MAT157 Problem Set 2

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1. a) $\forall x \in F,\ \exists y \in F : \forall z \in F,\ z \neq x,\ xy = 1 \implies yz \neq 1$
The negation:
$\exists x \in F,\ \forall y \in F : \exists z \in F, z \neq x, xy = 1$ and $yz = 1$
Plain English:
There exists an element x that for all y such that there exists z with $z \neq x$, $xy = 1$ and $yz = 1$

b) Let $a(x)$ be the angle sum of polygon x and let H be the set of all hyperbolic octagons.
$\forall x \in H,\ a(x) > \pi$
The negation:
$\exists x \in H,\ a(x) \leq \pi$
Plain English:
There exists a hyperbolic octagon with an angle sum less than or equal to π.

c) Let Q be the set of all flavors of quarks, $c(x)$ be the charge of quark x, and $m(x)$ be the mass of quark x.
$\forall x, y \in Q, c(x) = c(y)$ and $m(x) = m(y)$
The negation:
$\exists x, y \in Q, c(x) \neq c(y)$ op $m(x) \neq m(y)$
Plain English:
There exists two flavors of quarks that do not have the same charge or the same mass.

d) Let S be the set of students in this class, H be the set of all homework assignments, L be the set of all lectures, $h(x, y)$ be student x does homework y, $l(x, y)$ be student x goes to lecture y, and $f(x)$ be the percentage that student x gets on the final exam.
$\forall x \in S : h(x, y), \forall y \in H$ and $l(x, z), \forall z \in L \implies f(x) \geq 50$

The negation:
$\exists x \in S : (h(x, y), \forall y \in H$ and $l(x, z), \forall z \in L)$ and $f(x) < 50$
Plain English:
There exists a student who does all the homework and all the assignments but scores less than 50 percent on the final exam.

2. a)

Proof.

$\Rightarrow$
If $a = b$ then $|a - b| = |a - a| = 0 < \epsilon$.

$\Leftarrow$
We will take the contrapositive. If $a \neq b$ then $\exists x \in \mathbb{R} \setminus \{0\}$ such that $b = a + x$. Thus, $|a - b| = |a - (a + x)| = |x| > \epsilon$

$\square$

Proof.

b) Assume that for all rational numbers x, $a \geq x$ or $x \geq b$. Let $a = 0$ and $b = 2$, giving us $a < b$. Consider $x = 1$. Clearly x is rational, but $a < x$ and $x < b$. This contradicts our assumption that for all rational numbers x, $a \geq x$ or $x \geq b$

$\square$