1 时域卷积定理

定理 1 (时域卷积定理)**.** *若* $F_1(jw) = F[f_1(t)], F_2(jw) = F[f_2(t)]$, *则*

$$F\left[f_1(t) \star f_2(t)\right] = F_1(jw)F_2(jw)$$

证明.

$$\begin{aligned}
F\left[f_1(t) \star f_2(t)\right] &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[f_1(t) \star f_2(t)\right]e^{-jwt}dtd\tau \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_1(\tau)f_2(t-\tau)e^{-jwt}dtd\tau \\
&= \int_{-\infty}^{+\infty} f_1(\tau)d\tau\left[\int_{-\infty}^{+\infty} f_2(t-\tau)e^{-jwt}dt\right] \\
&= F_2(jw)\left[\int_{-\infty}^{+\infty} f_1(\tau)e^{-jw\tau}d\tau\right] \\
&= F_1(jw)F_2(jw)
\end{aligned}$$

□

2 频域卷积定理

定理 2 (频域卷积定理)**.**

$$F[f_1(t)f_2(t)] = \frac{1}{2\pi}F_1(jw) \star F_2(jw)$$

证明.

$$\begin{aligned}
F^{-1}\left[F_1(jw) \star F_2(jw)\right] &= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[F_1(jw) \star F_2(jw)\right]e^{jwt}dw \\
&= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[F_1(ju)F_2(j(w-u))\right]e^{jwt}dwdu \\
&= \int_{-\infty}^{+\infty} F_1(ju)du\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} F_2(j(w-u))e^{jwt}dw\right] \\
&= \int_{-\infty}^{+\infty} F_1(ju)e^{jut}du\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} F_2(j(w-u))e^{jwt}dw\right] \\
&= 2\pi f_2(t)\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} F_1(ju)e^{jut}du\right] \\
&= 2\pi f_1(t)f_2(t)
\end{aligned}$$

□