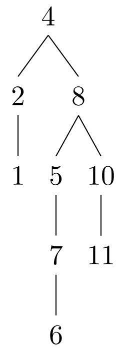
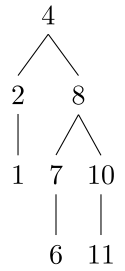
# 1 remove 函数的实现思路

我们使用一颗具体的二叉树来说明我们是如何实现 `remove()` 函数的.




按照提示，我们先构造了一个 `detachMin()` 函数用以查找以 t 为根的子树中的最小节点，返回这个节点，并从原子树中删除这个节点。显然，当要删除的节点具有两个子树时，通过这个函数返回的右子树最小节点将代替被删除节点。具体而言，我们如果需要删除以 `8` 为根的子树中的最小节点，`detachMin()` 函数会返回 `5`，并将 `7` 接到 `8` 的左节点处.

此时二叉树变成

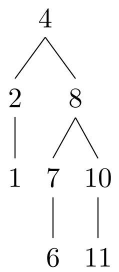



在理解了 `detachMin()` 的功能后，我们可以实现 `remove()` 的功能了，具体实现如下：

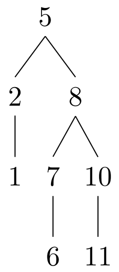
```
 BinaryNode *detachMin (BinaryNode *&t)
 {
if (t == nullptr) return nullptr;
if (t->left == nullptr)
{ BinaryNode *minNode = t;
t = t->right;
return minNode;
}
return detachMin(t->left);
 }
 void remove (const Comparable &x, BinaryNode *&t)
 {
 if (t == nullptr) return;
 if (x > t->element) remove(x, t->right);
 else if (x < t->element) remove(x, t->left);
 else
     {
     if (t->left == nullptr)
```

```
            {
            BinaryNode *oldNode = t;
            t = t->right;
            delete oldNode;
            }
        else if (t->right == nullptr)
            {
            BinaryNode *oldNode = t;
            t = t->left;
            delete oldNode;
            }
        else
            {
            BinaryNode *minNode = detachMin(t->right);
            minNode->left = t->left;
            minNode->right = t->right;
            delete t;
            t = minNode;
            }
        }
    }
```

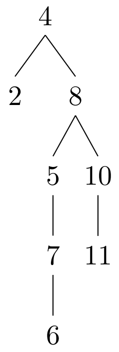
例如，当我们对前文的二叉树运行 `remove(4)` 时，`detachMin()` 函数先将二叉树化为



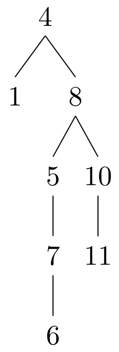

然后剩余的代码用 `5` 替代了根节点 `4`. 从而二叉树最后变成

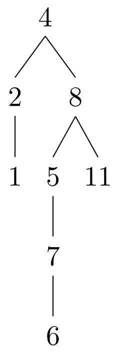


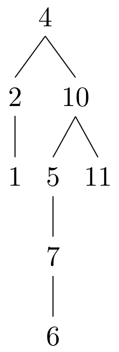

这仍然是一个 BST. 如果我们最初选择删除叶子 `1`，则二叉树变为




类似的，在最初分别选择删除 `2`（只有左子节点的节点）、`10`（只有右子节点的节点）`8`（有左右子节点的节点）后得到的二叉树分别为




## 2 测试程序

刚才的图示用代码表示如下

```cpp
void testBinarySearchTree() {
BinarySearchTree<int> bst;
```

```

    bst.insert(4);
    bst.insert(2);
    bst.insert(8);
    bst.insert(5);
    bst.insert(1);
    bst.insert(7);
    bst.insert(6);
    bst.insert(10);
    bst.insert(11);
    bst.printTree();
    BinarySearchTree<int> bst_1 = bst;
    BinarySearchTree<int> bst_2 = bst;
    BinarySearchTree<int> bst_3 = bst;
    BinarySearchTree<int> bst_4 = bst;
    BinarySearchTree<int> bst_5 = bst;
    BinarySearchTree<int> bst_6;

    std::cout << "Delete a node with no child : " << std::endl;
    bst_1.remove(1);
    bst_1.printTree();

    std::cout << "Delete a node with a left child : " << std::endl;
    bst_2.remove(2);
    bst_2.printTree();

    std::cout << "Delete a node with a right child : " << std::endl;
    bst_3.remove(10);
    bst_3.printTree();

    std::cout << "Delete a node with both right and left child : "<< std::endl;
    bst_4.remove(8);
    bst_4.printTree();
    std::cout << "Delete a node twice : "<< std::endl;
    bst_4.remove(8);
    bst_4.printTree();

    std::cout << "Delete a nonexisting node : "<< std::endl;
    bst.remove(111);
    bst.printTree();

    std::cout << "Delete a root node : "<< std::endl;
    bst_5.remove(4);
    bst_5.printTree();
```

```cpp

    std::cout << "Delete a node of an empty tree : " << std::endl;
    std::cout << "Before deletion : " << std::endl;
    bst_6.printTree();
    std::cout << "After deletion : " << std::endl;
    bst_6.remove(1);
    bst_6.printTree();
}
```

可以看到，我们还额外测试了对空树 remove() 的结果.

# 3 测试的结果

编译后，结果如下：

```
1
2
4
5
6
7
8
10
11
Delete a node with no child :
2
4
5
6
7
8
10
11
Delete a node with a left child :
1
4
5
6
7
8
10
11
Delete a node with a right child :
1
2
```

```
4
5
6
7
8
11
Delete a node with both right and left child :
1
2
4
5
6
7
10
11
Delete a node twice :
1
2
4
5
6
7
10
11
Delete a nonexisting node :
1
2
4
5
6
7
8
10
11
Delete a root node :
1
2
5
6
7
8
10
11
Delete a node of an empty tree :
```

```
Before deletion :
Empty tree
After deletion :
Empty tree
```

符合预期结果.