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ELSEVIER Applied Mathematics and Computation 163 (2005) 199–211
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

A numerical solution of the Burgers' equation using cubic B-splines

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Abstract

In the present paper numerical solutions of the one-dimensional Burgers' equation are obtained by a method based on collocation of cubic B-splines over finite elements. The accuracy of the proposed method is demonstrated by three test problems. The numerical results are found in good agreement with exact solutions. Time-space integration of the Burgers' equation yields a system of difference equation which is shown to be unconditionally stable.

1. Introduction

Consider the one-dimensional Burgers' equation

$$U_t + UU_x = \lambda U_{xx}, \quad a \leqslant x \leqslant b, \; t \geqslant 0 \tag{1}$$

with the initial condition

$$U(x,0) = f(x) \quad a \leqslant x \leqslant b \tag{2}$$

and the boundary conditions

$$U(a,t) = \beta_1, \quad U(b,t) = \beta_2, \tag{3}$$

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0096-3003/$ - see front matter
doi:10.1016/j.amc.2004.01.028

where $\lambda > 0$ is the coefficient of kinematic viscosity, and β_1, β_2 and $f(x)$ will be chosen in a later section.

Because of its similarity to the Navier–Stokes equation, Burgers' equation often arises in the mathematical modelling used to solve problems in fluid dynamics involving turbulence. Burgers' equation was first introduced by Bateman [1] when he mentioned it as worthy of study and gave its steady solutions. It was later treated by Burger [2] as a mathematical model for turbulence and after whom such an equation is widely referred to as Burgers' equation. Since then the equation has found applications in field as diverse as number theory, gas dynamics, heat conduction, elasticity, etc. Hopf [3] and Cole [4] showed independently that this equation can be transformed to the linear diffusion equation and solved exactly for an arbitrary initial condition. The exact solutions of the one dimensional Burgers' equation have been surveyed by Benton and Platzman [7]. In many cases these solutions involve infinite series which may converge very slowly for small values of the viscosity coefficient λ which correspond to steep wave fronts in the propagation of the dynamic wave-forms [5]. Many studies have been done on the numerical solutions of Burgers' equation to deal with solutions for the small values of λ. Some of the earlier numerical studies are documented as follows: a finite element method has been given by Caldwell et al. [15] to solve Burgers' equation by altering the size of the element at each stage using information from three previous steps. Moreover Caldwell and Smith [17] have discussed the comparison of a number of numerical approaches to the equation. Nguyen and Reynen [18] also suggested a space-time finite element method based on a least-square weak formulation using piecewise linear shape functions. A kind of finite element method based on weighted residual formulation was given by Varoğlu and Liam Finn [14] and demonstrated the high accuracy and the stability. Spline and B-spline functions together with some numerical technique have been used in getting the numerical solution of Burgers' equation recently. Rubin and Graves have used the spline function technique and quasi-linearisation for the numerical solution of the Burgers' equation in one space variable [8]. A cubic spline collocation procedure was developed for the Burgers' equation in the papers [10,19]. The implicit-finite difference schemes together with cubic splines interpolating space derivatives in the Burgers' equation has been proposed in the papers [11–13,22]. The B-spline Galerkin method and B-spline collocation methods have been setup for the numerical solution of the differential equations [20,21]. In addition to finite difference and finite element methods, some others methods exist in the literature.

In the present method, we have proposed a type of the cubic B-spline collocation procedure in which nonlinear term in the equation is linearized by using the form introduced by the Rubin and Graves [8]. For the numerical procedure, time derivative is discretized in the usual finite difference scheme. Solution and its principal derivatives over the subinterval are approximated by the combination of the cubic B-splines and unknown element parameters. Using the values of the cubic B-splines, nodal values and its derivatives at the knots are expressed in terms of element parameters. Placing nodal values and its derivatives in the Burgers' equation result in system consisting of $N+1$ equations for $N+3$ parameters. The resulting system can be solved with Thomas algorithm [6] after the boundary conditions are applied.

