Roll No. ..

1782

Printed Pages : 4

GSQ / M–19
MATHEMATICS
Paper–BM–361
Real and Complex Analysis

Time allowed : 3 hours] *[Maximum marks : 40*

Note : *Attempt **five** questions in all, selecting **one** question from each section. **Question No. 1 is compulsory.***

Compulsory Question

1. (a) Evaluate $\int_0^\infty x^3 e^{-x}\, dx$ 2

(b) Change the order of integration of $\int_0^a \int_{x^2/a}^{2a-x} xy\, dy\, dx$. 2

(c) Find the Fourier co-efficient a_n for the function

$f(x) = x \cos x \quad \text{in } [-\pi, \pi]$ 2

(d) Find the invariant points of the Mobius transformation $w = z^3$. 2

Section–I

2. (a) Prove that the functions

$u = \sin^{-1} x + \sin^{-1} y, \quad v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$

are functionally dependent. Also find the relation between them. 4

(b) Express $\int_0^1 x^m (1-x^n)^p\, dx$ in terms of Beta function and hence evaluate $\int_0^1 x^5 (1-x^3)^3\, dx$. 4

3. (a) Evaluate $\iiint_V x^2\, dx\, dy\, dz$, where V is the interior of sphere $x^2 + y^2 + z^2 = 1$. 4

(b) Show that the mass of an octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, the density at any point being $\rho = kxyz$ is $\frac{k a^2 b^2 c^2}{48}$. 4

Section–II

4. (a) Obtain a Fourier series expansion for the function $f(x) = |\cos x|$ in $(-\pi, \pi)$. 4

(b) Find the Fourier series expansion for the function f(x) in $(0, 2\pi)$ defined as :

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ 2\pi - x, & \pi < x < 2\pi \end{cases}$$

Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$$

5. (a) Find a series of cosines of multiples of x which will represent $f(x) = x \sin x$ in $(0, \pi)$. 4

(b) Find the Fourier series expansion for $f(x)$ if

$$f(x) = \begin{cases} 0, & -2 < x < -1 \\ k, & -1 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$$ 4

Section–III

6. (a) Prove that the function $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin. 4

(b) If $f(z)$ is a regular function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4 |f'(z)|^2$$ 4

7. (a) Show that the function $u(x, y) = e^x \cos y$ is harmonic. Determine its harmonic conjugate $v(x, y)$ and the analytic function $f(z) = u + iv$. 4

(b) Show that $f(z) = xy + iy$ is continuous everywhere but is not analytic. 4

Section–IV

8. (a) Find the image of the line $y - x + 1 = 0$ under the mapping $w = \frac{1}{z}$. 4

(b) Find the Mobius transformation which maps the points $z = 0, -1, i$ onto $w = i, 0, \infty$. Also find the image of the unit circle $|z| = 1$. 4

9. (a) Determine the region of the w-plane into which the region $\frac{1}{2} \le x \le 1$ and $\frac{1}{2} \le y \le 1$ is mapped by the transformation $w = z^2$. 4

(b) Find the Mobius transformation which maps the half plane $I(z) \ge 0$ into circle $|w| \le 1$. 4