



Data Mining ▷ Frequent Pattern Mining

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Outline

Introduction FR APRIORI Optimizations References

- 1 Introduction to FPM
- 2 Frequent itemsets
- 3 APRIORI
- 4 APRIORI optimizations
- 5 References



A very popular topic in computer science

Introduction FR APRIORI Optimizations References

Since 1993...

Among the most cited across computer science

- AIS [AIS93]
- APRIORI [AS94]... top 10 in data mining [WKRQ⁺07]

Thousands of papers

- many improvements
- many extensions

Thousands of applications

- works for examples×properties datasets
- e.g. basket×products, patient×symptoms, vehicle×defaults

⇒ Interesting surveys [Goe03, HCXY07].



A very popular topic in computer science

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Since 1993...

... but before

- GUHA
- CHARADE

⇒ But with APRIORI [AS94]: monotonicity property.



Roots: AIS [AIS93] & APRIORI [AS94]

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Find associations between products (Supermarket basket analysis)



- which products are frequently bought together?
- do some products influence the sales of other products?



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Roots: AIS [AIS93] & APRIORI [AS94]

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Find associations:

- co-occurrence of properties
- applications:
 - supermarket: basket×products
 - health care: patients×symptoms
 - web: pages× keywords
 - text: texts× words
 - quality: products×defaults
 - ...

id ₁	A ₁	A ₂	A ₃	A ₅	A ₆	A ₇	A ₈
id ₂	A ₂	A ₄	A ₅	A ₆			
id ₃	A ₁	A ₂	A ₃	A ₄	A ₆	A ₇	A ₈
id ₅	A ₂	A ₄	A ₅	A ₇	A ₈		
id ₆	A ₁	A ₂	A ₃	A ₆	A ₇		
id ₇	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈

	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈
id ₁	1	1	1	0	1	1	1	1
id ₂	0	1	0	1	1	1	0	0
id ₃	1	1	1	1	0	1	1	1
id ₄	1	1	1	0	0	1	1	1
id ₅	0	1	0	1	1	0	1	1
id ₆	1	1	1	0	0	1	1	0
id ₇	0	1	1	1	1	1	1	1

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Roots: AIS [AIS93] & APRIORI [AS94]

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A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈
1	1	1	0	1	1	1	1
0	1	0	1	1	1	0	0
1	1	1	1	0	1	1	1
1	1	1	0	0	1	1	1
0	1	0	1	1	0	1	1
1	1	1	0	0	1	1	0
0	1	1	1	1	1	1	1

⇒ Works for examples×properties datasets...monotonicity property [AS94]: great success.

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Boolean data bases

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Discrete attribute:

id ₁	P
id ₂	A
id ₃	P
id ₄	P
id ₅	E

	P	E	A
id ₁	1		
id ₂			1
id ₃	1		
id ₄	1		
id ₅		1	

Continuous attribute:

id ₁	1100
id ₂	0
id ₃	2200
id ₄	800
id ₅	3500

	[0..500]	[501..1000]	[1001..2500]	[2501..]
id ₁			1	
id ₂	1			
id ₃			1	
id ₄		1		
id ₅				1

⇒ One can always encode in binary attributes.

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Binary attributes and frequent itemsets

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A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈
1	1	1	0	1	1	1	1
0	1	0	1	1	1	0	0
1	1	1	1	0	1	1	1
1	1	1	0	0	1	1	1
0	1	0	1	1	0	1	1
1	1	1	0	0	1	1	0
0	1	1	1	1	1	1	1

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Binary attributes and frequent itemsets

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A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈
1	1	1	0	1	1	1	1
0	1	0	1	1	1	0	0
1	1	1	1	0	1	1	1
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0	1	0	1	1	0	1	1
1	1	1	0	0	1	1	0
0	1	1	1	1	1	1	1

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Binary attributes and frequent itemsets

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A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈
1	1	1	0	1	1	1	1
0	1	0	1	1	1	0	0
1	1	1	1	0	1	1	1
1	1	1	0	0	1	1	1
0	1	0	1	1	0	1	1
1	1	1	0	0	1	1	0
0	1	1	1	1	1	1	1

↔ Co-occurrence (presence and absence): works on '1'.

