particular, on the road mode sequences in which the transport assignment has been performed, this algorithm optimizes the costs looking for other feasible cheapest road services. Once the (heuristic) optimization of road services has been performed, the related solution is fixed.

After the phase involving the minimization of the road service costs, rail and maritime arcs can be optimized by means of `MRMST`. This phase needs a deeper investigation than the first one. Indeed, in the transportation mean assignment, one is willing to optimize the third objective function that was not taken into account in the path search phase (recall that there we optimized paths with respect to time and cost only). To this aim, we decided to implement the algorithm to work sequentially over a prefixed ordering of the shipments (a similar approach can be found in, e.g., [27]).

Table 4
The local search routine in MRSC

Step 1	For each sequence i do		
	Step 1.1	Let n_i be the number of arcs of sequence i	
	Step 1.2	Set $j = 1$	
	Step 1.3	Order road transportation means according to non-increasing cost values	
	Step 1.4	While $j \neq n_i$ do	
		Step 1.4.1	If $j = 1$, assign the cheapest feasible transportation mean, yet not assigned to the sequence, to arc j
		Step 1.4.2	Otherwise, assign the same transportation mean assigned to arc $j - 1$, if feasible
		Step 1.4.3	If it is not feasible, set $j = 1$ and goto Step 1.4
		Step 1.4.4	$j = j + 1$

In the tactical phase, the ordering of the demands is not performed on the basis of priorities (as will happen next in the operational phase), and the possible violation of the delivery time associated with a demand can be taken into account. More in detail, the ordering is made on the basis of not increasing values of the remaining time r_i (i is the shipment index) on the route once the fixed road traveling time (computable after the optimization performed in previous phase) has been subtracted from the overall available time (ties are *broken* arbitrarily).

Given the demand ordering, the algorithm starts from the first one and scans one by one the associated efficient paths, computed in the path search phase. For each efficient path, it considers the sequences to be served by rail and/or maritime services, finds the schedule and the corresponding transportation mean minimizing the waiting time at the logistic node, where the waiting time is the difference between the starting time of a service, with a prefixed timetable, and the availability time (*ready time*) of the vehicle at the platform, defined by the finishing time of the platform operations at the node.

The capacity of the logistic node is then considered; in fact, each logistic node has a limit on the quantity of the daily workable freight that implies that the ordering of the paths is an important issue to be considered since all the logistic services are organized as a FIFO (first in first out) queue and, therefore, the paths with a greater chance of generating penalty must be served first.

Once all the efficient paths have been considered for a certain shipment, the algorithm chooses the one that optimizes the third objective function and does not violate the delivery time window. In fact, in a shipment i, during the assignment of a rail/maritime service with timetable in a subpath (sequence), it is possible to make sharing by using a transportation mean (with positive residual capacity) assigned to the same sequence of the path selected for a shipment preceding i, in the ordered list of demands.

4.2. Operational phase

The operational phase works on the output of the tactical phase. Indeed, the network onto which demands of the operational phase will be routed is defined by the subgraph induced by all the routes found in the former phase. Moreover, additional rail and maritime services could be added to the starting ones, if, as explained in the previous section, these help in reducing the overall waiting time as well as to improve efficiency. We can say that the tactical problem goal is mainly that of selecting from the initial transport offer a subnetwork and a set of services operating on it, onto which the operational phase can be run satisfactorily.

From the implementation viewpoint, there are some differences between the algorithms used in the tactical and in the operational phase. Firstly, in the latter, the ordering of the shipments occurs in a preliminary analysis, where a delivery flag A is associated with shipments with high priority, and B refers to low priority shipments. Moreover, we note that, even though the shipments assigned to the operational phase can be different from those used to run the tactical phase (see e.g., [20]), in our case they are the same as those of the tactical phase. Finally, note that, differently from the tactical phase, in the operational phase delivery time windows cannot be violated.

5. Experimental results

In this section we present computational results. The presentation is divided into three subsections: two are devoted to the case study analysis, and the other to synthetic tests that we carried out to give further insight on the algorithm proposed.

