MAT240 Assignment 1

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September 14, 2021

3. a)

Proof.

Consider g and g' are both inverses of f.

$$g(f(x)) = x = g'(f(x)),\ \forall x \in X$$

Since x was chosen arbitrarily, $f(x)$ is just an arbitrary object in Y.
Thus, $g(y) = g'(y),\ \forall y \in Y$ and $g = g'$

$\square$

b) $f : \mathbb{R} \to \mathbb{R},\ f(x) = x^2$ is not invertible because it is not a bijection. This is because $1 \neq -1$ but $f(1) = f(-1)$ (This is shown by the problem below.)

c)

Proof.

$\Rightarrow$

We will first prove that f is a surjection.
By assumption $\forall y \in Y,\ f(g(y)) = y$ and because $g(y)$ is just an arbitrary element of X, this shows that $\forall y \in Y,\ \exists x \in X$ st $f(x) = y$

We will now show that f is an injection.
Assume that f is not an injection.
Thus, $\exists x, x' \in X$ st $x \neq x'$ and $f(x) = f(x')$
Then $g(f(x)) = g(f(x')) = x = x'$ and thus we have reached a contradiction.

Thus we have proven that f is bijective.

$\Leftarrow$

Because $f : X \to Y$ is a bijective function, then
$\forall y \in Y, \exists! x \in X$ st $f(x) = y$
Thus we can construct a function $g : Y \to X$ that maps all $y \in Y$ to the unique $x \in X$ st $f(x) = y$
Thus, by construction $g(f(x)) = x$ and $f(g(y)) = y$

$\square$

d) It does not follow that $f \circ g = I_Y$

Proof.

Consider the function $f : \{a\} \to \{1, 2\}$ such that $f(a) = 1$ and the function $g : \{1, 2\} \to \{a\}$ such that $g(1) = a$ and $g(2) = a$
Thus $(g \circ f)(a) = a$ and $(g \circ f) = I_X$ But $(f \circ g)(2) = 1$ so
$(f \circ g) \neq I_Y$
Thus we have constructed a counterexample.

$\square$

e) It does follow that $f \circ g = I_Y$ now.

Proof.

If $g \circ f = I_X$ then f is injective because if we assume f is not injective, then we result in the contradiction where
$g(f(x)) = g(f(x')) = x = x'$ when $x \neq x'$
Thus f is injective and surjective, making it a bijection, and by part c, this implies that it is invertible and there exists a g st $f \circ g = I_Y$

$\square$