COSC3106: Theory of Computing

Lec#2: Finite Automaton

School of Computer Science and Technology

- **By the end of this lecture, you should be able to…**
  - **Familiarize yourself with the state machine and state diagram**
  - **Understand then main concept of the finite automata and regular languages**
  - **Operations on Languages and DFA**

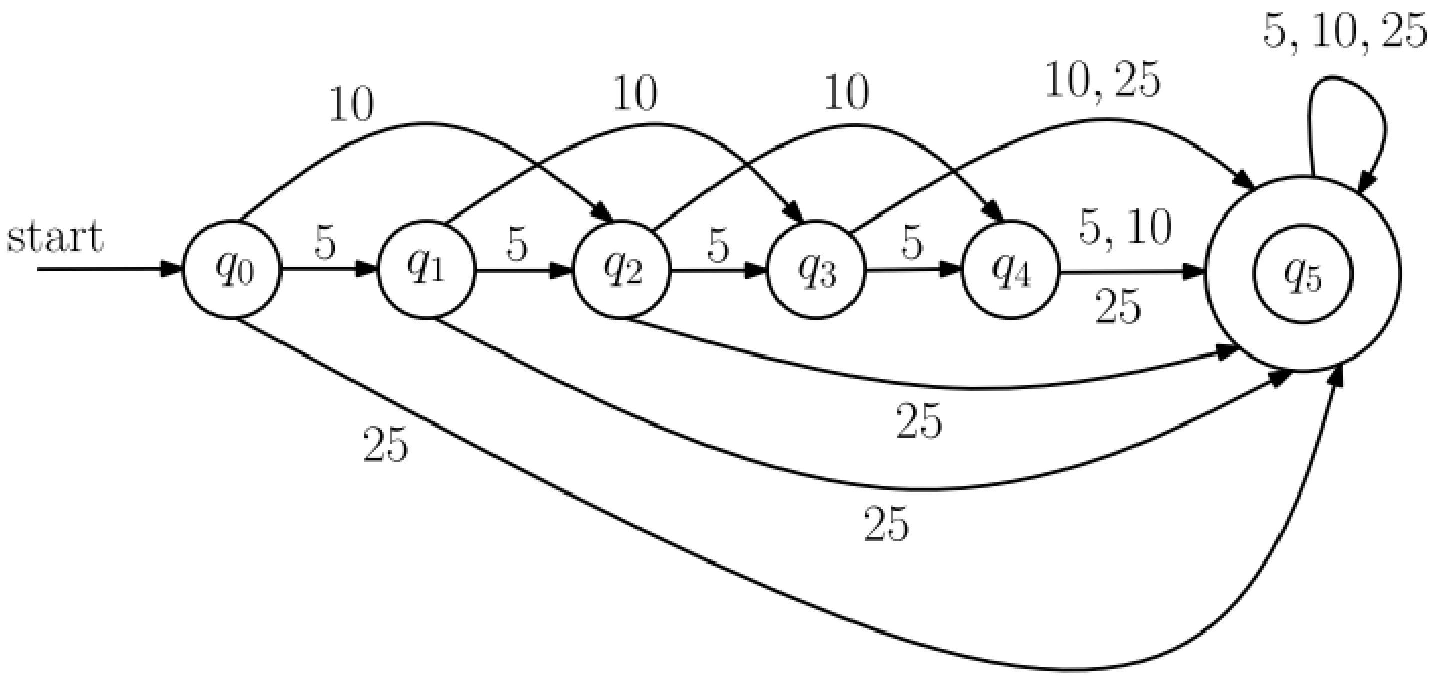
- Before we give a formal definition of a finite automaton, we consider an example in which such an automaton shows up in a natural way. Let us consider the problem of designing a "computer" that controls a toll gate.

- Ex: Controlling a toll gate:
  - When a car arrives at the toll gate, the gate is closed. The gate opens as soon as the driver has paid 25 cents.
  - Assume that we have only three coin denominations: 5, 10, and 25 cents. We also assume that no excess change is returned.
  - The question that to be answered: <u>At any moment, the machine must decide whether to open the gate</u>?
  - To make this decision; we need to define the states of the machine according to the sequence of coins inserted by the driver.

