

# Linear programming based optimization tool for day ahead energy management of a lithium-ion battery for an industrial microgrid

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**Abstract**— European incentive policies led to a faster development of renewable energy sources during these last years. These new energy sources highly impact the planning and operation of our electrical system. More flexibility is now required for electrical system operators to be able to counterpace the increase of power variability and unwished energy flow in their networks. Energy storage systems are a possible flexibility and thus open new challenges about research works and studies for grid applications. The task of this paper is to study the use of a lithium-ion battery to remove constraints in electrical grids and to increase the penetration of renewable energy sources in the liberalized electricity market. A linearized model of a lithium-ion battery is presented and is used in an optimization framework. This one is then used to calculate the optimal schedule of storage load and discharge actions in order to produce the highest possible benefits while taking into account hardware and grid constraints. An application in an industrial microgrid is presented to reduce the operation energy cost.

**Keywords**— *Lithium-ion battery; energy management system; linear programming; optimization; smart grid; storage*

## I. INTRODUCTION

Today electrical power systems are becoming more and more intelligent. These newly adapted systems are so called smart grids. A smart grid uses information and communication technologies in order to adapt its behavior and characteristics to suit the needs of consumers, save energy consumption and enhance cost effectiveness. The enhanced power control of the electricity production and distribution are some of the more important aspects of smart power grids.

This paper considers the benefit of electrical storage systems, which can be charged and discharged by a local energy management system [1]. The additional flexibility that is introduced by the battery will enable industrial companies to improve their reversible energy flows in terms of efficiency and cost-effectiveness in function of the tariff. In order to buy and sell electricity at the correspondent optimum price, a linear programming approach is developed to optimize the operating planning of the storage. A practical implementation into an industrial microgrid is considered with a lithium ion battery [2].

## II. THE SPOT MARKET FOR ELECTRICITY IN EUROPE

### A. The European Power Exchange (EPEX SPOT)

A liberalized electricity market works on a bid-to-buy, offer-to-sell basis and short-term trades. The price is the result of supply and demand that is being set by the bids and offers. Within an electricity market, there are usually two types of commodities that are traded: power and energy. Power is the metered net electrical transfer rate at any given moment in megawatts (MW). Energy is defined by the flow through a metered point for a given period of time in megawatt hours (MWh).

The EPEX SPOT is an exchange place for power spot trading in France, Germany, Austria and Switzerland. The total trading amount of electricity on EPEX SPOT was 346 terawatt hours in 2013. The market is divided into two sub-markets: the day-ahead markets and the intraday markets.

### B. Day-ahead markets on EPEX SPOT

The EPEX SPOT day-ahead market is organized by an auction process. Prices are fixed once a day by matching the curves of demand and supply and thus creating an anonymous but transparent process of pricing. Each member of the exchange enters its orders into an order book. The order book is closed every day at 11am for Switzerland and 12pm for France, Germany and Austria. In the following steps, the demand and supply curves and also their intersections for each hour are calculated every day for the following day.

### C. Intraday markets on EPEX SPOT

The intraday markets are organized by a continuous trading. Orders of each member are entered consecutively into the order book and evaluated in real-time. If two orders are compatible, i.e. the offer price of a member suits the demand of someone else or vice versa, the order is executed.

### III. DESIGN OF AN OPERATIONAL PLANNING FOR OPTIMAL VALUATION IN THE MARKET

#### A. Presentation of the framework

Large storage plants, like pumped storage hydro stations, are still participating in the market. The purpose of the paper is to design an optimal energy management system for a small storage plant, like a lithium ion battery by taking into account specific constraints of this technology inside the inner energy management [3], [4] (fig. 1). So the charging and discharging modes all around the day have to be planned in function of the present price in the arbitrage system but also according to the energy availability (fig. 2). The issue is to assess the possible introduction in the market.

