Optimal Scheduling of Critical Peak Pricing Considering Wind Commitment

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***Abstract*—Demand response has been widely implemented as one of the "virtual" control mechanisms to make peak load management more efficient and economic. One of the popular demand response programs is called critical peak pricing (CPP). Relatively simple pricing schemes and convenient implementation within current energy system metering infrastructure make it well accepted by many utilities and load-serving entities (LSEs). In this paper, we investigate the optimal scheduling of CPP events from the perspective of an LSE which has wind energy to sell into the day-ahead market. The goal is to minimize the total operational cost for the whole planning horizon, taking into account the energy purchasing cost, revenue from the CPP, and wind energy sales, as well as imbalance penalties due to wind energy over- and under-commitments. We propose a multi-stage stochastic mixed integer nonlinear programming model. In addition, we perform various analyses of both the special case of a single-stage problem and the general multi-stage problem analytically and experimentally. Our analysis leads to useful operational insights and policy implications on how to manage a renewable-integrated system more efficiently.**

***Index Terms*—Integer programming, optimal control, optimization methods, power generation scheduling, strategic planning, wind.**

NOMENCLATURE

In this section, we present the notation of the model. We shall explain them in more detail when we describe the model in Section II.

A. Parameters

T	Planning horizon.
Γ	Set of planning periods $\{1,\ldots,T\}$.
i	Index for period within Γ, $i=1,\ldots,T$.

Sources of randomness

w_i	Actual wind energy output.
p_i^L	Lower market price rate.
p_i^H	Higher market price rate.
p_i^w	Day-ahead wind energy price.
p_i^1	Non-participants' rate.
p_i^2	CPP participants' rate during non-critical hours and also their rate during critical hours if a CPP event is not scheduled.
P_i^u	Imbalance penalty for under-bidding wind energy.
P_i^d	Imbalance penalty for over-bidding wind energy.
Q_i	Total demand.
q_i^1	Non-participants' demand.
q_i^2	Participants' demand during non-critical peak hours.
q_i^c	Participants' demand during critical peak hours.

Other parameters

τ_i	Number of periods from the previous CPP event till period i.
N_i	Total number of CPP events called as of period i.
K	Threshold of incurring p_i^H.
u	Allowed upper band for over-committing wind.
d	Allowed lower band for under-committing wind.
Δq_i^c	Participants' load reduction.
q_i	Energy purchased from DAM.
$p_{\max}$	Maximum allowed CPP rate.
$N_{\max}$	Maximum number of critical peak events allowed.
$\tau_{\min}$	Minimum number of periods between two consecutive CPP events.
ϵ	Empirical stationary price elasticity of participants' integrated load.
f	Function of load reduction with respect to the critical peak rate and other parameters.

B. Decision Variables

x_i	Indicator for scheduling a CPP event, $x_i=1$ if an event scheduled for period i and $x_i=0$ otherwise.
y_i	CPP participants' rate during critical peak hours on a *CPP day* (i.e., on a day when a CPP event is called).
z_i	Wind energy committed into DAM.

I. INTRODUCTION

DEMAND response mechanisms have been introduced to encourage electricity customers to reduce their consumption at critical times, thereby reducing peak loads on electricity grids. In the past, utilities mainly used direct load control programs and time-of-use tariffs to induce residential peak load reduction. Direct load control allows the utilities to control large residential electrical appliances by providing monthly bill credits and such programs have been popular since the 1980s due to their convenience to implement within the current metering systems. Time-of-use generally implements a higher price during afternoon peak hours and a regular price for the other hours during the day. Real-time pricing is more effective in tracking the dynamic nature of market prices than both direct load control and time-of-use, but it has been considered not economically beneficial enough due to the complexity entailed for small electricity users [1].

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Another type of demand response program, whose characteristics are between direct load control, time-of-use, and real-time pricing, is called critical peak pricing (CPP). A CPP program offers the participating customers lower than alternative rates during non-critical hours and days but higher rates during critical hours. Such a pricing scheme induces the participating customers to reduce their electricity usage during critical hours, when the real-time market (RTM) price is high, so that the load-serving entities (LSEs) can avoid buying the extra energy from the RTM at a very high marginal cost. From the participants' standpoint, they benefit from paying a lower than alternative rate during non-critical days. Since there are usually many more non-critical days than critical event days, a cooperative participant can enjoy substantial cost savings on his/her electricity bill at the end of a billing cycle. Finally, from the social planner's perspective, CPP programs, or more generally demand response mechanisms, are intended to maximize socio-economic benefits by inducing energy conservation and peak load shifting.

The CPP programs have been successfully implemented among both commercial and residential customers, often during summers when the system-level demands and the wholesale electricity prices are very high. For example, in 2010, 9, 12, and 4 critical peak event days were scheduled by three electric companies on 1650, 4100, and 1350 customers, respectively, leading to 3.9%, 2.8%, and 5.26% reduction in the average event day load [2].

Critical peak event days can be selected based on environmental factors such as temperature, market prices, and/or electricity supply and demand conditions [3], and an event is usually announced one day ahead. Fig. 1 demonstrates the daily actual load and maximum temperature processes, which exhibit high correlations. This explains why in some existing programs CPP events are scheduled according to a temperature-based criterion.

The CPP, or in general demand response, can be viewed as supplying via "virtual" generation. The other source of supply that has increasing penetration in today's energy system is that from the renewable resources. In order to stimulate the renewable energy usage in the United States, a Renewable Portfolio Standard (RPS) was established in 2002 and has been accelerated since. The RPS program requires utilities and service providers to produce a target percentage level of energy from renewable resources. For example, RPS sets the target for California as 20% in 2010 and 33% by 2020, respectively.

In this paper, we are concerned with one specific type of renewable energy, namely wind energy. There are two major advantages of using wind energy compared to traditional sources: environmental friendliness and economy. One challenge, yet, is that the supply level of wind power is more difficult to forecast. Such uncertainty is due to the high variability of wind speeds and directions.

