Cardano's Formula for Cubic Equations

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Abstract

Girolamo Cardano, also known as Cardano, was an Italian doctor and mathematician best known for his work Ars Magna, the first Latin treatise on algebra. He described the strategies he had learned from Tartaglia for solving cubic and quartic problems.

1 Introduction to Cardano's Formula

Let P be the cubic equation:

$ax^3 + bx^2 + cx + d = 0$, $a \neq 0$

Then P has solutions:

$x_1 = S + T - \frac{b}{3a}$

$x_2 = -\frac{S+T}{2} - \frac{b}{3a} + \frac{i\sqrt{3}}{2}(S - T)$

$x_3 = -\frac{S+T}{2} - \frac{b}{3a} - \frac{i\sqrt{3}}{2}(S - T)$

where:

$S = \sqrt[3]{R + \sqrt{Q^3 + R^2}}$

$T = \sqrt[3]{R - \sqrt{Q^3 + R^2}}$

where:

$Q = \frac{3ac - b^2}{9a^2}$

$R = \frac{9abc - 27a^2d - 2b^3}{54a^3}$

The expression $D = Q^3 + R^2$ is called the discriminant of the equation.

Let $a, b, c, d, \in R$ Then:

- If $D > 0$, then one root is real and two are complex conjugates.
- If $D = 0$, then all roots are real, and at least two are equal.
- If $D < 0$, then all roots are real and unequal.

1.1 Some Examples

- $x^3 - 2x^2 - 5x + 6 = 0$
- $x^3 - 3x^2 + 4x - 2 = 0$