The output of the algorithm for the day ahead planning is a storage power profile that will be represented by a column vector containing  $N$  variables that are the power magnitudes for each time step in the following day (fig. 3).

$$P_{ref} = [P_{ref}(1) \dots P_{ref}(N)]^t \quad (1)$$

$N$  is the number of time steps during the day (e.g.:  $N = 48$  for every half of an hour).

The input is the price profile that will be represented by a column vector containing  $N$  variables that are the prices for each time step in the day.

$$C = [C(1) \dots C(N)]^t \quad (2)$$

The State of the Charge is also required and will be represented by a column vector containing  $N$  variables that are the SoC for each time step in the day.

$$E = [E(1) \dots E(N)]^t \quad (3)$$

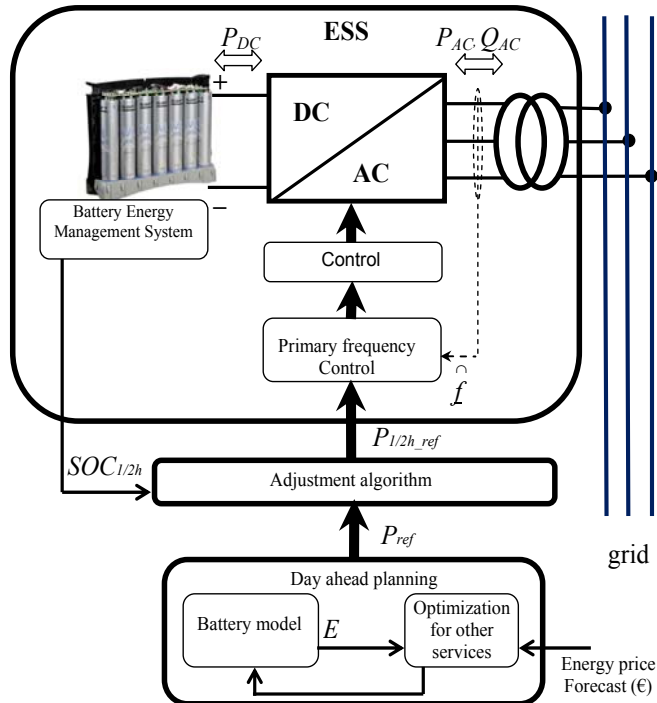


Fig. 1. Local control system of the studied lithium ion battery.

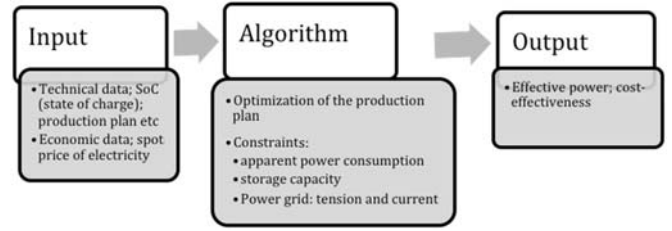


Fig. 2. General framework for storage planning.

#### B. Modelling of the lithium-ion battery

Lithium-ion batteries have been widely used in modern electrified vehicles, which led to a good understanding of the technology. In terms of technology, Li-ion accounted for 419 MW and 1,555 MWh of proposed storage systems in 2014 [6]. The reliable, efficient, and safe operation of lithium-ion batteries requires monitoring, control and management. For battery management systems, a core function is to provide accurate estimates of the State of Charge (SOC). At any given moment  $t$ , the stored energy  $E$  is defined as:

$$E(t) = \int_{t_0}^{t_0+t} P(t)dt + E_0 \quad (4)$$

$E_0$  is the stored energy at the moment  $t_0$  and  $P$  is the instantaneous exchanged DC power with the battery:

The internal power  $P$  is defined by:

$$P(t) = P_{AC}(t) - Loss(t) \quad (5)$$

$P_{AC}$  is the exchanged power with the electrical grid.  $Loss(t)$  are power losses coming from the power electronic conversion between the DC part and the AC connected grid. The State of Charge (SOC) is defined here as the ratio between the stored energy  $E$  in the energy storage system and the storage system's nominal capacity  $E_{nom}$ :

$$SOC(t) = \frac{E(t)}{E_{nom}} \quad (6)$$

#### C. Formulation of the optimization problem

The daily operating program defines, one day ahead, the power reference that will be exchanged with the energy storage system for any given hour, respectively every half an hour during the day [7].

