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Electricity time-of-use tariff with consumer behavior consideration

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ABSTRACT

This paper investigates an electricity time-of-use (TOU) tariff problem with the consideration of consumer behavior. Under the TOU tariff, we consider two periods: the peak and base periods. A two-level model is established to solve the TOU tariff problem: in the upper level, the producer determines the TOU tariff with the consideration of consumer behavior; in the lower level, the consumers respond to the TOU tariff through shifting some electricity consumption in the peak period to the base period. Using the traditional flat-rate (FR) tariff as a baseline, we verify the conditions under which the producer has incentives to adopt the TOU tariff. With the adoption of a general consumer transfer cost, we solve for the optimal TOU tariff under different situations. Our results demonstrate that proper adoption of the TOU tariff can create a win–win situation for both the producer and the consumers: the producer can increase its profit and the consumers can save their electricity cost. We further evaluate the effectiveness of the TOU tariff in terms of the peak demand reduction. Using a quadratic transfer cost, we obtain some managerial insights into the TOU tariff problem, and illustrate that the TOU tariff is always beneficial to the producer and the consumers under the quadratic transfer cost.

1. Introduction

Facing the challenge of energy shortage and opportunities from energy technology advancement, both producers and consumers are paying more concern to designing a more efficient pricing mechanism to improve the social welfare and promote energy efficiency. Time-of-use (TOU) tariff is an effective way to achieve these goals. Under the TOU tariff, the electricity price is higher in the peak period, but lower in the off-peak period. The TOU tariff aims to reduce the peak load and enhance the electricity supply security. Under the traditional flat-rate (FR) tariff, consumers do not have the incentive to reduce their electricity use during the peak period and to use electricity wisely. This could lead to a power crisis in the peak period, such as the California's power crisis during 2000 and 2001 (Faruqui and George, 2005). With the adoption of the TOU tariff, consumers have the incentives to shift part of their electricity use from the peak period to the off-peak period, so that the electricity load in the peak period can be reduced. As electricity load is reduced in the peak period, electricity companies can save high electricity generation costs at the peak period and avoid building new generation units for the peak period load. As a result, electricity companies have the opportunities to increase their net profits by adopting the TOU tariff. From the standpoint of the consumers, there are opportunities for them to save their electricity bills by shifting electricity use from the high-rate period (peak period) to the low-rate period (off-peak period).

Industries have paid more attention to the implication of the peak pricing schedule after the California's power crisis during 2000 and 2001. After the crisis, California's three investor-owned utilities conducted the well-known Statewide Pricing Pilot (SPP) experiment from July 2003 to December 2004. The SPP experiment conclusively shows that consumers reduce energy use ranging from 7.6% to 27% in the peak period in response to dynamic pricing (Faruqui and George, 2005), which is a kind of TOU pricing mechanism. Based on the SPP experiment, there are a number of studies focusing on the TOU pricing in the electricity market (e.g., Baskette et al., 2006; Herter et al., 2007; Herter, 2007). Also, some research has pointed out that the highest peak period accounts for a large proportion of the cost of energy (Faruqui and Sergici, 2010; Faruqui et al., 2010). Therefore, designing an efficient electricity pricing mechanism is a possible way to promote and control energy efficiency.

The TOU tariff has been tested and implemented in some European countries and some states in the U.S.A. in the past decade. For example, in the Gulf Power Select Program in Florida in the U.S.A., the reduction in electricity use during the critical peak period was up to 41% (Faruqui and Sergici, 2010). In Norway, there was an 8–9% reduction in electricity use at the peak period

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(Faruqui and Sergici, 2010). From the consumers' perspective, under the TOU tariff, they have incentives to save electricity bills by changing their electricity use from the peak period to the off-peak period or use energy efficient products. On the other hand, electricity load reduction in the peak period may enable electricity companies to avoid high electricity generation cost at the peak period and additional peak capacity installation. To get an in-depth understanding of the TOU tariff, we investigate the TOU tariff with the consideration of consumer behavior in this paper. We further evaluate the effect of the TOU tariff in terms of the increased profit for the producer, the cost saving for the consumers, and the peak load reduction. Using the traditional FR tariff as a baseline, we verify the conditions under which the producer and/or consumers benefit from the TOU tariff structure, and evaluate the effectiveness of the TOU tariff structure. Our results show that proper adoption of the TOU tariff can create a win–win situation for both the consumers and the producer.

In summary, the main contributions of this paper are listed below:

1. We properly formulate the TOU tariff considering both the producer and the consumers in a time horizon including the peak and off-peak periods. Our research fills a research gap in the literature about the TOU tariff by considering consumer behavior.
2. We formulate consumer behavior in response to different prices in different periods through introducing the transfer cost concept. It demonstrates that the consumer behavior of shifting electricity demand is based on consumer interests. Under a proper tariff design, the consumer behavior can benefit the performance of the whole system, i.e., reducing the peak load, lowering the risk of electricity shortage and securing the supply reliability.
3. We identify the conditions under which both the producer and consumers can benefit from the TOU tariff. More importantly, we provide a method to measure how much peak load can be reduced under the TOU tariff, compared with the traditional FR tariff. The peak load reduction enables the producer to secure a reliable supply and save costs from building an additional peak capacity.

2. Literature review

In the literature, there are extensive studies on electricity pricing, such as Doucet and Roland (1993), Crew et al. (1995), Borenstein and Holland (2005) and Chao (2010). Admitting the challenge of energy shortage worldwide, both the supply and demand sides in the electricity market are seeking a more efficient mechanism of pricing to deal with uncertain environments, particularly within the peak periods. Several studies investigated the peak pricing with the consideration of demand uncertainty, such as Brown and Johnson (1969), Carlton (1977), Meyer (1975), Crew and Kleindorfer (1978), Sherman and Visscher (1978) and Borenstein and Holland (2005); while other research studied the peak pricing under supply uncertainty, like Chao (1983), Coate and Panzar (1989) and Kleindorfer and Fernando (1993). Recently, Chao (2011a) provided a unified economic model on efficient pricing and investment in restructured electricity markets in one single pricing period. Despite the extensive studies about electricity pricing, most of these studies focused on pricing in the peak period only. These studies did not consider the impacts of consumer behavior on a shift of some electricity consumption from the peak hours to off-peak hours. Such consumption shift could be an effective way to address the efficiency of electricity pricing. Our research enriches this stream of research by considering both the peak and off-peak periods within a referred horizon to seek for the optimal tariff scheme.

It is observed that the structure of electricity market has a great impact on electricity market price and profit (see, e.g., Garcia et al., 2005; Bunn and Oliveira, 2008). A study conducted by PB power for The Royal Academy of Engineering (PB Power, 2004) reported different costs for different available technologies of electricity generation. With simulation approach, Banal-Estañol and Micola (2009) showed that the relationship between technological diversification of electricity generation and wholesale market prices is mediated by the supply-to-demand ratio. Chao (2011a) investigated electricity pricing through the establishment of a model with multiple technologies, including renewable and non-renewable technologies. Chao (2011b) studied the consumer baseline choice, focusing on administrative and contractual approaches. These studies suggested that in electricity market, the suppliers need to consider the costs of different technologies when designing an efficient tariff structure. Furthermore, the suppliers may be beneficial from the potential cost–savings by adjusting its supply structure.