In general, there are two markets for wind energy providers to participate in, the day-ahead market (DAM) and the RTM. We shall only consider the DAM in this paper. Wind energy providers use the recent day-ahead generation and price forecasts to decide on the amount of wind energy to commit into the DAM, i.e., a contract size. Due to the congestion conditions in regional grids and the required accuracy in matching supply with demand, wind farm owners are often required to pay imbalance penalty costs if the wind delivery amount for the next day is higher or lower than that specified in the contract. The imbalance penalty cost rate for over-generation may not be the same as that for under-generation, and the specifics of such a penalty mechanism may be quite different among different areas. For example, over-production will not incur any penalty in some places in the United States, in order to avoid the under-bidding strategy by wind farm owners [4].

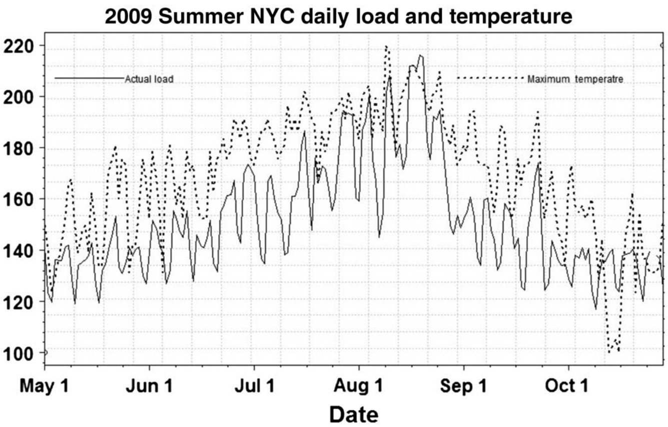

Fig. 1. Typical summer load and maximum temperature.

Because of the high volatility of wind energy generation, one reasonable imbalance penalty scheme, which we consider in this paper, is to impose a flexible "band" around the contracted amount. Specifically, when the actual wind energy supply exceeds (is lower than) the contract size by a certain factor, then an over-generation (under-generation) penalty cost is charged and no penalty cost is incurred as long as the actual delivery amount falls within the band.

There have been some works in the literature on or related to the implementation of CPP programs. Herter *et al.* [5] investigate participants' load response with respect to temperature. Piette *et al.* [6] study the automated critical peak test field in California in 2005 with the goal to evaluate how CPP automation can increase the participation rate and load-saving efficiency. This experiment also helps to find the control strategies for system-wide demand response. The critical peak price considered that there is a two-level fixed time interval pricing scheme. Herter [7] reviews the CPP programs with residential customers and discusses the relationship between the CPP benefits and satisfaction degree with residential income levels. Joo *et al.* [8] formulate the optimal critical day scheduling problem using the swing option evaluation approach and consider three different pricing dynamics. Zhang *et al.* [9] formulate the optimal CPP strategy problem with integer programming and consider the maximization of the entire savings of an LSE and customers. Black and Tyagi [10] investigate several potential problems with CPP programs when the participants' power consumption level is comparatively large and is not quite sensitive to the nominal time frame. Tyagi *et al.* [11] study the optimal event scheduling problem using a temperature-based threshold policy and propose a value iteration formulation.

As for the optimal wind energy contracting problem considering imbalance costs, Matevosyan and Soder [12] formulate the imbalance cost minimization problem for wind farm owners within the stochastic optimization framework and propose optimal bidding strategies under several imbalance cost structures. Piwko *et al.* [4] assume zero penalty cost for under-generation of wind power, which prevents the under-bidding strategy that would be harmful for the electricity market. Zhang [13] studies the optimal wind-bidding strategy considering both the imbalance penalty and allowed imbalance bandwidths.

Our goal is to develop a scientific approach to scheduling CPP events in the presence of renewable energy (specifically under wind energy penetration). To the best of our knowledge, this paper is the first study that examines these two aspects jointly. Specifically, the model we study in this paper considers an LSE as the decision maker with wind generation. The LSE sells a certain amount of wind energy in the DAM and also reserves a certain amount of wind energy to satisfy local demand. In addition, the LSE provides industry energy users with a CPP contract, in agreement that the participants will reduce load upon a CPP event notice. The objective is to jointly optimize, from the LSE's point of view, CPP scheduling (including event calling and critical hour pricing) and the amount of wind energy committed to the wind DAM, in order to minimize its total expected cost. We also consider the stochastic nature of wind power and thus variability of wind commitment strategies that depend on the imbalance penalties and allowed imbalance bands imposed on renewable providers.

The remainder of this paper is organized as follows. In the Nomenclature, we list the notation that we use for the optimization model formulation. In Section II, we present our stochastic optimization model for both the single period problem (Section II-A) and multi-period problem (Section II-B) in detail. In Section III, we provide some analytical and numerical results for the single-stage problem and discuss the qualitative insights yielded by our analysis. A case study of the multi-stage problem is provided in Section IV. Finally, we make some concluding remarks in Section V.

II. Model Formulation

In this section, we propose a general stochastic programming model for joint optimization of CPP event scheduling and wind energy commitments. We start by formulating the stochastic optimization problem for a single stage and subsequently extend the model to multiple stages.

A. Single-Stage Model

In this section, we present a single-stage (or single delivery-day) stochastic model for the joint optimal CPP scheduling and wind energy commitment problem. Adopting the notation introduced in the preceding section, we simply have $T = 1$, $\Gamma = \{1\}$, and $N_{\max} = 1$. Therefore, for the sake of simplicity, we omit the subscript (or period index) in the notation used for the single-stage problem in this section and also in the analysis in Section III.

