General expressions of bsplines of degree 0,1,2 and 3 in 1D
-
Another tool for ToFu

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Contents

1. General expression of the bsplines 2

2. Derivatives 3

A. Distributing the degree 3 polynoms 5

B. Distributing the degree 3 polynoms - bis 6

1. General expression of the bsplines

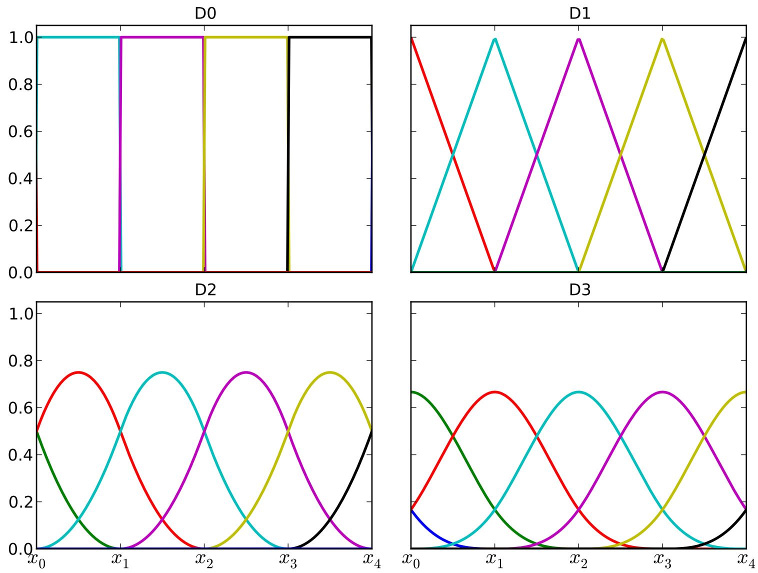

Figure 1.1: Bsplines of degrees 0, 1, 2 and 3

A b-spline $b_{d,0}$ of degree d living from x_0 is:$b_{d,0} = \frac{x-x_0}{x_{0+d}-x_0} b_{d-1,0} + \frac{x_{0+d+1}-x}{x_{0+d+1}-x_{0+1}} b_{d-1,1}$
Hence:

$$b_{0,0} = \begin{cases} 1 & \text{, if} \quad x \in [x_0, x_1[\\ 0 & \text{, else} \end{cases}$$

$$b_{1,0} = \begin{cases} \frac{x-x_0}{x_1-x_0} & \text{, if} \quad x \in [x_0, x_1[\\ \frac{x_2-x}{x_2-x_1} & \text{, if} \quad x \in [x_1, x_2[\end{cases}$$

$$b_{2,0} = \begin{cases} \frac{(x-x_0)^2}{(x_2-x_0)(x_1-x_0)} & \text{, if} \quad x \in [x_0, x_1[\\ \frac{(x-x_0)(x_2-x)}{(x_2-x_0)(x_2-x_1)} + \frac{(x-x_1)(x_3-x)}{(x_2-x_1)(x_3-x_1)} & \text{, if} \quad x \in [x_1, x_2[\\ \frac{(x_3-x)^2}{(x_3-x_2)(x_3-x_1)} & \text{, if} \quad x \in [x_2, x_3[\end{cases}$$

$$b_{3,0} = \begin{cases} \frac{(x-x_0)^3}{(x_3-x_0)(x_2-x_0)(x_1-x_0)} & \text{, if} \quad x \in [x_0, x_1[\\ \frac{x-x_0}{x_3-x_0}\left(\frac{(x-x_0)(x_2-x)}{(x_2-x_0)(x_2-x_1)} + \frac{(x-x_1)(x_3-x)}{(x_2-x_1)(x_3-x_1)}\right) + \frac{x_4-x}{x_4-x_1}\left(\frac{(x-x_1)^2}{(x_3-x_1)(x_2-x_1)}\right) & \text{, if} \quad x \in [x_1, x_2[\\ \frac{x-x_0}{x_3-x_0}\left(\frac{(x_3-x)^2}{(x_3-x_2)(x_3-x_1)}\right) + \frac{x_4-x}{x_4-x_1}\left(\frac{(x-x_1)(x_3-x)}{(x_3-x_1)(x_3-x_2)} + \frac{(x-x_2)(x_4-x)}{(x_3-x_2)(x_4-x_2)}\right) & \text{, if} \quad x \in [x_2, x_3[\\ \frac{(x_4-x)^3}{(x_4-x_3)(x_4-x_2)(x_4-x_1)} & \text{, if} \quad x \in [x_3, x_4[\end{cases}$$