2. Collocation method

The region $[a,b]$ is partitioned into uniformly sized finite elements of length h by the knots x_j such that $a = x_0 < x_1 < \cdots < x_N = b$. Let $\phi_m(x)$ be cubic B-splines with knots at the points x_m, $m = 0,\dots,N$. The set of splines $\{\phi_{-1},\phi_0,\phi_1,\dots,\phi_N,\phi_{N+1}\}$ forms a basis for functions defined over $[a,b]$. Thus, an approximation $U_N(x,t)$ to the exact solution $U(x,t)$ can be expressed in terms of the cubic B-splines as trial functions:

$$U_N(x,t) = \sum_{m=-1}^{N+1} \delta_m(t)\phi_m(x), \tag{4}$$

where δ_m are time dependent quantities to be determined from boundary conditions and collocation form of the differential equations.

Cubic B-splines ϕ_m with the required properties are defined by relationship [9]

$$\phi_m(x) = \frac{1}{h^3}\begin{cases} (x-x_{m-2})^3 & [x_{m-2},x_{m-1}],\\ h^3+3h^2(x-x_{m-1})+3h(x-x_{m-1})^2-3(x-x_{m-1})^3 & [x_{m-1},x_m],\\ h^3+3h^2(x_{m+1}-x)+3h(x_{m+1}-x)^2-3(x_{m+1}-x)^3 & [x_m,x_{m+1}],\\ (x_{m+2}-x)^3 & [x_{m+1},x_{m+2}],\\ 0 & \text{otherwise}, \end{cases} \tag{5}$$

where $h = x_{m+1} - x_m$, $m = -1,\dots,N+1$. The variation of $U_N(x,t)$ over typical element $[x_m,x_{m+1}]$ is given by

$$U_N(x,t) = \sum_{j=m-1}^{m+2} \delta_j(t)\phi_j(x). \tag{6}$$

Using trial function (4) and cubic splines (5), the values of U, U', U'' at the knots are determined in terms of the element parameters δ_m by

$$
\begin{aligned}
U_m &= U(x_m) = \delta_{m-1} + 4\delta_m + \delta_{m+1}, \\
U'_m &= U'(x_m) = \frac{3}{h}(\delta_{m+1} - \delta_{m-1}), \\
U''_m &= U''(x_m) = \frac{6}{h^2}(\delta_{m-1} - 2\delta_m + \delta_{m+1}),
\end{aligned} \tag{7}
$$

where the symbols $'$ and $''$ denote first and second differentiation with respect to x, respectively.

At the knots, an approximate solution U_m for the Burgers' equation

$$U_t + UU_x - \lambda U_{xx} = 0 \tag{8}$$

can be obtained by considering the solution of

$$(U_t)_m^n + (1-\theta)f_m^n + \theta f_m^{n+1} = 0, \tag{9}$$

where $(f)_m^n = (UU_x)_m^n - \lambda(U_{xx})_m^n$. If time derivative is discretized in the usual finite difference way, we have

$$U_m^{n+1} - U_m^n + (1-\theta)\Delta t f_m^n + \theta\Delta t f_m^{n+1} = 0. \tag{10}$$

The nonlinear term in Eq. (10) may be linearized by using the following term [8]

$$(UU_x)_m^{n+1} = U_m^{n+1}(U_x)_m^n + U_m^n(U_x)_m^{n+1} - U_m^n(U_x)_m^n. \tag{11}$$

Substitution of the approximate values of the nodal values U, the first and second derivatives U_x, U_{xx} given by the Eqs. (7) at the knots in Eq. (10) yields following difference equation with the variables δ

$$
\begin{aligned}
&\delta_{m-1}^{n+1}\left(1 + \theta\Delta t L_2 - \frac{3\theta\Delta t L_1}{h} - \lambda\frac{6\theta\Delta t}{h^2}\right) + \delta_m^{n+1}\left(4 + 4\theta\Delta t L_2 + \lambda\frac{12\theta\Delta t}{h^2}\right) \\
&\quad + \delta_{m+1}^{n+1}\left(1 + \theta\Delta t L_2 + \frac{3\theta\Delta t L_1}{h} - \lambda\frac{6\theta\Delta t}{h^2}\right) \\
&\quad = L_1 - (1-\theta)\Delta t[L_1L_4 + L_3L_2 - L_3L_4 - \lambda L_5] + \theta\Delta t L_1 L_2,
\end{aligned} \tag{12}
$$