5.1. The case study

The network considered in our case study from Italy has 3488 nodes and 7200 arcs. In what follows, the

Table 5
Demand elements considered in the case study

ID	Quantity	PTW_1	PTW_2	DTW_1	DTW_2	ID	Quantity	PTW_1	PTW_2	DTW_1	DTW_2
1	72	0	1440	7200	10,080	35	44	0	1440	8640	11,520
2	234	0	1440	5760	8640	36	96	0	1440	5760	8640
3	49	0	1440	5760	8640	37	60	0	1440	8640	11,520
4	84	0	1440	5760	8640	38	216	0	1440	8640	11,520
5	54	0	1440	4320	7200	39	162	0	1440	8640	11,520
6	91	0	1440	7200	10,080	40	63	0	1440	5760	8640
7	91	0	1440	4320	7200	41	77	0	1440	4320	7200
8	30	0	1440	5760	8640	42	72	0	1440	4320	7200
9	128	0	1440	8640	11,520	43	50	0	1440	8640	11,520
10	112	0	1440	5760	8640	44	288	0	1440	8640	11,520
11	48	0	1440	4320	7200	45	136	0	1440	7200	10,080
12	56	0	1440	8640	11,520	46	80	0	1440	4320	7200
13	96	0	1440	8640	11,520	47	55	0	1440	5760	8640
14	192	0	1440	7200	10,080	48	119	0	1440	7200	10,080
15	288	0	1440	5760	8640	49	40	0	1440	4320	7200
16	99	0	1440	5760	8640	50	60	0	1440	5760	8640
17	63	0	1440	4320	7200	51	104	0	1440	8640	11,520
18	144	0	1440	5760	8640	52	88	0	1440	8640	11,520
19	66	0	1440	4320	7200	53	200	0	1440	7200	10,080
20	36	0	1440	4320	7200	54	54	0	1440	2880	5760
21	24	0	1440	2880	5760	55	48	0	1440	4320	7200
22	77	0	1440	5760	8640	56	288	0	1440	4320	7200
23	72	0	1440	8640	11,520	57	200	0	1440	5760	8640
24	104	0	1440	5760	8640	58	60	0	1440	4320	7200
25	54	0	1440	5760	8640	59	70	0	1440	4320	7200
26	288	0	1440	8640	11,520	60	20	0	1440	4320	7200
27	153	0	1440	5760	8640	61	170	0	1440	8640	11,520
28	49	0	1440	4320	7200	62	77	0	1440	5760	8640
29	56	0	1440	8640	11,520	63	135	0	1440	4320	7200
30	42	0	1440	5760	8640	64	160	0	1440	7200	10,080
31	48	0	1440	5760	8640	65	54	0	1440	4320	7200
32	50	0	1440	4320	7200	66	49	0	1440	5760	8640
33	232	0	1440	8640	11,520	67	234	0	1440	5760	8640
34	88	0	1440	4320	7200	68	60	0	1440	7200	10,080

main characteristics of the considered case study are reported.

- There are road and rail modes.
- Trucks have 35 ton capacity, trains 625 ton, and boats 1250 ton.
- The number of freight types is 180.
- The number of demands to be served is 68.
- There are 13 different (starting) rail services with a prefixed timetable.
- There are 130 different fees associated with transportation means.
- Each demand is associated with either 2 or 3 mandatory nodes, and at most 2 forbidden nodes.
- The overall planning horizon is 9 days.
- Each logistic platform has a daily working capacity equal to 500,000 ton, and is open daily from 8:00 a.m. to 5:00 p.m.
- Quantities to be delivered range from 20 to 288 ton.

In Table 5, for each ID demand, we show the quantity to be routed, the pickup time window $[PTW_1, PTW_2]$, and the delivery time window $[DTW_1, DTW_2]$.

We first report our findings on the tactical phase. Table 6 shows the results achieved by the proposed algorithm for the 68 shipments of the case study.

In the table columns indicate:

- ID is the demand identifier.
- Time is the total trip time (in minutes).
- Cost is the total cost of the trip (in Euros).
- Q is the amount of freight moved from the origin to the destination (in tons).
- RC is the average residual capacity, i.e., the average unused capacity over the route (measured in tons).

