Population Dynamics Systems
A Study of Predator-Prey Interactions

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Abstract

In this project, we explore the complex dynamics of a 2D cellular automaton predator-prey system featuring three distinct types of actors: herbivores, low-level predators and top-level predators. We investigate the system's equilibrium dependence on various parameters such as grid size, initial populations, reproduction rates and starvation rates, aiming to examine what factors lead to a stable coexistence of all three species. Further, we analyze the impact of initial parameters on the long-term survival of the actors and identify that a large enough initial herbivore and top-level predator population, as well as a not too big low-level predator population, are benefical for creating a stable system.

Keywords: cellular automaton, predator-prey relationships, complex systems, principal component analysis (PCA), clustering

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1 Introduction

1.1 Problem

1.1.1 Problem definition

We will study a 2D cellular automaton predator-prey system with three types of actors and analyze the dependence of the equilibrium on the parameters.

1.1.2 Objective

For a predator-prey system, we can in the context of this project define ***stability*** of the system as when all three species – herbivores, low-level predators and top-level predators – survive for a long enough period of time, for instance for 500 iterations of the system, as an indication of long-term beneficial coexistence in the ecosystem. The purpose of this project is hence to answer questions related to the stability of the predator-prey system, such as:

- To what extent do initial and system parameters such as population numbers, reproduction rates and starvation rates determine whether a system becomes stable over time?
- What parameters are needed to achieve a stable system? Do they share common characteristics or are there multiple archetypes of parameters that create prosperous systems?
- Do some parameters matter more than others?
- Given initial parameters that generate a stable system, is a certain size of the system required to guarantee stability?
- What do the optimal initial conditions mean in the domain context of ecological interactions between the three species?

1.2 Model

1.2.1 Predator-Prey Systems

Predator-prey systems are fundamental concepts in ecology, representing the dynamic interactions between species in an ecosystem. In our model, we consider a simplified yet potentially insightful representation of these systems. Our predator-prey system consists of three primary actor types:

1. **Top-level Predator**: this species is at the top of the ecosystem food chain and preys on the low-level predator.

2. **Low-level Predator**: is eaten by the top-level predator and feeds on the herbivore.

3. **Herbivore**: at the lowest trophic level, herbivores serve as the food for the low-level predator and reproduces organically, i.e. not needing external food, normally represented by a reproduction rate.

We will study the way these three species interact over time, subject to certain initial and system conditions.

1.2.2 Cellular Automaton

To simulate and investigate the behavior of our predator-prey system, we employ the cellular automaton framework. Cellular automata consist of a grid of cells, where each cell can exist in a finite number of states. The state of a cell evolves over discrete time steps based on predefined rules that govern interactions with neighboring cells. In our model, the grid represents the spatial landscape where our actors interact, and in every iteration, each cell can be in one of four states: herbivore, low-level predator, top-level predator or empty.

The cellular automaton framework allows us to model the dynamics of our predator-prey system, explore the consequences of various initial conditions and parameters, and gain insights into the equilibrium state of the ecosystem.

2 Methods

2.1 Choice of discretization and integrator

In the context of simulating complex ecological systems, the choice of discretization and integrator plays a crucial role in accurately capturing the dynamics of the predator-prey interactions within our cellular automaton model. We have carefully considered and implemented the following discretization and integration techniques to ensure the reliability and fidelity of our simulations:

2.1.1 Spatial discretization

The spatial discretization involves dividing the simulated ecosystem into a grid of discrete cells, with each cell representing a specific area within the environment. This grid-based approach allows us to model the spatial distribution of species and interactions among them. The grid size is one of the parameters we vary in our simulations. By adjusting the grid size, we explore the impact of spatial resolution on the dynamics and stability of the ecosystem.

2.1.2 Temporal discretization

We let one unit of time equal one iteration update of the grid.

2.1.3 Integration scheme and the rules governing the system

To update the state of the ecosystem at each time step, we employ an integration scheme that defines how the interactions between species are computed and how the ecosystem evolves over time. Our cellular automaton model utilizes a synchronous update scheme, where all cells in the grid are updated simultaneously based on the current state of their neighbors. This synchronous approach ensures that interactions and state transitions occur consistently across the entire ecosystem, contributing to the stability and predictability of the simulation.