Or (see B for details):

$$b_{2,0} = \begin{cases} \frac{x^2-2xx_0+x_0^2}{(x_2-x_0)(x_1-x_0)} & \text{, if} \quad x \in [x_0, x_1[\\ \frac{-x^2(x_3+x_2-x_1-x_0)+2x(x_3x_2-x_1x_0)-(x_3x_2x_0-x_2x_1x_0+x_3x_2x_1-x_3x_1x_0)}{(x_2-x_1)(x_2-x_0)(x_3-x_1)} & \text{, if} \quad x \in [x_1, x_2[\\ \frac{x^2-2xx_3+x_3^2}{(x_3-x_2)(x_3-x_1)} & \text{, if} \quad x \in [x_2, x_3[\end{cases}$$

$$b_{3,0} = \begin{cases} \frac{x^3-3x^2x_0+3xx_0^2-x_0^3}{(x_3-x_0)(x_2-x_0)(x_1-x_0)} & \text{, if} \quad x \in [x_0, x_1[\\ \frac{x^3A+x^2B+xC+D}{(x_4-x_1)(x_3-x_1)(x_3-x_0)(x_2-x_1)(x_2-x_0)} & \text{, if} \quad x \in [x_1, x_2[\\ \frac{x^3A'+x^2B'+xC'+D'}{(x_4-x_2)(x_4-x_1)(x_3-x_2)(x_3-x_1)(x_3-x_0)} & \text{, if} \quad x \in [x_2, x_3[\\ \frac{-x^3+3x^2x_4-3xx_4^2+x_4^3}{(x_4-x_3)(x_4-x_2)(x_4-x_1)} & \text{, if} \quad x \in [x_3, x_4[\end{cases}$$

2. Derivatives

By noting $\partial_n b_{d,0}$ the n-th derivative of b-spline $b_{d,0}$, where $n \le d$:

$$\partial_1 b_{1,0} = \begin{cases} \frac{1}{x_1-x_0} & \text{, if} \quad x \in [x_0, x_1[\\ \frac{-1}{x_2-x_1} & \text{, if} \quad x \in [x_1, x_2[\end{cases}$$

$$\partial_1 b_{2,0} = \begin{cases} \frac{2(x-x_0)}{(x_2-x_0)(x_1-x_0)} & \text{, if} \quad x \in [x_0, x_1[\\ \frac{-2x(x_3+x_2-x_1-x_0)+2(x_3x_2-x_1x_0)}{(x_2-x_1)(x_2-x_0)(x_3-x_1)} & \text{, if} \quad x \in [x_1, x_2[\\ \frac{-2(x_3-x)}{(x_3-x_2)(x_3-x_1)} & \text{, if} \quad x \in [x_2, x_3[\end{cases}$$

$$\partial_2 b_{2,0} = \begin{cases} \frac{2}{(x_2-x_0)(x_1-x_0)} & \text{, if} \quad x \in [x_0, x_1[\\ \frac{-2(x_3+x_2-x_1-x_0)}{(x_2-x_1)(x_2-x_0)(x_3-x_1)} & \text{, if} \quad x \in [x_1, x_2[\\ \frac{2}{(x_3-x_2)(x_3-x_1)} & \text{, if} \quad x \in [x_2, x_3[\end{cases}$$

$$\partial_1 b_{3,0} = \begin{cases} \frac{3(x-x_0)^2}{(x_3-x_0)(x_2-x_0)(x_1-x_0)} & \text{, if} \quad x \in [x_0, x_1[\\ \frac{3x^2A+2xB+C}{(x_4-x_1)(x_3-x_1)(x_3-x_0)(x_2-x_1)(x_2-x_0)} & \text{, if} \quad x \in [x_1, x_2[\\ \frac{3x^2A'+2xB'+C'}{(x_4-x_2)(x_4-x_1)(x_3-x_2)(x_3-x_1)(x_3-x_0)} & \text{, if} \quad x \in [x_2, x_3[\\ \frac{-3(x_4-x)^2}{(x_4-x_3)(x_4-x_2)(x_4-x_1)} & \text{, if} \quad x \in [x_3, x_4[\end{cases}$$