Table 6
Main characteristics of the shipments

ID	Time	Cost	Q	RC	RoN	Mm	Oc	ID	Time	Cost	Q	RC	RoN	Mm	Oc
1	7200	738	72	33	4	No	3	35	*	*	*	*	*	*	*
2	5765	3948	234	11	6	No	1	36	*	*	*	*	*	*	*
3	5760	342	49	21	3	No	2	37	*	*	*	*	*	*	*
4	5760	723	84	21	3	No	1	38	8640	3061	216	219	8	Yes	4
5	4325	360	54	16	6	No	2	39	*	*	*	*	*	*	*
6	7200	852	91	14	6	No	3	40	5765	444	63	7	6	No	2
7	4325	762	91	14	6	No	2	41	4323	678	77	28	4	No	2
8	5760	133	30	5	3	No	2	42	*	*	*	*	*	*	*
9	*	*	*	*	*	*	*	43	8640	212	50	245	8	Yes	4
10	5763	1280	112	28	4	No	2	44	8640	6059	288	75	10	Yes	5
11	5765	384	48	22	6	No	2	45	*	*	*	*	*	*	*
12	8640	420	56	155	8	Yes	5	46	4325	696	80	25	6	No	2
13	8640	671	96	265	8	Yes	4	47	5760	366	55	15	5	No	2
14	7200	2916	192	18	3	No	2	48	7200	1360	119	21	4	No	3
15	*	*	*	*	*	*	*	49	5765	352	40	30	6	No	2
16	5760	813	99	6	5	No	2	50	5760	386	60	10	5	No	2
17	4323	396	63	4	4	No	2	51	*	*	*	*	*	*	*
18	5760	1805	144	31	5	No	5	52	*	*	*	*	*	*	*
19	4325	408	66	4	6	No	2	53	7200	386	60	10	6	No	3
20	4325	288	36	34	6	No	2	54	*	*	*	*	*	*	*
21	*	*	*	*	*	*	*	55	4323	336	48	22	4	No	2
22	5760	681	77	28	3	No	2	56	*	*	*	*	*	*	*
23	*	*	*	*	*	*	*	57	8643	3264	200	10	4	No	2
24	*	*	*	*	*	*	*	58	4323	384	60	10	4	No	2
25	5763	408	54	16	4	No	2	59	*	*	*	*	*	*	*
26	*	*	*	*	*	*	*	60	*	*	*	*	*	*	*
27	5760	1895	153	22	5	No	2	61	*	*	*	*	*	*	*
28	5763	388	49	21	4	No	2	62	*	*	*	*	*	*	*
29	8640	420	56	106	10	Yes	5	63	4323	1368	135	5	4	No	2
30	5760	314	42	28	5	No	2	64	7200	2110	169	15	4	No	3
31	5760	338	48	22	5	No	2	65	4323	360	54	16	4	No	2
32	4325	344	50	20	6	No	2	66	5765	388	49	21	6	No	2
33	8640	3285	232	203	8	Yes	4	67	*	*	*	*	*	*	*
34	4323	744	88	17	4	No	2	68	7200	444	60	10	6	No	3

- RoN is the number of nodes in the route.
- Mm indicates whether more than one mode is used.
- Oc is the number of operator changes, i.e., the number of different operators involved in a certain transport ID.

Note that the entry "*" associated with a shipment means that some constraints have been violated in the route. In order to have an idea on the degree of infeasibility of these solutions, in Table 7, we show the results, only for these shipments, assuming to have all soft constraints; in particular, we highlight in columns "Time" and "VMN" the percentage of extra time needed with respect to the maximum allowed delivery time DTW_2, and the number of mandatory nodes that have not been included in the route, respectively.

Referring to the latter aspect, we say that, for those few cases (four times) for which this happens, the routes use a small amount of extra time, i.e., about 1% of the maximum deliverable time allowable, while forcing the usage of the mandatory nodes would have caused more than 8% of extra time. Therefore, in such a situation, the decision maker can have a considerable penalty cost reduction towards the commitment, if the mandatory node constraint can be relaxed.

The analysis on the results of the tactical phase gave the information that 12 more timetable rail services should be activated to optimize the system.

With these results, we run the operational phase in which, as described in Section 4, we considered also the priority of the demands (reported in Table 8; "A" means higher, "B" means lower) to cope with the

Table 7
Results for the shipments with violated constraints

ID	Time (%)	Cost	Q	RC	RoN	Mm	Oc	VMN
9	5	2512	128	145	5	Yes	5	0
15	4	3455	288	105	4	Yes	5	0
21	2	523	24	124	6	Yes	6	0
23	1	634	72	15	7	No	3	1
24	6	1568	104	145	8	Yes	2	0
26	7	3470	288	115	8	Yes	7	0
35	2	445	44	12	6	No	2	0
36	2	892	96	105	7	Yes	5	0
37	1	712	60	10	3	No	3	1
39	2	2725	162	5	3	No	4	0
42	2	882	72	124	8	Yes	6	0
45	3	1665	136	8	5	No	4	0
51	2	1559	104	105	6	Yes	5	0
52	1	819	88	12	7	No	2	1
54	1	746	54	25	5	No	2	0
56	1	3780	288	25	4	No	2	0
59	2	657	70	125	7	Yes	5	0
60	2	358	20	11	5	No	3	0
61	2	2890	170	14	6	No	3	1
62	3	625	77	80	6	Yes	4	0
67	2	3215	234	85	6	Yes	4	0
Average	1.5							