From the standpoint of electricity producer, it is crucial to design an efficient tariff mechanism to reduce the demands in the peak hours. As the peak load is reduced, the producer can save the cost in installing extra capacity in the peak hours and reduce the shortage risk in the peak hours. Pineau and Zaccour (2007) studied a two-period electricity market with multiple firms under transferable demands between the peak and base periods. Their study highlighted the importance of the tariff structure. However, their study did not consider specific consumer behavior from the consumers' perspective. Triki and Violi (2009) proposed a two-stage retail pricing scheme in an open market with the use of stochastic programming. Colon (2010) investigated how to develop time-of-use tariff structures for residential and small commercial consumers in Ireland's electricity market. Using some numerical examples, Chao (2010) provided a comprehensive discussion about price-responsive demand management for a smart grid world, including benefits and barriers of price-responsive demand. However, these studies mainly focus on the supplier's perspective and do not consider the consumers' specific responses when designing the electricity tariff. Furthermore, there is little theoretical research to formulate and evaluate the benefits of the TOU tariff with the consideration of consumer behavior.

From the consumers' perspective, there are chances to save their electricity bills while guaranteeing the electricity use. This could be achieved through adjusting the electricity use from a high-price period to a low-price period. Spector et al. (1995) studied the responses from business consumers about time-of-use electricity rates. Nogales and Conejo (2006) addressed a problem of day-ahead electricity price forecasting by building a time-series model. Herter (2007) showed that high-use consumers respond significantly more in kW reduction than low-use consumers, while low-use consumers save significantly more on the percentage reduction of the annual electricity bills than that of the high-use consumers. Recently, the application of the TOU tariff has given rise to a debate about fairness to consumers (e.g., Faruqui, 2010; Hogan, 2010; Alexander, 2010). The debate emphasizes that successful design and implementation of the TOU tariff should take account of consumer responses to and benefits from the TOU tariff.

As we discussed above, most studies on electricity tariff focus on either the producer's perspective or consumers' perspective. There is little research investigating the participation of both sides of suppliers and consumers, as well as the interaction between the two sides. In fact, facing different tariffs, the consumers may have different behavior to respond to. For example, under a time-dependent tariff, consumers will try to avoid the peak hours due to the high price. The consumer responses will certainly change the demand pattern, which in turn influences the producer's expected profit and the efficiency of the pricing mechanism. Differing from these studies, our research integrates consumer responses to different tariffs into the tariff design. Therefore, the interaction between the electricity supply and consumer demand can be investigated. In other words, the mechanism proposed in this paper takes account of the participation of both the producer and the consumers.

3. Model features

We consider two periods of electricity use: the peak period and the base period. Two types of tariff structures are considered: the traditional FR tariff and the TOU tariff. The FR tariff charges at the same electricity price all the time, whereas the TOU tariff imposes different prices at different time periods within a reference period, e.g., a day. Under the TOU tariff, a higher price is charged in the peak period, while a lower price is charged in the base period. Under the traditional FR tariff, consumers do not have the incentives to shift their consumption from the peak period to the base period. Under the TOU tariff, because of different prices in different periods, it is possible for the consumers to save their electricity costs through shifting part of their consumption from the peak period to the base period. Figs. 1 and 2 demonstrate the FR tariff and the TOU tariff, respectively.

Consider that there are two types of technologies, base technology and peak technology, i.e., technology i, $i \in \{b,p\}$. Here the subscripts b and p are used to represent base and peak, respectively. Define q_b and q_p to be the total demand of electricity (in megawatt hour, MWh) in the base and peak periods, respectively. In this paper, we consider a reliable supply and all demands will be met. Let $k_i \geq 0$ denote the installed capacity (in megawatt, MW) for technology i, $i \in \{b,p\}$. The average unit operating cost of technology i is denoted by β_i, $i \in \{b,p\}$; and the average unit capacity cost of technology i is denoted by c_i, $i \in \{b,p\}$. The base capacity is used all the time, while the peak capacity is adopted only during the peak period. Let the unit electricity prices (\$/MWh) of the base period and the peak period be p_b and p_p, respectively. Define T to be the total time of the reference period, e.g., $T=8760$ hours in a year. The peak period is a fraction of the entire reference period. Denote τ to be the percentage of time in which the peak capacity is used, e.g., $t=\tau T$ is the number of peak-load hours per year. As illustrated in Fig. 3, $\tau=(t_2-t_1)/T$.

It is reasonable that the sum of unit capacity cost and unit production cost of technology p is not less than that of technology b; otherwise, the producer will install the technology p in the base period. Therefore, we assume that $\beta_p+c_p \geq \beta_b+c_b$ throughout this paper. As the peak period is relatively much shorter than the base period, we also assume that $\tau(\beta_p+c_p)<\beta_b+c_b$. We summarize the notations in Table 1.

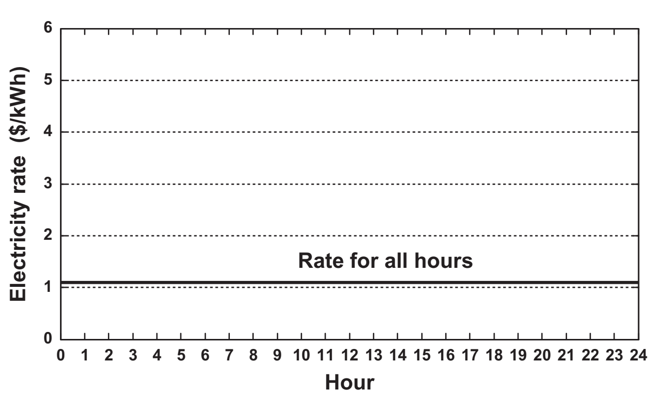

Fig. 1. Illustration of the flat-rate (FR) tariff.

In the following discussion, we normalize $T=1$. The production of electricity in the peak and base periods is restricted by

$$\begin{cases} 0 \leq q_b \leq (1-\tau)k_b \\ 0 \leq q_p \leq \tau(k_b+k_p) \end{cases}. \tag{1}$$

Following Pineau and Zaccour (2007), the capacity costs for the two installed technologies are $W_b=c_bk_b$ and $W_p=c_pk_p\tau$. The production costs for the base and peak periods are $V_b=\beta_bq_b$ and

$$V_p=\begin{cases} \beta_bq_p & \text{if } 0 \leq q_p \leq \tau k_b \\ \beta_b\tau k_b+\beta_p(q_p-\tau k_b) & \text{if } \tau k_b<q_p \leq \tau(k_b+k_p) \end{cases}$$

respectively. Furthermore, when the installed capacity of technology b is able to produce enough electricity to meet the demand,

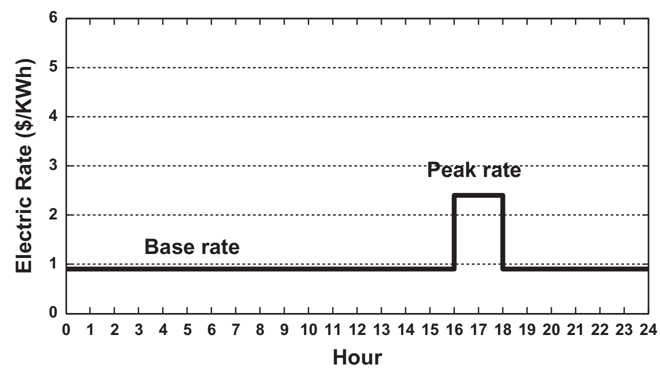

Fig. 2. Illustration of the time-of-use (TOU) tariff.