Consider an LSE that serves its customer demand and also has wind energy (e.g., generated by a wind farm owned by the LSE) to sell into the DAM. The LSE can choose to schedule a CPP event by charging the participating customers higher energy prices than otherwise applied rates during critical peak hours, and in response to the event scheduling, the participants will reduce their demand during those hours. Note that LSEs are regulated as alluded to in Section I, they may schedule only a limited number of CPP events during each season or year, and also the energy markup rate for critical peak hours must be within a reasonable range (see [14] for a concrete example).

For a single delivery day, the LSE needs to decide whether to schedule a CPP event or not x, the rate for CPP participants during critical hours on a CPP day y, as well as the amount of wind energy to commit to sell in the DAM z, subject to uncertainty in customer demand (or load), wind energy supply, and market prices.

Specifically, the first source of randomness is demand. The total demand Q consists of three portions: q^1 due to non-participants of the CPP program, q^2 due to CPP participants during non-critical hours, and q^c due to CPP participants during critical peak hours.

The second source of randomness is the realized wind energy supply w. We assume an imbalance band penalty mechanism with constant allowed upper bandwidth u, constant allowed lower bandwidth d, and the corresponding imbalance penalty rates P^u and P^d, respectively. That is, for each unit of realized wind energy supply exceeding $(1+u)z$, a penalty of P^u is charged on the LSE and for each unit of shortage below $(1-d)z$, a penalty of P^d is imposed.

Note that P^u and P^d must be greater than the wind energy settlement price p^w to prevent intentional over- or under-commitment and these three rates are highly correlated with other price parameters (p^L, p^H, p^1, p^2). Throughout this paper, we assume that the LSE's energy purchasing cost has a piecewise linear convex form, or more specifically, the cost of a units of energy is given by $p^L \min\{a, K\} + p^H (a-K)^+$ for a constant K, where $p^L < p^H$ and p^L (p^H) denote the average cost for purchasing from the market, a unit of energy supply above (below) threshold K. This assumption is made without loss of generality since the cost is typically convex in the purchasing quantity and most convex functions can be well approximated by a piecewise linear function. Also, the retail price for CPP participants during non-critical hours and during critical hours on a non-CPP day p_2 is typically lower than non-participants' price p_1. These seven rate parameters constitute the third source of uncertainty in the model.

Note that if a CPP event is called or $x = 1$, then CPP participants will reduce their load during critical hours by the amount of Δq^c and the magnitude of this load reduction is a function of the pre-CPP load during these hours q^c, the chosen CPP rate y, the otherwise applied rate p^2, and the elasticity coefficient ϵ, i.e., $\Delta q^c = f(q^c, y, p^2, \epsilon)$. In particular, an independent system operator or LSE's usually estimate the function $f(\cdot)$ from empirical experiences and history data, and often $\epsilon = \epsilon(S)$, where S is some exogenous environmental variable.

Therefore, the actual total demand, with the incentivizing effect of a possible CPP event taken into consideration, can simply be written as $Q - x \cdot \Delta q^c$. We further assume that the LSE uses all of its reserved wind energy $(w-z)^+$ to meet the demand and purchase energy from the market to satisfy any remaining unmet demand $q = q(x, y, z) = Q - x \cdot \Delta q^c - (w-z)^+$.

The objective of the LSE is then to choose the triple (x^*, y^*, z^*) that minimizes the expected cost function $\mathbb{E}_{\boldsymbol{\xi}}[C(x,y,z)]$, where

$$\begin{aligned} C(x,y,z) = {} & p^L \min\{q(x,y,z), K\} + p^H[q(x,y,z) - K]^+ \\ & + [w - (1+u)z]^+ P^u + [(1-d)z - w]^+ P^d \\ & - [p^w z + p^1 q^1 + p^2 q^2 + xy(q^c - \Delta q^c) \\ & + (1-x)p^2 q^c] \end{aligned} \quad (1)$$

and the expectation is taken with respect to the random vector $\boldsymbol{\xi}\{w, p^L, p^H, p^w, p^1, p^2, P^u, P^d, Q, q^1, q^2, q^c\}$.

Since $x = 0$ or 1, the minimization of $\mathbb{E}_{\boldsymbol{\xi}}[C(x,y,z)]$ boils down to comparing $EC(1)$ and $EC(0)$, where $EC(x) := \min_{(y,z)} \mathbb{E}_{\boldsymbol{\xi}}[C(x,y,z)]$, and by this definition we can represent the expectation of the cost saving due to a CPP event, say ΔC, as

$$\mathbb{E}_{\boldsymbol{\xi}}[\Delta C] = EC(0) - EC(1). \quad (2)$$

Note that if $K = \infty$, then the terms with z in the expression of $C(x,y,z)$ can be separated out from those with x and y, and so the optimal contract size z^* may simply be determined independently, in a similar manner as in [13].

For a finite K value (as is the case in practice), joint coordination of CPP scheduling and wind energy commitment does have significant impacts on the LSE's operating costs. Specifically, if the scheduling of a CPP event and the wind energy commitment is decided separately, the optimal triple $(x^\bullet, y^\bullet, z^\bullet)$ is obtained by solving $\min_{(x,y)} \mathbb{E}_{\boldsymbol{\xi}_a}[\pi_1(x,y)]$ and $\min_z \mathbb{E}_{\boldsymbol{\xi}_b}[\pi_2(z)]$, respectively, where

$$\begin{aligned} \pi_1(x,y) = {} & p^L \min\{q_s(x,y), K\} + p^H[q_s(x,y) - K]^+ \\ & - [p^1 q^1 + p^2 q^2 + xy(q^c - \Delta q^c) + (1-x)p^2 q^c] \end{aligned} \quad (3)$$

with $q_s(x,y) := Q - x \cdot \Delta q^c$, and

$$\pi_2(z) = [w - (1+u)z]^+ P^u + [(1-d)z - w]^+ P^d - p^w z \quad (4)$$

with $\boldsymbol{\xi}_a = \{p^L, p^H, p^1, p^2, Q, q^1, q^2, q^c\}$ and $\boldsymbol{\xi}_b = \{w, p^w, P^u, P^d\}$, respectively. Therefore, we may express the expected *profit degradation* (or equivalently cost increase) caused by managing CPP scheduling and wind energy commitments separately as

$$\begin{aligned} & \mathbb{E}_{\boldsymbol{\xi}}[\text{profit degradation}] \\ & \quad = (\mathbb{E}_{\boldsymbol{\xi}_a}[\pi_1(x^\bullet, y^\bullet)] + \mathbb{E}_{\boldsymbol{\xi}_b}[\pi_2(z^\bullet)]) - \mathbb{E}_{\boldsymbol{\xi}}[C(x^*, y^*, z^*)]. \end{aligned} \quad (5)$$