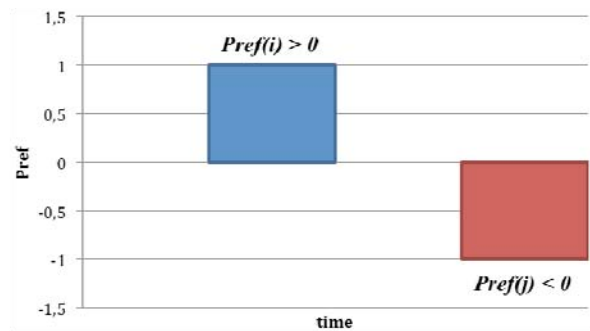


Fig. 3. Analysis of a programmed power profile.

In following formulations, a positive sign for the power is equivalent to an injection of power into the power grid, while a negative sign will be equivalent to the absorption of power from the power grid and loading the battery.

#### IV. DAY AHEAD OPTIMIZATION ALGORITHM OF THE STORAGE UNIT

##### A. The objective function

The objective is to calculate storage power references (in each time step) that maximize the financial gains from the trading while taking into account equipment and microgrid constraints [8] (fig. 4).

In the mathematical theory of linear programming, any maximization problem may be formulated as a minimization problem [9], [10]. The resulting objective function is expressed as:

$$f(P_{ref}) = -T_{step} \left( \sum_{k=1}^N C(k) \cdot P_{ref}(k) \right) \quad (7)$$

$T_{step}$  is the duration of the time step

$P_{ref}$  is the exchanged reference power with the electric grid during the considered time step, with the following assumption:

$$P_{ref}(k) = P(k \cdot T_{step}) \quad (8)$$

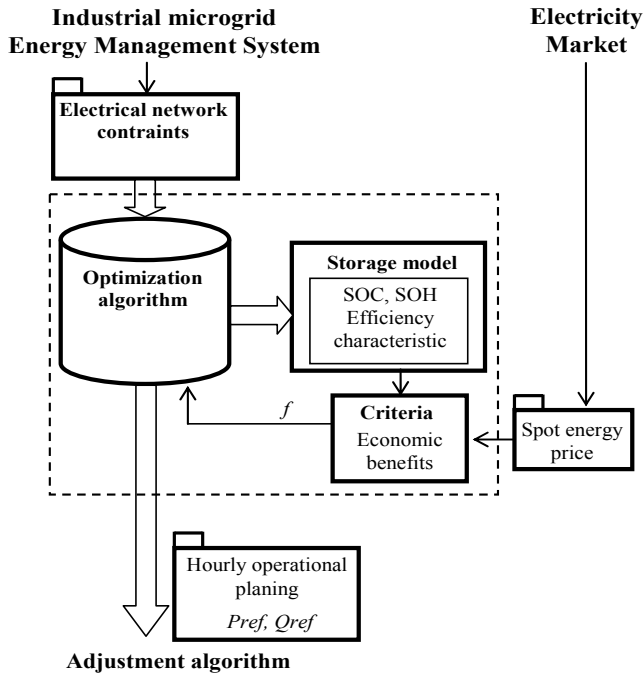


Fig. 4. Optimization framework for the day ahead planning.

$C(k)$  is the spot price of electricity at 12:00 (day-ahead electricity market) for that specific time step (in *Euro/MWh*) from the European Power Exchange (EPEX) Spot. The used example of price time variation is shown on fig. 5.

The impact of the storage operation on the price variation and the intraday price fluctuations is considered negligible.