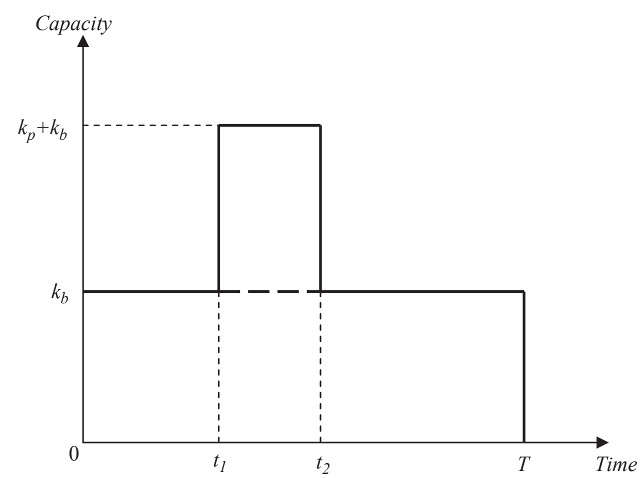

Fig. 3. Capacity used in the two periods.

Table 1
Notations used throughout this paper.

Notations	
p_b	Unit electricity price of the base period
p_p	Unit electricity price of the peak period
k_i	The installed capacity for technology i, $i \in \{b, p\}$
β_i	Unit operating cost of technology i, $i \in \{b, p\}$
c_i	Unit capacity cost of technology i, $i \in \{b, p\}$
T	The total time of the reference period
τ	The percentage of time in which the peak capacity is used
q_b	The total demand in the base period
q_p	The total demand in the peak period

the producer will not need to install the technology p, i.e.,

$$k_p \begin{cases} =0 & \text{if } 0 \le q_p \le \tau k_b \\ >0 & \text{if } \tau k_b < q_p \le q_T \end{cases}, \quad \text{where } q_T = \tau(k_b + k_p). \tag{2}$$

Let p_0 be the electricity price under the FR tariff structure. The demands in the base and peak periods are q_{b0} and q_{p0}, respectively. Therefore, the total cost for the consumers is

$$C_{T0} = p_0(q_{b0} + q_{p0}). \tag{3}$$

It is observed that the average electricity demand (i.e., kWh/hour) in the peak period is much higher than that in the base period
(e.g., Herter et al., 2007). Therefore, we assume that $q_{p0}/\tau > q_{b0}/(1-\tau)$.

Let $k_{i0} \ge 0$ denote the installed capacity for technology i under the FR tariff structure, $i \in \{b, p\}$. The total profit of the producer is

$$\Pi_0 = p_0(q_{b0} + q_{p0}) - \beta_b k_{b0} - c_b k_{b0} - \beta_p k_{p0}\tau - c_p k_{p0}\tau, \tag{4}$$

where

$$k_{p0} \begin{cases} =0 & \text{if } 0 \le q_{p0} \le \tau k_{b0} \\ >0 & \text{if } \tau k_{b0} < q_{p0} \le \tau(k_{b0} + k_{p0}) \end{cases}.$$

It is also noted that $q_{b0} \le (1-\tau)k_{b0}$ and $q_{p0} \le \tau(k_{b0} + k_{p0})$. Under the assumption $q_{p0}/\tau > q_{b0}/(1-\tau)$ and $\tau(\beta_p + c_p) < \beta_b + c_b$, it can be proved that $k_{b0} = q_{b0}/(1-\tau)$ and $k_{p0} = q_{p0}/\tau - q_{b0}/(1-\tau)$ for maximizing the producer's profit.

To evaluate the performance of the TOU tariff, we use the FR tariff as a baseline throughout this paper. In the following discussion, we will compare the TOU tariff and the FR tariff.

We consider the TOU tariff under which the electricity price is p_i, $i \in \{b, p\}$, for the base and peak periods. To successfully implement the TOU tariff, the producer needs to ensure that the consumers will not pay more under the TOU tariff than that under the FR tariff when the consumers do not shift any electricity use; otherwise, the producer will face tremendous resistance from the consumers. Also, the producer does not want to reduce its total charge with the adoption of the TOU tariff in such case. Therefore, it is acceptable for both the producer and the consumers that the total electricity charge, $\sum_{i \in \{b,p\}} p_i q_i$, under the TOU tariff is the same as that under the FR tariff, when the consumers do not shift any electricity consumption. This consideration can be represented as below:

$$p_b q_{b0} + p_p q_{p0} = p_0(q_{b0} + q_{p0}). \tag{5}$$

4. Model formulation

To design the TOU tariff, we establish a two-level model in this section. In the upper level, the producer decides on the electricity prices for the peak period and the base period, respectively, with the consideration of consumer behavior in the lower level. In the lower level, the consumers respond to the given prices in different periods by determining their shifted electricity consumption.

4.1. Consumer behavior

With the adoption of the TOU tariff, consumers have the incentives to shift their consumption from the peak period to the base period if they can save some electricity cost. Meanwhile, there is also a shift cost (cost for the shift) when electricity use is shifted. For example, under the FR tariff, some consumers wash their clothes in the peak period; with different prices in different periods under the TOU tariff, some of these consumers may shift their washing of clothes to the base period. Such shifted demand is expected to reduce the total of electricity cost and shift cost, while satisfying the total electricity demand. As consumers need to arrange all resources prepared for the changes, there may be a cost occurred. We refer such a cost associated with the shifting of electricity use as the transfer cost, or shift cost, in this paper. Obviously, the transfer cost is an increasing function of the amount of electricity shifted to the base period. Define $q_0 \ge 0$ to be the total shifted demand of the consumers from the peak period to the base period. As electricity is very necessary for people, we assume the electricity demand is inelastic in this paper, i.e., the change of pricing structure will not change the total demand. Then, the production in the periods will become $q_b = q_{b0} + q_0$ and $q_p = q_{p0} - q_0$. The transfer cost is denoted as $C_s = C_s(q_0)$ which is assumed to be continuously differentiable. It is reasonable to assume that the transfer cost is convex increasing in the amount of the shifted consumption. More precisely, we assume $C_s^{(1)}(q_0) > 0$ and $C_s^{(2)}(q_0) > 0$. The transfer cost equals zero when the shifted consumption is zero, i.e., $C_s(0) = 0$. The characteristics of the transfer cost reflect the following intuitive understanding: (1) More shifted demand leads to more transfer cost for the consumers. (2) The difficulty of shifting electricity use increases as the shifted demand increases. In other words, the consumers can easily shift a small amount of electricity use, but are difficult to shift too much electricity use. This is reasonable in practice. (3) When there is no shifted electricity use, no transfer cost occurs. We further assume that $C_s^{(3)}(q_0) \ge 0$. Given the prices of the peak and base periods, p_b and p_p, the objective of the consumers is to minimize the total cost, i.e.,

$$\text{Min } C(q_0) = p_b(q_{b0} + q_0) + p_p(q_{p0} - q_0) + C_s(q_0)$$

$$\text{s.t. } 0 \le q_0 \le q_{p0}. \tag{6}$$

The total cost in the objective function is composed of three parts: the total consumption cost in the base period, the total consumption cost in the peak period, and the total transfer cost associated with the shifted demand q_0. The constraint requires that the shifted consumption cannot exceed the total demand in the peak period. Proposition 1 below provides the optimal total shifted consumption of the consumers under various situations.