where

$$
\begin{aligned}
&L_1 = \delta_{m-1}^n + 4\delta_m^n + \delta_{m+1}^n, \quad L_2 = \frac{3}{h}(\delta_{m+1}^n - \delta_{m-1}^n), \\
&L_3 = \delta_{m-1}^{n-1} + 4\delta_m^{n-1} + \delta_{m+1}^{n-1}, \quad L_4 = \frac{3}{h}(\delta_{m+1}^{n-1} - \delta_{m-1}^{n-1}), \\
&L_5 = \frac{6}{h^2}(\delta_{m-1}^n - 2\delta_m^n + \delta_{m+1}^n).
\end{aligned}
$$

The system (12) consists of $N+1$ linear equations in $N+3$ unknowns $\mathbf{d}^n = (\delta_{-1}^n, \delta_0^n, \delta_1^n, \ldots, \delta_N^n, \delta_{N+1}^n)$. To obtain a unique solution to this system we need two additional constraints. These are obtained from the boundary conditions $U(x_0) = \beta_1$ and $U(x_N) = \beta_2$. Imposition of boundary conditions enables us to elimination of parameters δ_{-1}, δ_{N+1} from the system (12) with help of equations:

$$\begin{aligned} U(x_0) &= \delta_{-1}^{n+1} + 4\delta_0^{n+1} + \delta_1^{n+1} = \beta_1 \Rightarrow \delta_{-1}^{n+1} = \beta_1 - (4\delta_0^{n+1} + \delta_1^{n+1}), \\ U(x_N) &= \delta_{N-1}^{n+1} + 4\delta_N^{n+1} + \delta_{N+1}^{n+1} = \beta_2 \Rightarrow \delta_{N+1}^{n+1} = \beta_2 - (\delta_{N-1}^{n+1} + 4\delta_N^{n+1}), \end{aligned} \tag{13}$$

so the system (12) is reduced to $(N + 1) \times (N + 1)$ matrix system, which can be solved by using the Thomas algorithm.

Note that in the system (12) $\theta = 0$ gives explicit scheme, $\theta = 1$ gives a fully implicit scheme and $\theta = 0.5$ result in mixed scheme of the Crank–Nicholson. We used to Crank–Nicholson approach by taking $\theta = 0.5$. The time evolution of the approximate solution $U_N(x, t)$ is determined by the time evolution of the vector $\mathbf{d}^n$ which is found repeatedly by solving the recurrence relation, once the initial vectors $\mathbf{d}^0$, $\mathbf{d}^1$ have been computed from the initial conditions.

Initial vector of parameters $\mathbf{d}^0$ can be obtained from the initial condition and boundary values of the derivatives of the initial condition. So the following relations at the knots x_m are used

1. $(U_x)_N(x_0, 0) = U_x(x_0, 0)$,
2. $U_N(x_j, 0) = U(x_j, 0)$, $j = 0, \ldots, N$,
3. $(U_x)_N(x_N, 0) = U_x(x_N, 0)$.

The above equations yield a tridiagonal matrix system whose solution can be found with the Thomas algorithms. The first time derivatives $\mathbf{d}^1$ is determined from matrix system (12) by putting $\theta = 0$. Nodal values U and its first and second derivatives can recovered from Eqs. (7) at each time steps.

3. The stability analysis

The nonlinear term UU_x of the Burgers' equation is linearized by taking U as a constant g. Substituting the Fourier mode $\delta_m^n = \xi^n e^{i\beta mh}$, where β is the mode number and h is the element size, into linearized form of Eq. (12), we have

$$\xi = \frac{a - ib}{c + id}, \tag{14}$$

where

$$a = 2\cos\beta h + 4 - (1-\theta)\frac{12}{h^2}\lambda\Delta t(1 - \cos\beta h),$$

$$b = (1-\theta)\frac{6}{h}g\Delta t\sin\beta h,$$

$$c = 2\cos\beta h + 4 - \theta\frac{12}{h^2}\lambda\Delta t(1-\cos\beta h),$$

$$d = \theta\frac{6}{h}g\Delta t\sin\beta h.$$

Taking the modulus of Eq. (14) gives $|\xi| \leqslant 1$, we find that the difference scheme (12) is unconditionally stable for $\theta \in [\frac{1}{2}, 1]$.