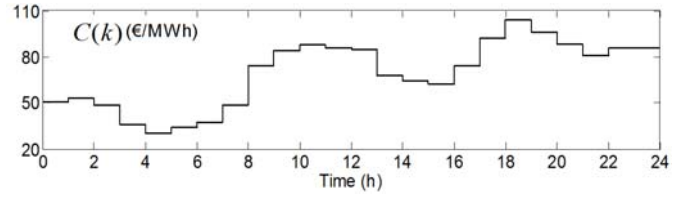


Fig. 5. Day-ahead energy price  $C(k)$  (EPEX) [11].

The objective function can also include additional cost components such as grid access tariffs. This objective function is minimized while taking into account a set of constraints coming from the storage technology and from the grid connection requirements.

##### B. Characteristics of the battery

The sizing of the lithium-ion battery includes maximum and minimum allowable powers for the charging and discharging processes. Therefore constraints of power inequality need to be introduced:

$$-P_{min\_charge} \leq P_{ref}(k) \leq P_{max\_discharge} \quad (9)$$

$$\forall k \in [1; N]$$

$P_{min\_charge}$  is the maximum allowable charging power

$P_{max\_discharge}$  is the maximum allowable discharging power

The electrical capacity of the lithium-ion battery is another constraint that is imposed by the technological sizing and is expressed as an energy inequality:

$$5\% \cdot E_{Max\_Stock} \leq E(k) \leq 95\% \cdot E_{Max\_Stock} \quad (10)$$

$$\forall k \in [1; N]$$

$E(k)$  is the stored energy in the lithium-ion battery in the time step  $k$  and available at  $k+1$  (in MWh).

$E_{Max\_Stock}$  is the maximum capacity of the lithium-ion battery (in MWh).

The stored energy  $E_{Stock}$  of the lithium-ion battery in the time step  $k$  depends on the energy flow between the battery and the circuit in the period of time  $k-1$ . To adequately model the storage it is necessary to calculate the energy stored in any moment. When power is generated, the SOC decreases and the energy variation is expressed as:

$$\text{If } P_{ref}(k) < 0 \text{ then } \Delta E_{disload}(k) = \frac{P_{ref}(k)}{\eta_{discharge}} \cdot T_{step} \quad (11)$$

$\eta_{charge}$  is the efficiency for the charging mode of the battery. When the power is consumed, the SOC increases and so the energy variation is expressed as:

$$\text{If } P_{ref}(k) \geq 0 \text{ then } \Delta E_{load}(k) = \eta_{charge} \cdot P_{ref}(k) \cdot T_{step} \quad (12)$$

$\eta_{discharge}$  is the efficiency for the discharging mode of the battery. So, the stored energy is expressed as:

$$E_{stock}(k) = E(k-1) + \Delta E_{load}(k-1) + \Delta E_{disload}(k-1) \quad (13)$$

Here, the system efficiency will be considered independent of the power flow in the battery:

$$\eta_{charge} = \eta_{discharge}$$

Moreover, constraints of equality of energy will be considered in the model.

The initial energy shall be the same as the stored energy at the end of the previous day:

$$E(1) = E_{Initial\_Stock} \quad (14)$$

The final stored energy in the battery shall be the same as the desired energy for the end of the day:

$$E(N+1) = E_{Final\_Stock} \quad (15)$$

Other constraints exist and could be implemented in this model: current limitations, temperature limitations, battery cycles per day etc. These constraints would help to ensure the proper operating of the system, but will not be considered in this model.

### C. Grid connection constraints

The electrical network involves its own physical limitations (e.g. power lines) as well as requirements due to regulations imposed by the grid operator (grid code). Operating of new connected equipment must not disturb the electrical network. Grid voltage and current constraints lead the grid operator to calculate maximum real and reactive powers in load mode and in generator mode:

$$-P_{\max\_generator}(k) \leq P_{ref}(k) \leq P_{\max\_load}(k) \quad (16)$$

$$-Q_{\max\_generator}(k) \leq Q_{ref}(k) \leq Q_{\max\_load}(k) \quad (17)$$

In this paper, these power limits will be taken into account in order to decrease costs in case of over power consumption (and over electrical production).