Proposition 1. *Under the TOU tariff, given the electricity prices*, i.e., *p_p in the peak period and p_b in the base period, and the shift cost $C_s = C_s(q_0)$*, (i) *when $C_s^{(1)}(q_{p0}) \le p_p - p_b$, the optimal shifted consumption $q_0 = q_{p0}$*; (ii) *when $C_s^{(1)}(0) < p_p - p_b < C_s^{(1)}(q_{p0})$, the optimal shifted consumption q_0 uniquely satisfies $C_s^{(1)}(q_0) = p_p - p_b$, $q_0 \in (0, q_{p0})$; and* (iii) *when $p_p - p_b \le C_s^{(1)}(0)$, the optimal shifted consumption $q_0 = 0$.* □

Proposition 1 indicates that consumers will shift electricity consumption only when the (unit) electricity price difference is larger than the marginal transfer cost at $q_0 = 0$, i.e., $C_s^{(1)}(0) < p_p - p_b$. If the base price is smaller enough than the peak price, the consumers will shift all peak consumption to the base period. When the difference of the prices at the peak and base periods is moderate, the consumers will shift part of their peak period consumption. In this case, the marginal transfer cost at the shifted consumption equals the unit price difference between the peak and base periods.

Under the TOU tariff, after consumers shifting their electricity consumption, the production in the base and peak periods are: $q_b = q_{b0} + q_0$ and $q_p = q_{p0} - q_0$, respectively. The sum of the consumers' electricity cost and shift cost is $C_{TT} = p_b(q_{b0} + q_0) + p_p(q_{p0} - q_0) + C_s(q_0)$. The cost saving for the consumers is $\Delta C_T = C_{TT} - C_{T0} = p_b(q_{b0} + q_0) + p_p(q_{p0} - q_0) + C_s(q_0) - p_0 q_T$, where $q_T = q_{b0} + q_{p0}$. $\Delta C_T < 0$ means the consumers pay less cost under the TOU tariff.

4.2. Producer's decision

From the producer's perspective, the application of the TOU tariff should increase its profit, so that the producer has the incentive to implement the TOU tariff. Under the TOU tariff, the producer's profit can be represented as:

$$\text{Max } \Pi_T(p_b,p_p,k_b,k_p)=p_b(q_{b0}+q_0)+p_p(q_{p0}-q_0)-V_b-V_p-W_b-W_p.$$

$$\text{s.t. } q_b=q_{b0}+q_0,$$
$$q_p=q_{p0}-q_0,$$
$$0\le q_b\le(1-\tau)k_b,$$
$$0\le q_p\le\tau(k_b+k_p),$$
$$p_bq_{b0}+p_pq_{p0}=p_0(q_{b0}+q_{p0}),$$
$$q_0=\begin{cases} q_{p0}, & C_s^{(1)}(q_{p0})\le p_p-p_b \\ (C_s^{(1)})^{-1}(p_p-p_b), & C_s^{(1)}(0)<p_p-p_b<C_s^{(1)}(q_{p0}), \\ 0, & p_p-p_b\le C_s^{(1)}(0) \end{cases}$$
$$k_p\begin{cases} =0 & \text{if} 0\le q_p\le\tau k_b \\ >0 & \text{if} \tau k_b<q_p\le q_T \end{cases},$$
$$k_i\ge0,\ i\in\{b,p\}. \tag{7}$$

In (7), V_b and V_p are operating costs for technologies b and p, respectively; W_b and W_p are capacity costs for technologies b and p, respectively. The objective function is the total profit of the producer over the entire reference period, where the sum of the first two terms is the total revenue over the entire reference period. The shifted electricity demand q_0 is derived from Proposition 1. The constraints $0\le q_b\le(1-\tau)k_b$ and $0\le q_p\le\tau(k_b+k_p)$ indicate that the production quantity in each period cannot exceed the capacity installed. The constraint

$$k_p\begin{cases} =0 & \text{if } 0\le q_p\le\tau k_b \\ >0 & \text{if } \tau k_b<q_p\le q_T \end{cases}$$

is from (2).

Define $\Delta\Pi=\Pi_T-\Pi_0$ as the profit difference of the producer under the TOU and FR tariff structures. The producer has the incentive to implement the TOU tariff only when the profit increases, i.e., $\Delta\Pi>0$. To facilitate the presentation, we define a cost indicator $A=\left((\beta_p+c_p)-(\beta_b+c_b)\right)/(1-\tau)$ and a demand indicator $\overline{q}_0=(1-\tau)q_{p0}-\tau q_{b0}>0$. Under the assumption $\beta_p+c_p\ge\beta_b+c_b$, we have $A\ge0$. The cost indicator A is increasing in the costs of technology p and decreasing in the costs of technology b. When the unit cost difference between the technology p and technology b increases, the value of A increases. The demand indicator $\overline{q}_0$ is increasing in the peak period demand under FR tariff and decreasing in the base period demand under the FR tariff. We will prove later that A and $\overline{q}_0$ are two important indicators of the optimal TOU scheme. Because $\Pi_0$is a constant, the producer's maximization problem (7) is equivalent to the following maximization problem.

$$\text{Max } \Delta\Pi(p_b,p_p,k_b,k_p)=\Pi_T(p_b,p_p,k_b,k_p)-\Pi_0,$$
$$\text{s.t. } q_b=q_{b0}+q_0,$$
$$q_p=q_{p0}-q_0,$$
$$0\le q_b\le(1-\tau)k_b,$$
$$0\le q_p\le\tau(k_b+k_p),$$
$$p_bq_{b0}+p_pq_{p0}=p_0(q_{b0}+q_{p0}),$$
$$q_0=\begin{cases} q_{p0}, & C_s^{(1)}(q_{p0})\le p_p-p_b \\ (C_s^{(1)})^{-1}(p_p-p_b), & C_s^{(1)}(0)<p_p-p_b<C_s^{(1)}(q_{p0}), \\ 0, & p_p-p_b\le C_s^{(1)}(0) \end{cases}$$
$$k_p\begin{cases} =0 & \text{if} 0\le q_p\le\tau k_b \\ >0 & \text{if} \tau k_b<q_p\le q_T \end{cases},$$
$$k_i\ge0,\ i\in\{b,p\}. \tag{8}$$

5. Discussions and analyses

According to Proposition 1, the consumer behavior under the pricing scheme (p_b,p_p) can be classified into three situations as summarized in Table 2.

The producer will take the consumer behavior into account to design the tariff structure. In the following, we analyze the three situations, respectively.

