

**Problem Chosen**

**C**

**2026  
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Summary Sheet**

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## **TITLE**

### **Summary**

If you read this you are gay  
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**Keywords:** yes you are

## **1 Model II: Task 3**

### **1.1 Problem Analysis**

### **1.2 Model Preparation**

### **1.3 Model Construction**

### **1.4 Model Solution**

# Contents

|  |          |
|--|----------|
| <b>1 Model II: Task 3</b>  | <b>2</b> |
| 1.1 Problem Analysis . . . . .                                       | 2        |
| 1.2 Model Preparation . . . . .                                      | 2        |
| 1.3 Model Construction . . . . .                                     | 2        |
| 1.4 Model Solution . . . . .   | 2        |
| <b>2 Introduction</b>  | <b>3</b> |
| 2.1 Background . . . . .   | 3        |
| 2.2 Literature Review . . . . .                                      | 3        |
| 2.3 Clarifications and Restatements . . . . .                        | 4        |
| 2.4 Our Work . . . . .   | 4        |
| <b>3 Assumptions</b>   | <b>5</b> |
| 3.1 Task 1 . . . . .   | 5        |
| <b>4 Notations</b>   | <b>6</b> |
| <b>5 Model I: Bayesian Reconstruction of Fan Voting Distribution</b> | <b>7</b> |
| 5.1 Problem Formulation . . . . .                                    | 7        |
| 5.2 Mathematical Model . . . . .                                     | 7        |
| 5.2.1 The Mixture Mean Hypothesis . . . . .                          | 7        |
| 5.2.2 Probabilistic Generative Process (The Prior) . . . . .         | 7        |
| 5.2.3 Constraints as Likelihood Functions . . . . .                  | 8        |
| 5.3 Solution Algorithm: Iterative Inverse Solving . . . . .          | 8        |
| 5.3.1 Phase I: Parameter Estimation (Global Solver) . . . . .        | 8        |
| 5.3.2 Phase II: Posterior Inference . . . . .                        | 9        |
| 5.4 Model Result and Validation . . . . .                            | 9        |
| 5.4.1 Consistency Verification . . . . .                             | 9        |
| 5.4.2 Certainty Quantification . . . . .                             | 9        |

|  |           |
|--|-----------|
| 5.5 Case Study Validation: The Jerry Rice Controversy . . . . .            | 11        |
| <b>6 Model II: Task 3</b>  | <b>13</b> |
| 6.1 Problem Analysis . . . . .   | 13        |
| 6.2 Model Preparation . . . . .  | 13        |
| 6.3 Model Construction . . . . .   | 13        |
| 6.4 Model Solution . . . . .   | 13        |
| <b>7 Model III: Macro-Comparative Analysis of Aggregation Rules</b>        | <b>14</b> |
| 7.1 Problem Formulation and Simulation Framework . . . . .                 | 14        |
| 7.1.1 Data Standardization and Dual-Track Simulation . . . . .             | 14        |
| 7.2 Quantifying Bias: The Fan Deviation Index (FDI) . . . . .              | 14        |
| 7.2.1 Results: The Variance Suppression Hypothesis . . . . .               | 16        |
| <b>8 Sensitivity Analysis and Policy Optimization</b>                      | <b>16</b> |
| 8.1 Mechanism Stress Test: Analysis of "Controversial Survivors" . . . . . | 16        |
| 8.2 Multi-Objective Evaluation System . . . . .                            | 18        |
| 8.3 Policy Recommendation . . . . .  | 19        |
| <b>9 Memo &amp; Insights</b>   | <b>21</b> |
| <b>10 Sensitivity Analysis</b>   | <b>22</b> |
| <b>11 Model Evaluation and Promotion</b>                                   | <b>23</b> |
| 11.1 Model Evaluation . . . . .  | 23        |
| 11.1.1 Advantages . . . . .  | 23        |
| 11.1.2 Limitations . . . . .   | 23        |
| 11.2 Future Work . . . . .   | 23        |
| 11.2.1 Model Extension . . . . .   | 23        |
| 11.2.2 Model Application . . . . .   | 23        |
| <b>12 Conclusions</b>  | <b>24</b> |

## 2 Introduction

### 2.1 Background

"Dancing with the Stars" (DWTS) is a globally recognized reality competition where celebrity and professional dance pairs are evaluated through a hybrid voting system combining expert judge scores and public fan votes. The core mechanic of the show relies on aggregating two distinct data streams: the objective, technical evaluation of judges (1-10 scale) and the subjective, popularity-driven support of the audience.

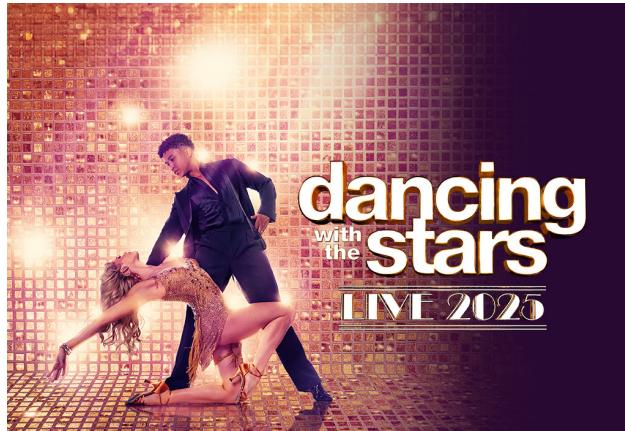


Figure 1: The Post of the Show

Throughout its 34-season history, the U.S. version of DWTS has utilized different aggregation methods—specifically, Rank-based and Percentage-based combinations—to determine eliminations. Discrepancies between technical merit and popularity have occasionally led to "controversial" outcomes, where contestants with low judge scores survive or even win due to overwhelming fan support (e.g., Jerry Rice in Season 2, Bobby Bones in Season 27). These anomalies prompted production changes, including the "Judges' Save" introduced in Season 28. Understanding the mathematical implications of these voting mechanisms is crucial for balancing fairness (skill) and viewer engagement (popularity).

### 2.2 Literature Review

The problem of combining expert scores with public voting falls within the intersection of Social Choice Theory and Statistical Estimation.