Before extending the above formulation to multiple stages, we note that even for this single-stage model, closed-form analytical results are difficult to derive. For a general distribution of $\boldsymbol{\xi}$, the evaluation of $EC(x)$ and the minimization of $\mathbb{E}_{\boldsymbol{\xi}}[C(x,y,z)]$ need to be performed numerically or via Monte Carlo simulation, and so does the calculation of (2) and (5). In Section III, we shall conduct further analysis under additional distributional assumptions on $\boldsymbol{\xi}$.

B. Multi-Stage Stochastic Programming Model

For the multi-stage joint optimal CPP scheduling and wind energy commitment problem, we propose the following multi-stage stochastic mixed integer nonlinear programming (SMINLP) model:

$$\min_{(x_i,y_i,z_i)_{i\in\Gamma}} \sum_{i\in\Gamma} \mathbb{E}_{\boldsymbol{\xi}_i}[C_i(x_i,y_i,z_i)] \quad (6)$$

where

$$\sum_{i\in\Gamma} x_i \le N_{\max} \quad (7)$$

and for any $i \in \Gamma$

$$\begin{aligned} \Delta q_i^c &= f(q_i^c, y_i, p_i^2, \epsilon) && (8) \\ C_i(x_i,y_i,z_i) &= p_i^L \min\{q_i(x_i,y_i,z_i), K\} + p_i^H[q_i(x_i,y_i,z_i) - K]^+ \\ &\quad + [w_i - (1+u)z_i]^+ P_i^u + [(1-d)z_i - w_i]^+ P_i^d \\ &\quad - [p_i^w z_i + p_i^1 q_i^1 + p_i^2 q_i^2 + x_i y_i (q_i^c - \Delta q_i^c) \\ &\quad + (1-x_i) p_i^2 q_i^c] && (9) \\ q_i(x_i,y_i,z_i) &= Q_i - x_i \cdot \Delta q_i^c - (w_i - z_i)^+ && (10) \\ \tau_{i+1} &= \tau_i + 1 - x_i \tau_i, \quad \text{with } \tau_1 = \tau_{\min} && (11) \\ \tau_i &\ge \tau_{\min} x_i && (12) \\ x_i &\in \{0,1\} && (13) \\ y_i &\in [p_i^2, p_{\max}] && (14) \\ z_i &\ge 0 && (15) \\ \boldsymbol{\xi}_i &= \{w_i, p_i^L, p_i^H, p_i^w, p_i^1, p_i^2, P_i^u, P_i^d, Q_i, q_i^1, q_i^2, q_i^c\}. && (16) \end{aligned}$$

The objective function and most of the constraints in the above formulation are natural extensions of those in the single-stage model introduced in the preceding section. Equation (7) ensures that the LSE can schedule at most $N_{\max}$ CPP events across the T periods. Equation (11) provides a recursive definition of τ_i's, whose values depend on the choice of x_i's. Equation (12) guarantees that a CPP event is scheduled for period i only if $\tau_i \ge \tau_{\min}$.

We recommend that this multi-stage model is used as a rolling horizon planning tool. Specifically, depending on the availability of computing power and other practical constraints, the LSE should resolve the model every day or every few days using updated price and demand forecasts.

C. Scenario-Based Formulation

Although the random vector $\boldsymbol{\xi}$ is usually defined on a continuous probability space, we can always discretize the space and obtain a sufficiently good approximating space $(\Omega, \mathcal{F}, \mathcal{P})$ with a finite support, i.e., $\Omega = (\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_K)$ with probabilities $P_1, \ldots, P_K$. Then the uncertainty in the objective function (6) can be represented using scenarios that realize each $\boldsymbol{\xi}_k \in \Omega, \quad k = 1, \ldots, K$. The decision in scenario $\boldsymbol{\xi}_k \in \Omega$ is denoted as $\{x_{ik}, y_{ik}, z_{ik}\}$. The multi-stage SMINLP (6)–(16)

can be reformulated as the following equivalent deterministic MINLP:

$$\min \sum_{\boldsymbol{\xi}_k \in \Omega} P_k \left[\sum_{i=1}^{T} C_i(x_{ik}, y_{ik}, z_{ik}) \right]$$

$$\text{such that for any } i \in \Gamma \text{ and } k = 1, \ldots, K \tag{17}$$

$$\sum_{j=1}^{i} x_{jk} \le N_{\max} \tag{18}$$

$$q_{ik} = Q_{ik} - x_{ik} \cdot \Delta q_{ik}^c - (w_{ik} - z_{ik})^+$$

$$x_{(t-1)k} - x_{tk} \le 1 - x_{ik} \tag{19}$$

$$\forall t \in [i - \tau_{\min} + 1, i - 1], \quad i \in \Gamma / \{1\} \tag{20}$$

$$x_{ik} = \frac{\left[\sum_{\boldsymbol{\xi}_u \in \Omega_i^k} P_u x_{iu} \right]}{\sum_{\boldsymbol{\xi}_u \in \Omega_i^k} P_u} \tag{21}$$

$$x_{ik} \in \{0, 1\} \tag{22}$$

$$y_{ik} \in [p_i^2, p_{\max}] \tag{23}$$

$$z_{ik} \ge 0 \tag{24}$$

where Ω_i^k is the set of scenarios that are indistinguishable from scenario $\boldsymbol{\xi}_k$ up to time step i. Equation (19) can be considered as the coupling constraint. Equations (18) and (20) are the dynamic constraints. Equation (21) is the nonanticipativity constraint requiring all the scenarios with the same realization up to time step i to have the same decision as well.