$$\partial_2 b_{3,0} = \begin{cases} \frac{6(x-x_0)}{(x_3-x_0)(x_2-x_0)(x_1-x_0)} & \text{, if} \quad x \in [x_0, x_1[\\ \frac{6xA+2B}{(x_4-x_1)(x_3-x_1)(x_3-x_0)(x_2-x_1)(x_2-x_0)} & \text{, if} \quad x \in [x_1, x_2[\\ \frac{6xA'+2B'}{(x_4-x_2)(x_4-x_1)(x_3-x_2)(x_3-x_1)(x_3-x_0)} & \text{, if} \quad x \in [x_2, x_3[\\ \frac{6(x_4-x)}{(x_4-x_3)(x_4-x_2)(x_4-x_1)} & \text{, if} \quad x \in [x_3, x_4[\end{cases}$$

$$\partial_3 b_{3,0} = \begin{cases} \frac{6}{(x_3-x_0)(x_2-x_0)(x_1-x_0)} & \text{, if} \quad x \in [x_0, x_1[\\ \frac{6A}{(x_4-x_1)(x_3-x_1)(x_3-x_0)(x_2-x_1)(x_2-x_0)} & \text{, if} \quad x \in [x_1, x_2[\\ \frac{6A'}{(x_4-x_2)(x_4-x_1)(x_3-x_2)(x_3-x_1)(x_3-x_0)} & \text{, if} \quad x \in [x_2, x_3[\\ \frac{-6}{(x_4-x_3)(x_4-x_2)(x_4-x_1)} & \text{, if} \quad x \in [x_3, x_4[\end{cases}$$

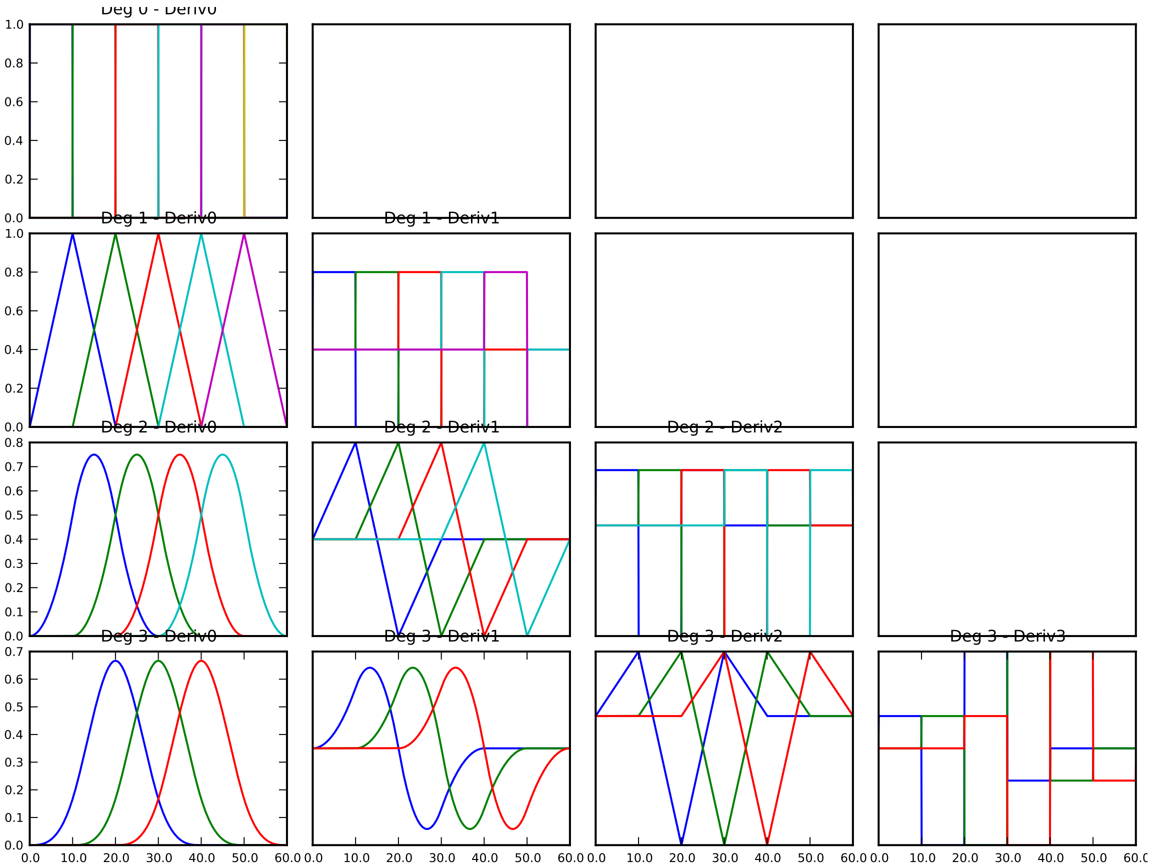

Figure 2.1: Bsplines of degrees 0, 1, 2 and 3 and their derivatives D0, D1, D2 and D3