Proposition 2. *Situation-I leads to* $q_0=q_{p0}$, $\Delta\Pi=-q_{p0}C_s^{(1)}(q_{p0})-((\beta_b-\beta_p-c_p)/(1-\tau))\overline{q}_0-q_{p0}((c_b)/(1-\tau))$, *and the optimal solution is* $p_b=p_0-(q_{p0}/q_T)C_s^{(1)}(q_{p0})$ and $p_p=p_0+(q_{b0}/q_T)C_s^{(1)}(q_{p0})$. *So,* $p_p-p_b=C_s^{(1)}(q_{p0})$. □

Proposition 3. *Situation-III leads to* $q_0=0$, $\Delta\Pi=0$, *and an optimal solution is* $p_b=p_0-(q_{p0}/q_T)C_s^{(1)}(0)$ *and* $p_p=p_0+(q_{b0}/q_T)C_s^{(1)}(0)$. *So,* $p_p-p_b=C_s^{(1)}(0)$. □

Propositions 2 and 3 indicate that the optimal solutions for Situation-I and Situation-III lie at the boundaries of Situation-II, i.e., $p_p-p_b=C_s^{(1)}(q_{p0})$ for Situation-I and $p_p-p_b=C_s^{(1)}(0)$ for Situation-III. Therefore, Situation-I and Situation-III are dominated by Situation-II. The optimal results for Situation-II, which are also the optimal results over the three situations, are provided in Theorem 1 below.

Theorem 1. *Under the TOU tariff, with given cost parameters* β_i, c_i, $i\in\{b,p\}$, *we have* $(k_b,k_p)=((q_{b0}+q_0)/(1-\tau),(q_{p0}-q_0)/\tau-(q_{b0}+q_0)/(1-\tau))$ *and*

(i) when $A\le C_s^{(1)}(0)$, the optimal $q_0=0$, the producer will not implement the TOU tariff;
(ii) when $C_s^{(1)}(0)<A<\overline{A}$, the optimal q_0 satisfies the condition $C_s^{(1)}(q_0)+q_0C_s^{(2)}(q_0)=A$, the optimal TOU tariff scheme is $(p_b,p_p)=(p_0-(q_{p0}/q_T)C_s^{(1)}(q_0),p_0+(q_{b0}/q_T)C_s^{(1)}(q_0))$, the increased profit is $\Delta\Pi=q_0^2C_s^{(2)}(q_0)$ and the consumers' cost saving is $q_0C_s^{(1)}(q_0)-C_s(q_0)$;
(iii) when $\overline{A}\le A$, the optimal $q_0=\overline{q}_0$, the optimal TOU tariff scheme is $(p_b,p_p)=(p_0-(q_{p0}/q_T)C_s^{(1)}(\overline{q}_0),p_0+(q_{b0}/q_T)C_s^{(1)}(\overline{q}_0))$, the increased profit is $\Delta\Pi=\overline{q}_0[-C_s^{(1)}(\overline{q}_0)+A]$ and the consumers' cost saving is $\overline{q}_0C_s^{(1)}(\overline{q}_0)-C_s(\overline{q}_0)$;

where $A=\left((\beta_p+c_p)-(\beta_b+c_b)\right)/(1-\tau)$, $\overline{A}=C_s^{(1)}(\overline{q}_0)+\overline{q}_0C_s^{(2)}(\overline{q}_0)$ and $\overline{q}_0=(1-\tau)q_{p0}-\tau q_{b0}>0$. □

Theorem 1 provides the producer's optimal TOU tariff scheme (p_b,p_p) with the consideration of consumer behavior under three different situations. There are two thresholds of A that determine the optimal scheme (p_b,p_p). The producer will have the incentive to implement the TOU tariff only when A is larger than the marginal transfer cost at zero shifted consumption; otherwise, the producer will keep using the FR tariff. From the producer's perspective, designing the tariff scheme needs to consider its related factors under different scenarios. In scenario $A\in(C_s^{(1)}(0),\overline{A})$, the shifted consumption q_0 satisfies a specific equation $C_s^{(1)}(q_0)+$

Table 2
Consumer behavior under different situations.

Situation	Condition	Consumer behavior
I	$C_s^{(1)}(q_{p0})\le p_p-p_b$	Shift all peak electricity use in the peak period
II	$C_s^{(1)}(0)\le p_p-p_b\le C_s^{(1)}(q_{p0})$	Shift part of the peak electricity use
III	$p_p-p_b\le C_s^{(1)}(0)$	No electricity use is shifted

$q_0 C_s^{(2)}(q_0) = A$. Therefore, q_0 is determined by the operating and capacity costs β_i, c_i, $i \in \{b,p\}$. In scenario $A \in [\overline{A},\infty)$, the shifted demand $q_0 = \overline{q}_0$ is independent of A and determined by the peak and base demands under the FR tariff, i.e., q_{p0} and q_{b0}. Therefore, for the design of the TOU tariff, when $A \in (C_s^{(1)}(0),\overline{A})$, the costing environment is the major concern of the producer; while when $A \in [\overline{A},\infty)$, the consumer demands are the main concerns. It is noted that when the shifted consumption $q_0 = \overline{q}_0$, the demands in the peak and base periods are $q_b = (1-\tau)q_T$ and $q_p = \tau q_T$, respectively. Hence, under the TOU tariff, we have $k_b = q_b/(1-\tau) = q_p/\tau$. This indicates that the base and peak periods have the same installed capacity. In other words, the producer fully utilizes the installed capacity of technology b and does not need to install technology p during the peak period.

From the perspective of the producer and social welfare, another concern of implementing the TOU tariff scheme is the reduction in the peak period demand for electricity. This is because when the peak period demand is very high, the producer will have to install additional high cost capacity to ensure a reliable supply. Also, the risk of electricity shortage becomes very high as the peak period demand increases a lot. Therefore, apart from the profit drive, the reduction of peak period demand is also an important reason of adopting the TOU tariff. The TOU tariff's effectiveness of reducing peak period demand can be measured by $\delta = (q_0/q_{p0}) \cdot 100\%$. Proposition 4 provides the δ under different situations.

Proposition 4. *With given cost parameters β_i, c_i, $i \in \{b,p\}$, under the optimal TOU tariff scheme (p_b,p_p),*

(i) when $C_s^{(1)}(0) < A < \overline{A}$, then $\delta = (q_0/q_{p0}) \cdot 100\%$, where q_0 satisfies $C_s^{(1)}(q_0) + q_0 C_s^{(2)}(q_0) = A$;
(ii) when $\overline{A} \le A$, then $\delta = [1-\tau-\tau(q_{b0}/q_{p0})] \cdot 100\%$. □

The optimal TOU tariff scheme derived from Theorem 1 leads to Proposition 5 directly.

Proposition 5. *With given cost parameters β_i, c_i, $i \in \{b,p\}$, under the optimal TOU tariff scheme (p_b,p_p), we have $p_p - p_b = C_s^{(1)}(q_0)$, where q_0 is consumers' shifted demand.* □

Although the shifted consumption q_0 is affected by the values of A and $\overline{A}$, the marginal transfer cost of q_0 always equals the difference between the peak and base prices as long as the producer adopts the TOU tariff, i.e., $p_p - p_b = C_s^{(1)}(q_0)$. This equation bridges the producer's designed TOU tariff scheme and the consumer behavior.