Table 8
Priorities of the demands in the operational phase

ID_ship.	Priority	ID_ship.	Priority	ID_ship.	Priority	ID_ship.	Priority
1	A	35	B	18	A	52	A
2	A	36	B	19	B	53	A
3	A	37	B	20	B	54	A
4	B	38	A	21	B	55	A
5	B	39	A	22	A	56	A
6	A	40	B	23	A	57	A
7	B	41	A	24	A	58	A
8	A	42	B	25	A	59	A
9	B	43	A	26	A	60	A
10	B	44	B	27	A	61	A
11	B	45	A	28	A	62	A
12	B	46	A	29	A	63	B
13	B	47	A	30	A	64	B
14	A	48	A	31	A	65	B
15	A	49	A	32	A	66	B
16	A	50	A	33	A	67	B
17	B	51	A	34	A	68	A

routine `MRMTS`. All the demands were served on time independently on their priority.

We conclude this section noting that computing times are very limited: in particular, a run of the algorithm on the network with 68 demands takes about 5 min.

5.2. *A different analysis on the algorithm*

To give a further insight on how the algorithm works, we show its performance when the multi-objective problem is scalarized introducing a min-sum objective function with weights λ, δ, and β, associated with the

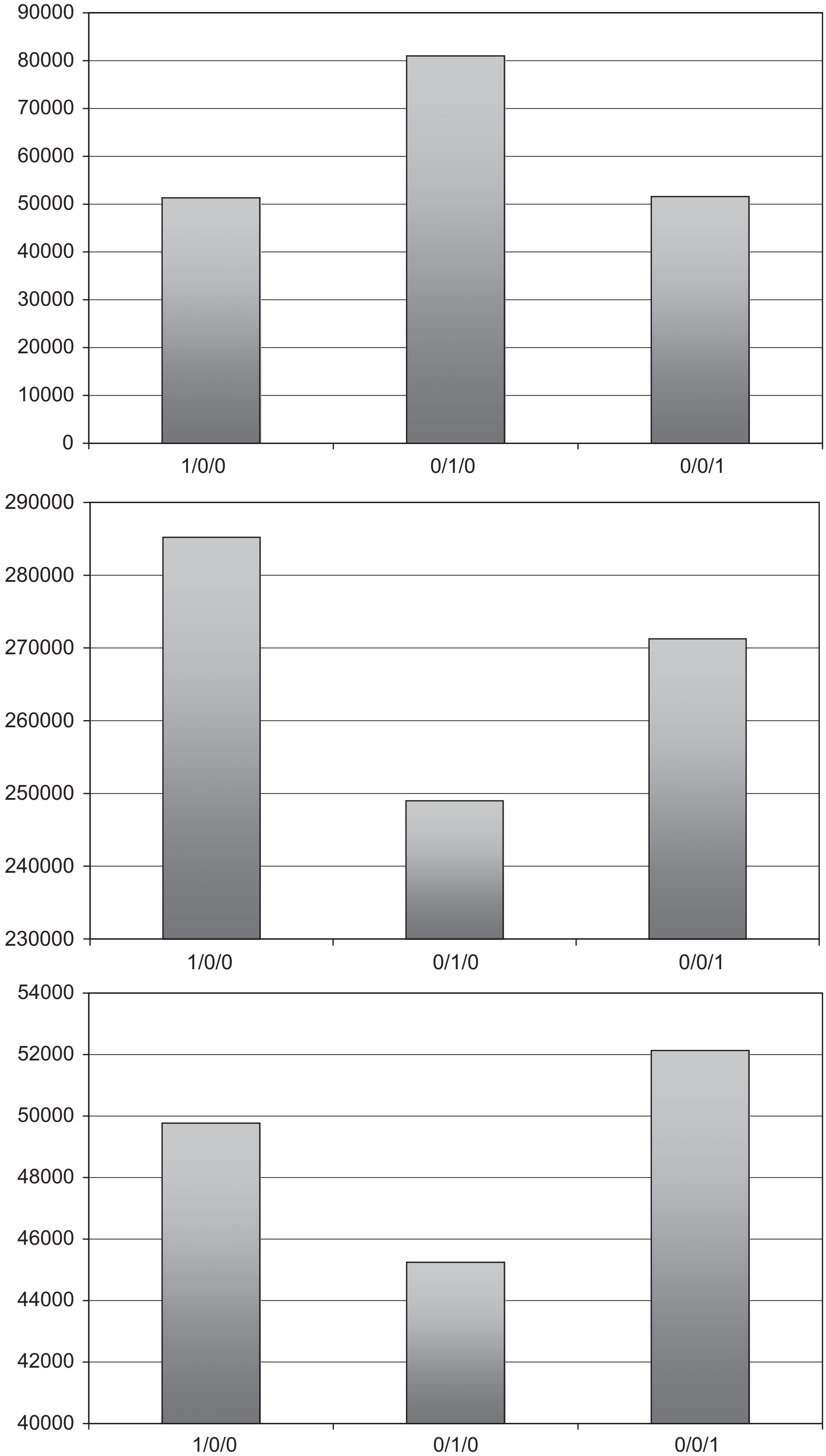

Fig. 2. Total cost (upper figure), total time (middle figure) and total waiting time (lower figure) profiles for different values of the parameters $\lambda/\delta/\beta$.

Table 9
Tests on different network sizes

No. of shipments	Nodes	Density	Transportation means	Number of violated constraints	Total number of constraints	(1 − solution_cost/ sum_route_costs) (%)
10	50	0.2	10	2	4186	84
10	50	0.2	15	0	5328	83
10	50	0.4	10	1	7818	82
10	50	0.4	15	0	10,012	81
15	100	0.2	20	3	20,458	85
15	100	0.2	25	3	25,158	83
15	100	0.4	20	4	37,584	81
15	100	0.4	25	3	55,124	79
20	150	0.2	25	4	74,894	78
20	150	0.2	30	3	87,444	80
20	150	0.4	25	4	95,422	80
20	150	0.4	30	3	101,456	75
25	200	0.2	30	5	154,668	74
25	200	0.2	35	5	175,286	77
25	200	0.4	30	7	285,342	74
25	200	0.4	35	6	301,250	75
30	250	0.2	35	7	212,778	75
30	250	0.2	40	7	265,412	71
30	250	0.4	35	8	421,144	72
30	250	0.4	40	7	451,292	74
35	300	0.2	40	8	381,294	74
35	300	0.2	45	8	421,928	73