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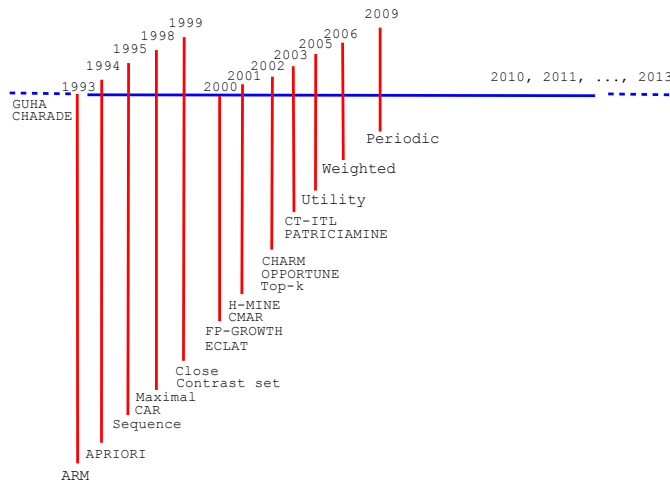
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Some key works

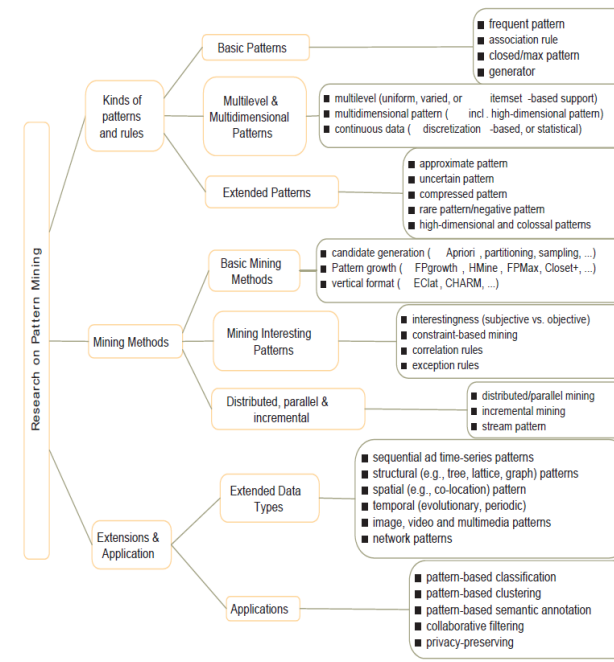
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[HKP11]

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Mining Association Rules

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Formally [AIS93, AS94]

- $\mathcal{I} = \{i_1, i_2, \dots, i_m\}$: a set of binary attributes, called **items**
- \mathcal{D} : a data base of **transactions**, where:
 - each transaction t is a set of items, $t \subseteq \mathcal{I}$
 - each transaction t is represented as a binary vector ($t[k] = 1$ if t contains i_k , and $t[k] = 0$ otherwise)
 - each transaction t has an unique associated identifier
- A a set of items (**itemset**) in \mathcal{I} :
 - t satisfies A if $\forall i_k \in A, t[k] = 1$ (t contains A i.e. $A \subseteq t$)

Association rule [AS94]

An association rule is an **implication** of the form $A \rightarrow B$, where $A \subset \mathcal{I}$, $B \subset \mathcal{I}$, and $A \cap B = \emptyset$ (in [AIS93] B is a single item).

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Mining Association Rules

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Confidence and support of $A \rightarrow B$ (in \mathcal{D})

- $A \rightarrow B$ holds with **confidence** c
if $c\%$ of transactions in \mathcal{D} that contain A also contain B .
- $A \rightarrow B$ has **support** s
if $s\%$ of transactions in \mathcal{D} contain $A \cup B$.

