Machine Learning Formulas

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Contents

1 线性回归(Linear Regression)

1.1 当训练集X只有1项时

$$X = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 1 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{(n+1)*1} \tag{1}$$

$$\theta = \begin{pmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{pmatrix}_{(n+1)*1} \tag{2}$$

$$y = y \tag{3}$$

$$\begin{aligned} h_\theta(x) &= \theta^T X = X^T \theta \\ &= \begin{pmatrix} 1 & x_1 & x_2 & \dots & x_n \end{pmatrix} \begin{pmatrix} \theta_0 \\ \theta_1 \\ \dots \\ \theta_n \end{pmatrix} \\ &= \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n \end{aligned} \tag{4}$$

1.2 当训练集X有m项时

$$
\begin{aligned}
X &= \begin{pmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \\ \vdots \\ x^{(m)} \end{pmatrix} \\
&= \begin{pmatrix} x_0^{(1)} & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & \dots & x_n^{(1)} \\ x_0^{(2)} & x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & \dots & x_n^{(2)} \\ x_0^{(3)} & x_1^{(3)} & x_2^{(3)} & x_3^{(3)} & \dots & x_n^{(3)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_0^{(m)} & x_1^{(m)} & x_2^{(m)} & x_3^{(m)} & \dots & x_n^{(m)} \end{pmatrix} \\
&= \begin{pmatrix} 1 & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & \dots & x_n^{(1)} \\ 1 & x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & \dots & x_n^{(2)} \\ 1 & x_1^{(3)} & x_2^{(3)} & x_3^{(3)} & \dots & x_n^{(3)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_1^{(m)} & x_2^{(m)} & x_3^{(m)} & \dots & x_n^{(m)} \end{pmatrix}_{m*(n+1)}
\end{aligned} \tag{5}
$$

$$
\theta = \begin{pmatrix} \theta^{(0)} \\ \theta^{(1)} \\ \theta^{(2)} \\ \theta^{(3)} \\ \vdots \\ \theta^{(n)} \end{pmatrix}_{(n+1)*1} \tag{6}
$$

$$
y = \begin{pmatrix} y^{(1)} \\ y^{(2)} \\ y^{(3)} \\ \vdots \\ y^{(m)} \end{pmatrix}_{m*1} \tag{7}
$$

1.2.1 Cost Function

1. 数值形式：

$$
J(\theta) = \frac{1}{2m}\left[h_\theta(x^{(i)}) - y^{(i)}\right]^2 \tag{8}
$$

2. 矩阵形式：

$$
J(\theta) = \frac{1}{2m}\left[h_\theta(x) - y\right]^T\left[h_\theta(x) - y\right] \tag{9}
$$

1.2.2 梯度下降

1. 数值形式

$$\frac{\partial J(\theta)}{\partial \theta_j} = \frac{1}{m}\left[h_\theta(x^{(i)}) - y^{(i)}\right] x_j^{(i)} \tag{10}$$

迭代方式:

$$\theta_j := \theta_j - \alpha\frac{1}{m}\left[h_\theta(x^{(i)}) - y^{(i)}\right] x_j^{(i)} \tag{11}$$

2. 矩阵形式

$$\nabla J(\theta) = \frac{1}{2m}X^T\left[h_\theta(x) - y\right] \tag{12}$$

迭代方式:

$$\theta := \theta - \alpha\frac{1}{m}X^T\left[h_\theta(x) - y\right] \tag{13}$$

1.3 Feature Normalization

$$x_i = \frac{x_i - \mu}{\sigma} \tag{14}$$

或

$$x_i = \frac{x_i - \mu}{max - min} \tag{15}$$

1.4 公式法求解（Normal Equation）

$$\theta = (X^TX)^{-1}X^Ty \tag{16}$$

2 逻辑回归(Logistic Regression)

2.1 当只有2个类别时，使用1个分类器

2.1.1 预测函数

1. 数值形式

$$h_\theta(x) = \frac{1}{1 + e^{\theta^T x}} \tag{17}$$

2. 矩阵形式

$$h_\theta(X) = \frac{1}{1 + e^{X\theta^T}} \tag{18}$$

2.1.2 Cost Function

1. 数值形式

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \left[-y^{(i)} log h_\theta(x^{(i)}) - (1 - y^{(i)}) log(1 - h_\theta(x^{(i)})) \right] \tag{19}$$

2. 矩阵形式

$$J(\theta) = \frac{1}{m} \left[-y^T \log h_\theta(x) - (1 - y^T) \log (1 - h_\theta(x) \right] \tag{20}$$

2.1.3 梯度下降

1. 数值形式

$$\frac{\partial J(\theta)}{\partial \theta_j} = \frac{1}{m} \sum_{i=1}^{m} \left[h_\theta(x^{(i)}) - y^{(i)} \right] x_j^{(i)} \tag{21}$$