2.2 Rules governing the system

A key aspect of our cellular automaton model is the set of rules that govern the interactions and state transitions of the species within the grid. These rules are essential for the integration of the system and play a pivotal role in determining the dynamics of the predator-prey interactions. The integration process of the system is encapsulated in the function `simulate_step`, which updates the state of the grid based on these rules:

2.2.1 Integration function

The `simulate_step` function is designed to process each cell of the grid in a single time step. Here is an overview of its logic:

1. **Initialization of Starvation counters**: at the beginning, we initialize the starvation counters for predators, which keep track of how many iterations a predator can survive without food.

2. **Random cell processing**: to reduce bias in the simulation, the order of cell processing is randomized. This ensures that no particular pattern or direction influences the behavior of the actors.

3. **Species-specific rules**:

    - **Herbivores**: if a cell contains a herbivore, there is a probability (based on the reproduction rate) that it reproduces into an adjacent empty cell.
    - **Low-level predators**: these predators need to find adjacent herbivores to eat. If they find one, they move to that cell, eat the herbivore, and reset their starvation counter. They also have a chance to

reproduce in an adjacent empty cell. If they do not find food, their starvation counter increases, and they die if this counter exceeds their starvation limit.

- **Top-level predators**: similar to low-level predators, but they prey on low-level predators. Their reproduction and starvation mechanics follow the same logic.

4. **State update**: the state of each cell is updated based on the rules above. This includes changes due to movement, reproduction, eating, and starvation.

5. **Grid update**: after processing all cells, the new state of the grid is returned, representing the ecosystem at the next time step.

This function is the core of our simulation, dictating how the different species interact with each other and their environment. The randomness in cell processing and the probabilistic nature of reproduction and starvation introduce a level of unpredictability, reflecting the inherent complexity and variability of natural ecosystems.

2.3 Simulation setup

In our simulations, we vary several key parameters to investigate their effects on the stability and dynamics of the predator-prey system. The parameters include:

- **Grid size**: the width of the square spatial grid representing the ecosystem.

- **Initial population densities**: for the initial state of the system, we randomize starting probabilities for the herbivores, low-level predators, top-level predators respectively, such that the probabilities add to one. For each cell, it is then randomly assigned an initial state given the probabilities.

- **Reproduction rates**: The rates at which herbivores, low-level predators, and top-level predators reproduce.

- **Starvation limits**: the limits at which low-level predators and top-level predators starve if they cannot find prey, or more precisely how many iterations they can survive without food.

By systematically varying these parameters, we hope to gain valuable insights into the factors influencing ecosystem stability.

The variation of these parameters is controlled within specific limits. The grid size is chosen to explore different spatial resolutions from width 1 cell to width 50 cells, allowing us to examine the impact of varying levels of detail in our simulations. Initial population densities for herbivores, low-level predators, top-level predators and empty cells are given as random probabilities of occupying each cell in the grid.

Finally, reproduction rates for each species are randomly selected from a uniform distribution between 0 and 1, given the lack of domain knowledge about real ecosystems. Similarly, starvation limits for low-level predators and top-level predators are assigned random values within the range of 1 to 50, encompassing a broad spectrum of survival thresholds.

For the first study, we will set grid size constant to 50 cells and attempt to find sets of system parameters that generate systems that reach equilibrium. The idea is to try 5 000 possible choices for the initial parameters and see how many iterations each set of parameters lasts with all three species alive. If the system lasts 500 iterations, we consider it a stable system and stop the calculation. The choice of 500 iterations as a limit for stability is rather arbitrary, but the thought behind it is that it is much longer than the maximum starvation limit (50 iterations), ensuring that the system survives multiple cycles, while not making the limit too high as to make calculations too time-intensive.

After having found parameters that generates a stable system for grid size 50, we will keep the system parameters constant and vary the grid size, to see if there is a minimum required size for the system to reach equilibrium. The extreme case is grid size 1, which consists of a single cell, and already in iteration 0 has failed to sustain all three species.

This systematic approach enables us to investigate the conditions that promote the long-term coexistence of all three actor types in our predator-prey ecosystem model, shedding light on the complex dynamics governing ecological interactions.