Proposition 6. *With given cost parameters β_i, c_i, $i \in \{b,p\}$, under the optimal TOU tariff scheme (p_b,p_p),* we have $(1-\tau)p_b + \tau p_p < p_0$. □

Proposition 6 shows that the average weighted price under the TOU tariff is less than the electricity price under the FR tariff. This encourages consumers to shift their electricity use from the high price period to the low price period. The reduced average weighted price does not necessarily mean that the producer's profit will decrease. According to our previous discussion, as the consumers shift their peak consumption, the producer could benefit from the total reduced cost of technology p. This explains why proper adoption of the TOU tariff could create a win–win situation for both the producer and the consumers.

5.1. The case with quadratic transfer cost

In the following, we consider a particular case in which the consumers have a quadratic transfer cost to get some managerial insights, i.e., $C_s = C_s(q_0) = c_s q_0^2$, where $c_s > 0$ is a positive constant parameter. With the first and second derivatives, we can get $C_s^{(1)}(q_0) = 2c_s q_0$ and $C_s^{(2)}(q_0) = 2c_s$. Also, we obtain that $C_s^{(1)}(q_0) + q_0 C_s^{(2)}(q_0) = 4c_s q_0$ and $C_s^{(1)}(\overline{q}_0) + \overline{q}_0 C_s^{(2)}(\overline{q}_0) = 4c_s[(1-\tau)q_{p0} - \tau q_{b0}]$. According to our analyses of the general situation, we can derive the optimal TOU tariff scheme under quadratic transfer cost as stated in Proposition 7.

Proposition 7. *Under the TOU tariff with quadratic transfer cost, given cost parameters β_i, c_i, $i \in \{b,p\}$ and $c_s > 0$, we have $(k_b,k_p) = ((q_{b0}+q_0)/(1-\tau),(q_{p0}-q_0)/\tau - (q_{b0}+q_0)/(1-\tau))$ and*

(i) when $A < 4c_s\overline{q}_0$, the optimal $q_0 = A/4c_s$; the maximum increased profit is $\Delta\Pi = A^2/8c_s$; the total cost saving for the consumers is $A^2/16c_s$; the optimal TOU tariff scheme is $\begin{cases} p_b = p_0 - (A/2)\cdot(q_{p0}/q_T) \\ p_p = p_0 + (A/2)\cdot(q_{b0}/q_T) \end{cases}$, where $q_T = q_{b0} + q_{p0}$;

(ii) when $A \ge 4c_s\overline{q}_0$, the optimal $q_0 = \overline{q}_0$, the maximum increased profit is $\Delta\Pi = -2c_s\overline{q}_0^2 + A\overline{q}_0$; the total cost saving for the consumers is $c_s\overline{q}_0^2$; the optimal TOU tariff scheme is $\begin{cases} p_b = p_0 - (q_{p0}/q_T)2c_s\overline{q}_0 \\ p_p = p_0 + (q_{b0}/q_T)2c_s\overline{q}_0 \end{cases}$, where $\overline{q}_0 = (1-\tau)q_{p0} - \tau q_{b0} > 0$. □

With quadratic transfer cost $C_s(q_0) = c_s q_0^2$, the marginal transfer cost with zero shifted consumption is 0. i.e., $C_s^{(1)}(0) = 0$. Since $A > 0$, the producer will have the incentive to adopt the TOU tariff under quadratic transfer cost. Proposition 7 provides the optimal TOU tariff scheme (p_b,p_p). It shows that with quadratic transfer cost, the price difference between the peak and base periods is $p_p - p_b = A/2$, which is determined by the costs of technology b and technology p; or the price difference between the peak and base periods is $p_p - p_b = 2c_s\overline{q}_0$, which is determined by the demand indicator and the shift cost parameter. Under the situation $A < 4c_s\overline{q}_0$, the producer's increased profit $\Delta\Pi(q_0)$ is double of the total cost reduction for the consumers, i.e., $\Delta\Pi(q_0) = 2\Delta C_T$; whereas under the situation $A \ge 4c_s\overline{q}_0$, the producer's increased profit $\Delta\Pi(q_0)$ is larger than the consumers' total cost saving, i.e., $\Delta\Pi(q_0) > \Delta C_T$. Therefore, with quadratic transfer cost, both the producer and consumers benefit from the TOU tariff scheme for all possible situations. The increased profit encourages the producer to adopt the TOU tariff, while the electricity cost saving ensures that the TOU tariff is acceptable (or welcomed) by the consumers. As a result, the successful implementation of the TOU tariff creates a win–win situation.

6. Conclusions

In this paper, we investigate the electricity time-of-use (TOU) pricing problem with the consideration of consumer behavior. Taking account of the consumers' response to different pricing, we establish a two-level model to derive the optimal TOU tariff scheme under different situations. In the upper level, the producer aims at increasing its profit through adopting the TOU tariff. In the lower level, the consumers respond to the different pricing by shifting their electricity consumption to save their electricity cost. With a general transfer cost associated with the consumers' shifting consumption, we bridge the producer's tariff design and consumer behavior. Using the traditional FR tariff as a baseline, we verify the conditions under which the TOU tariff is beneficial to the producer and the consumers. We analytically derive the optimal tariff scheme under different costing environments. Our analysis demonstrates that a proper adoption of the TOU tariff may create a win–win situation for both the producer and consumers: the producer increases its profit while the consumers save their electricity cost.

The two thresholds of the cost indicator A divide the optimal TOU tariff scheme into three situations. The producer has the incentives to adopt the TOU tariff only when the cost indicator A is larger than the marginal transfer cost at zero shifted consumption. Depending on the costing and demand environment, the major influential factors to the optimal TOU tariff scheme are different. Our results suggest that when the cost indicator A is between the two thresholds, the costs of technology p and technology b are the main concerns of the producer when designing the TOU scheme. When the cost indicator A is very large, the peak and base demands are the key factors that the producer needs to consider. We further examine the effectiveness of the TOU tariff in terms of the peak load reduction. This benefit can help the producer avoid additional capacity installation and reduce the shortage risk when peak demand becomes very high. We further consider a particular case with a quadratic transfer cost. It suggests that both the producer and consumers can always benefit from the TOU tariff under the quadratic transfer cost.

In this paper, we assume all consumers are homogenous, and, therefore, we suppose they have the same transfer cost. In reality, some consumers may be more price sensitive than others. Therefore, different consumers may have different transfer cost functions, which lead to different consumer behaviors. Considering multi-group consumers under the TOU tariff is an interesting direction of future research. From the supply perspective, competition may have some effects on the TOU tariff design and implementation. Investigating the TOU tariff in a competitive market is another interesting direction for future research.