A. DISTRIBUTING THE DEGREE 3 POLYNOMS

$$
\begin{aligned}
& \frac{x-x_0}{x_3-x_0}\left(\frac{(x-x_0)(x_2-x)}{(x_2-x_0)(x_2-x_1)}+\frac{(x-x_1)(x_3-x)}{(x_2-x_1)(x_3-x_1)}\right)+\frac{x_4-x}{x_4-x_1}\left(\frac{(x-x_1)^2}{(x_3-x_1)(x_2-x_1)}\right)\\
= & \frac{(x-x_0)^2(x_2-x)(x_3-x_1)+(x-x_0)(x-x_1)(x_3-x)(x_2-x_0)}{(x_3-x_0)(x_3-x_1)(x_2-x_0)(x_2-x_1)}+\frac{-x^3+x^2(2x_1+x_4)-x(x_1^2+2x_1x_4)+x_1^2x_4}{(x_4-x_1)(x_3-x_1)(x_2-x_1)}\\
= & \frac{1}{(x_3-x_1)(x_2-x_1)}\left(\frac{(x-x_0)^2(x_2-x)(x_3-x_1)+(x-x_0)(x-x_1)(x_3-x)(x_2-x_0)}{(x_3-x_0)(x_2-x_0)}+\frac{-x^3+x^2(2x_1+x_4)-x(x_1^2+2x_1x_4)+x_1^2x_4}{(x_4-x_1)}\right)\\
= & \frac{1}{(x_3-x_1)(x_2-x_1)}\left(\frac{(x^2x_2-2xx_0x_2+x_0^2x_2-x^3+2x^2x_0-xx_0^2)(x_3-x_1)+(x^2x_3-x(x_0+x_1)x_3+x_0x_1x_3-x^3+x^2(x_0+x_1)-xx_0x_1)(x_2-x_0)}{(x_3-x_0)(x_2-x_0)}+\frac{-x^3+x^2(2x_1+x_4)-x(x_1^2+2x_1x_4)+x_1^2x_4}{(x_4-x_1)}\right)\\
= & \frac{1}{(x_3-x_1)(x_2-x_1)}\left(\frac{(-x^3+x^2(2x_0+x_2)-x(x_0^2-2x_0x_2)+x_0^2x_2)(x_3-x_1)+(-x^3+x^2(x_0+x_1+x_3)-x(x_0x_1+x_0x_3+x_1x_3)+x_0x_1x_3)(x_2-x_0)}{(x_3-x_0)(x_2-x_0)}+\frac{-x^3+x^2(2x_1+x_4)-x(x_1^2+2x_1x_4)+x_1^2x_4}{(x_4-x_1)}\right)\\
= & \frac{1}{(x_3-x_1)(x_2-x_1)}\left(\frac{x^3(-x_3+x_1-x_2+x_0)+x^2(2x_3x_0+x_3x_2-2x_1x_0-x_2x_1+x_2x_0+x_2x_1+x_3x_2-x_0^2-x_1x_0-x_3x_0)-x(x_3x_0^2-2x_3x_2x_0-x_1x_0^2+2x_2x_1x_0+(x_2x_1x_0+x_3x_2x_0+x_3x_2x_1-x_1x_0^2-x_3x_0^2-x_3x_1x_0))+x_3x_2x_0^2-x_2x_1x_0^2+x_3x_2x_1x_0-x_3x \cdots}{(x_3-x_0)(x_2-x_0)}\right.\\
= & \frac{1}{(x_3-x_1)(x_2-x_1)}\left(\frac{x^3(-x_3+x_1-x_2+x_0)+x^2(x_3x_0+2x_3x_2-3x_1x_0+x_2x_0-x_0^2)-x(-x_3x_2x_0-2x_1x_0^2+3x_2x_1x_0+x_3x_2x_1-x_3x_1x_0)+x_3x_2x_0^2-x_2x_1x_0^2+x_3x_2x_1x_0-x_3x_1x_0^2}{(x_3-x_0)(x_2-x_0)}+\frac{-x^3+x^2(2x_1+x_4)-x(x_1^2+2x_1x_4)+x_1^2x_4}{(x_4-x_1)}\right)\\
= & \frac{[x^3(-x_3+x_1-x_2+x_0)+x^2(x_3x_0+2x_3x_2-3x_1x_0+x_2x_0-x_0^2)-x(-x_3x_2x_0-2x_1x_0^2+3x_2x_1x_0+x_3x_2x_1-x_3x_1x_0)+x_3x_2x_0^2-x_2x_1x_0^2+x_3x_2x_1x_0-x_3x_1x_0^2](x_4-x_1)+[-x^3+x^2(2x_1+x_4)-x(x_1^2+2x_1x_4)+x_1^2x_4](x_3-x_0)(x_2-x_0)}{(x_4-x_1)(x_3-x_1)(x_3-x_0)(x_2-x_1)(x_2-x_0)}\\
= & \frac{[x^3(-x_4x_3+x_4x_1-x_4x_2+x_4x_0+x_3x_1-x_1^2+x_2x_1-x_0x_1)+x^2(x_4x_3x_0+2x_4x_3x_2-3x_4x_1x_0+x_4x_2x_0-x_4x_0^2-x_3x_1x_0-2x_3x_1x_2+3x_1^2x_0-x_2x_1x_0+x_1x_0^2)-x(-x_4x_3x_2x_0-2x_4x_1x_0^2+3x_4x_2x_1x_0+x_4x_3x_2x_1-x_4x_3x_1x_0+x_3x_2x_1x_0+2x_1^2x_0^2-3x \cdots}{}\\
= & \frac{x^3(-x_4x_3+x_4x_1-x_4x_2+x_4x_0+x_3x_1-x_1^2+x_2x_1-x_0x_1-x_3x_2+x_3x_0+x_2x_0-x_2^2)+x^2(x_4x_3x_0+2x_4x_3x_2-3x_4x_1x_0+x_4x_2x_0-x_4x_0^2-x_3x_1x_0-2x_3x_1x_2+3x_1^2x_0-x_2x_1x_0+x_1x_0^2+2x_3x_2x_1-2x_3x_1x_0-2x_2x_1x_0+2x_2^2x_1+x_4x_3x_2-x_4x_3x_0-x_4x_2 \cdots}{}\\
= & \frac{x^3(-x_4x_3+x_4x_1-x_4x_2+x_4x_0+x_3x_1+x_2x_1-x_1x_0-x_3x_2+x_3x_0+x_2x_0-x_2^2-x_1^2)+x^2(3x_4x_3x_2-3x_4x_1x_0-x_4x_0^2-3x_3x_1x_0+3x_1^2x_0-3x_2x_1x_0+x_1x_0^2+2x_2^2x_1+x_4x_2^2)-x(-x_4x_3x_2x_0-2x_4x_1x_0^2+x_4x_2x_1x_0+3x_4x_3x_2x_1-3x_4x_3x_1x_0+2x_4x_2^2x_1 \cdots}{(x_4-x_1)(x_3-x_1)(x_3-x_0)(x_2-x_1)(x_2-x_0)}\\
= & \frac{x^3A+x^2B+xC+D}{(x_4-x_1)(x_3-x_1)(x_3-x_0)(x_2-x_1)(x_2-x_0)}
\end{aligned}
$$