4. The test problems

We now obtain numerical solutions of Burgers' equation for three standard problems. To measure the accuracy of the numerical method between numerical and exact ones we compute the weighted 1-norm $|e|_1$ defined by

$$|e|_1 = \frac{1}{N}\sum_{i=1}^{N-1}\frac{|U(x_i,t_j) - U_{i,j}|}{|U(x_i,t_j)|}$$

(a) The first problem has initial condition

$$U(x,0) = \sin(\pi x), \quad 0 \leqslant x \leqslant 1 \tag{15}$$

and boundary values

$$U(0,t) = U(1,t) = 0, \quad t \geqslant 0. \tag{16}$$

The theoretical solution of this problem was expressed as an infinite series by Cole [4]

$$U(x,t) = \frac{4\pi\lambda\sum_{j=1}^{\infty} jI_j\left(\frac{1}{2\pi\lambda}\right)\sin(j\pi x)\exp(-j^2\pi^2\lambda t)}{I_0\left(\frac{1}{2\pi\lambda}\right) + 2\sum_{j=1}^{\infty} I_j\left(\frac{1}{2\pi\lambda}\right)\cos(j\pi x)\exp(-j^2\pi^2\lambda t)}, \tag{17}$$

where I_j are the modified Bessel functions. This problem gives the decay of sinusoidal disturbance.

Numerical solutions obtained by the present method can be compared with exact solutions in Table 1 for $\lambda = 1$, time step $\Delta t = 0.00001$ and various space steps at time $t = 0.1$. The solutions become more accurate with the smaller space steps. So numerical predictions are seen reasonably in good agreement with exact values. In the same table, magnitude of the error is shown by calculating the $|e|_1$ norm. With corresponding parameters used in the paper [23], pointwise convergence of the proposed algorithm is better than that of finite difference schemes especially when the smaller space steps are used. So that present cubic B-spline collocation algorithm compares favorably with both explicit finite difference and exact explicit finite difference scheme.

The further comparison of the numerical results is made with exact ones for the different viscosity coefficients λ shown in the Table 2. The smaller values,

Table 1
Comparison of results $t = 0.1$ for $\lambda = 1$, $\Delta t = 0.00001$ and various mesh sizes

x	$h = 0.1$	$h = 0.05$	$h = 0.025$	$h = 0.0125$	$h = 0.00625$	Exact
0.1	0.10888	0.10937	0.10949	0.10952	0.10953	0.10954
0.2	0.20847	0.20945	0.20969	0.20975	0.20977	0.20979
0.3	0.28992	0.29138	0.29175	0.29184	0.29186	0.29190
0.4	0.34537	0.34726	0.34773	0.34785	0.34788	0.34792
0.5	0.36859	0.37080	0.37136	0.37149	0.37153	0.37158
0.6	0.35589	0.35823	0.35881	0.35896	0.35900	0.35905
0.7	0.30696	0.30914	0.30969	0.30983	0.30986	0.30991
0.8	0.22552	0.22722	0.22765	0.22776	0.22778	0.22782
0.9	0.11942	0.12036	0.12060	0.12065	0.12067	0.12069
$\lvert e\rvert_1$ (present)	0.00734	0.00095	0.00014	0.00003	0.00001	
$\lvert e\rvert_1$ ([23]-exp.)	0.007571	0.002025	0.000555	0.000177		
$\lvert e\rvert_1$ ([23]exact-exp.)	0.007278	0.0011885	0.000448	0.000077		

Table 2
Comparison of results at different times $\lambda = 1.0$, 0.1 and 0.01 with $h = 0.0125$ and $\Delta t = 0.0001$