迭代方式:

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} \left[h_\theta(x^{(i)}) - y^{(i)} \right] x_j^{(i)} \tag{22}$$

2. 梯矩阵形式

$$\nabla J(\theta) = \frac{1}{m} X^T \left[h_\theta(x) - y \right] \tag{23}$$

迭代方式:

$$\theta := \theta - \alpha \frac{1}{m} X^T \left[h_\theta(x) - y \right] \tag{24}$$

2.2 当只有k个类别时，使用k个分类器

2.3 避免过拟合

1. 线性回归

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}[h_\theta(x^{(i)}) - y^{(i)}]^2 + \lambda\frac{1}{2m}\sum_{j=1}^{n}\theta_j^2 \tag{25}$$

$$\theta_j = \theta_j(1 - \alpha\frac{\lambda}{m}) - \frac{\alpha}{m}\sum_{i=1}^{n}\left[h_\theta(x) - y\right]x_j^{(i)} \tag{26}$$

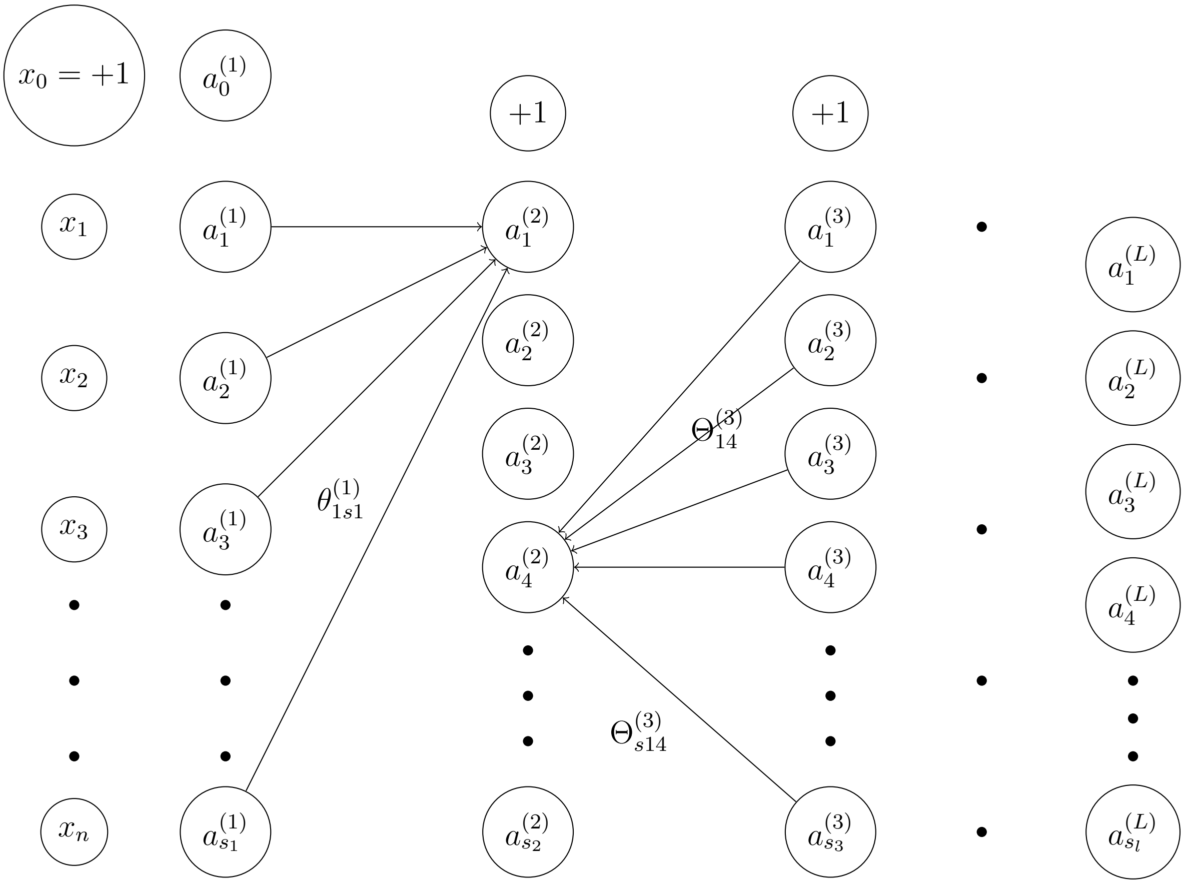
2. 逻辑回归

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\left[-y^{(i)}logh_\theta(x^{(i)}) - (1 - y^{(i)})log(1 - h_\theta(x^{(i)}))\right] + \lambda\frac{1}{2m}\sum_{j=1}^{n}\theta_j^2 \tag{27}$$

$$\theta_j = \theta_j - \alpha\left[\frac{1}{m}\sum_{i=1}^{m}[h_\theta(x^{(i)}) - y^{(i)}]x_j^{(i)} + \frac{\lambda}{m}\theta_j\right] \tag{28}$$