where
$$
\begin{cases}
A & =-x_4x_3+x_4x_1-x_4x_2+x_4x_0+x_3x_1+x_2x_1-x_1x_0-x_3x_2+x_3x_0+x_2x_0-x_2^2-x_1^2\\
B & =3x_4x_3x_2-3x_4x_1x_0-x_4x_0^2-3x_3x_1x_0+3x_1^2x_0-3x_2x_1x_0+x_1x_0^2+2x_2^2x_1+x_4x_2^2\\
C & =-x_4x_3x_2x_0-2x_4x_1x_0^2+x_4x_2x_1x_0+3x_4x_3x_2x_1-3x_4x_3x_1x_0+2x_4x_2^2x_1+x_3x_2x_1x_0+2x_1^2x_0^2-4x_2x_1^2x_0+x_2^2x_1^2\\
D & =x_4x_3x_2x_0^2-x_4x_2x_1x_0^2+x_4x_3x_2x_1x_0-x_4x_3x_1x_0^2-x_3x_2x_1x_0^2+x_2x_1^2x_0^2-x_3x_2x_1^2x_0+x_3x_1^2x_0^2+x_4x_3x_2x_1^2-x_4x_3x_1^2x_0-x_4x_2x_1^2x_0+x_4x_2^2x_1^2
\end{cases}
$$