III. Analysis: Single-Stage Problem

In this section, we perform various analyses of our single-stage model. Our focus is on examining the policy implications of different system characteristics and highlighting the key cost-benefit tradeoffs. In particular, due to the complicated nature of the joint optimization problem, we shall isolate and make some additional assumptions on the three sources of randomness (i.e., demand, wind energy supply, and price) in our model and put emphasis on qualitative operational insights as opposed to technicalities.

Throughout the remainder of this paper, we assume that the responsive load process has a (truncated) linear form (as used in [15]). Specifically,

$$\Delta q_i^c = f(q_i^c, y_i, p_i^2, \epsilon) := \epsilon \cdot q_i^c \cdot \frac{y_i - p_i^2}{p_i^2} \tag{25}$$

where the elasticity coefficient ϵ equals the percentage of CPP participants' load reduction due to each percentage point's increase in the critical peak hour rate. In this section, we restrict ourselves to the single-stage model proposed in Section II-A.

First, we assume in this section that the wind energy supply is the only source of uncertainty, i.e., $\boldsymbol{\xi} = \{w\}$. We further assume that the cost rate threshold value K is moderate, i.e., such that $\min\{q(x, y, z), K\} = K$ always holds. In this case, we have derived the following sufficient condition on the price characteristics for the optimality of scheduling CPP.

Proposition 1: Let $r := p_{\max}/p^2$. If

$$p^H > \frac{p_{\max}[1 - \epsilon(r-1)] - p^2}{\epsilon(r-1)} \tag{26}$$

it is always optimal to schedule a CPP event, i.e., $x^* = 1$.

Proof: Let $z_0^* = \arg\min_z \mathbb{E}_{\boldsymbol{\xi}}[C(0, 0, z)]$, we have

$$EC(0) - EC(1) \ge \mathbb{E}_{\boldsymbol{\xi}}[C(0, 0, z_0^*)] - \mathbb{E}_{\boldsymbol{\xi}}[C(1, p_{\max}, z_0^*)] > 0 \tag{27}$$

where the inequality holds since $EC(1) \le \mathbb{E}_{\boldsymbol{\xi}}[C(1, p_{\max}, z_0^*)]$ by the definition of $EC(\cdot)$. The desired condition then easily follows by solving inequality (27). ■

Proposition 1 suggests that when the higher energy cost purchasing cost rate p^H exceeds a certain level, it is always beneficial to schedule a CPP event. Note that p^H is highly correlated with, or simply can be viewed as a proxy for, the *RTM price*, since the way that many LSEs purchase energy from the market is that they first buy from the DAM, a base quantity at the day-ahead price and then satisfy their remaining needs from the RTM, which usually has a higher average price and greater volatility (see e.g., [16]). Exactly for the same reason, we have made the realistic assumption that $\min\{q(x, y, z), K\} = K$ always holds.

Next, we derive the analytical characterization of the optimal contract size. We denote $\phi(\cdot)$ and $\Phi(\cdot)$ as the standard normal probability density and cumulative distribution functions, respectively.

Proposition 2: Let $w \sim \mathbb{N}(\mu, \sigma^2)$. The optimal contract size z^* solves

$$p^H\Phi(\eta) + [-(1+u)P^u\Phi(\alpha)] + (1-d)P^d\Phi(\beta) - p^w = 0 \tag{28}$$

where $\alpha = (\mu - (1+u)z)/\sigma$, $\beta = ((1-d)z - \mu)/\sigma$, and $\eta = (\mu - z)/\sigma$.

Proof: Under the above assumption, the objective function has the following explicit form:

$$\begin{aligned} \mathbb{E}_{\boldsymbol{\xi}}[C(x, y, z)] &= \mathbb{E}_w[C(x, y, z)] \\ &= p^L K + p^H(Q - x\Delta q^c \\ &\qquad - (\mu - z)\Phi(\eta) - \sigma\phi(\eta) - K) \\ &\quad + [\mu - (1+u)z]\Phi(\alpha)P^u + \sigma\phi(\alpha)P^u \\ &\quad + [(1-d)z - \mu]\Phi(\beta)P^d + \sigma\phi(\beta)P^d \\ &\quad - [p^w z + p^1 q^1 + p^2 q^2 + xy(q^c - \Delta q^c) \\ &\qquad + (1-x)p^2 q^c]. \end{aligned} \tag{29}$$

Equation (28) simply follows from setting to zero the derivative with respect to z. ■

We remark that one can utilize the above characterization of z^* to answer some interesting questions. For example, the sensitivity analysis of z^* can be performed by either numerically solving (28) under different parameter settings or applying the implicit function theorem (e.g., see [17]) to analytically derive the partial derivative of z^* with respect to the various parameters.

Our next result examines and compares the two operating levels, i.e., "virtual" generation from a CPP event and the real generation of wind energy.

Proposition 3: If

$$p^H < \left[q^c\epsilon(r-1) - \frac{\sigma}{\sqrt{2\pi}}\right]^{-1} \cdot \{p^2q^c + p^w\mu + p^u\varphi(u) + p^2q^c + p^2q^c + p^w\mu + p^u\varphi(u) + p^d\varphi(d) - p_{\max}q^c[1-\epsilon(r-1)]\} \quad (30)$$

with $\varphi(x) := x\mu\Phi(-x\mu/\sigma) + \sigma\phi(-x\mu/\sigma)$, then $\mathbb{E}_w[C(0,0,\mu)] < \min_y \pi_1(1,y)$, where $\pi_1(x,y)$ is defined in (3).