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Appendix

Proof of proposition 1

Differentiating $C(q_0)$ twice with respect to q_0, we have $C^{(1)}(q_0)=(p_b-p_p)+C_s^{(1)}(q_0)$ and $C^{(2)}(q_0)=C_s^{(2)}(q_0)>0$. Therefore, $C(q_0)$ is convex in q_0. When $q_0=0$, $C^{(1)}(0)=(p_b-p_p)+C_s^{(1)}(0)$; when $q_0=q_{p0}$, $C^{(1)}(q_{p0})=(p_b-p_p)+C_s^{(1)}(q_{p0})$. There are three situations:

(i) If $C_s^{(1)}(q_{p0})\le p_p-p_b$, then the optimal $q_0=q_{p0}$;
(ii) If$C_s^{(1)}(0)<p_p-p_b<C_s^{(1)}(q_{p0})$, then the optimal q_0 uniquely satisfies $C_s^{(1)}(q_0)=p_p-p_b$, and $q_0\in(0,q_{p0})$;
(iii) If $p_p-p_b\le C_s^{(1)}(0)$, then the optimal $q_0=0$. Then, the results follow. □

Proof of proposition 2

Under Situation-I where $C_s^{(1)}(q_{p0})\le p_p-p_b$, the optimal $q_0=q_{p0}$. Therefore, the demands at the base and peak periods are $q_b=q_{b0}+q_0=q_T\le(1-\tau)k_b$ and $q_p=q_{p0}-q_0=0$, respectively. The producer's profit under the TOU tariff is $\Pi_T(p_b,p_p,k_b,k_p)=p_bq_T-\beta_bq_T-c_bk_b$. Since $k_b\ge q_T/(1-\tau)$, take $k_b=q_T/(1-\tau)$ to maximize Π_T. Moreover, due to $p_bq_{b0}+p_pq_{p0}=p_0q_T$ and $C_s^{(1)}(q_{p0})\le p_p-p_b$, we have $p_b=p_0-(q_{p0}/q_T)C_s^{(1)}(q_{p0})$ and $p_p=p_0+(q_{b0}/q_T)C_s^{(1)}(q_{p0})$ to maximize Π_T, leading

$\Pi_T=q_T\left[p_0-\frac{q_{p0}}{q_T}C_s^{(1)}(q_{p0})-\beta_b-\frac{c_b}{1-\tau}\right]$. Therefore,

$$\begin{aligned}\Delta\Pi&=(p_b-p_0)q_T+\frac{1}{1-\tau}q_{b0}(\tau\beta_b-\tau\beta_p-\tau c_p)-q_{p0}(\beta_b+\frac{c_b}{1-\tau}-\beta_p-\frac{\tau c_p}{\tau})\\&=(p_b-p_0)q_T+(\tau\beta_b-\tau\beta_p-\tau c_p)(\frac{q_{b0}}{1-\tau}-\frac{q_{p0}}{\tau})-q_{p0}\frac{c_b}{1-\tau}\\&=-q_{p0}C_s^{(1)}(q_{p0})-\frac{\beta_b-\beta_p-c_p}{1-\tau}\bar{q}_0-q_{p0}\frac{c_b}{1-\tau}.\ \square\end{aligned}$$

Proof of proposition 3

Under Situation-III where $p_p-p_b\le C_s^{(1)}(0)$, the consumers will not shift their peak electricity use to the base period, i.e., the optimal $q_0=0$.

The producer's profit under the TOU tariff is $\Pi_T(p_b,p_p,k_b,k_p)=p_bq_{b0}+p_pq_{p0}-\beta_bk_{b0}-c_bk_{b0}-\beta_pk_{p0}\tau-c_pk_{p0}\tau$. The profit difference will be $\Delta\Pi=0$. An optimal solution is $p_b=p_0-(q_{p0}/q_T)C_s^{(1)}(0)$ and $p_p=p_0+(q_{b0}/q_T)C_s^{(1)}(0)$. Therefore, the results follow. □

Proof of theorem 1

Under Situation-II where $C_s^{(1)}(0)\le p_p-p_b\le C_s^{(1)}(q_{p0})$, by Proposition 1, the optimal shifted electricity use q_0 satisfies the first-order condition, i.e., $C_s^{(1)}(q_0)=p_p-p_b$, and $q_0\in[0,q_{p0}]$. Together with (5), we have $\begin{cases}p_bq_{b0}+p_pq_{p0}=p_0(q_{b0}+q_{p0})\\p_p-p_b=C_s^{(1)}(q_0)\end{cases}$. Therefore, the pricing at the base and peak periods are $\begin{cases}p_b=p_0-(q_{p0}/q_T)C_s^{(1)}(q_0)\\p_p=p_0+(q_{b0}/q_T)C_s^{(1)}(q_0)\end{cases}$. For the consumers, the cost saving is$-\Delta C_T=(p_p-p_b)q_0-C_s(q_0)=C_s^{(1)}(q_0)q_0-C_s(q_0)$.

Regarding the installed capacity in the peak period, there are two cases:

Case (a). $q_p\le k_b\tau$. Then $k_p=0$, and the producer's profit under the TOU tariff is $\Pi_T=p_b(q_{b0}+q_0)+p_p(q_{p0}-q_0)-\beta_bq_T-c_bk_b$, subject to the capacity constraints:

$\begin{cases}q_b\le(1-\tau)k_b\\q_p\le\tau(k_b+k_p)=\tau k_b\end{cases}$, which is equivalent to $\begin{cases}k_b\ge q_b/(1-\tau)\\k_b\ge q_p/\tau\end{cases}$.
Although we have the assumption $q_{p0}/\tau\ge q_{b0}/(1-\tau)$, we cannot directly decide the relationship between $q_b/(1-\tau)$ and q_p/τ. Therefore, we have to consider two subcases: Subcase (a.1) $q_b/(1-\tau)\ge q_p/\tau$(i.e., $q_0\ge\bar{q}_0$), and Subcase (a.2) $q_b/(1-\tau)\le q_p/\tau$ (i.e.,$q_0\le\bar{q}_0$). We then consider these two cases as follows.

Subcase (a.1). $q_0\ge\bar{q}_0$. With full utilization of capacity, we have $k_b=q_b/(1-\tau)$ in this subcase. Then the profit difference is

$$\begin{aligned}\Delta\Pi&=q_0\left[(p_b-p_p)-\frac{c_b}{1-\tau}\right]+(\frac{q_{b0}}{1-\tau}-\frac{q_{p0}}{\tau})[\tau\beta_b-\tau\beta_p-\tau c_p]\\&=q_0\left[-C_s^{(1)}(q_0)-\frac{c_b}{1-\tau}\right]+(\frac{q_{b0}}{1-\tau}-\frac{q_{p0}}{\tau})[\tau\beta_b-\tau\beta_p-\tau c_p].\end{aligned}$$

By taking the first derivative of $\Delta\Pi$with respect to q_0, we have $\Delta\Pi^{(1)}(q_0)=-C_s^{(1)}(q_0)-c_b/(1-\tau)-q_0C_s^{(2)}(q_0)<0$. It implies that $\Delta\Pi$ is decreasing in q_0. Therefore, the optimal $q_0=\bar{q}_0$.