Similarly
$$
\frac{x-x_0}{x_3-x_0}\left(\frac{(x_3-x)^2}{(x_3-x_2)(x_3-x_1)}\right)+\frac{x_4-x}{x_4-x_1}\left(\frac{(x-x_1)(x_3-x)}{(x_3-x_1)(x_3-x_2)}+\frac{(x-x_2)(x_4-x)}{(x_3-x_2)(x_4-x_2)}\right)=\frac{x^3A'+x^2B'+xC'+D'}{(x_4-x_2)(x_4-x_1)(x_3-x_2)(x_3-x_1)(x_3-x_0)}
$$
where we simply substitute (x_0,x_1,x_2,x_3,x_4) in A,B,C,D by (x_4,x_3,x_2,x_1,x_0):

$$
\begin{cases}
A' & =-x_0x_1+x_0x_3-x_0x_2+x_0x_4+x_1x_3+x_2x_3-x_3x_4-x_1x_2+x_1x_4+x_2x_4-x_2^2-x_3^2\\
B' & =3x_0x_1x_2-3x_0x_3x_4-x_0x_4^2-3x_1x_3x_4+3x_3^2x_4-3x_2x_3x_4+x_3x_4^2+2x_2^2x_3+x_0x_2^2\\
C' & =-x_0x_1x_2x_4-2x_0x_3x_4^2+x_0x_2x_3x_4+3x_0x_1x_2x_3-3x_0x_1x_3x_4+2x_0x_2^2x_3+x_1x_2x_3x_4+2x_3^2x_4^2-4x_2x_3^2x_4+x_2^2x_3^2\\
D' & =x_0x_1x_2x_4^2-x_0x_2x_3x_4^2+x_0x_1x_2x_3x_4-x_0x_1x_3x_4^2-x_1x_2x_3x_4^2+x_2x_3^2x_4^2-x_1x_2x_3^2x_4+x_1x_3^2x_4^2+x_0x_1x_2x_3^2-x_0x_1x_3^2x_4-x_0x_2x_3^2x_4+x_0x_2^2x_3^2
\end{cases}
$$

B. Distributing the degree 3 polynoms - bis

$$
\begin{aligned}
& \tfrac{x-x_0}{x_3-x_0}\left(\tfrac{(x-x_0)(x_2-x)}{(x_2-x_0)(x_2-x_1)}+\tfrac{(x-x_1)(x_3-x)}{(x_2-x_1)(x_3-x_1)}\right)+\tfrac{x_4-x}{x_4-x_1}\left(\tfrac{(x-x_1)^2}{(x_3-x_1)(x_2-x_1)}\right)\\
= & \tfrac{(x-x_0)^2(x_2-x)(x_3-x_1)(x_4-x_1)+(x-x_0)(x-x_1)(x_3-x)(x_2-x_0)(x_4-x_1)+(x_4-x)(x-x_1)^2(x_3-x_0)(x_2-x_0)}{(x_4-x_1)(x_3-x_1)(x_3-x_0)(x_2-x_1)(x_2-x_0)}\\
= & \tfrac{(x^2-2xx_0+x_0^2)(x_2-x)(x_3-x_1)(x_4-x_1)+(x^2-x(x_0+x_1)+x_0x_1)(x_3-x)(x_2-x_0)(x_4-x_1)+(x^2-2xx_1+x_1^2)(x_4-x)(x_3-x_0)(x_2-x_0)}{(x_4-x_1)(x_3-x_1)(x_3-x_0)(x_2-x_1)(x_2-x_0)}\\
= & \tfrac{(x^2x_2-2xx_2x_0+x_2x_0^2-x^3+2x^2x_0-xx_0^2)(x_3-x_1)(x_4-x_1)+(x^2x_3-xx_3(x_0+x_1)+x_0x_1x_3-x^3+x^2(x_0+x_1)-xx_0x_1)(x_2-x_0)(x_4-x_1)+(x^2x_4-2xx_4x_1+x_4x_1^2-x^3+2x^2x_1-xx_1^2)(x_3-x_0)(x_2-x_0)}{(x_4-x_1)(x_3-x_1)(x_3-x_0)(x_2-x_1)(x_2-x_0)}\\
= & \tfrac{1}{(x_4-x_1)(x_3-x_1)(x_3-x_0)(x_2-x_1)(x_2-x_0)}\left[\begin{array}{l}
-x^3((x_3-x_1)(x_4-x_1)+(x_2-x_0)(x_4-x_1)+(x_3-x_0)(x_2-x_0))\\
+x^2((x_2+2x_0)(x_3-x_1)(x_4-x_1)+(x_3+x_0+x_1)(x_2-x_0)(x_4-x_1)+(x_4+2x_1)(x_3-x_0)(x_2-x_0))\\
+x(-x_0(2x_2+x_0)(x_3-x_1)(x_4-x_1)-(x_3(x_0+x_1)+x_0x_1)(x_2-x_0)(x_4-x_1)-x_1(2x_4+x_1)(x_3-x_0)(x_2-x_0))\\
+x_2x_0^2(x_3-x_1)(x_4-x_1)+x_0x_1x_3(x_2-x_0)(x_4-x_1)+x_4x_1^2(x_3-x_0)(x_2-x_0)
\end{array}\right]\\
= & \tfrac{Ax^3+Bx^2+Cx+D}{(x_4-x_1)(x_3-x_1)(x_3-x_0)(x_2-x_1)(x_2-x_0)}
\end{aligned}
$$