Note that $\min_y \pi_1(1,y)$ gives the optimal cost during a CPP event day in a conventional system without any wind energy penetration. Therefore, Proposition 3 states that, under certain conditions, the simplistic policy of committing into the DAM exactly the forecasted mean value of the wind energy supply can lead to a lower operating cost than that in a conventional system with the CPP "virtual" generation.

Proof of Proposition 3: Under the assumptions made in this section, we have that

$$\mathbb{E}_w[C(0,0,\mu)] = p^LK + p^H\left(Q - \frac{\sigma}{\sqrt{2\pi}} - K\right) + P^u\varphi(u) + P^d\varphi(d) - (p^wz + p^1q^1 + p^2q^2 + p^2q^c)$$

and

$$\pi_1(1,y) = p^LK + p^H(Q - \Delta q^c - K) - [p^1q^1 + p^2q^2 + y(q^c - \Delta q^c)].$$

Then using (25) and the fact that $\arg\min_y \pi_1(1,y) \le p_{\max}$, we conclude from some simple calculations that $\min_y \pi_1(1,y) > \mathbb{E}_w[C(0,0,\mu)]$, when (30) holds. ■

IV. Case Study: Multi-Stage Model

In this section, we present illustrative examples to show how the proposed multi-stage stochastic programming model can be used to jointly optimize the CPP event scheduling and wind energy commitments. As is well known, multi-stage sequential decision-making problems, either in the form of SMINLP or its dynamic programming formulation, remain one of the most difficult problem classes in the (stochastic) optimization literature. Solving such problems analytically, even approximatively, often requires methods that exploit problem-specific structures. We shall not attempt to address such algorithmic challenges in our analysis and rather we focus on discussing operational insights and policy implications.

Consider the renewable-integrated CPP scheduling problem during a typical summer week. Let the planning horizon $T = 7$. The minimum interval days between two consecutive critical event days are $\tau_{\min} = 2$. From 11 A.M. to 3 P.M. are critical peak hours, and at most $N_{\max} = 2$, CPP events can be scheduled. The notification of a CPP event is sent out to participants by 3 P.M. on the previous day. The day-ahead notification allows many large electricity users to pre-cool their systems.

TABLE I
Optimal CPP Scheduling Policy

$(r_{\max}, \epsilon)$	y_1/p_1^L	y_2/p_2^L	y_3/p_3^L	y_4/p_4^L	y_5/p_5^L	y_6/p_6^L	y_7/p_7^L
(3, 0.1)	0	0	3	0	0	3	0
(5, 0.1)	0	0	4.4	0	0	4.5	0
(5, 0.01)	0	0	5	0	0	5	0

TABLE II
Temperature-Based CPP Schedule

Day	1	2	3	4	5	6	7
Temperature (°F)	92	90	88	84	89	93	94
Schedule (1/0)	1	0	0	0	0	1	0

A. Modeling Random Sources

The load process $\{Q_i\}_{i=1}^T$ is modeled based on the historical load data published on Market and Operational Data in NYISO [18]. $\{q_i^1\}_{i=1}^T$, $\{q_i^2\}_{i=1}^T$, and $\{q_i^c\}_{i=1}^T$ are each assumed to be a constant proportion of Q_i's. The price process $\{p_i^L\}_{i=1}^T$ is generated from the published DAM locational marginal pricing. Other price parameters are assumed to be some constant multiples of p_i^L's. We further assume $\{w_i\}_{i=1}^T$ to be a Gaussian process.

B. Scenario Tree Construction

Let $\mathcal{N}$ be the set of nodes in the scenario tree, starting from root node at time $i = 1$ to the leaf node at time $i = T$, $\mathcal{N}_i$ be the set of nodes on time step i for all scenarios, and $\mathcal{N}_b$ be the branching point of the binomial tree, at which point the decision of CPP is made. At each branching point, we say that the node is branching up if the load goes higher and branching down otherwise. Thus, we have $K = 2^T$ scenarios, and each scenario has equal probability $\mathrm{Prob}_k = 1/K, k = 1, \ldots, K$.

We generate a large number of load series $\{Q_i\}$ with T time steps. Let the sample mean and standard deviation for each time step be μ_i^Q and σ_i^Q, respectively, for $i = 1, \ldots, T$. Let q_i^n be the load value at each node $n \in \mathcal{N}_i$, and satisfies

$$q_{in} = \mu_i^Q + \sum_{i=1}^T w_{in}2^{-(T+1-i)/2}\sigma_{[1+2(i-1)]}^Q \quad (31)$$

where $w_{in} = I(\hat{n}\text{ branching up}) - I(\hat{n}\text{ branching down})$ with $\hat{n} = \max\{n : t(n) \le 2\cdot(i-1)+1, n \in \mathcal{N}_b\}$, and I is the indicator function.

Equation (31) guarantees that $\{q_{in}\}$ has approximately the same mean and variance as the simulated sample for each i.

C. Optimal CPP Scheduling Policy

We compare our optimal CPP scheduling policy to the existing temperature-based scheduling policy for one typical summer week in southern California [19]. Define $r_{\max}$ as the ratio between $p_{\max}$ and the mean of p_i^L's (which we assume to be equal in this set of experiments).

Table I shows the optimal CPP event scheduling policy as well as the ratio between the optimal CPP price rate and the lower market rate under different choices of $r_{\max}$ and ϵ values and 10% wind penetration level, and Table II shows the existing temperature-based scheduling policy (1 indicating CPP selected for that day and 0 not selected).