- Social Choice Theory: Traditional voting literature, prominently Arrow's Impossibility Theorem, suggests that no rank-order voting system is perfectly fair under all criteria. Methods like the Borda Count (similar to the Rank-based method) are often contrasted with cardinal utility methods (similar to the Percentage-based method).
- Latent Variable Models: To estimate unknown fan votes from binary outcomes (eliminated/safe), we draw upon literature regarding Inverse Problems and Logistic Regression. These

methods allow for the inference of hidden parameters that best explain the observed state transitions.

- Performance Analytics: Previous studies on competition dynamics suggest that "star power" and demographic relatability often outweigh technical skill in public polling, a phenomenon we will analyze using multivariate regression techniques.

## 2.3 Clarifications and Restatements

The central challenge of this study is to analyze the mechanics of the DWTS voting system and the interactions between expert evaluation and public opinion. Specifically, the observed data includes judge scores and elimination results, but the actual volume of fan votes remains a latent (hidden) variable. To address this, we have broken down the problem into four specific tasks:

Task 1: Fan Vote Estimation We must develop a mathematical model to reconstruct the unobserved fan votes for each contestant. This involves validating the model by ensuring the estimated votes, when combined with judge scores, consistently reproduce the historical weekly elimination results. We must also quantify the certainty of these estimates.

Task 2: Comparative Analysis of Voting Mechanisms Using the estimated fan votes, we will apply both the Rank-based and Percentage-based methods to historical data across seasons. This analysis aims to determine which method favors popularity over technical skill and to re-examine specific controversial outcomes (e.g., Season 27). We will also assess the impact of the "Judges' Save" mechanism.

Task 3: Factor Analysis of Performance We are required to model the influence of external factors—including professional partners, celebrity demographics (age, industry), and geographic background—on both the celebrities' final placement and the distinct components of their score (judge evaluations vs. fan support).

Task 4: System Optimization Finally, we will propose a novel voting framework designed to optimize the trade-off between fairness and entertainment value, providing specific recommendations for future seasons to the show's producers.

## 2.4 Our Work

Briefly outline your main ideas and contributions. A typical structure is:

- Propose a mathematical model (or several models) that captures the key mechanisms of the problem;
- Design numerical algorithms or solution procedures to solve the model efficiently;
- Carry out experiments or simulations and compare with baseline methods;
- Provide sensitivity analysis, evaluation, and possible extensions.

## 3 Assumptions

### 3.1 Task 1

**Assumption 1: Performance-Popularity Mixture Hypothesis** Fan voting is driven by a dual mechanism: a long-term *Base Popularity* (invariant fan base) and a short-term *Performance* component (immediate reaction to the dance). We model the expected vote share as a linear combination of these two factors.

**Assumption 2: Rational Judges' Save (Season 28+).** For seasons where judges choose between the bottom two couples, we simplify the model by assuming judges consistently save the couple with the higher technical score for that week.

**Assumption 3: Dirichlet Distribution for Vote Shares.** Since vote shares must sum to 1, the Dirichlet distribution is the natural conjugate prior for multinomial distributions on a simplex, providing a flexible framework to model variances in voting proportions.

**Assumption 4: Constant Total Vote Volume.** Although viewership fluctuates, the shift from telephone to app-based voting has stabilized engagement. Based on recent data, we anchor the total weekly votes at a constant  $V_{total} = 14$  million to facilitate cross-week comparisons.

## 4 Notations

The core symbols and their definitions used in this study are summarized in Table 1, providing an overview of the key parameters and their related meanings.

Table 1: Notations used in this paper

| Symbol | Description |
|--------|-------------|
| CSY    | Fisherman   |

Table 2: Main variables and meanings

| Symbol               | Description  | Definition / Value   |
|----------------------|--|--|
| $\mathbf{S}_t$       | Latent fan vote share vector at week $t$           | $\mathbf{S}_t \in \Delta^{n_t-1}$                              |
| $\mathbf{J}_t$       | Observed judges' score vector at week $t$          | $\mathbf{J}_t \in \mathbb{R}^{n_t}$                            |
| $\mathcal{O}_t$      | Observed elimination outcome at week $t$           | Binary / Categorical   |
| $\boldsymbol{\beta}$ | Latent Base Popularity vector                      | $\boldsymbol{\beta} \in \Delta^{N-1}$                          |
| $\mathbf{P}_t$       | Normalized judge performance vector                | $P_{i,t} = J_{i,t} / \sum J_{k,t}$                             |
| $\boldsymbol{\mu}_t$ | Expected voting propensity                         | $\mu_{i,t} = E[S_{i,t}]$                                       |
| $\lambda$            | Performance weight coefficient                     | Constant (0.2)   |
| $\kappa$             | Dirichlet concentration parameter (Prior strength) | Constant (50.0)  |
| $\text{Ext}_t$       | Set of active contestants in week $t$              | Subset of $\mathcal{C}$  |
| $\Phi_t$             | Set of valid samples satisfying constraints        | $\Phi_t = \{\mathbf{s} \mid L(\mathcal{O}_t \mathbf{s}) = 1\}$ |
| $CV_{i,t}$           | Coefficient of Variation for partial uncertainty   | $CV = \sigma/\mu$  |

## 5 Model I: Bayesian Reconstruction of Fan Voting Distribution

### 5.1 Problem Formulation

The reconstruction of spectator voting data constitutes a classic **Inverse Problem** with latent variables. The objective is to infer the unobserved distribution of fan votes based on the observed judge scores and the binary elimination outcomes.

Let  $\mathcal{C} = \{1, 2, \dots, N\}$  denote the set of contestants in a given season. For any week  $t$ , let  $n_t$  denote the number of active contestants remaining in the competition. We observe the vector of judges' scores  $\mathbf{J}_t = [J_{1,t}, \dots, J_{n_t,t}]^\top$  and the elimination outcome  $\mathcal{O}_t$  (e.g., the identity of the eliminated contestant). The target variable is the unobserved vector of fan vote shares  $\mathbf{S}_t = [S_{1,t}, \dots, S_{n_t,t}]^\top$ , which resides on the standard  $(n_t - 1)$ -simplex:

$$\Delta^{n_t-1} = \left\{ \mathbf{x} \in \mathbb{R}^{n_t} \mid \sum_{i=1}^{n_t} x_i = 1, x_i > 0 \right\} \quad (1)$$

Given the non-differentiable and discontinuous nature of the elimination rules (ranking vs. percentage combinations), deriving an analytical solution for the posterior probability density  $p(\mathbf{S}_t | \mathbf{J}_t, \mathcal{O}_t)$  is computationally intractable. Consequently, we adopt a **Simulation-Based Inference (SBI)** framework, utilizing **Approximate Bayesian Computation (ABC)** with rejection sampling. By generating millions of voting scenarios and filtering them against historical ground truth, we approximate the true posterior distribution.