3 Results and analysis

3.1 Ecosystem simulation and grid visualization

Figure 1 displays an example of the grid representation of our ecosystem simulation. The grid shows the distribution of different species, including herbivores, low-level predators, top-level predators, and empty cells over time. This simulation helps us study the dynamics of the ecosystem over time.

3.2 Parameter simulation

3.2.1 Characteristics of stable ecosystems

We want to study the stability of simulated ecosystems. Stability, in this context, is quantified by the lifespan of the ecosystem, with longer lifespans indicating more stable conditions. In this case, simulations were ran at most until iteration 500.

To distill our multi-dimensional data into a more comprehensible form and enable visualization, we employed Principal Component Analysis (PCA). PCA is a dimensionality reduction technique that transforms the data into a set of orthogonal components that capture the most variance, allowing us to visualize the results in a two-dimensional plot.

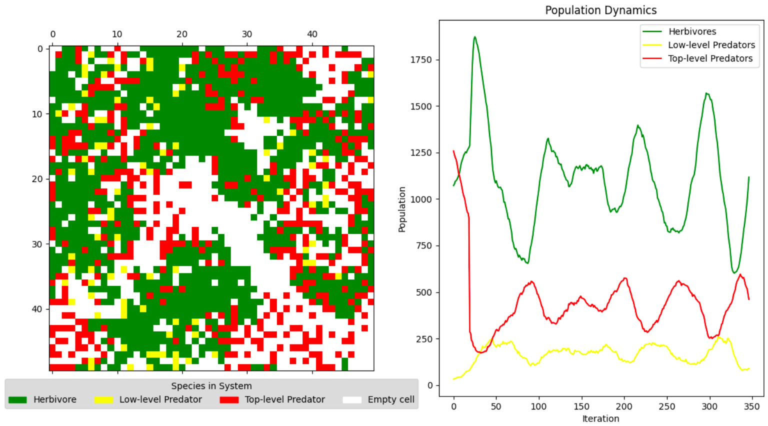

Figure 1: Ecosystem Simulation Grid.

Before running the PCA analysis, the data was pre-processed by normalizing it, ensuring that each parameter contributes equally to the result, irrespective of their original scales.

The PCA plot, shown in Figure 2, visualizes the distribution of our simulations with respect to the two principal components (dimensions PCA1 and PCA2) that account for the most variance in the data. Each point represents a single simulation run, colored according to its lifespan.

We can see that most simulations fail quite fast, as indicated by the many yellow dots. Of those who do not initially fail, most seem to become stable at least until iteration 500, while the color scale indicates that not a lot of simulations fail in the middle (approximately between iteration 50 and 450).

We also identify that the simulations that become stable seem to be gathered in a cluster where PCA1 is between -3 and 2, and PCA2 is between -2 and 1. There are large regions where all simulations fail fast (where there are only yellow points). This indicates that there are some regions of parameter sets that are more successful than others, and that it is not completely random.

However, we note that there are multiple cases where there are yellow and purple points right next to each other, indicating chaotic behaviour, as small differences in input parameters leads to great differences in lifespan. This is a result of the stochastic nature of the initial probabilities; just because herbivores have a 50% initial probability of occupying each individual cell does not mean that exactly 50% of all cells will initially be herbivores, nor are the initial locations of herbivore cells constant.

To further interpret the PCA results, we examine the PCA loading plot shown in Figure 3. This plot illustrates the contribution of each simulation parameter to the two principal components. Parameters that are more influential in determining the variance captured by the principal components are