3 神经网络图例

3.1 神经网络示意图



3.2 神经网络－前向算法

3.2.1 X、θ、Θ、z、a

1. X

$$X = \begin{pmatrix} (x^{(1)})^T \\ (x^{(2)})^T \\ (x^{(3)})^T \\ \vdots \\ (x^{(m)})^T \end{pmatrix} = \begin{pmatrix} x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & \dots & x_n^{(1)} \\ x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & \dots & x_n^{(2)} \\ x_1^{(3)} & x_2^{(3)} & x_3^{(3)} & \dots & x_n^{(3)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{(m)} & x_2^{(m)} & x_3^{(m)} & \dots & x_n^{(m)} \end{pmatrix} = \begin{pmatrix} x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & \dots & x_n^{(1)} \\ x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & \dots & x_n^{(2)} \\ x_1^{(3)} & x_2^{(3)} & x_3^{(3)} & \dots & x_n^{(3)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{(m)} & x_2^{(m)} & x_3^{(m)} & \dots & x_n^{(m)} \end{pmatrix} \Rightarrow (m, n) \tag{29}$$

2. $a^{(1)}$

$$a^{(1)} = X \Rightarrow (m, n) \tag{30}$$

3. $\Theta^{(1)}$

$$\Theta^{(1)} = \begin{pmatrix} \theta_{10}^{(1)} & \theta_{11}^{(1)} & \theta_{12}^{(1)} & \dots & \theta_{1,s_1}^{(1)} \\ \theta_{20}^{(1)} & \theta_{21}^{(1)} & \theta_{22}^{(1)} & \dots & \theta_{2,s_1}^{(1)} \\ \theta_{30}^{(1)} & \theta_{31}^{(1)} & \theta_{32}^{(1)} & \dots & \theta_{3,s_1}^{(1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \theta_{s_20}^{(1)} & \theta_{s_21}^{(1)} & \theta_{s_22}^{(1)} & \dots & \theta_{s_2,s_1}^{(1)} \end{pmatrix} \Rightarrow (s_2, s_1 + 1) = (s_2, n + 1) \tag{31}$$

4. $z^{(2)}$

给$a^{(1)}$的每个数据均添加上$a_0=1$后与$\Theta^{(1)}$计算,得到$z^{(2)\text{注[1]}}=(1,a^{(1)})(\Theta^{(1)})^T$

$$
\begin{aligned}
z^{(2)} &= a^{(1)}(\Theta^{(1)})^T \Rightarrow (m,n+1)*(n+1,s_2) \\
&= \begin{pmatrix} 1 & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & \dots & x_n^{(1)} \\ 1 & x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & \dots & x_n^{(2)} \\ 1 & x_1^{(3)} & x_2^{(3)} & x_3^{(3)} & \dots & x_n^{(3)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_1^{(m)} & x_2^{(m)} & x_3^{(m)} & \dots & x_n^{(m)} \end{pmatrix} \begin{pmatrix} \theta_{10}^{(1)} & \theta_{11}^{(1)} & \theta_{12}^{(1)} & \dots & \theta_{1,n}^{(1)} \\ \theta_{20}^{(1)} & \theta_{21}^{(1)} & \theta_{22}^{(1)} & \dots & \theta_{2,n}^{(1)} \\ \theta_{30}^{(1)} & \theta_{31}^{(1)} & \theta_{32}^{(1)} & \dots & \theta_{3,n}^{(1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \theta_{s_2,0}^{(j)} & \theta_{s_2,1}^{(j)} & \theta_{s_2,2}^{(j)} & \dots & \theta_{s_2,n}^{(1)} \end{pmatrix}^T \\
&= \begin{pmatrix} 1 & x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & \dots & x_n^{(1)} \\ 1 & x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & \dots & x_n^{(2)} \\ 1 & x_1^{(3)} & x_2^{(3)} & x_3^{(3)} & \dots & x_n^{(3)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_1^{(m)} & x_2^{(m)} & x_3^{(m)} & \dots & x_n^{(m)} \end{pmatrix} \begin{pmatrix} \theta_{10}^{(1)} & \theta_{20}^{(1)} & \theta_{30}^{(1)} & \dots & \theta_{s_2,0}^{(1)} \\ \theta_{11}^{(1)} & \theta_{21}^{(1)} & \theta_{31}^{(1)} & \dots & \theta_{s_2,1}^{(1)} \\ \theta_{12}^{(1)} & \theta_{22}^{(1)} & \theta_{32}^{(1)} & \dots & \theta_{s_2,2}^{(1)} \\ \theta_{13}^{(1)} & \theta_{23}^{(1)} & \theta_{33}^{(1)} & \dots & \theta_{s_2,3}^{(1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \theta_{1,n}^{(1)} & \theta_{2,n}^{(1)} & \theta_{3,n}^{(1)} & \dots & \theta_{s_2,n}^{(1)} \end{pmatrix} \\
&= \begin{pmatrix} z^{(2)(1)} \\ z^{(2)(2)} \\ z^{(2)(3)} \\ \vdots \\ z^{(2)(m)} \end{pmatrix} \\
&= \begin{pmatrix} z_1^{(2)(1)} & z_2^{(2)(1)} & z_3^{(2)(1)} & \dots & z_{s_2}^{(2)(1)} \\ z_1^{(2)(2)} & z_2^{(2)(2)} & z_3^{(2)(2)} & \dots & z_{s_2}^{(2)(2)} \\ z_1^{(2)(3)} & z_2^{(2)(3)} & z_3^{(2)(3)} & \dots & z_{s_2}^{(2)(3)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z_1^{(2)(m)} & z_2^{(2)(m)} & z_3^{(2)(m)} & \dots & z_{s_2}^{(2)(m)\text{注[2]}} \end{pmatrix} \Rightarrow (m,n+1)*(n+1,s_2)=(m,s_2)
\end{aligned} \tag{32}
$$