Problem of mining association rules [AS94]

To generate all association rules that have support and confidence greater than the user-specified minimum support and minimum confidence respectively.

Syntactic constraints were introduced in [AIS93].

Example: Supermarket: basket \times products

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Which products are frequently bought together?



Mining Association Rules

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How to solve the problem of mining association rules?

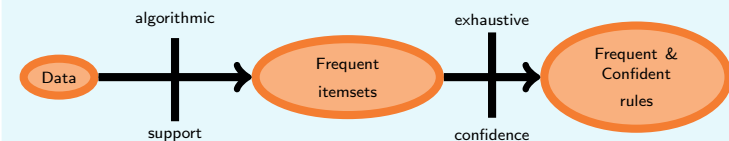
- very exhaustive
 - generate all possible rules
 - count their supports and compute confidence
 - but... $\mathcal{O}(3^n)$
- more clever
 - first, find all frequent itemsets
 - second, split every frequent itemset I in two parts A and B , such that $A \rightarrow B$ is confident

Mining Association Rules

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How to find all frequent itemsets?

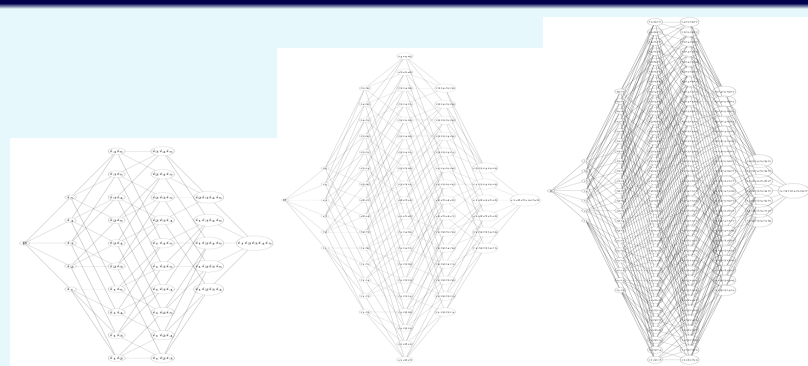
- very exhaustive
 - generate all possible itemsets, count their support
 - but... $\mathcal{O}(2^n)$
- very clever
 - APRIORI [AS94]
 - key-point: **downward-closure** property of support (also called **anti-monotonicity**)
 - all subsets of a frequent itemset are also frequent
 - all supersets of an infrequent itemset are infrequent



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Poset of itemsets



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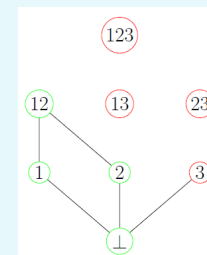


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APRIORI [AS94]

- an itemset is called a **candidate** itemset if all of its subsets are known to be frequent
- so iteratively find frequent itemsets with cardinality from 1 to k (k-itemset); **level-wise search**



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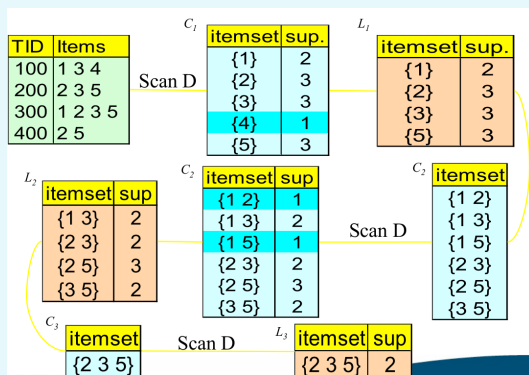
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Example (Rastogi and Shim)



⇒ Problem: every count step needs a (very) costly scan over the complete database.