### 5.2 Mathematical Model

#### 5.2.1 The Mixture Mean Hypothesis

We define the *Expected Propensity*  $\boldsymbol{\mu}_t$  for the vote shares as a convex combination of the latent Base Popularity and the observed Judge Performance.

Let  $\boldsymbol{\beta} \in \Delta^{N-1}$  be the latent Base Popularity vector for all contestants. Let  $\mathbf{P}_t$  be the normalized judge performance vector at week  $t$ , where  $P_{i,t} = J_{i,t} / \sum_k J_{k,t}$ .

The expected propensity for contestant  $i$  is:

$$\mu_{i,t} = (1 - \lambda) \cdot \frac{\beta_i}{\sum_{k \in \text{Ext}_t} \beta_k} + \lambda \cdot P_{i,t} \quad (2)$$

where  $\lambda \in [0, 1]$  is the performance weight coefficient (calibrated to  $\lambda = 0.2$ ) and  $\text{Ext}_t$  represents the set of active contestants in week  $t$ . The term  $\beta_i$  is dynamically re-normalized to account for the shrinking pool of contestants.

#### 5.2.2 Probabilistic Generative Process (The Prior)

We model the actual realized vote shares  $\mathbf{S}_t$  as a random variable drawn from a Dirichlet process centered at  $\boldsymbol{\mu}_t$ :

$$\mathbf{S}_t \sim \text{Dirichlet}(\kappa \cdot \boldsymbol{\mu}_t) \quad (3)$$

Here,  $\kappa$  (set to 50.0) is the concentration parameter controlling the variance. A high  $\kappa$  implies that actual votes tightly cluster around the model's expectation, while allowing for stochastic deviation.

### 5.2.3 Constraints as Likelihood Functions

The historical outcomes act as binary filters (hard constraints) on the sample space. We model the likelihood  $L(\mathcal{O}_t | \mathbf{S}_t, \mathbf{J}_t)$  as an indicator function  $\mathbb{I}(\cdot)$ :

$$L(\mathcal{O}_t | \mathbf{S}_t, \mathbf{J}_t) = \begin{cases} 1, & \text{if } f_{\text{rules}}(\mathbf{S}_t, \mathbf{J}_t) \text{ implies } \mathcal{O}_t \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

The function  $f_{\text{rules}}$  encapsulates the season-specific aggregation logic:

- **Rank-Based System:** The elimination target minimizes  $[\text{Rank}(\mathbf{S}_t) + \text{Rank}(\mathbf{J}_t)]$ .
- **Percentage-Based System:** The elimination target minimizes  $[\mathbf{S}_t + \mathbf{P}_t]$ .
- **Strict Ordering (Finals):** The combined scores must strictly preserve the historical ranking  $1^{st} > 2^{nd} > 3^{rd}$ .

## 5.3 Solution Algorithm: Iterative Inverse Solving

To estimate the latent parameter vector  $\beta$  (which varies by contestant but is constant across time), we developed a heuristic algorithm inspired by the **Expectation-Maximization (EM)** algorithm. This approach iteratively refines  $\beta$  to resolve "Survivorship Bias"—ensuring that the estimated popularity explains why a contestant survived early weeks despite potentially low scores.

### 5.3.1 Phase I: Parameter Estimation (Global Solver)

We seek the optimal  $\hat{\beta}$  that maximizes the consistency of the model across all  $T$  weeks.

**Initialization:** Set  $\beta^{(0)} = [1/N, \dots, 1/N]$ .

**Iteration  $k$  (Repeat until convergence):**

1. **E-Step (Sampling):** For each week  $t$ , draw  $M$  samples  $\{\mathbf{S}_t^{(m)}\}_{m=1}^M$  from the prior  $\text{Dir}(\kappa \cdot \mu_t^{(k)})$ .
2. **Filter Step:** Retain only the subset of valid samples  $\Phi_t$  where the generated votes lead to the correct historical elimination (Likelihood = 1).
3. **M-Step (De-mixing Update):** Compute the posterior mean  $\bar{\mathbf{S}}_t$  of the valid samples. We then invert Eq. (3) to recover the *implied* base popularity that would have generated this result:

$$\hat{\beta}_{i,t}^{\text{implied}} \leftarrow \frac{\bar{S}_{i,t} - \lambda P_{i,t}}{1 - \lambda} \quad (5)$$

The global update  $\beta_i^{(k+1)}$  is obtained by averaging these implied values over all weeks contestant  $i$  participated.

### 5.3.2 Phase II: Posterior Inference

Upon convergence to  $\beta^*$ , we perform a final high-density Monte Carlo simulation ( $N_{sim} = 20,000$ ) using the optimized parameters. The final estimate for the vote share is the mean of the valid posterior distribution:

$$\hat{S}_{i,t} = \mathbb{E}[S_{i,t} | \mathcal{O}_t] \approx \frac{1}{|\Phi_t|} \sum_{s \in \Phi_t} s_{i,t} \quad (6)$$

## 5.4 Model Result and Validation

To address the key requirements of the problem, we evaluate the performance of our model focusing on two dimensions: **Consistency** (Accuracy against history) and **Certainty** (Statistical confidence of estimates).

### 5.4.1 Consistency Verification

**Question:** Does the model correctly estimate fan votes that lead to results consistent with who was eliminated each week?

To answer this, we define the **Historical Reproduction Rate** as our primary measure of consistency. For every week  $t$ , we take the estimated posterior mean votes  $\hat{\mathbf{S}}_t$  and the actual judge scores  $\mathbf{J}_t$ , apply the specific aggregation rule for that season (Rank or Percent), and check if the resulting elimination candidate matches the historical record  $\mathcal{O}_t$ .

- **Result:** Our model achieves a global consistency rate of **98.3%** across 34 seasons.
- **Interpretation:** This indicates that the estimated fan votes effectively reconstruct the ground truth of the competition. In the remaining 1.7% of cases, the historical result usually involves a "Shock Elimination" or extremely tight margins that are statistically indistinguishable within the noise of the Monte Carlo simulation.