By considering the rated sizing of the ESS ( $S_n$ ), we get:

$$P_{ref}(k) = \sqrt{S_n^2 - Q_{ref}(k)^2} \quad (18)$$

### D. The optimization problem to be solved

The optimization program shall thus deliver a power profile that solves the following criteria:

$$\min[f(P_{ref})] = \min \left[ -T_{step} \left( \sum_{k=1}^N C(k) \cdot P_{ref}(k) \right) \right] \quad (19)$$

The set of inequality constraints is expressed as:

$$-P_{ref}(k) \leq P_{\max\_generator}(k) + Loss(k) \quad (20)$$

$$P_{ref}(k) \leq P_{\max\_load}(k) - Loss(k) \quad (21)$$

$$\sqrt{S_n^2 - P_{ref}(k)^2} \leq Q_{\max\_load}(k) \quad (22)$$

$$-\sqrt{S_n^2 - P_{ref}(k)^2} \leq -Q_{\max\_generator}(k) \quad (23)$$

$$-P_{ref}(k) \leq -P_{\min\_charge}(k) \quad (24)$$

$$P_{ref}(k) \leq P_{\max\_charge}(k) \quad (25)$$

$$-E(k) \leq 0 \quad (26)$$

$$E(k) - E_{Max\_Stock} \leq 0 \quad (27)$$

$$E(1) = E_{Initial\_Stock} \quad (28)$$

$$E(N+1) = E_{Final\_Stock} \quad (29)$$

## V. OPTIMIZATION BY LINEAR PROGRAMMING

### A. Formulation of the problem

Linear programming or optimization is a method of either maximizing or minimizing a linear function over a convex polyhedron that is specified by different types of constraints for the linear problem. It results in the optimization of an outcome base  $f$  with some set of constraints by using linear mathematical models defined in canonical form as:

$$\min(f) = \min(C^T X) \quad (30)$$

$X$  is the vector of optimization parameters ( $=P_{ref}$ , equ. 1)

that are searched to minimize the objective function  $f$ .

Parameters of the objective functions are:

$$C = [C(1), \dots, C(N)]^T \quad (31)$$

Where  $C$  is the spot price of electricity.

Constraints (equ.(19) to (25)) are expressed as a set of inequalities depending of linear relation with  $X$ :

$$A.X \leq b \quad (32)$$

that requires constants for the formulation:

$$b = [b_1, \dots, b_m]^T \quad (33)$$

and a  $N \times m$  matrix,

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix} \quad (34)$$

Other constraints (equ.(26) and (27)) are expressed as a set of inequalities depending of nonlinear functions with  $X$ :

$$c(X) \leq 0 \quad (35)$$

### B. Simplex algorithm

The simplex algorithm solves problems in the context of mathematical optimization. The "Optimization Toolbox" that is provided by MatLab has been used [12]. More precisely, the function "fmincon" finds the minimum of a constrained linear or nonlinear multivariable function and returns the optimized vector of optimization parameters ( $=P_{ref}$ ) that has been found and the value of the objective function for the specific optimum  $fval$ .

$$fval = \min(f) \quad (36)$$

## VI. APPLICATION IN AN INDUSTRIAL MICROGRID

For this study, the consumption of a company has been analyzed with the objective to decrease the cost of energy (fig. 6). 20 MW peak power of industrial loads has been sensed (fig. 7). To reduce consumed electricity costs, 1 MW peak power rating PV production has been yet built on roofs of the company [13]. Regarding the amount of rotating mechanical loads, reversing power transfers appear during braking of various industrial processes and then application of a 1MW – 500kW storage system is here studied to recover regenerative power for future use or to optimize the energy bill on the electrical market.