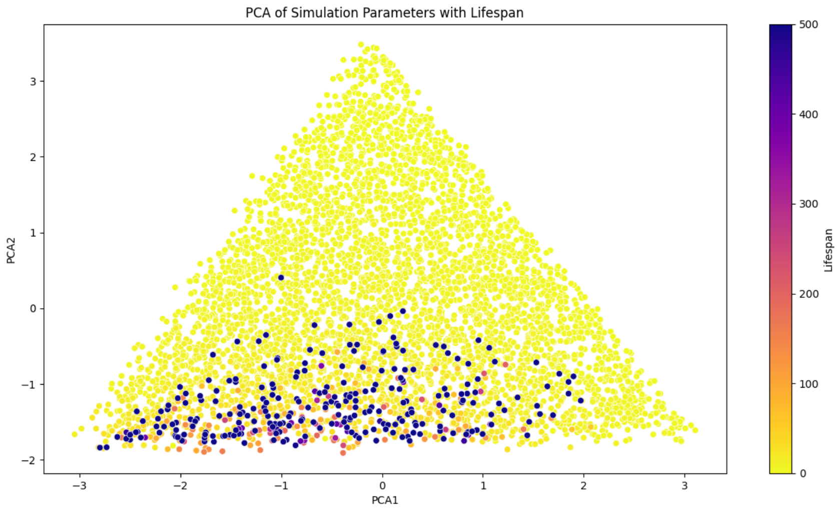

Figure 2: PCA of simulation parameters with lifespan. The color bar represents the lifespan of the ecosystems.

represented by longer vectors.

Since the three longest vectors are the initial probabilites of the three species, these explain a lot of the variance in lifespan and are important for the stability of the system. This seems intuitively reasonable, since most simulations fail very fast, and the initial probabilities should probably matter a lot for those, as the other parameters in the system (such as starvation and reproduction rates) don't have time to play out to a great extent in simulations that fail fast.

Further, a correlation analysis was conducted to measure the linear relationships between each parameter and the ecosystem's lifespan. The correlation coefficients, depicted in Figure 4, range from -1 to 1. A value closer to 1 indicates a strong positive correlation, suggesting that as the parameter value increases, so does the lifespan. Conversely, a value closer to -1 indicates a negative relationship.

From the correlation analysis, we see that a high initial herbivore probability contributes most positively to the lifespan, whereas a high initial probability for the low predator and a high starvation rate for the top predator contribute most negatively.

Finally, a centroid was calculated to represent the average parameter values of the most stable ecosystems, those with the highest lifespan. The centroid provides insights into the parameter values associated with long-term stability, as depicted in Table 1.

The simulation distributions were uniform between 0 and 1 for the reproduction rates, and uniform between 1 and 50 for the starvation rates. If the

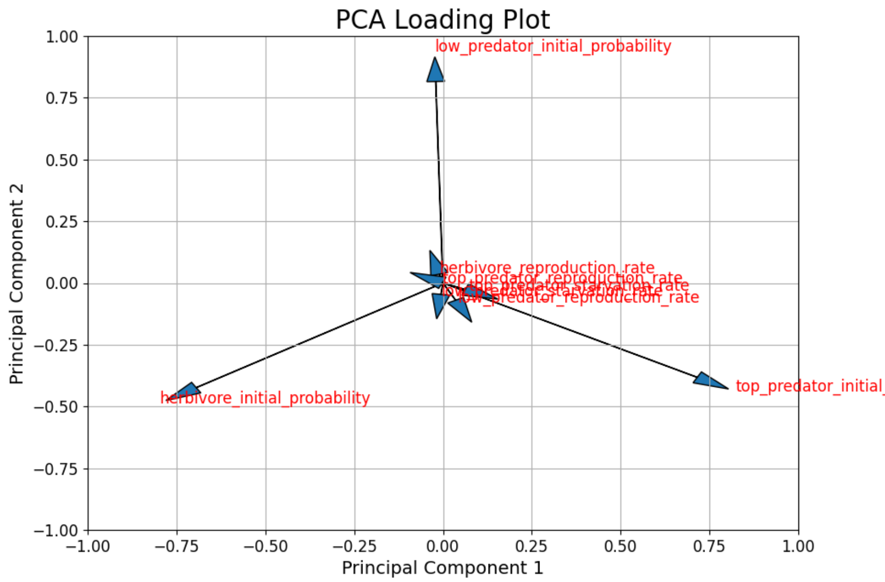

Figure 3: PCA loading plot showing the contribution of each parameter to the principal components. The direction and length of the arrows indicate how each parameter influences the component axes.

lifespan was independent of the parameters, we would expect the centroid to be located in $\frac{1}{3}$ for the initial probabilities, in 0.5 for the reproduction rates, and at around 25.5 for the starvation rates. We note that the only parameters clearly deviating from this are the initial probabilities, again indicating that they are the most important.