5. $a^{(2)}$

$$a^{(2)}=g(z^{(2)}) \Rightarrow (m,s_2) \tag{33}$$

6. 一般式

注[1]从$a^{(1)}$得到$a^{(2)}$需要经过sigmoid()函数，后续的从$a^{(j)}$得到$a^{(j+1)}$均需要经过sigmoid()函数

后续同理：

$$
\begin{aligned}
\Theta^{(2)} &= \begin{pmatrix} \theta_{10}^{(2)} & \theta_{11}^{(2)} & \theta_{12}^{(2)} & \dots & \theta_{1,s_2}^{(2)} \\ \theta_{20}^{(2)} & \theta_{21}^{(2)} & \theta_{22}^{(2)} & \dots & \theta_{2,s_2}^{(2)} \\ \theta_{30}^{(2)} & \theta_{31}^{(2)} & \theta_{32}^{(2)} & \dots & \theta_{3,s_2}^{(2)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \theta_{s_30}^{(2)} & \theta_{s_31}^{(2)} & \theta_{s_32}^{(2)} & \dots & \theta_{s_3,s_2}^{(2)} \end{pmatrix} \Rightarrow (s_3, s_2+1) \\
z^{(3)} &= (1, a^{(2)})(\Theta^{(2)})^T \Rightarrow (m, s_2+1) * (s_2+1, s_3) = (m, s_3) \\
a^{(3)} &= g(z^{(3)}) \Rightarrow (m, s_3) \\
&\vdots \\
a^{(j)} &= g(z^{(j-1)}) \Rightarrow (m, s_j) \\
\Theta^{(j)} &= \begin{pmatrix} \theta_{10}^{(j)} & \theta_{11}^{(j)} & \theta_{12}^{(j)} & \dots & \theta_{1,s_j}^{(j)} \\ \theta_{20}^{(j)} & \theta_{21}^{(j)} & \theta_{22}^{(j)} & \dots & \theta_{2,s_j}^{(j)} \\ \theta_{30}^{(j)} & \theta_{31}^{(j)} & \theta_{32}^{(j)} & \dots & \theta_{3,s_j}^{(j)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \theta_{s_{j+1}0}^{(j)} & \theta_{s_{j+1}1}^{(j)} & \theta_{s_{j+1}2}^{(j)} & \dots & \theta_{s_{j+1},s_j}^{(j)} \end{pmatrix} \Rightarrow (s_{j+1}, s_j+1) \\
z^{(j+1)} &= (1, a^{(j)})(\Theta^{(j)})^T \Rightarrow (m, s_j+1) * (s_j+1, s_{j+1}) = (m, s_3) \\
a^{(j+1)} &= g(z^{(j+1)}) \Rightarrow (m, s_{j+1})
\end{aligned} \tag{34}
$$

3.2.2 y

$$
y = \begin{pmatrix} y^{(1)} \\ y^{(2)} \\ y^{(3)} \\ \vdots \\ y^{(m)} \end{pmatrix}_{m*1} \tag{35}
$$

为进行矩阵运算，要将其转化为如下形式:注[3]

$$
y = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & 0 & 0 \end{pmatrix}_{m*s_L}{}^{\text{注[4]}} \tag{36}
$$

注[3]y所对应的值所在的索引位置值为1，其他位置均为0