x	t	$\lambda = 1$ Numerical	$\lambda = 1$ Exact	$\lambda = 0.1$ Numerical	$\lambda = 0.1$ Exact	$\lambda = 0.01$ Numerical	$\lambda = 0.01$ Exact
0.25	0.4	0.01357	0.01357	0.30890	0.30889	0.34192	0.34191
	0.6	0.00189	0.00189	0.24075	0.24074	0.26897	0.26896
	0.8	0.00026	0.00026	0.19569	0.19568	0.22148	0.22148
	1.0	0.00004	0.00004	0.16258	0.16256	0.18819	0.18819
	3.0	0.00000	0.00000	0.02720	0.02720	0.07511	0.07511
0.50	0.4	0.01923	0.01924	0.56965	0.56963	0.66071	0.66071
	0.6	0.00267	0.00267	0.44723	0.44721	0.52942	0.52942
	0.8	0.00037	0.00037	0.35925	0.35924	0.43914	0.43914
	1.0	0.00005	0.00005	0.29192	0.29192	0.37442	0.37442
	3.0	0.00000	0.00000	0.04019	0.04021	0.15018	0.15018
0.75	0.4	0.01362	0.01363	0.62538	0.62544	0.91027	0.91026
	0.6	0.00189	0.00189	0.48715	0.48721	0.76725	0.76724
	0.8	0.00026	0.00026	0.37385	0.37392	0.64740	0.64740
	1.0	0.00004	0.00004	0.28741	0.28747	0.55605	0.55605
	3.0	0.00000	0.00000	0.02976	0.02977	0.22483	0.22481

especially $\lambda < 10^{-3}$, we use, the worse numerical solutions are obtained. The agreement between results from the presented schemes and the exact solution appears very satisfactory for high viscosity λ. A graphical solution with parameters $\lambda = 1,\ h = 0.1,\ \Delta t = 0.01$ is illustrated at some times in Fig. 1. It is known from outcome of the Miller's paper [5] that exact values are not practical to make comparison for the small values of $\lambda < 10^{-3}$ because of slow convergence of the Fourier series solution. Solutions are plotted for the smaller values of $\lambda = 10^{-4}$ at sometimes in Fig. 2. The graph shows development of a

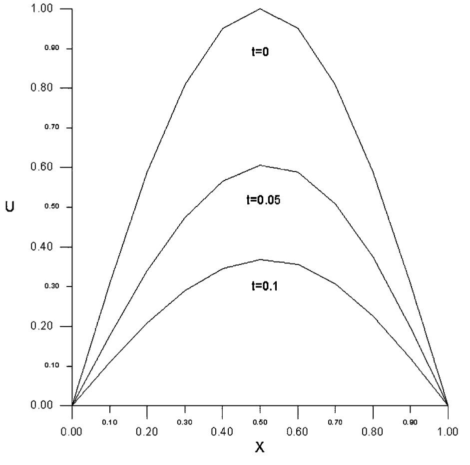

Fig. 1. Solution. for $\lambda = 1$, $h = 0.1$, $\Delta t = 0.01$.

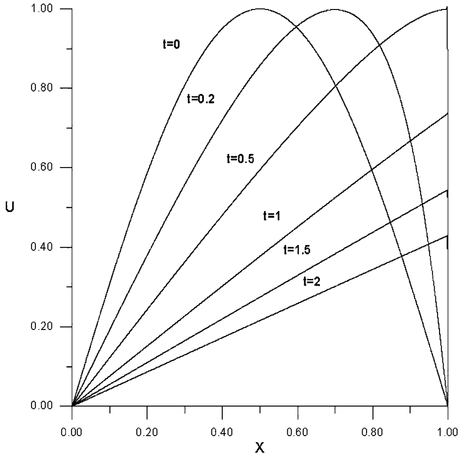

Fig. 2. $\lambda = 10^{-4}$, $h = 10^{-4}$, $\Delta t = 0.01$.

sharp front near $x = 0$ at early times and afterwards the amplitude of sharp front starts to decay while steepness of the solution near the right hand boundary $x = 1$ increase sharply. This steepness in the solution has been dealt with taking the smaller space steps. Otherwise oscillation at the boundary $x = 1$ takes place during time advances.

(b) As the second test example, we consider particular solution of Burgers' equation

$$U(x,t) = \frac{\frac{x}{t}}{1 + \sqrt{\frac{t}{t_0}} \exp\left(\frac{x^2}{4\lambda t}\right)}, \quad t \geqslant 1, \; 0 \leqslant x \leqslant 1, \tag{18}$$

where $t_0 = \exp(\frac{1}{8\lambda})$.

