MAT157 Assignment 1

Nicolas Coballe

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5. a) We will prove this using contradiction.

Proof.

Assume all functions $f : A \to B$ are not bijective. Then, consider $A := \mathbb{R}$ and $f : A \to A,\ f(x) = \frac{1}{x}$ if $x \neq 0$ and $f(x) = 0$ otherwise. f is injective because it only maps zero to itself and strictly decreases on the interval $(-\infty, 0)$ and $(0, \infty)$ We will show this by taking $0 < x < y \implies 0 < \frac{x}{y} < 1 \implies 0 < \frac{1}{x} < \frac{1}{y}$. A similar argument works for $x < y < 0$. Because f is strictly decreasing on intervals $(-\infty, 0)$ and $(0, \infty)$ and the images of $f((-\infty, 0))$ and $f((0, \infty))$ are clearly disjoint, f must be an injection. Now let $B := f(A)$ and thus, $f(A) \subseteq B \subseteq A$. Now consider the map $h : A \to B,\ f(x) = \frac{1}{x}$ if $x \neq 0$ and $f(x) = 0$ otherwise. We have already proven that this function is injective and because $h(A) = B,\ h$ is surjective. Thus h is bijective, a contradicition.

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