The observed initial probabilities in the simulation, particularly the high number of herbivores, the relatively few low-level predators, and the significant number of top-level predators, reflect an interesting dynamic. The abundance of herbivores, constituting the base of the food chain, is essential for sustaining the ecosystem. The lower initial probability of low-level predators might be interpreted as a mechanism to prevent the rapid depletion of herbivores, thereby maintaining a balance between populations.

Interestingly, the relatively high initial probability of top-level predators seems counterintuitive, as it could risk over-predation on the low-level predators. This higher probability might reflect a scenario where the top-level predators, not being in direct competition with herbivores, can coexist in larger numbers, provided there are enough low-level predators to sustain them. Such a system, with abundant herbivores and a large population of top-level predators, seem to lead to a relatively stable ecosystem as long as the intermediate trophic level (low-level predators) is sufficiently maintained.

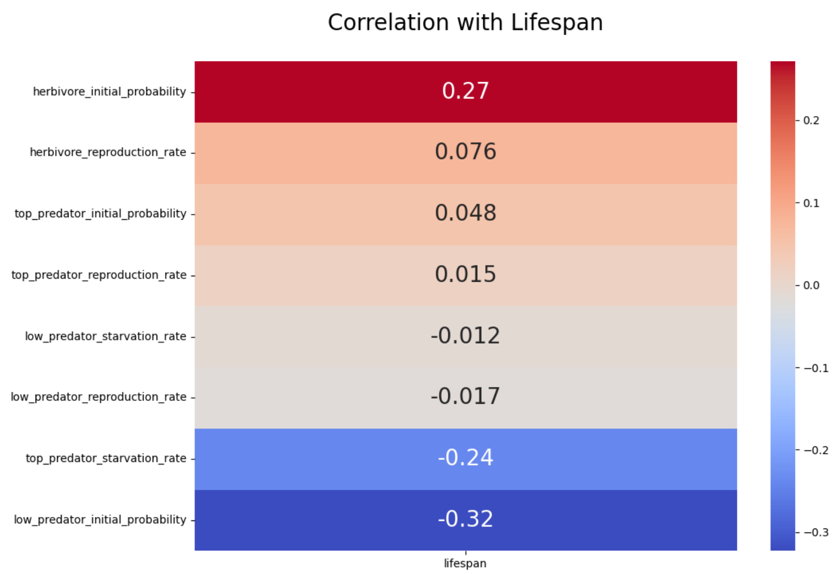

Figure 4: Correlation coefficients for each parameter with respect to lifespan. Positive values indicate a tendency for the parameter to increase lifespan, whereas negative values suggest an inverse relationship.

3.2.2 Minimum grid size for stability

Once we have found a set of parameters that generates a stable system, we can keep those parameters constant but vary the grid size, to see what grid size is needed to achieve stability. The parameters that were identified to generate a stable system (reaching 500 iterations) for a grid size of 50×50 are presented in Table 2.

In this case, the limit for stability was set to 100 iterations, and for each simulated grid size (every tenth integer between 1 and 61), 100 simulations were ran. In figure 5, the results for probability of stability, as well as the mean lifespan, are plotted by grid size.

From the plot, we conclude that the chance of reaching stability is for small grid sizes zero, then monotonically increases with grid size and seems to converge to 95-100% at around grid size 60.

4 Conclusion

Looking back at the questions we initially asked, we can now answer them.

Table 1: Simulation parameters for the centroid

Parameter	Value
Herbivore Initial Probability	0.551
Low Predator Initial Probability	0.092
Top Predator Initial Probability	0.357
Herbivore Reproduction Rate	0.485
Low Predator Reproduction Rate	0.522
Top Predator Reproduction Rate	0.499
Low Predator Starvation Rate	26.294
Top Predator Starvation Rate	26.055

Table 2: Simulation parameters for the stable system

Parameter	Value
Initial Herbivore Probability	0.449
Initial Low Predator Probability	0.014
Initial Top Predator Probability	0.537
Herbivore Reproduction Rate	0.285
Low Predator Reproduction Rate	0.209
Top Predator Reproduction Rate	0.989
Low Predator Starvation Limit	5
Top Predator Starvation Limit	19