35	300	0.4	40	9	778,198	73
35	300	0.4	45	8	859,092	72
40	350	0.2	50	10	792,842	70
40	350	0.2	55	9	812,932	69
40	350	0.4	50	10	1,345,022	69
40	350	0.4	55	8	145,268	68
45	400	0.2	55	9	1,056,984	68
45	400	0.2	60	8	1,146,482	67
45	400	0.4	55	10	2,045,910	65
45	400	0.4	60	7	2,102,944	65
50	450	0.2	60	11	1,392,030	62
50	450	0.2	65	10	1,498,222	64
50	450	0.4	60	11	2,912,026	60
50	450	0.4	65	11	3,942,730	58
55	500	0.2	65	12	1,789,284	57
55	500	0.2	70	11	1,844,298	55
55	500	0.4	65	12	3,619,200	56
55	500	0.4	70	12	3,967,212	56
Average	275			7		72

three objective functions, respectively. This implies that the problem reduces to find a route with the lowest value of the (unique) objective function for each origin–destination pair, respectively, of type pt_1, pt_2 and pt_3.

Since our objective functions are heterogeneous in terms of range of values that they can assume, we scalarized the multi-objective problem considering only the three parameter combinations leading to pure objective functions, i.e., $\lambda/\delta/\beta = 1/0/0$, $0/1/0$ and $0/0/1$. Fig. 2 gives the results of this analysis referring to the operational phase.

The upper chart depicts the total cost of the routes found, i.e., the sum of the costs (in Euros) related to the 68 shipments; the middle chart shows the total time associated with the routes found (in minutes); finally, the lower chart reports the total waiting time over the routes (in minutes).

In the upper chart, it can be noted that, as soon as cost or transportation mean sharing objective is concerned, the total route cost reaches the lowest values. In particular, we observe that when the pure cost objective is concerned the route cost is minimized thanks to an ad hoc "route and transportation means" choice; while, when a pure transportation mean sharing objective is considered, the route cost is minimized because of an optimized capacity utilization and transportation mean sharing.