Initial condition is obtained from Eq. (18) when $t = 1$ is used. Boundary conditions are $U(0,t) = U(1,t) = 0$. Analytical solution represents shock-like solutions of the one-dimensional Burgers' equation. Figs. 3 and 4 illustrate the propagation of shock for $\lambda = 0.005$, $h = 0.02$, $\Delta t = 0.1$ and $\lambda = 0.0005$, $h = 0.005$, $\Delta t = 0.01$ at some different times, respectively. Both numerical and analytical solutions are plotted in the same figures and can not distinguishable. These graphs agree with those reported in the paper [18]. We have the initial shock for $\lambda = 0.005$ which is sharp and the center of shock moves toward the right with changes of sharpness through smoothness as time progress. With smaller $\lambda = 0.0005$, initial shock is very steep, advances in the center of the shock to the right is less than the case of $\lambda = 0.005$ and during time progression, steepness remains almost unchanged. A further verification of the present method is illustrated by tabulating results for different values of space steps in Table 3.

(c) The exact solution of Burgers' equation [16] is

$$U(x,t) = \frac{\alpha + \mu + (\mu - \alpha)\exp\eta}{1 + \exp\eta}, \quad 0 \leqslant x \leqslant 1, \; t \geqslant 0, \tag{19}$$

where $\eta = \frac{\alpha(x - \mu t - \gamma)}{\lambda}$, α, μ and γ are constants, when the boundary conditions are

$$U(0,t) = 1 \quad \text{and} \quad U(1,t) = 0.2, \; t \geqslant 0 \tag{20}$$

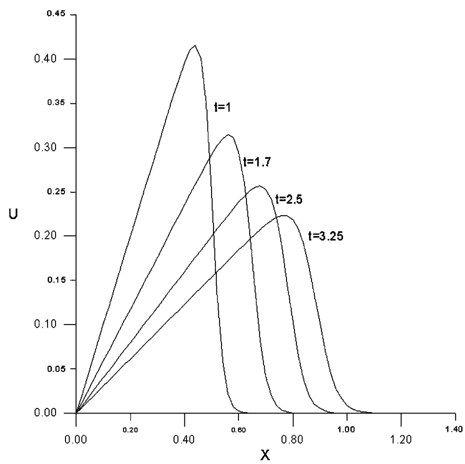

Fig. 3. Solutions for $\lambda = 0.005$.

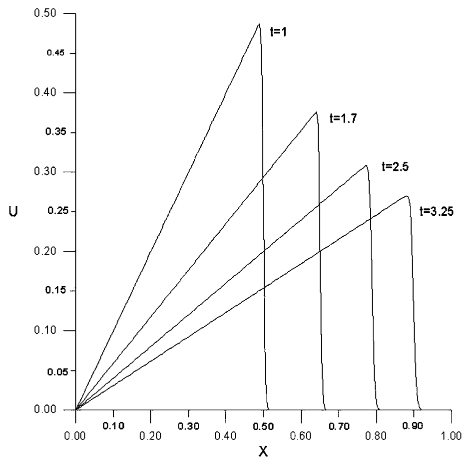

Fig. 4. Solutions for $\lambda = 0.0005$.

Table 3
Comparison of result at different times for $\lambda = 0.0005$ and $[a, b] = [0, 1]$ with $h = 0.005$ and $\Delta t = 0.01$

x	$t = 1.7$ Numerical	$t = 1.7$ Exact	$t = 2.5$ Numerical	$t = 2.5$ Exact	$t = 3.25$ Numerical	$t = 3.25$ Exact
0.1	0.05883	0.05882	0.04000	0.04000	0.03077	0.03077
0.2	0.11765	0.11765	0.08000	0.08000	0.06154	0.06154
0.3	0.17648	0.17647	0.12001	0.12000	0.09231	0.09231
0.4	0.23531	0.23529	0.16001	0.16000	0.12308	0.12308
0.5	0.29414	0.29412	0.20001	0.20000	0.15385	0.15385
0.6	0.35296	0.35294	0.24001	0.24000	0.18462	0.18462
0.7	0.00000	0.00000	0.28001	0.28000	0.21539	0.21538
0.8	0.00000	0.00000	0.00811	0.00977	0.24616	0.24615
0.9	0.00000	0.00000	0.00000	0.00000	0.12358	0.12435