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Strategies

- reduce the number of candidate itemsets
- reduce the number of transactions
- reduce the number of comparisons

Count step needs a (very) costly scan over \mathcal{D} : optimizations

- partition [SON95]: partition database, and mine each part separately (relative support instead of absolute support), union of all frequent itemsets of all parts are a superset of all frequent itemsets in \mathcal{D} , extra pruning step
- sampling [Toi96]: run APRIORI on small sample of \mathcal{D} , correct result
- Dynamic Itemset Counting [BMUT97]: interrupt algorithm after every x transactions and already generate larger candidates if possible

Many research

- for sparse/dense data, for many/few items
- to improve the counting step
- to read efficiently the database
- to generate efficiently the candidates
- to prune the candidates
- to manage efficiently the ordering of items
- ...

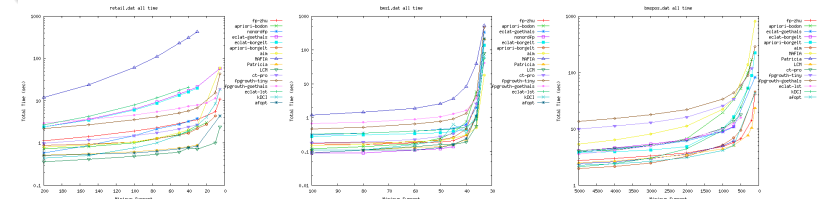
To improve the counting step if \mathcal{D} fits in memory

- ECLAT [Zak00]
- FP-GROWTH [HPY00]
- ...

⇒ Differ in counting strategy and how \mathcal{D} is represented in memory.

Many optimizations exist!

- no winner, it depends on implementation, on data



<http://fimi.ua.ac.be/experiments/>

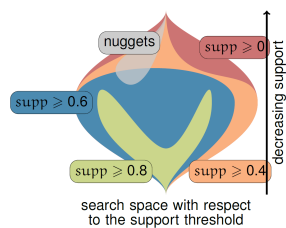
⇒ ...Implementation matters!

Concise representations of frequent itemsets

- an itemset is maximal frequent if none of its immediate supersets is frequent [Jr.98]
- an itemset is frequent closed if none of its immediate supersets has the same support as the itemset [PBT99]
- $FIM \subset CLO \subset MAX$

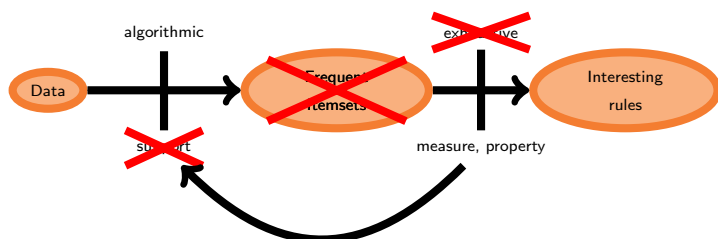
↔ Closed and maximal frequent itemsets are typically by orders of magnitude fewer itemsets than all frequent itemsets. However, all frequent itemsets can be induced from these itemsets and thus algorithms mining closed and maximal frequent itemsets are often more efficient.

Next step: use of the good interestingness measure (without support)?



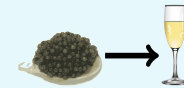
finding frequent itemsets is costly
can we avoid this steep?
can we reach directly interesting rules?

⇒ algorithmic properties of measures [LBLL09, BLL11]

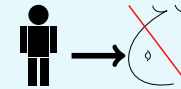


Main issues: complexity and quality

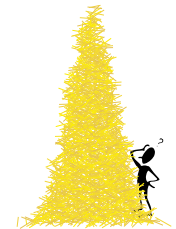
- large number of itemsets, of rules
... most of them **uninteresting**
- some infrequent patterns may be lost: **nuggets**



- some frequent patterns may be true but **well known/obvious**



- ... **invalid** patterns... **surprising** patterns...



How to select the *good* ones?

↔ Interestingness measures [LMVL08].

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