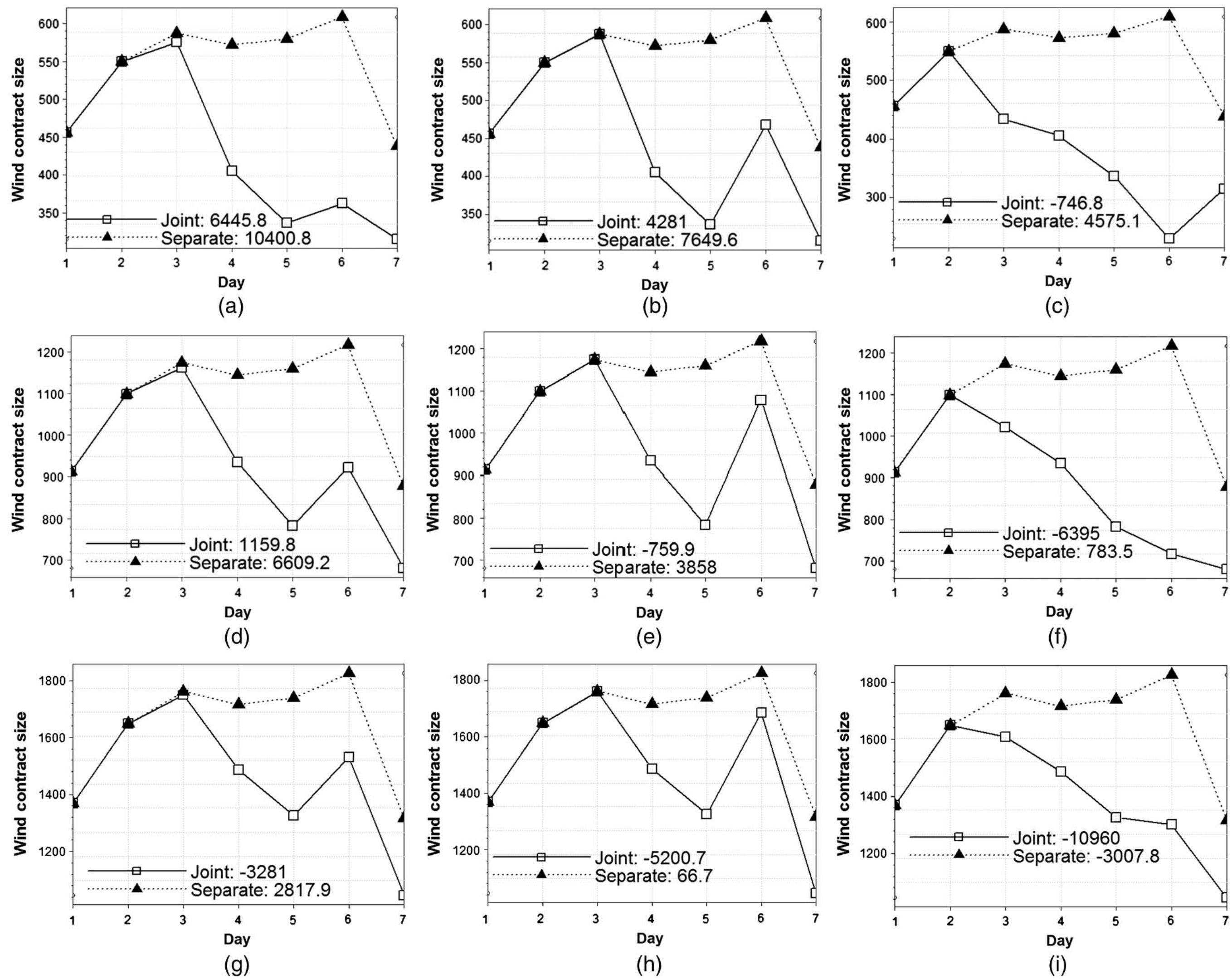

Fig. 2. Joint versus separate optimization: optimal wind contract and total cost comparison: (a) $r_{\max} = 3$, $\epsilon = 0.1$; (b) $r_{\max} = 5$, $\epsilon = 0.1$; (c) $r_{\max} = 5$, $\epsilon = 0.01$; (d) $r_{\max} = 3$, $\epsilon = 0.1$; (e) $r_{\max} = 5$, $\epsilon = 0.1$; (f) $r_{\max} = 5$, $\epsilon = 0.01$; (g) $r_{\max} = 3$, $\epsilon = 0.1$; (h) $r_{\max} = 5$, $\epsilon = 0.1$; (i) $r_{\max} = 5$, $\epsilon = 0.01$.

From Table I, we can find that the optimal CPP events are scheduled on Wednesday and Saturday for this selected summer week, with all the other days scheduled as non-critical days (indicated with 0 in Table I). On the scheduled CPP event day, the ratios of CPP participants' prices to non-participants' prices are higher when the participants' price elasticity gets smaller, while other conditions stay the same, as shown in rows 2 and 3 in Table I. In addition, the allowed maximum CPP rate is hit for both the row 1 case and the row 3 case in Table I, but not for the row 2 case. For $\epsilon = 0.1$, setting $r_{\max} = 5$ is more appropriate than $r_{\max} = 3$, in order to reflect the critical condition changes on different selected CPP event days. For example, for the row 2 case, Wednesday is less critical compared to Saturday in this case due to the more integrated wind resources [see upper line in Fig. 2 (a)–(i)], and thus the optimal critical price rate is lower (4.4 compared to 4.5). Therefore, for the row 3 case in Table I with $\epsilon = 0.01$, a good CPP contract should set the maximum allowed rate $r_{\max}$ higher than 5.

From Table II, we can find that temperature-based scheduling policy selects the first and the sixth days of the week since the temperatures of these two days exceed the pre-determined threshold (92 °F), and the interval between these two days satisfies the minimum allowed interval ($\tau_{\min}$) constraint. The cost associated with this policy is non-optimal (an average of 17% higher) to our CPP policy for each scenario of $(r_{\max}, \epsilon)$ listed in Table I.