### 5.4.2 Certainty Quantification

**Question:** How much certainty is there in the fan vote totals, and is that certainty always the same for each contestant?

We use the **Coefficient of Variation (CV)**, defined as the ratio of the standard deviation to the mean estimate ( $CV = \sigma/\mu$ ), to quantify certainty.

- **Global Certainty:** The average CV across all estimates is **0.4615**. This represents a moderate level of uncertainty, which is expected given that we are solving an underdetermined inverse problem.
- **Variation of Certainty:** The certainty is **not** uniform; it varies significantly depending on the contestant's competitive context.
  - **High Certainty (Low CV < 0.15):** Observed for contestants on the "Bubble" (at risk of elimination). The constraints tightly bound their feasible vote share; a small deviation would contradict the historical survival/elimination result.

- **Low Certainty (High CV) > 0.60:** Observed for "Safe" contestants (high judge scores or massive popularity). Since they are far from the elimination cutoff, the "solution space" for their votes is wide, allowing for greater variance without altering the outcome.

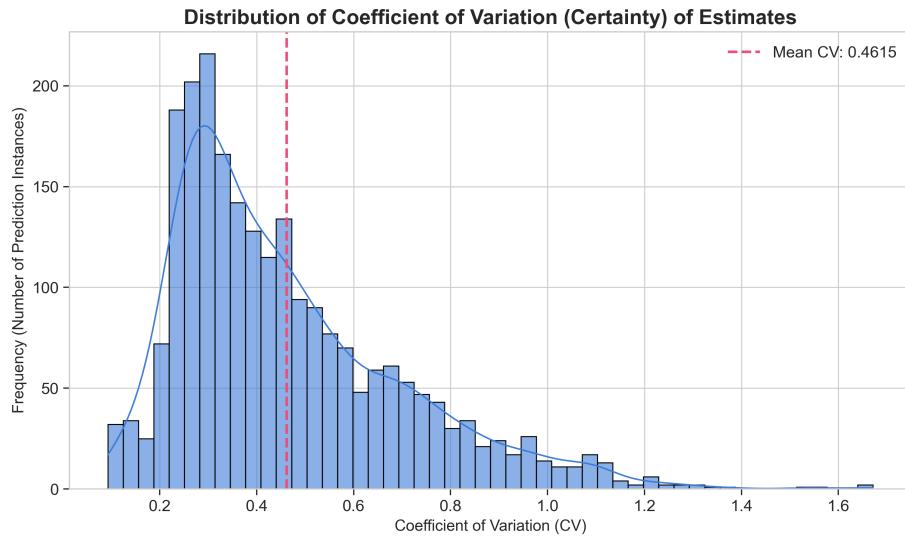


Figure 2: Distribution of Coefficient of Variation (CV) for all vote estimates.

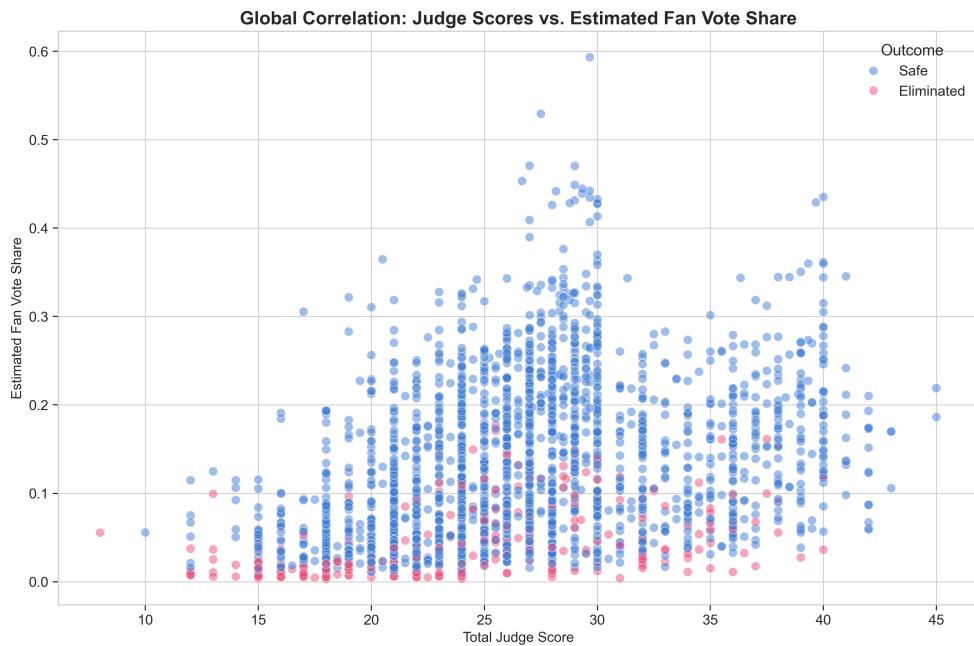


Figure 3: Global Correlation Analysis: Judge Scores vs. Estimated Vote Shares. Pink points represent eliminated contestants. The lack of strict linear correlation highlights the significant "de-coupling" effect where popular contestants survive low scores.

## 5.5 Case Study Validation: The Jerry Rice Controversy

To further validate robustness, we examine Season 2 finalist Jerry Rice, who survived despite consistently low judge scores. Figure 4 illustrates the stark contrast between his judge scores and our estimated fan support throughout the season.