Subcase (a.2). $q_0 \le \bar{q}_0$. With full utilization of capacity, we have $k_b = q_p/\tau$ in this subcase. Then the profit difference is

$$\Delta\Pi = q_0\left[(p_b - p_p) + \frac{c_b}{\tau}\right] + \left(\frac{q_{b0}}{1-\tau} - \frac{q_{p0}}{\tau}\right)\left[\tau\beta_b + c_b - \tau\beta_p - \tau c_p\right]$$
$$= q_0\left[-C_s^{(1)}(q_0) + \frac{c_b}{\tau}\right] + \left(\frac{q_{b0}}{1-\tau} - \frac{q_{p0}}{\tau}\right)\left[\tau\beta_b + c_b - \tau\beta_p - \tau c_p\right].$$

By taking the first and second derivatives of $\Delta\Pi$ with respect to q_0, we have $\Delta\Pi^{(1)}(q_0) = -C_s^{(1)}(q_0) + c_b/\tau - q_0 C_s^{(2)}(q_0)$ and $\Delta\Pi^{(2)}(q_0) = -2C_s^{(2)}(q_0) - q_0 C_s^{(3)}(q_0) < 0$. Therefore, $\Delta\Pi$ is concave in q_0, and we have the following results:

(i) when $c_b/\tau \le C_s^{(1)}(0)$, the optimal $q_0 = 0$, the producer will not implement the TOU tariff;
(ii) when $C_s^{(1)}(0) < c_b/\tau < C_s^{(1)}(\bar{q}_0) + \bar{q}_0 C_s^{(2)}(\bar{q}_0)$, the optimal q_0 satisfies the condition $C_s^{(1)}(q_0) + q_0 C_s^{(2)}(q_0) = c_b/\tau$;
(iii) when $C_s^{(1)}(\bar{q}_0) + \bar{q}_0 C_s^{(2)}(\bar{q}_0) \le c_b/\tau$, the optimal $q_0 = \bar{q}_0$.

Note that in Subcase (a.1), the optimal solution of q_0 lies at the boundary of the condition for Subcase (a.2), i.e., $q_0 = \bar{q}_0$. Therefore, Subcase (a.1) is dominated by Subcase (a.2). By combining Subcases (a.1) and (a.2), we have the results that, if $q_p \le k_b\tau$, then $k_p = 0$, $k_b = q_p/\tau$, and the optimal q_0 is determined by (i), (ii), and (iii) above.

Case (b). $\tau k_b \le q_p \le q_T$. Then $k_p \ge 0$, and the producer's profit under the TOU tariff is $\Pi_T = p_b(q_{b0} + q_0) + p_p(q_{p0} - q_0) - \beta_b k_b - c_b k_b - \beta_p k_p \tau - c_p k_p \tau$, which is decreasing in k_b and k_p. Similarly, we cannot directly decide the relationship between $q_b/(1-\tau)$ and q_p/τ, and we have to consider two subcases: Subcase (b.1) $q_b/(1-\tau) > q_p/\tau$and Subcase (b.2) $q_b/(1-\tau) \le q_p/\tau$. We then consider these two cases as follows.

Subcase (b.1). $q_b/(1-\tau) > q_p/\tau$. Recall that in Case (b), we have $\tau k_b \le q_p$, so $k_b \le q_p/\tau < q_b/(1-\tau)$. On the other hand, we have the following capacity constraints: $k_b \ge q_b/(1-\tau)$ and $k_b + k_p \ge q_b/\tau$. Therefore, there is a contradiction in this subcase, and we need not consider this subcase in the following analysis.

Subcase (b.2). $q_b/(1-\tau) \le q_p/\tau$. Similarly, we have the constraints: $k_b \ge q_b/(1-\tau)$ and $k_b + k_p \ge q_b/\tau$. And in this subcase with the assumption $\tau(\beta_p + c_p) < \beta_b + c_b$, the optimal combination of k_b and $k_p$that maximize $\Pi_T$is $k_b = q_b/(1-\tau)$ and $k_p = q_p/\tau - q_b/(1-\tau)$.

Then the profit difference is

$$\Delta\Pi = q_0\left[(p_b - p_p) - \frac{\beta_b + c_b}{(1-\tau)} + \frac{\beta_p + c_p}{(1-\tau)}\right] = q_0\left[-C_s^{(1)}(q_0) + A\right],$$

where $A = \left((\beta_p + c_p) - (\beta_b + c_b)\right)/(1-\tau) \ge 0$. It is noted that A is independent of q_0. With the first and second derivatives, we have $\Delta\Pi^{(1)}(q_0) = -C_s^{(1)}(q_0) + A - q_0 C_s^{(2)}(q_0)$ and $\Delta\Pi^{(2)}(q_0) = -2C_s^{(2)}(q_0) - q_0 C_s^{(3)}(q_0) < 0$. Note that $q_0 \le \bar{q}_0$ in this subcase. Together with $q_0 \ge 0$, we have $0 \le q_0 \le \bar{q}_0$. When $q_0 = 0$, $\Delta\Pi^{(1)}(0) = -C_s^{(1)}(0) + A$. When $q_0 = \bar{q}_0$, $\Delta\Pi^{(1)}(\bar{q}_0) = -C_s^{(1)}(\bar{q}_0) + A - \bar{q}_0 C_s^{(2)}(\bar{q}_0)$. There are three sub-cases: (1) when $A \le C_s^{(1)}(0)$, the optimal $q_0 = 0$; (2) when $C_s^{(1)}(0) < A < C_s^{(1)}(\bar{q}_0) + \bar{q}_0 C_s^{(2)}(\bar{q}_0)$, the optimal q_0 satisfies the condition $C_s^{(1)}(q_0) + q_0 C_s^{(2)}(q_0) = A$; and (3) when$C_s^{(1)}(\bar{q}_0) + \bar{q}_0 C_s^{(2)}(\bar{q}_0) \le A$, the optimal $q_0 = \bar{q}_0$.

Note that in Case (a), $k_b = q_p/\tau$, which is on the boundary of the condition for Case (b). Thus, Case (a) is dominated by Case (b). Combining Cases (a) and (b), we have the following conclusions:

(i) when $A \le C_s^{(1)}(0)$, the optimal $q_0 = 0$, the producer will not implement the TOU tariff;
(ii) when $C_s^{(1)}(0) < A < C_s^{(1)}(\bar{q}_0) + \bar{q}_0 C_s^{(2)}(\bar{q}_0)$, then the optimal q_0 satisfies the condition $C_s^{(1)}(q_0) + q_0 C_s^{(2)}(q_0) = A$, the increased profit is $\Delta\Pi(q_0) = q_0^2 C_s^{(2)}(q_0)$ and the consumers' total cost saving is $q_0 C_s^{(1)}(q_0) - C_s(q_0)$;
(iii) when $C_s^{(1)}(\bar{q}_0) + \bar{q}_0 C_s^{(2)}(\bar{q}_0) \le A$, then the optimal $q_0 = \bar{q}_0$, the increased profit is $\Delta\Pi(q_0) = \bar{q}_0[-C_s^{(1)}(\bar{q}_0) + A]$ and the consumers' total cost saving is $\bar{q}_0 C_s^{(1)}(\bar{q}_0) - C_s(\bar{q}_0)$.

Proof of proposition 6

Based on Theorem 1, we have $(1-\tau)p_b + \tau p_p = p_0 + (1/q_T)C_s^{(1)}(q_0)[\tau q_{b0} - (1-\tau)q_{p0}]$. As $(q_{p0}/\tau) > (q_{b0}/1-\tau)$, we have $\tau q_{b0} - (1-\tau)q_{p0} < 0$. Therefore, $(1-\tau)p_b + \tau p_p < p_0$. □

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