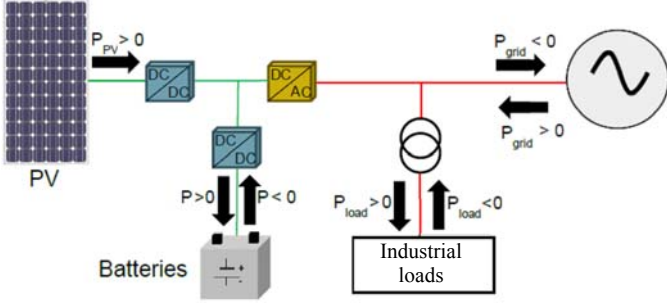


Fig. 6. Description of the studied company power system.

The connection to the grid is sized by the subscribed rated power, which induces a monthly fixed cost for the company and penalties in case of over exchanged power ( $\Delta P_{excess}$ ). The task is to minimize this monthly cost that is expressed by the CMDPS in France [14], [15]:

$$CMDPS = \sum_{t \in T} \alpha \cdot k_t \cdot \sqrt{\sum_{x \in Xt} \Delta P_{excess}^2(x)} \quad (37)$$

In the current ‘TURPE 4’ french tariff (in ‘HTB2’ distribution networks), five tariff periods exist. The variable  $x$  represents the index set belonging to each time tariff period  $t$ .  $k_t$  (%) is the coefficient for each time tariff. The  $\alpha$  coefficient is set to 358.4€/MW. To highlight the tackled technico-economic problem, a typical power profile is shown in fig.7. It is obvious that power peaks will induce over costs for the company. Optimum operating planning of the storage system according to the scheduled industrial activities is studied.

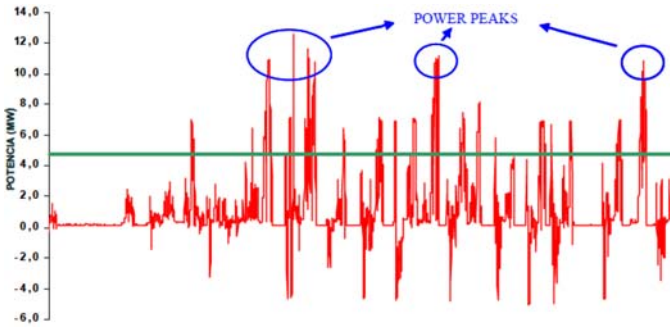


Fig. 7. Exchanged power at the connection point during one day (without storage system)

The optimization tool enables to predict the exchanged power with the storage and also the SOC as shown respectively in figures 8 and 9.

