Roll No. ..... **Total Pages : 05**

GSQ/M-20 **1745**

MATHEMATICS

BM-363

Dynamics

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

Compulsory Question

1. (a) A particle moves in a plane, its velocities parallel to the axes of x and y being $u + ey$ and $v + ex$ respectively. Show that it moves in a conic section. **1½**

(b) A particle moving with S.H.M. of period 12 seconds travels 10 cm from the position of rest in 2 seconds. Find the amplitude, the maximum velocity and the velocity at the end of 2 seconds. **2**

(c) A body of mass 25 gms is acted upon by a constant force. It acquires a velocity of 2 cm/sec. in 5 seconds from rest. Find, how large is the force acting. **2**

(d) Two balls are projected from the same point in directions inclined at 60° and 30° to the horizontal. If they attain the same height, what is the ratio of their velocities of projection ? **1½**

(e) Write Differential Equation of central orbit in polar form. **1**

Unit I

2. (a) Find the expressions for tangential and normal components of acceleration of particle moving along a plane curve. **4**

(b) A passenger travelling in a train with velocity 90 km/hr on a straight level track observes that another train which is 180 m long and moving constant takes 4 seconds to pass by. What is the velocity of passing train ? **4**

3. (a) A particle is describing S.H.M. of period T along a straight line. If v be its speed when at a distance x from the mean and a is the amplitude, show that $v^2T^2 = 4\pi^2\left(a^2 - x^2\right)$. **4**

(b) An elastic string of natural length l and modulus of elasticity λ has one end fixed at a point O on a smooth horizontal table. A particle of mass m is attached to the other end A pulled to the position B, where AB = a and then let go. Discuss the motion. **4**

Unit II

4. (a) A mass of 10 kg falls freely a distance of 10 m from rest and is then brought to rest after penetrating through 1 m in sand. Find the average force exerted by the sand on it. **4**

(b) Two masses m_1 and m_2 ($m_1 > m_2$) are suspended by a light inextensible and flexible string which passes over a smooth fixed and light pulley. To find the motion of the system, the tension in the string and pressure on pulley. **4**

5. (a) Show that in any displacement of a particle, the change in the K.E. is equal to work done by the impressed forces acting on the particle. **4**

(b) A train of mass M lbs is ascending a smooth incline of 1 in n and when the velocity of train is v ft/sec, its acceleration is f ft/sec^2. Prove that the effective horse power of engine is $\frac{\mathrm{M}v(nf+g)}{550\,ng}$. **4**

Unit III

6. (a) A small bead is projected with any velocity along a smooth circular wire under the action of force

varying inversely as the fifth power of distance from a centre of force situated on the circumference. Prove that pressure on wire is constant. **4**

(b) A particle is projected with velocity 'u' from the lowest point and moves along the inside of a smooth vertical circle. Discuss the motion. **4**

7. (a) A bomber is flying at a constant horizontal velocity of 210 km/hr at a height of 1000 metres above the ground towards the point directly above the target. At what angle of sight should a bomb be dropped so as to hit the target ? **4**

(b) If R is the maximum range on an inclined plane through the point of projection of a particle and T the corresponding time of flight, show that $R = \frac{1}{2} g T^2$. **4**

Unit IV

8. (a) Prove that central orbit is always a plane curve. **4**

(b) A particle describes the equiangular spiral $r = ae^{\theta \cot \alpha}$ under a force to the pole. Find the law of force. **4**

9. (a) If a planet were suddenly stopped in its orbit when at a distance '*a*' from the sun, show that it would fall in the sun in time $\frac{\sqrt{2}\pi a^{3/2}}{4\sqrt{\mu}}$ which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution. **4**

(b) A particle moves on a smooth sphere under no force except the pressure of the surface. Show that its path is given by the equation $\cot\theta = \cot\beta\cos\phi$, where θ and ϕ are its angular co-ordinates. **4**