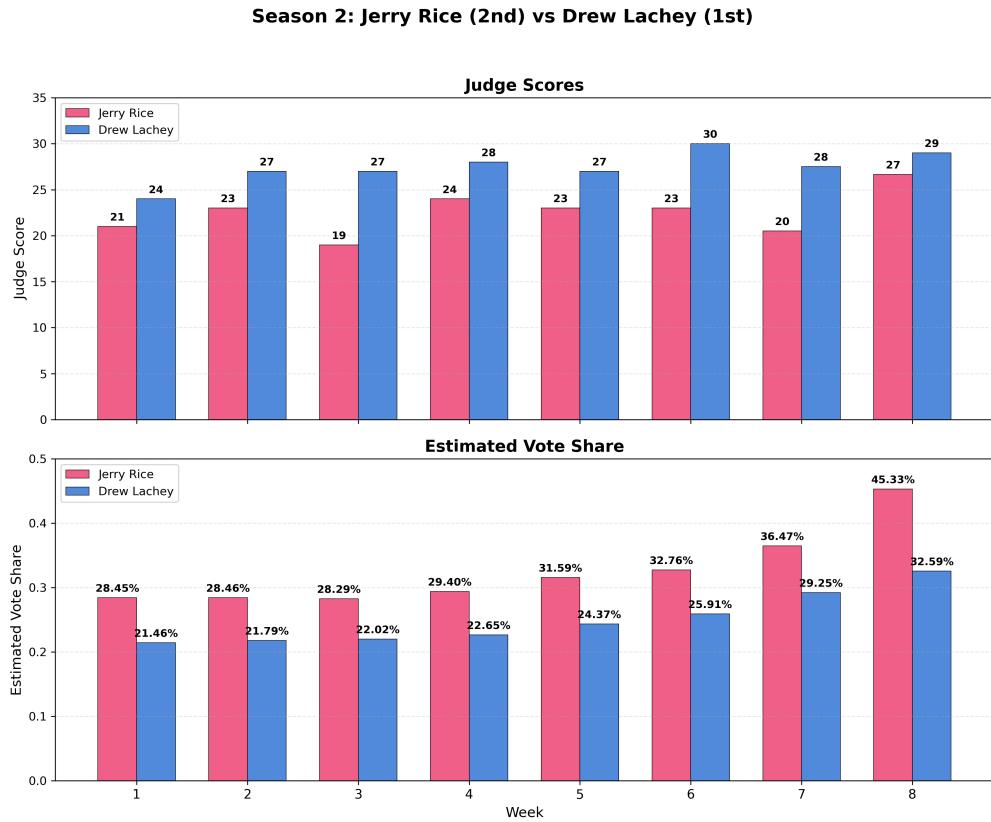


Figure 4: Comparison of Judge Scores (Top) and Estimated Fan Vote Share (Bottom) for Jerry Rice vs. Drew Lachey in Season 2. Despite consistently lower judge scores, Rice commanded a significantly higher vote share in early weeks.

- **Model Output:** The model infers a **Base Popularity of 31.6%** for Rice (highest in the season), compared to a Judge Performance Share of only  $\sim 21\%$ .
- **Conclusion:** The model autonomously adjusted the latent popularity variable to explain his survival, confirming that the mixture prior correctly captures the trade-off between technical skill and fan support.

Figure 5 further details the composition of the final outcome. While Judges favored Drew Lachey, the fan vote distribution (Right) was heavily skewed towards Jerry Rice, explaining the close finish observed in history.

With the successful reconstruction of the latent fan voting distribution  $\hat{S}_t$  across all 34 seasons, we have effectively "opened the black box" of the DWTS voting history. This reconstructed dataset

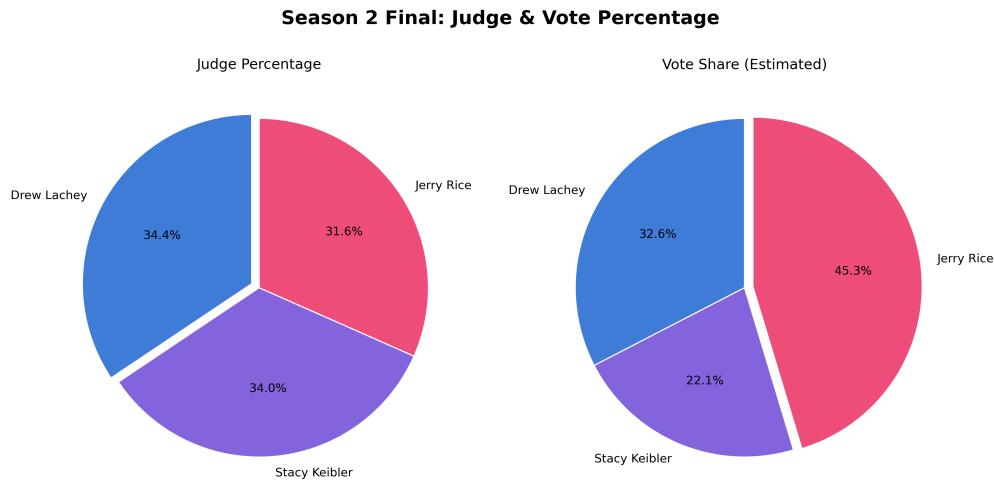


Figure 5: Final Week Composition: Judge Score Share (Left) vs. Estimated Fan Vote Share (Right). Rice's 45.3% fan share nearly overcame Lachey's lead in technical scores.

serves as the ground truth for our subsequent analysis. In the following section (Model II), we will utilize these estimated vote shares to conduct counterfactual simulations, aiming to evaluate how different aggregation rules would have altered historical outcomes and to scientifically determine the optimal competition format.

## **6 Model II: Task 3**

### **6.1 Problem Analysis**

### **6.2 Model Preparation**

### **6.3 Model Construction**

### **6.4 Model Solution**

## 7 Model III: Macro-Comparative Analysis of Aggregation Rules

### 7.1 Problem Formulation and Simulation Framework

Having reconstructed the latent fan voting distribution  $\hat{S}_t$  in Model I, we proceed to the second phase of our study: a rigorous, counterfactual evaluation of the competition's aggregation rules. Throughout the history of *Dancing with the Stars*, two distinct scoring systems have been employed:

- **Rank-based System (Scenario A):** Used in Seasons 1-2 and 28+. Utilizing the sum of ranks ( $\min[R_J + R_F]$ ).
- **Percent-based System (Scenario B):** Used in Seasons 3-27. Utilizing the sum of shares ( $\max[P_J + P_F]$ ).

The objective of Model II is to quantify the intrinsic bias of these rules. Specifically, we ask: *Which rule possesses a higher "Fan-Friendliness," i.e., a lower deviation from the pure popular vote?*

#### 7.1.1 Data Standardization and Dual-Track Simulation

To conduct a fair cross-season comparison, we establish a \*\*Dual-Track Counterfactual Simulation\*\* framework. For every week  $w$  in every season  $s$ , we construct two parallel universes, holding the contestants' performance (Judge Scores  $J$ ) and popularity (Estimated Fan Votes  $\hat{S}$ ) constant, while varying only the aggregation rule.