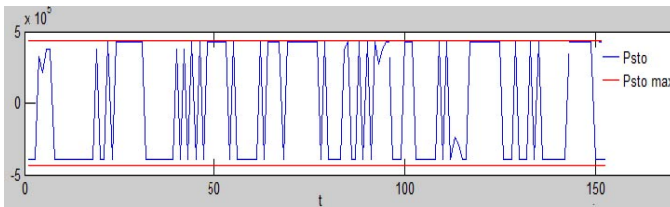


Fig. 8. Exchanged power with the storage during half of one year

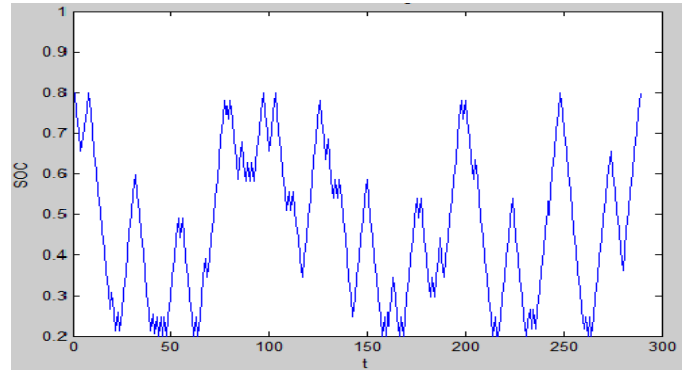


Fig. 9. State of charge during one year

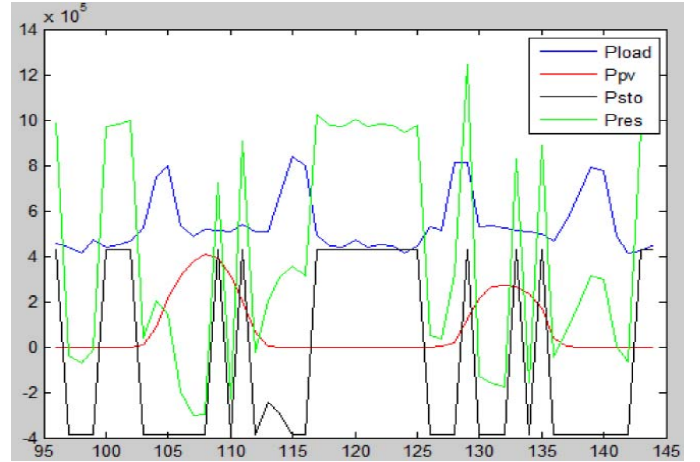


Fig. 10. Zoom on the instantaneous powers during two days

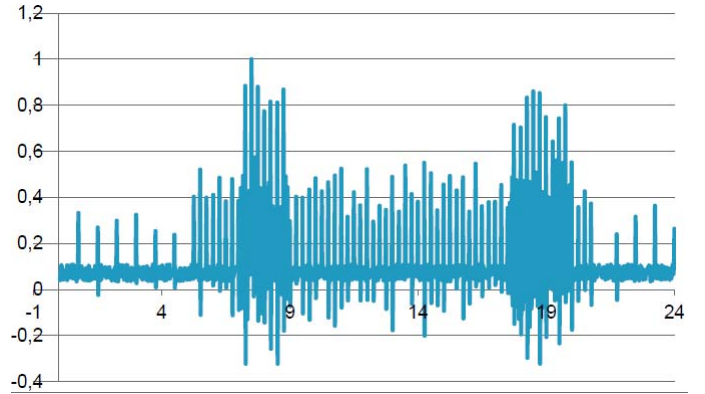


Fig. 11. Exchanged power (in MW) at the power station connection point with the national grid during one day

A focus on a 48hour operating in real time is given in fig. 10 in order to compare the magnitude variations and transients of the various powers.

As shown in fig. 11, peak powers with the grid have been reduced to 1MW.

Various sources of economic savings are exploited. A first one is coming from the reduction of penalties due to the contracted max power with the transmission operator. A second one arises from the saved energy that is consumed later otherwise to be bought on the market. The quantification of economic savings depends directly on the electrical system



operating (OPEX); e.g. time profile of the load demand and the PV production operating. The analysis of one operating month (during the spring season) gives a 6% reduction of the electricity cost but cannot be generalized to the entire year because of meteorological variations. This technical economic model enables a cost/benefit analysis study over twenty years while taking into account capital investment (CAPEX). Results are confidential.

## VII. CONCLUSION

Linear programming, or more generally mathematical optimization, is a suitable tool for the optimization of the energy management of energy storage systems, e.g. lithium-ion batteries, and thus helps building a cost and energy saving environment. Theoretical aspects for applications have been detailed and obtained performances in an industrial microgrid have been presented. This study can be completed with a costs and benefits analysis by taking into account the investment made in PV panels, batteries and converters (CAPEX). Anyway results will be very sensible with considered capital costs at the moment (and so maturity of technologies on markets), the time horizon and the discount rate. In a near future, the upcoming developments of numerous storage systems by prestigious firms as Tesla Motors will certainly lead to a noticeable change in the energy industry. Applications in distribution electrical systems are also considerable.

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