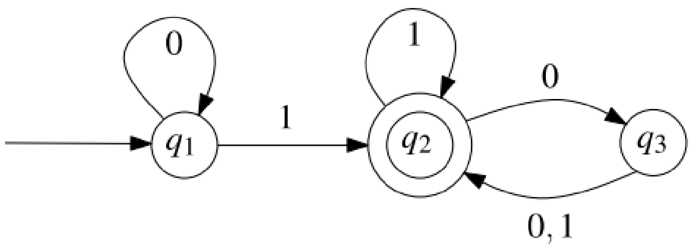
- States: The machine is in one of the following six states, at any moment during the process:
  - q_0
  - q_5
  - q_{10}
  - q_{15}
  - q_{20}
  - $q_{\geq 25}$
  - That can be represented using a directed graph called state diagram.

- State Diagram:
  - The vertices of the graph ◯ are the states, and the directed edges ⟶ are the transitions.
  - Note that the machine (or computer) only has to remember which state it is in at any given time.

- **State Diagram:**



- Let us look at another example. Consider the following state diagram:



  - Edges are labeled by 0’s and 1’s
- Ex#1: If the input string: 1101:

$$q_1 - 1 \rightarrow q_2 - 1 \rightarrow q_2 - 0 \rightarrow q_3 - 1 \rightarrow q_2$$

  - Result: Accept

- Ex#2: If the input string: 0101010

$$q_1 - 0 \rightarrow q_1 - 1 \rightarrow q_2 - 0 \rightarrow q_3 - 1 \rightarrow q_2 - 0 \rightarrow q_3 - 1 \rightarrow q_2 - 0 \rightarrow q_3$$

  - Result: Reject

- Ex#3: $\varepsilon \in \Phi$

$$q_1$$

- Machines start in an initial state, so if an input string is empty (ε) it will only be in its initial state.
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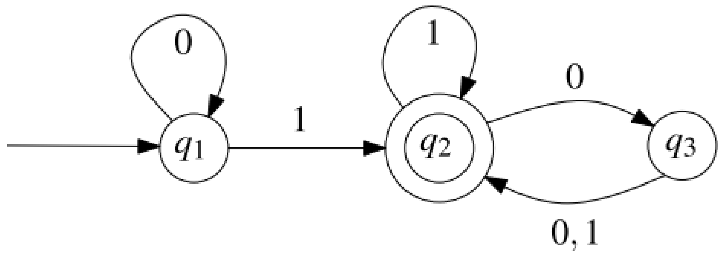
- Let us look at another example. Consider the following state diagram:



- What is the form of the set of binary strings that are accepted by this FA?
    - This FA (machine) accepts every binary string that ends with a 1.
    - Every binary string that there are an even number of 0s following the rightmost 1, is accepted by this machine.

- Language: Set of all strings that are accepted.

- Now, we can come to the formal definition of a finite automaton . . . . .

- **Definition: A finite automaton (FA) is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$, where**

  - Q **: finite set of states,**
  - Σ **: called the alphabet; is a finite set of symbols,**
  - $\delta : Q \times \Sigma \to Q$**: Transition function,**
  - $q_0 \in Q$**: start state,**
  - $F \subset Q$**: accept state/states.**

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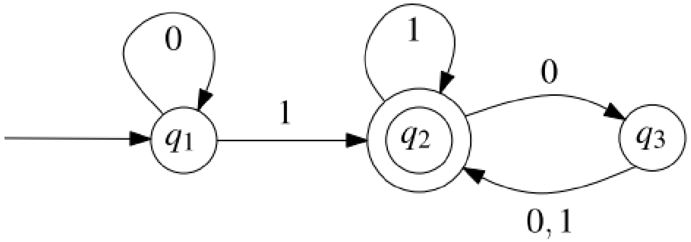
- **Represent the following state diagram as a formal FA?**



- $Q = \{q_1, q_2, q_3\}$**, finite set of states**
- $\Sigma = \{0, 1\}$,
- $q_0 = \{q_1\}$,
- $F = \{q_2\}$

$\delta =$

	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	q_2	q_2

Language of the finite automaton:

- Input string: $w = w_1, w_2, w_3, \ldots \ldots, w_n,$
- Sequnce of states: $r_0, r_1, r_2, \ldots \ldots, r_n,$
- $r_0 = q,$
- For $i = 0, 1, 2, \ldots \ldots, n-1$: $r_{i+1} = \delta(r_i, w_{i+1}),$
- If $r_n \in F$: accept w.
- If $r_n \notin F$: reject w.
- Special case: if $n = 0 \quad \Rightarrow \quad w = \varepsilon \ \equiv Accept \leftrightarrow r_0 \in F$

The Language M: $L(M) = \{w\text{: } M \text{ accepts } w\}$

- Language A (set of strings over Σ) is regular if $\exists$ a finite automaton M such that $L(M) = A$.