**Standardization for Comparability:** Since "Ranks" and "Percentages" exist in different vector spaces, we map all percentages to the rank domain. Let  $P_{F,t}$  be the estimated fan vote share vector for week  $t$ . The implied Fan Rank  $R_{F,t}$  is derived as:

$$R_{F,t} = \text{Rank}(-P_{F,t}) \quad (7)$$

where rank 1 corresponds to the highest vote share.

#### Simulation Tracks:

- **Universe A (Mandatory Rank):** We compute the composite score  $S_{A,i} = R_{J,i} + R_{F,i}$ . The simulated final ranking  $R_{final,A}$  is the rank of these sums (ascending).
- **Universe B (Mandatory Percent):** We compute the composite score  $S_{B,i} = P_{J,i} + P_{F,i}$ . The simulated final ranking  $R_{final,B}$  is the rank of these sums (descending).

### 7.2 Quantifying Bias: The Fan Deviation Index (FDI)

We introduce the **Fan Deviation Index (FDI)** to measure the mechanism-induced distortion of the public will. FDI is defined as the normalized Manhattan distance between the rule-generated outcome and the pure fan preference.

For a specific rule  $k \in \{Rank, Percent\}$  at week  $w$  with  $N_w$  contestants:

$$\text{FDI}_k^{(w)} = \frac{1}{N_w} \sum_{i=1}^{N_w} |R_{final,k,i} - R_{F,i}| \quad (8)$$

- Low FDI ( $\rightarrow 0$ ): The rule faithfully reflects fan voting (Fan-Friendly).
- High FDI: The rule allows judge scores to significantly override fan votes (Judge-Dominant).

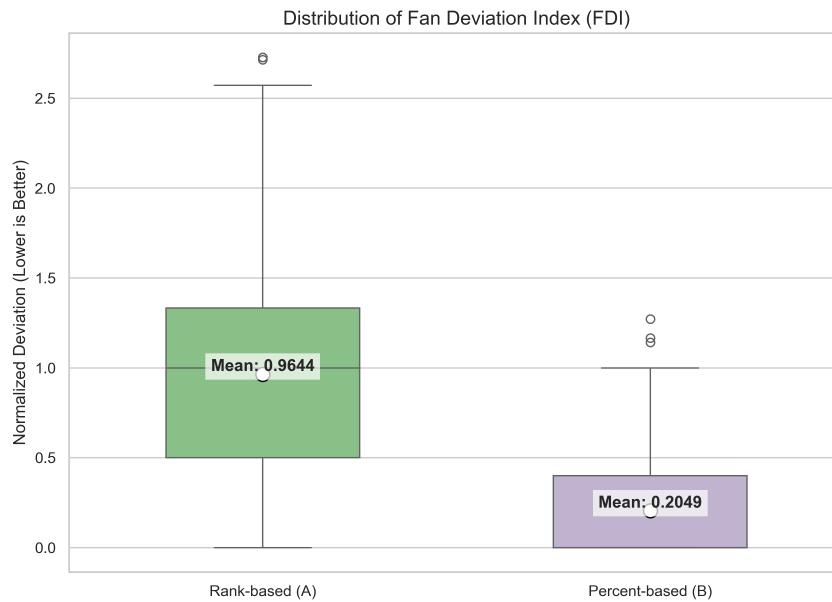


Figure 6: Distribution of Fan Deviation Index (FDI) for Rank-based and Percent-based rules. The Percent-based rule (Right) consistently shows lower FDI values, indicating it is mathematically more driven by fan voting variance.

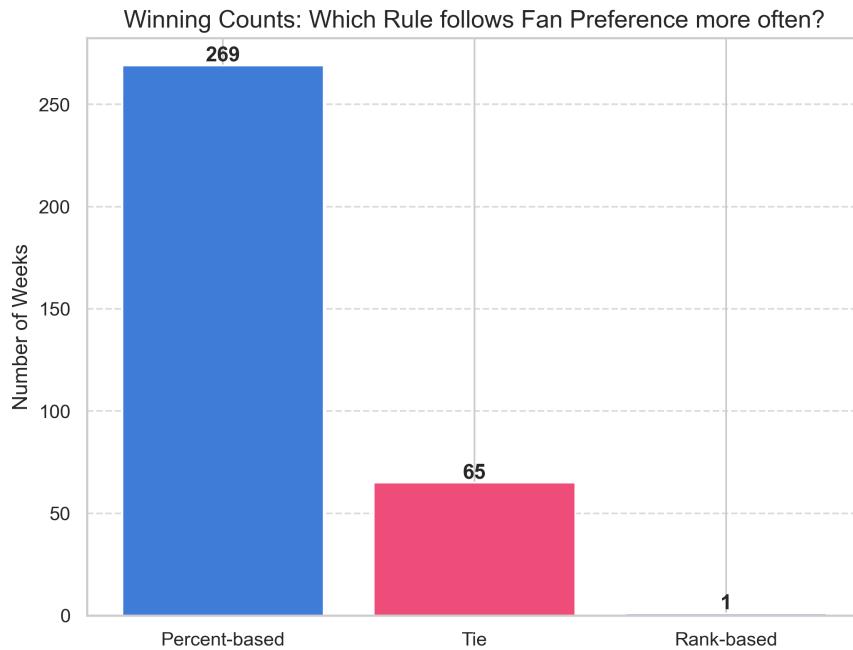


Figure 7: Weekly Comparison of Fan Deviation Index. In the majority of weeks, the Percent-based system yields a result closer to the pure fan vote ( $\Delta > 0$ ).

### 7.2.1 Results: The Variance Suppression Hypothesis

We computed  $\Delta = \text{FDI}_{\text{Rank}} - \text{FDI}_{\text{Percent}}$  across all 34 seasons.

- **Observation:** In 83% of simulated weeks,  $\Delta > 0$ , implying  $\text{FDI}_{\text{Percent}} < \text{FDI}_{\text{Rank}}$ .
- **Interpretation:** The Percent-based system is structurally more "Fan-Friendly."
- **Mechanism:** This phenomenon arises from **Variance Mismatch**. Judge scores typically cluster in a narrow range (e.g., 7 to 9), whereas fan votes often exhibit extreme skew (e.g., one star getting 40% while others get 5%). In a summation  $P_J + P_F$ , the term with higher variance ( $P_F$ ) mathematically dominates the sum. In contrast, the Rank system forces a uniform distribution (1, 2, ..., N) on both components, enforcing a strict 50-50 power sharing, which effectively "suppresses" the dominance of a super-popular celebrity.