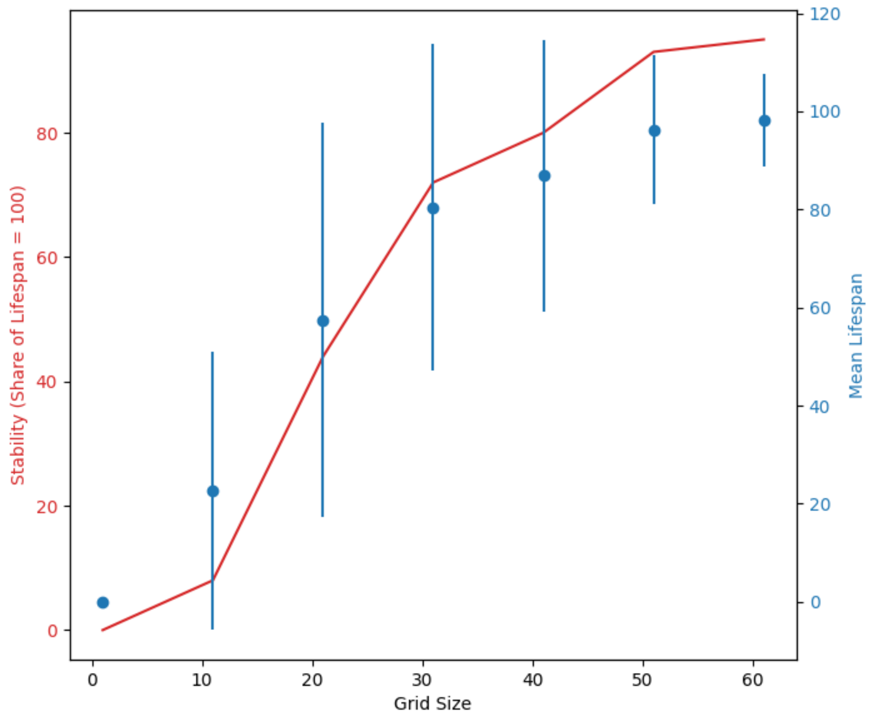

Figure 5: Probability of Ecosystem Stability and Mean Lifespan Across Different Grid Sizes.

4.1 Impact of initial and system parameters on stability

Our comprehensive simulations have shown that initial and system parameters, including population numbers, reproduction rates, and starvation rates, are critical in determining the stability of the predator-prey system. The balance between these parameters dictates the immediate survival prospects and long-term sustainability of the ecosystem. Particularly, the initial population densities of species are pivotal in setting the stage for the dynamics that follow.

4.2 Parameters for achieving a stable system

The study identified specific parameter sets conducive to stable ecosystems. These parameters generally revolved around a higher initial probability of herbivores and well-balanced reproduction and starvation rates for all species. While there is some variability, the successful parameter sets shared common traits, indicating that multiple archetypes can lead to prosperous systems. This variability underscores the complexity and adaptability of ecological systems.

4.3 Relative importance of different parameters

Our analysis underscores that not all parameters equally influence the system's stability. Initial population densities, particularly of herbivores, emerged as crucial in the early stages of ecosystem development. In contrast, reproduction rates and starvation limits might play more significant roles in the long-term dynamics, influencing the equilibrium and sustainability of the ecosystem.

4.4 System size requirement for stability

The research clearly indicates that system size, or grid size, significantly impacts stability. A minimum grid size is necessary to support the balanced interaction of species and prevent rapid extinction events. Larger grid sizes facilitate more robust population dynamics and contribute to the overall resilience and stability of the ecosystem.

4.5 Ecological interpretation of optimal initial conditions

The optimal initial conditions in our simulation reveal a balanced ecosystem where a high herbivore population supports the predatory layers, with low initial numbers of low-level predators preventing rapid prey depletion. The substantial presence of top-level predators, sustained by adequate low-level predators, suggests a stable equilibrium without direct competition for herbivores. However, this model simplifies the complexities of real ecosystems, which are influenced by a wider range of environmental and biological factors.

5 Code

The Python code used for simulations and data analysis, as well as video visualizations, is available on GitHub.