Differently from what happens for the cost analysis (see the middle chart), the total route time reaches the lowest value only with the pure time objective minimization, and is very sensitive to the parameter setting. As it can be inferred, the worst results are achieved in correspondence to both cost and transportation mean sharing objectives minimization.

Finally, looking at the lower chart, we see that the total waiting time is not very sensitive to the parameter setting.

5.3. Further tests on synthetic instances

A final test of the algorithm has been conducted on synthetic instances. We generated at random networks with sizes from 50 to 500 nodes, and variable density and number of transportation means. In Table 9, besides the number of shipments, nodes, density, transportation means and total number of constraints, we report the average number of violated constraints (delivery time windows exceeded and mandatory nodes not respected) and the transportation mean sharing index values. Computational results collected are very encouraging; indeed, the proposed approach allows one to find a solution for which the number of violated constraints is very limited, and a high value of the transportation mean sharing index is achieved (i.e., 72% on average).

6. Conclusions

In this paper, we have studied a complex vehicle routing problem in a multimodal network, where besides arc capacities and time windows, additional constrains (i.e., mandatory and forbidden nodes) are considered. Furthermore, a multi-objective optimization scenario, where the travel time, the operative cost and a transportation mean sharing index have to be simultaneously optimized, has been considered.

Our study has been motivated by a real-life application in Italy, where a logistic operator has to serve a number of freight shipment requests from its clients, defining routes with appropriate mode and transportation mean composition, to carry out commodities from their given origins to their given destinations.

To address the problem under consideration, a heuristic algorithm has been defined and implemented. The computational results, collected on the case study and on a set of randomly generated instances, are satisfactory and show the effectiveness of the proposed approach.

Appendix A. Martins' algorithm

Martins' algorithm [23] uses a multiple labeling approach. Each node $i \in V$ is associated with several labels, and the lth label contains the objective values (say r) and two pointers. The label can be represented as $[c_1^{P_{si}}, \ldots, c_r^{P_{si}}, j, l_1]_l$, where $c_h^{P_{si}}$ is the length of the path P_{si} from source s to node i, for $h=1,\ldots,r$, $j \neq i$ is some node of G, and l_1 indicates a certain label of node j for which $c_{h,l}^{P_{si}} = c_{h,l_1}^{P_{sj}} + c_h^{(i,j)}$, where $c_{h,l}^{P_{si}}$ is the hth component of the lth label of node i.

At each iteration, there are two kinds of labels, permanent ones and temporary ones. The algorithm selects a temporary label in a node i, converts it in permanent and updates all the labels of successors j of i, for each $(i, j) \in A$; then, it deletes all the labels that represent a dominated path P_{sj}. The algorithm stops when it runs out of temporary labels, and each permanent label represents an unique efficient path.

The node selection step is made considering, among all the labels in each node, the lexicographically smallest one. We recall that for some node i, a label

Table 10
Martins' algorithm

Step 1	Assign the temporary label $[(0,0,\ldots,0),-,-]_1$ to node s	
Step 2	If the set of temporary labels is empty go to Step 5. Otherwise, among all the temporary labels determine the lexicographically smallest one. Let it be the lth label associated with node i. Set this label as a permanent one	
Step 3	While some node $j \in V$ exists, such that $(i, j) \in A$, execute	
	Step 3.1	$c_k^{P_{sj}} = c_{k,l}^{P_{si}} + c_k^{(i,j)}$ for every $k=1,2,\ldots,r$ and let $[(c_1^{P_{sj}}, c_2^{P_{sj}}, \ldots, c_r^{P_{sj}}), i, l]_\xi$ be a new temporary label of node j
	Step 3.2	Among all the temporary labels of node j, delete all label representing a dominated path from s to j
Step 4	Return to Step 2	
Step 5	Find the non-dominated paths from s to t. For that, the two pointers of each label must be used	
Step 6	Stop	

$[c_1^{P_{si}}, \ldots, c_r^{P_{si}}, -, -]_\xi$ is said to be lexicographically smaller than a label $[c_1^{P'_{si}}, \ldots, c_r^{P'_{si}}, -, -]_\delta$ if $c_{1,\xi}^{P_{si}} = c_{1,\delta}^{P'_{si}}, \ldots, c_{k-1,\xi}^{P_{si}} = c_{k-1,\delta}^{P'_{si}}$, $c_{k,\xi}^{P_{si}} < c_{k,\delta}^{P'_{si}}$ holds for some $k \in \{1, \ldots, r\}$.

In Table 10, Martins' algorithm is sketched.

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