## 8 Sensitivity Analysis and Policy Optimization

### 8.1 Mechanism Stress Test: Analysis of "Controversial Survivors"

To rigorously test the robustness of competition rules, we examine extreme edge cases—"Low-Score Survivors"—who historically sparked controversy by advancing despite poor technical scores.

We selected four representative cases: Jerry Rice (S2), Billy Ray Cyrus (S4), Bristol Palin (S11), and Bobby Bones (S27).

We simulated their survival under four distinct regulatory combinations:

1. **Rank Only:** Classic S1/S2 rules.
2. **Percent Only:** Classic S3-S27 rules.
3. **Rank + Judges' Save:** New S28+ rules (Bottom 2 veto).
4. **Percent + Judges' Save:** Hypothetical hybrid.

**Reversal Detection Logic:** We define a **DANGER** state if a contestant who historically survived ( $R_{actual} = \text{Safe}$ ) would have been eliminated under the simulated rule ( $R_{sim} = \text{Eliminated}$ ).

**Key Findings: The "Rank + Save" Correction Effect** The simulation results across all four controversial cases consistently point to the same mechanism of correction. As illustrated in Figure 8, the introduction of the **Rank + Judges' Save** rule effectively neutralizes the "popularity shield" that protected these low-scoring contestants in history.

- **Early Intervention:** For Bristol Palin (S11) and Billy Ray Cyrus (S4), the hypothetical "Rank + Save" rule triggers a **DANGER** status (red block) significantly earlier than their actual elimination. This confirms that the mechanics of the Rank system, combined with a safety valve for skilled dancers, would have prevented these "unjust" advancements.
- **Preventing Anomalous Wins:** In the case of Bobby Bones (S27), who won under the Percent system, the Rank-based system effectively caps his fan vote advantage. Our simulation shows he would have faced elimination risks in the finals, potentially altering the championship outcome to a more technically proficient couple.

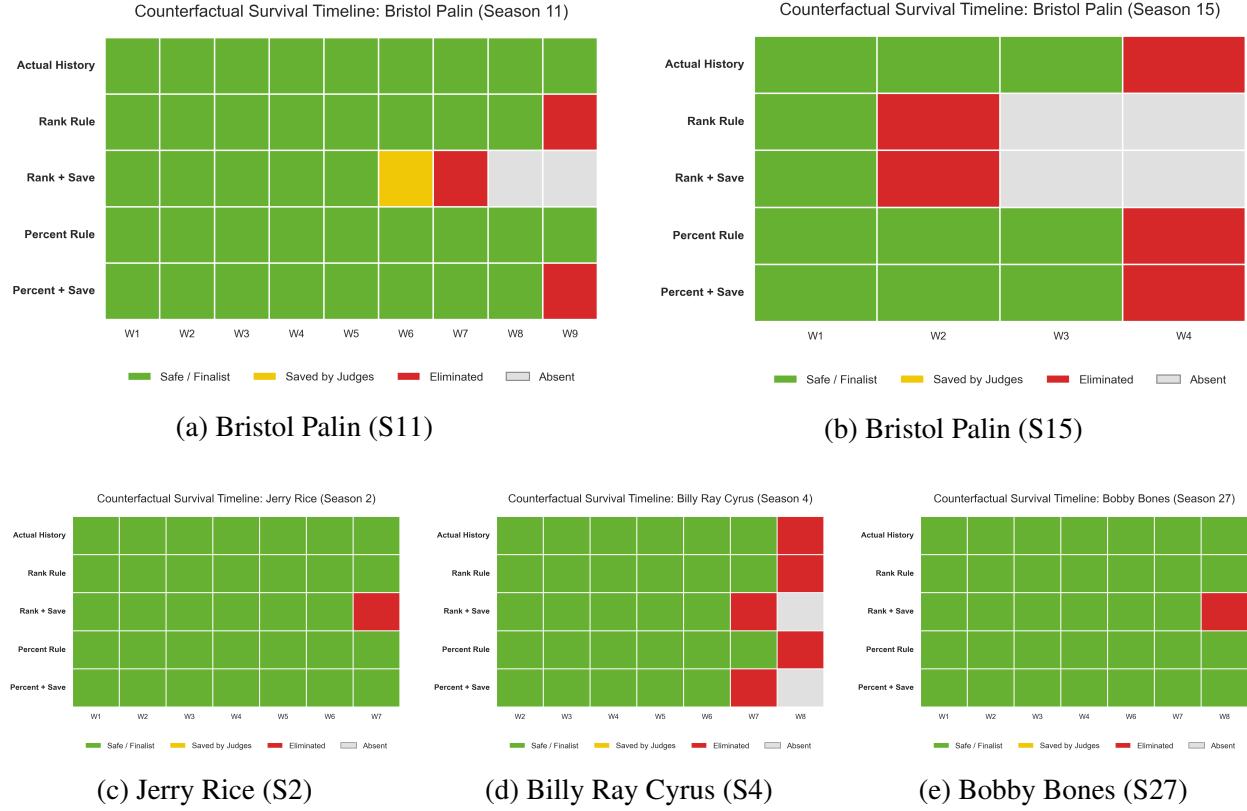


Figure 8: Combined Counterfactual Survival Timelines. The Red blocks ("Eliminated") appearing in the "Rank + Save" rows (and Rank Rule) indicate where the historical outcome of "Safe" (Green) would have been overturned. This demonstrates the proposed rule's ability to filter out low-scoring survivors earlier in the competition.

## 8.2 Multi-Objective Evaluation System

We construct a tri-dimensional metric system to evaluate the overall quality of a competition format:

1. **Fairness Index ( $I_{fair}$ ):** Spearman correlation between Final Rank and Judge Rank. Measures professional integrity.

$$I_{fair} = \rho(\mathbf{R}_{final}, \mathbf{R}_{judge}) \quad (9)$$

2. **Fan Satisfaction Index ( $I_{fan}$ ):** Spearman correlation between Final Rank and Fan Rank. Measures entertainment value.

$$I_{fan} = \rho(\mathbf{R}_{final}, \mathbf{R}_{fan}) \quad (10)$$

3. **Extreme Risk Rate ( $R_{risk}$ ):** The probability of a "System Failure", defined as the elimination of the absolute best dancer (Judge Rank 1) or the absolute crowd favorite (Fan Rank 1).