D. Joint Versus Separate Optimization

We next compare the jointly optimal solution and cost values obtained from our model and those of a separate optimization model, in which the CPP event scheduling and wind energy commitment are decided separately. Specifically, as we have already discussed in Section II-A, the separately optimal $\{x_i^\bullet, y_i^\bullet\}_{i=1}^T$ are determined from (6) to (16) excluding all the terms associated with w_i's and z_i's and the separately optimal $\{z_i^\bullet\}_{i=1}^T$ (similar to the problem considered in [13]) is obtained by solving for each $z_i^\bullet$ individually from

$$\min_z \mathbb{E}_{\boldsymbol{\xi}_{b,i}}[\pi_{2,i}(z)] \tag{32}$$

where the subscript i is appended to denote day i and $\boldsymbol{\xi}_{b,i}$ and $\pi_{2,i}(z)$ are defined similarly as in (4) for the single-stage problem.

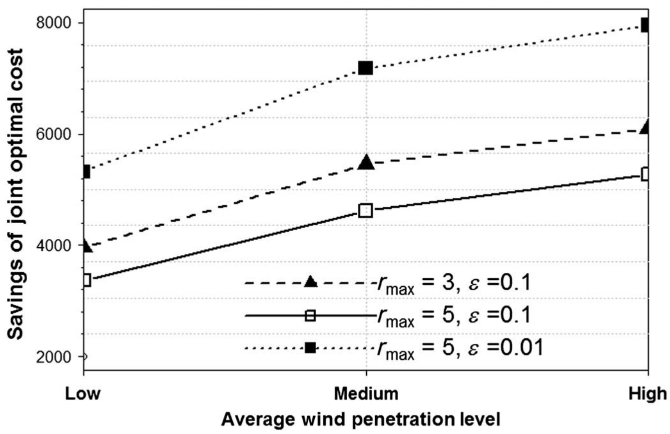

Fig. 3. Cost savings of joint optimizations.

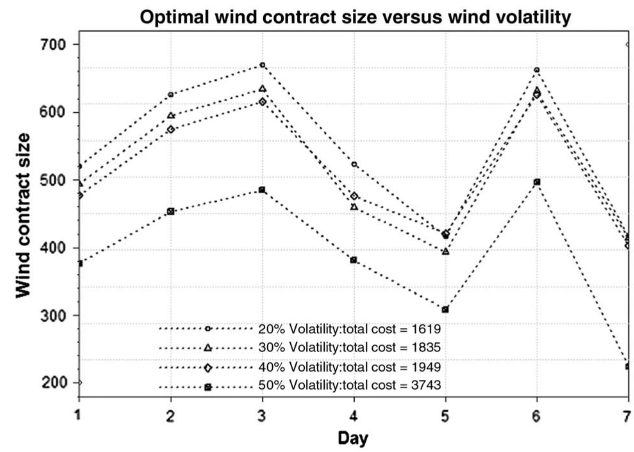

Fig. 4. Optimal wind contract size versus wind volatility.

Fig. 2(a)–(i) compares the optimal wind contract size and total cost between the joint and separate optimization models under different allowed maximum CPP price rates, elasticities, and *wind penetration levels*, where we formally define the wind penetration level as the ratio between the average wind energy supply and the average load. Fig. 2(a)–(c) illustrates the results under a low (10%) wind energy penetration level, Fig. 2(d)–(f) displays the case of a medium (20%) wind energy penetration level, and Fig. 2(g)–(i) shows the examples under a high (30%) wind energy penetration level. Fig. 3 further demonstrates the optimal cost difference between separate and joint optimization in various cases with different ($r_{\max}$, ϵ) and wind penetration levels. All these results indicate that our joint optimization model leads to lower operating costs, due to the wind energy penetration into local grids, and such cost savings can be quite substantial in some cases.

E. Sensitivity Analysis With Respect to Wind Volatility and Penetration Levels

In the remainder of this section, we explore how the optimal solution and cost objective function value change with the wind energy volatility and penetration levels.

Fig. 2(a)–(i) shows that as the wind energy penetration level increases, the optimal contract size increases, as expected. In addition, the LSE's total cost decreases, or more specifically, turns into negative (i.e., profiting) from positive. Under the same wind energy penetration level, the amount of wind energy reserved for local demand increases as the participants' elasticity decreases, because the marginal cost for incentivizing participants to reduce load outweighs the imbalance penalty from under-committing wind energy.

Fig. 4 shows that as the wind volatility level, formally defined as the coefficient of variation of w_i's, increases, the optimal wind contract size decreases. This shows that as the uncertainty level of wind output increases, the chance of incurring higher wind imbalance penalties in both directions increases, and hence the LSE would like to be more conservative in wind energy commitments into the DAM. Also shown in the figure is the intuitive fact that the optimal total cost rises as the volatility level increases. From our experiments, we have also observed that the optimal wind contract size is increasing in the allowed imbalance bands u and d.

A good policy for the LSE is to choose the energy sources in the order of increasing marginal cost, which is similar to the classic economic dispatch problem. Specifically, the LSE may use the "virtual generation" source to meet demand first, until the marginal cost exceeds that from reserving wind for local demand or reaches the maximum CPP rate $p_{\max}$, and then use wind energy to satisfy the remaining demand.

V. Concluding Remarks

In this paper, we have studied the problem of optimal scheduling of CPP events with day-ahead wind energy commitments from the perspective of an LSE. We have proposed a multi-stage SMINLP model and performed various analyzes on both the single-stage special case and the general multi-stage model. Our results lead to useful insights on how to manage a CPP program in a renewable-integrated system.

Due to the complicated nature of the general multi-stage stochastic programming model, our analysis is by no means exhaustive. In fact, many further analytical (such as the solution methodology for our SMINLP model) and experimental (such as additional sensitivity analysis with respect to other system characteristics) studies can be conducted based on our model. We leave these studies for future work.

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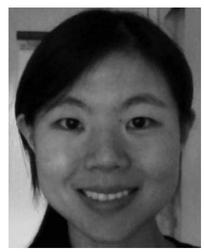

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