Advanced Control for Robotics: Homework #1

Shang Yangxing

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1 ODE and Its Simulation

1.1 Equation of Pendulum Motions

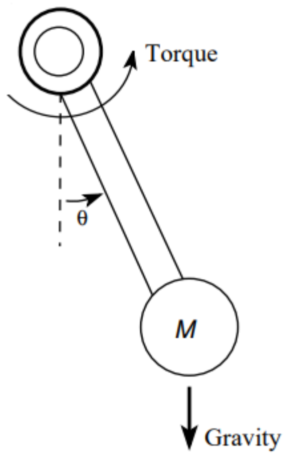

Figure 1: pendulum model

By applying the Newton's law of dynamics, a pendulum with no external force can be formulated as:

$$ml^2\ddot{\theta} + ml^2\alpha\dot{\theta} + mgl\sin\theta - T = 0. \tag{1}$$

in which,
m is mass of the ball
l is length of the rod
α is the damping constant
g is the gravitational constant
θ is angle measured between the rod and the vertical axis
T is torque of the joint, which is also the control input u
to a system of two first order equation by letting $x_1 = \theta$, $x_2 = \dot{\theta}$:

$$\dot{x_1} = x_2, \quad \dot{x_2} = -\frac{g}{l}\sin x_1 - \alpha x_2 + \frac{T}{ml^2}. \tag{2}$$

Written in standard state-space form:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{g}{l}\sin x_1 - \alpha x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} T \tag{3}$$

$$\boldsymbol{y} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \boldsymbol{x} \tag{4}$$

1.2 Simulation of Pendulum

2 Matrix calculus

3 Inner product

4 Some linear algebra

5 Gradient Flow