Roll No. .... **Total Pages : 04**

GSQ/M-20 **1743**

MATHEMATICS
BM-361
Real and Complex Analysis

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

Compulsory Question

1. (a) Evaluate : **2**

$$\int_0^\infty e^{-a^2x^2}\,dx$$

(b) Find the coefficient of magnification and angle of rotation at $z = 3 + i$ for the conformal transformation $w = z^2$. **2**

(c) Show that the function : **2**

$$v(x, y) = e^{-x}(x \sin y - y \cos y)$$

is harmonic.

(d) Define Fourier series for even functions. **2**

Section I

2. (a) Show that the functions $u = x^2 + y^2 + z^2$, $v = xy - xz - yz$, $w = x + y - z$ are functionally dependent. Also find the relation connecting them. **4**

(b) Prove that : **4**

$$\int_0^\infty \frac{x^{m-1} - x^{n-1}}{(1+x)^{m+n}}\, dx = 0$$

3. (a) Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy \, dx$ by changing the order of integration. **4**

(b) Evaluate : **4**

$$\iiint_V z\left(x^2 + y^2 + z^2\right) dxdydz,$$

where $V = \left\{(x, y, z) : 0 \le z \le h, x^2 + y^2 \le a^2\right\}$.

Section II

4. (a) If the series $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$ converges uniformly to a function 'f' on $[-\pi, \pi]$, then prove that it is the Fourier series for 'f' on $[-\pi, \pi]$. **4**

(b) Find the Fourier series expansion of the function $f(x) = x - x^2$ in $[-\pi, \pi]$. **4**

5. (a) Obtain $f(x) = x$ as Half range sine series in $0 < x < 2$. **4**

(b) Find the Fourier expansion for the function : **4**

$$f(x) = \begin{cases} a & \text{for } 0 < x < \pi \\ -a & \text{for } \pi < x < 2\pi \end{cases}$$

Section III

6. (a) Determine the stereographic projection of the points $z = x + iy$ of extended complex plane on the sphere of radius $\frac{1}{2}$ and centre $\left(0, 0, \frac{1}{2}\right)$ in $\mathbf{R}^3$. **4**

(b) Prove that $f(z) = \overline{z}$ is nowhere differentiable but continuous everywhere in complex plane. **4**

7. (a) Show that the function $u(x, y) = x^3 - 3xy^2$ is harmonic and find the corresponding analytic function. **4**

(b) Prove that the function $f(z) = \begin{cases} \frac{(\overline{z})^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not analytic at the origin, although the Cauchy-Riemann equations are satisfied at that point. **4**

Section IV

8. (a) Determine the region in the w-plane corresponding to the region bounded by the lines $x = 0$, $y = 0$, $x = 2$, $y = 1$ in the z-plane mapped under the transformation $w = z + (1 - 2i)$. **4**

(b) Find the fixed points and normal forms of the Mobius transformation $w = \frac{z}{z-2}$. **4**

9. (a) Find the bilinear transformation which maps the points $z = 0, -1, i$ onto $w = i, 0, \infty$. Also, find the image of the unit circle $|z| = 1$. **4**

(b) Prove that the image of $|z + 3i| = 6$ under the transformation $f(z) = \frac{1}{z}$ is $u^2 + v^2 = \frac{1}{27}(1 - 6v)$. **4**