Study Unit 10

Activity 10-6

Write down the negations of the following in a useful form.

(a) The negation of $\forall x \in \mathbb{Z}^+, x > 3$:

$\neg (\forall x \in \mathbb{Z}^+, x > 3)$

$\equiv \exists x \in \mathbb{Z}^+, \neg (x > 3)$

$\equiv \exists x \in \mathbb{Z}^+, x \leq 3$

(b) The negation of $\exists x \in \mathbb{R}, 2x = x^2$:

$\exists x \in \mathbb{R}, 2x = x^2)$

$\equiv \forall x \in \mathbb{R}, \neg (2x = x^2)$

$\equiv \forall x \in \mathbb{R}, 2x \neq x^2$

(c) The negation of $\forall x \in \mathbb{Z}, (x > 0) \vee (x^2 > 0)$:

$\neg [\forall x \in \mathbb{Z}, (x > 0) \vee (x^2 > 0)$

$\equiv \exists x \in \mathbb{Z}, \neg [(x > 0) \vee (x^2 > 0)]$

$\equiv \exists x \in \mathbb{Z}, \neg (x > 0) \wedge \neg (x^2 > 0)$

$\equiv \exists x \in \mathbb{Z}, (x \leq 0) \wedge (x^2 \leq 0)$

(d) The negation of $\exists y \in \mathbb{Z}^+, (y \leq 10) \wedge (y \neq 0)$:

$\neg [\exists y \in \mathbb{Z}^+, (y \leq 10) \wedge (y \neq 0)]$

$\equiv \forall y \in \mathbb{Z}^+, \neg [(y \leq 10) \wedge (y \neq 0)]$

$\equiv \forall y \in \mathbb{Z}^+, \neg (y \leq 10) \vee \neg (y \neq 0)$

$\equiv \forall y \in \mathbb{Z}^+, (y > 10) \vee (y = 0)$

(e) The negation of $\exists x \in A, P(x) \wedge Q(x)$:

$\neg (\exists x \in A, P(x) \wedge Q(x))$

$\equiv \forall x \in A, \neg (P(x) \wedge Q(x))$

$\equiv \forall x \in A, \neg P(x) \vee \neg Q(x)$

(f) $\forall x \in \mathbb{Z}^+, (x \leq 3) \rightarrow (x^3 \geq 1)$

$\neg (\forall x \in \mathbb{Z}^+, (x \leq 3) \rightarrow (x^3 \geq 1))$

$\equiv \neg (\forall x \in \mathbb{Z}^+, \neg (x \leq 3) \vee (x^3 \geq 1))$

$\equiv \exists x \in \mathbb{Z}^+, \neg (\neg (x \leq 3) \vee (x^3 \geq 1))$

$\equiv \exists x \in \mathbb{Z}^+, \neg \neg (x \leq 3) \wedge \neg (x^3 \geq 1)$

$\equiv \exists x \in \mathbb{Z}^+, (x \leq 3) \wedge (x^3 < 1)$