where
$$
\begin{cases}
A &= -((x_3-x_1)(x_4-x_1)+(x_2-x_0)(x_4-x_1)+(x_3-x_0)(x_2-x_0))\\
B &= (x_2+2x_0)(x_3-x_1)(x_4-x_1)+(x_3+x_0+x_1)(x_2-x_0)(x_4-x_1)+(x_4+2x_1)(x_3-x_0)(x_2-x_0)\\
C &= -(x_0(2x_2+x_0)(x_3-x_1)(x_4-x_1)+(x_3(x_0+x_1)+x_0x_1)(x_2-x_0)(x_4-x_1)+x_1(2x_4+x_1)(x_3-x_0)(x_2-x_0))\\
D &= x_2x_0^2(x_3-x_1)(x_4-x_1)+x_0x_1x_3(x_2-x_0)(x_4-x_1)+x_4x_1^2(x_3-x_0)(x_2-x_0)
\end{cases}
$$

Similarly
$$
\frac{x-x_0}{x_3-x_0}\left(\frac{(x_3-x)^2}{(x_3-x_2)(x_3-x_1)}\right)+\frac{x_4-x}{x_4-x_1}\left(\frac{(x-x_1)(x_3-x)}{(x_3-x_1)(x_3-x_2)}+\frac{(x-x_2)(x_4-x)}{(x_3-x_2)(x_4-x_2)}\right)=\frac{x^3A'+x^2B'+xC'+D'}{(x_4-x_2)(x_4-x_1)(x_3-x_2)(x_3-x_1)(x_3-x_0)}
$$
where we simply substitute (x_0,x_1,x_2,x_3,x_4) in A,B,C,D by (x_4,x_3,x_2,x_1,x_0) and notice that the denominator is the opposite of the substituted version:

$$
\begin{cases}
A' &= (x_1-x_3)(x_0-x_3)+(x_2-x_4)(x_0-x_3)+(x_1-x_4)(x_2-x_4)\\
B' &= -((x_2+2x_4)(x_1-x_3)(x_0-x_3)+(x_1+x_4+x_3)(x_2-x_4)(x_0-x_3)+(x_0+2x_3)(x_1-x_4)(x_2-x_4))\\
C' &= x_4(2x_2+x_4)(x_1-x_3)(x_0-x_3)+(x_1(x_4+x_3)+x_4x_3)(x_2-x_4)(x_0-x_3)+x_3(2x_0+x_3)(x_1-x_4)(x_2-x_4)\\
D' &= -(x_2x_4^2(x_1-x_3)(x_0-x_3)+x_4x_3x_1(x_2-x_4)(x_0-x_3)+x_0x_3^2(x_1-x_4)(x_2-x_4))
\end{cases}
$$