and initial condition is used for the exact solution at $t = 0$. We choose parameters to be $\alpha = 0.4$, $\mu = 0.6$ and $\gamma = 0.125$ so that comparison can be made with Refs. [16,21]. The solution represents a travelling wave, initially situated at $x = \gamma$, moving to the right with speed μ. Calculation is performed with the parameters of both $h = 1/18$, $\Delta t = 0.001$ and $h = 1/36, \Delta t = 0.025$ for the viscosity number $\lambda = 0.01$. The numerical results are compared with the analytic solution and the results presented in the papers [16,21] known as methods of compact differencing technique (CD), Petrov–Galerkin technique (SGA), product approximation version of SGA and quadratic B-spline collocation technique (QBGT) in Table 4. It is seen that the agreement between the numerical and the exact solution appears satisfactorily. The QBGT has pro-

Table 4
Comparison of the numerical results with analytical results at time $t = 0.5$ and $\lambda = 0.01$, ($k = \Delta t = 0.001$)

x	Present ($h = \frac{1}{36}$) $\Delta t = 0.025$	Present ($h = \frac{1}{18}$) k	PGM [16] ($h = \frac{1}{18}$) k	PY [16] ($h = \frac{1}{18}$) k	CD [16] ($h = \frac{1}{18}$) k	QBGT [21] ($h = \frac{1}{36}$) k	Exact
0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.056	1.000	1.000	1.000	1.000	1.030	1.000	1.000
0.111	1.000	1.000	1.000	1.000	0.990	1.000	1.000
0.167	1.000	1.000	1.000	1.000	0.973	1.000	1.000
0.222	1.000	1.000	0.998	0.999	1.009	1.000	1.000
0.278	0.999	0.996	0.911	0.997	1.004	0.999	0.998
0.333	0.986	0.994	0.970	0.982	0.986	0.985	0.980
0.389	0.850	0.835	0.862	0.850	0.696	0.847	0.847
0.444	0.448	0.461	0.461	0.444	0.360	0.452	0.452
0.500	0.236	0.240	0.159	0.171	0.228	0.238	0.238
0.556	0.204	0.199	0.300	0.286	0.203	0.204	0.204
0.611	0.200	0.199	0.194	0.197	0.200	0.200	0.200
0.667	0.200	0.200	0.213	0.211	0.200	0.200	0.200
0.722	0.200	0.200	0.211	0.210	0.200	0.200	0.200
0.778	0.200	0.200	0.188	0.190	0.200	0.200	0.200
0.833	0.200	0.200	0.201	0.207	0.200	0.200	0.200
0.889	0.200	0.200	0.191	0.193	0.200	0.200	0.200
0.944	0.200	0.200	0.203	0.202	0.200	0.200	0.200
1.000	0.200	0.200	0.200	0.200	0.200	0.200	0.200

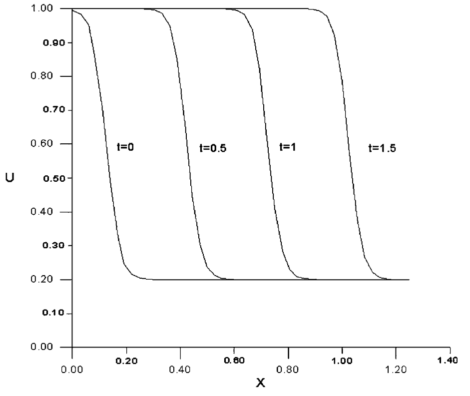

Fig. 5. Solutions at different times for $\lambda = 0.01$.

duced a little better results that ones we obtained. From Table 4 it is seen that for $\lambda = 0.01$ the agreement between the numerical and the exact solution appears very satisfactorily. Numerical propagation of the initial wave is also drawn at different times in Fig. 5.

We have described a type of a cubic B-spline collocation method to obtain the numerical solution of the Burgers' equation. According to the three test problems, the comparison of the calculations with the analytic solution shows that a cubic B-spline collocation method is capable of solving Burgers' equation accurately. The further comparison of the presented numerical results with the method mentioned in this paper shows that cubic collocation scheme has provided a little worse results than the a type of B-spline collocation method referenced [21] but better than the finite difference schemes. The proposed method is easy to implement and requires no any inner iteration or corrector to deal with the nonlinear term of the Burgers' equation.

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