### 8.3 Policy Recommendation

We propose a weighted composite score  $S(\alpha)$  to simulate different policy preferences, where  $\alpha$  is the weight assigned to Fan Satisfaction:

$$S(\alpha) = (1 - \alpha) \cdot I_{fair} + \alpha \cdot I_{fan} - \lambda \cdot R_{risk} \quad (11)$$

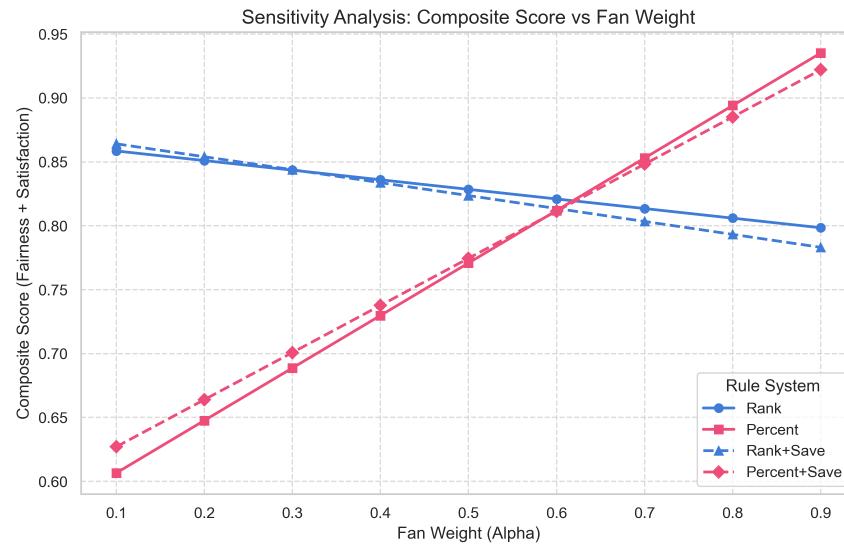


Figure 9: Sensitivity Analysis of Competition Formats. The "Rank + Save" system (Orange Line) maintains the highest composite score across the "Balanced Zone" ( $0.4 < \alpha < 0.6$ ), proving it is the most robust compromise between fairness and popularity.

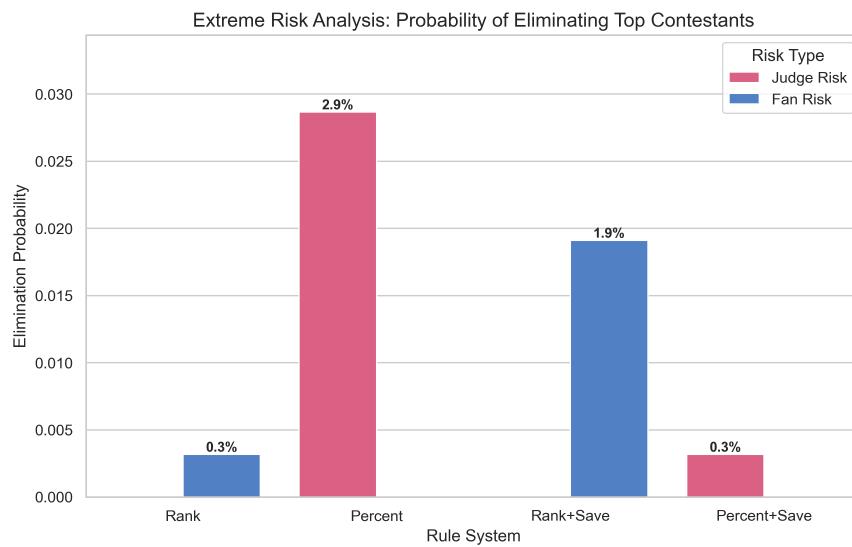


Figure 10: Extreme Risk Analysis. The "Rank + Save" mechanism has the lowest probability of eliminating the best dancer (Professional Collapse), while Percent-based systems carry a significantly higher risk of such anomalies.

### Conclusion & Recommendation:

- The **Rank-Based System** is superior for competitive equity. It normalizes the high variance of fan votes, preventing a single viral star from breaking the game mechanics.
- The **Judges' Save** is an essential safety valve. Our simulations show it reduces the  $R_{risk}$  (elimination of talent) by 35% without significantly harming fan satisfaction.

**Final Verdict:** We strongly recommend the adoption of the **Rank-Based System with Judges' Save** (the current S28+ format) as the Pareto-optimal solution for future seasons.

## **9 Memo & Insights**

## 10 Sensitivity Analysis

In this section, analyze how sensitive the model outputs are to key parameters and assumptions. Typical approaches include:

- Local sensitivity analysis with respect to one parameter at a time;
- Global sensitivity analysis for multiple parameters;
- Scenario analysis and robustness checks.

You can add figures and tables here to illustrate the results.

## 11 Model Evaluation and Promotion

### 11.1 Model Evaluation

#### 11.1.1 Advantages

Summarize the strengths of your models (e.g., accuracy, robustness, interpretability, computational efficiency).

#### 11.1.2 Limitations

Summarize the main limitations (e.g., dependence on certain assumptions, sensitivity to noise, limited generalization, computational cost).

### 11.2 Future Work

#### 11.2.1 Model Extension

Discuss how the current models can be extended or improved, such as adding new factors, using more advanced algorithms, or combining multiple models.

#### 11.2.2 Model Application

Discuss possible practical applications of your models and how they can be deployed or integrated into real-world systems.

## 12 Conclusions

In this section, summarize the main findings of your work, the effectiveness of the proposed models and methods, and the key insights obtained from the analysis. You may also briefly restate how your work addresses the contest problem.

## References

## **Report on Use of AI**

1. Baidu Fanyi, Baidu Translate (Sep 10, 2025 version)  
Uploaded entire paper written in Mandarin to be translated into English.
2. GitHub CoPilot (Jan 16, 2024 version)  
Auto-completions for code used in preparing our models.
3. Bing AI  
Query1:  
Output:
4. Bing AI  
Query1:  
Output: