COS 529 Assignment 4: Problem Set

Due 11:59pm 11/21/2025 ET

Collaboration policy This assignment must be completed individually. No collaboration is allowed.

Submission Submit a pdf file typeset with LaTeX on Canvas. Handwritten submissions are not allowed.

AI Use Policy

- For homework assignments, help from AI is disallowed if and only if the same help is disallowed from a classmate.
  - Allowed: discussing concepts and course materials with AI/classmate
  - Disallowed: ask AI/classmate to write code or solutions for you
  - Disallowed: ask AI/classmate to review or debug your code

Question 1 (10 Points)

Let $Q(X)$ be a joint distribution defined by a graphical model on random variables $X = (X_1, X_2, \ldots, X_N)$. Let $1, 2, 3, \ldots, K$ be the values each random variable can take. Denote $U = \{1, \ldots, K\}^N$ as the state space of the graphical model. Consider a Markov chain $Y^{(0)}, Y^{(1)}, \ldots, Y^{(t)}, \ldots$ where $Y^{(t)}$ is a random variable that takes values from U. The transition from $Y^{(t)}$ to Y^{t+1} is given by the following procedure:

- Pick an integer i uniformly randomly from $1, \ldots, N$.
- Sample $X_i \sim Q(X_i | X_{-i} = Y_{-i}^{(t)})$.
- Set $Y^{(t+1)} \leftarrow (Y_1^{(t}, \ldots, Y_{i-1}^{(t)}, X_i, Y_{i+1}^{(t)}, \ldots, Y_N^{(t)})$.

Here, X_{-i} is a shorthand for $X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_N$. Show that $Q(X)$ is a stationary distribution of this Markov chain.

Question 2 (10 Points)

Q2.1 (5 Points): The linear map $Ad_T : \mathfrak{se}(3) \to \mathfrak{se}(3)$, where $T \in SE(3)$, is defined as $Ad_T(\xi^\wedge) = T\xi^\wedge T^{-1}$. Here, T is represented as a 4×4 matrix:

$$\begin{pmatrix} R & t \\ 0 & 1 \end{pmatrix}$$

where $R \in SO(3)$ and $t \in \mathbb{R}^3$.

Ad_T can be alternatively represented as a 6×6 matrix A, such that $A\xi = (T\xi^\wedge T^{-1})^\vee$ for any $\xi \in \mathbb{R}^6$. Derive the expression of A in terms of R and t. Hint: use the fact that cross product is invariant under any rotation R, that is, $Rx \times Ry = R(x \times y)$. You must show the derivation steps.

Q2.2 (5 Points): Let $L(\xi) = \log(T \exp(\xi^\wedge) T_1 T_2^{-1})^\vee$, where $\xi \in \mathfrak{se}(3)$ and is represented as a vector in $\mathbb{R}^6$. Derive the expression of $\frac{\partial L(\xi)}{\partial \xi}|_{\xi=0}$. In your expression, you can directly use the left Jacobian $J_l(\xi)$ and the right Jacobian $J_r(\xi)$ of $SE(3)$, and you can use log, exp, Ad as primitives without expanding them. You must show the derivation steps.