Independent Component Analysis

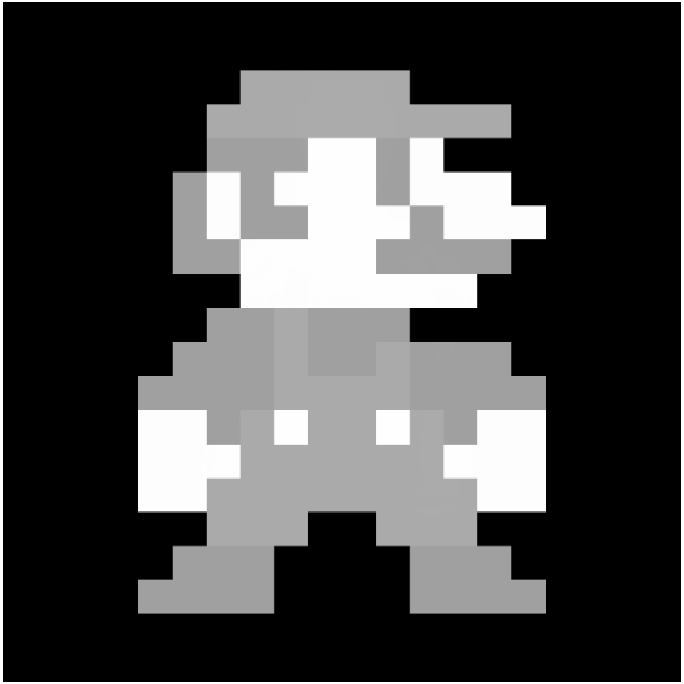
Course: Mathematical Algorithms for Artificial Intelligence

Yashpreet Goyal **Priyansh Abhishek Poddar** **Yash Sharma** **Shubhaditya B. Bechan**

Department of Artificial Intelligence and Machine Learning
R.V. College of Engineering, Bengaluru

January 26, 2026

STEGANOGRAPHY



Look at this image and tell what you can see?

Goal: Recover Independent Sources

We actually have hidden information in that simple looking image

The Cocktail Party Problem

- Multiple unknown source signals are mixed together
- We observe only the mixtures
- Goal: recover the original independent sources

The Cocktail Party Problem

- Multiple unknown source signals are mixed together
- We observe only the mixtures
- Goal: recover the original independent sources

Key Idea

Blind Source Separation using statistical independence

What is PCA?

Principal Component Analysis (PCA)

PCA is a dimensionality reduction technique that reduces the number of features in a dataset while preserving the most important information.

What is PCA?

Principal Component Analysis (PCA)

PCA is a dimensionality reduction technique that reduces the number of features in a dataset while preserving the most important information.

It transforms complex datasets by converting correlated variables into a smaller set of uncorrelated components called principal components.

WHAT IS PCA?

Principal Component Analysis (PCA)

PCA is a dimensionality reduction technique that reduces the number of features in a dataset while preserving the most important information.

It transforms complex datasets by converting correlated variables into a smaller set of uncorrelated components called principal components.

Key Limitation for ICA Problems

PCA removes second-order correlations, but does not ensure statistical independence — which is required for source separation.

What is Independent Component Analysis (ICA)?

Independent Component Analysis (ICA)

Independent Component Analysis (ICA) is a technique used to separate mixed signals into their independent, non-Gaussian components.

What is Independent Component Analysis (ICA)?

Independent Component Analysis (ICA)

Independent Component Analysis (ICA) is a technique used to separate mixed signals into their independent, non-Gaussian components.

Its aim is to find a linear transformation of the observed data that maximizes statistical independence among the recovered components.

What is Independent Component Analysis (ICA)?

Independent Component Analysis (ICA)

Independent Component Analysis (ICA) is a technique used to separate mixed signals into their independent, non-Gaussian components.

Its aim is to find a linear transformation of the observed data that maximizes statistical independence among the recovered components.

Core Idea

Unlike PCA, ICA looks beyond correlation and uses higher-order statistics to achieve true source separation.

Why ICA is Non-Trivial

- Mixing matrix A is unknown
- Source distributions are unknown
- Only assumption: statistical independence

Linear Mixing Model

We assume observed signals are linear mixtures of independent sources.

$$\mathbf{x}(t) = A\mathbf{s}(t) \tag{1}$$

- $\mathbf{s}(t)$ — unknown source vector
- A — unknown mixing matrix
- $\mathbf{x}(t)$ — observed mixtures

Linear Mixing Model

We assume observed signals are linear mixtures of independent sources.

$$\mathbf{x}(t) = A\mathbf{s}(t) \tag{1}$$

- $\mathbf{s}(t)$ — unknown source vector
- A — unknown mixing matrix
- $\mathbf{x}(t)$ — observed mixtures

Goal: Estimate an unmixing matrix W such that

$$\mathbf{y}(t) = W\mathbf{x}(t) \approx \mathbf{s}(t) \tag{2}$$

ICA Assumptions

ICA relies on three main assumptions:

1. Sources are statistically independent
2. At most one source is Gaussian
3. Mixing process is linear and stationary

ICA Assumptions

ICA relies on three main assumptions:

1. Sources are statistically independent
2. At most one source is Gaussian
3. Mixing process is linear and stationary

Why Non-Gaussianity Matters

Gaussian variables are fully described by mean and variance. Independence cannot be identified using only second-order statistics.

PCA vs ICA Comparison

Focus on Variance	Focuses on maximizing the variance	Doesn't focus on the issue of variance among the data points
Orthogonality	Focuses on the mutual orthogonality property of the principal components	Doesn't focus on the mutual orthogonality of the components
Independence	Doesn't focus on the mutual independence of the components	Focuses on the mutual independence of the components

Non-Gaussianity Principle

Central Limit Theorem Insight

A sum of independent random variables is more Gaussian than the original variables.

Non-Gaussianity Principle

Central Limit Theorem Insight

A sum of independent random variables is more Gaussian than the original variables.

Therefore:

- Mixed signals are more Gaussian
- Original sources are maximally non-Gaussian

ICA finds directions that maximize non-Gaussianity.

FastICA Algorithm

1. Center the data (zero mean)
2. Whiten the data (remove correlations)
3. Core Principles of ICA
4. Maximizing non Gaussianity using Kurtosis and Negentropy
5. Repeat until convergence.

Kurtosis

$$\text{kurt}(y) = \mathbb{E}[y^4] - 3(\mathbb{E}[y^2])^2 \quad (3)$$

- Zero for Gaussian variables
- Non-zero indicates non-Gaussianity

Negentropy

$$J(y) = H(y_{\text{gauss}}) - H(y) \tag{4}$$

- Based on information theory
- Always non-negative
- Zero only for Gaussian distributions

APPLICATIONS OF ICA

- Blind source separation (audio signals)
- EEG / MEG brain signal analysis
- Financial time-series separation
- Image feature extraction

Limitations

- Cannot determine scaling or order of sources
- Fails if multiple Gaussian sources exist
- Sensitive to noise and model mismatch

Summary

- ICA separates mixed signals using independence
- Relies on higher-order statistics
- Key idea: maximize non-Gaussianity
- Widely used in signal processing and ML