Study Unit 4

Activity 4-6

Using if and only if statements, write out a proof in words for each of the following identities, where X, Y and W are arbitrary subsets of a universal set U:

1(a) $(X')' = X$

It seems we should be able to produce a cast-iron proof.

$x \in (X')'$
iff $x \notin X'$
iff $x \in X$.
Therefore $(X')' = X$.

(b) $X - (Y \cap W) = (X - Y) \cup (X - W)$

$x \in X - (Y \cap W)$
iff $x \in X$ and $x \notin (Y \cap W)$
iff $x \in X$ and $(x \in Y'$ or $x \in W')$
iff $(x \in X$ and $x \in Y')$ or $(x \in X$ and $x \in W')$
iff $x \in (X - Y)$ or $x \in (X - W)$
iff $x \in (X - Y) \cup (X - W)$

Thus $X - (Y \cap W) = (X - Y) \cup (X - W)$ for all subsets X, Y and W of U.

(c) $X \cap (Y \cap W) = (X \cap Y) \cap W$

$x \in X \cap (Y \cap W)$
iff $x \in X$ and $x \in (Y \cap W)$
iff $x \in X$ and $(x \in Y$ and $x \in W)$
iff $(x \in X$ and $x \in Y)$ and $x \in W$
iff $x \in (X \cap Y)$ and $x \in W$
iff $x \in (X \cap Y) \cap W$.

We can conclude that $X \cap (Y \cap W) = (X \cap Y) \cap W$ for all subsets X, Y and W of U.

(d) $X \cap (Y \cup W) = (X \cap Y) \cup (X \cap W)$

$x \in X \cap (Y \cup W)$
iff $x \in X$ and $x \in (Y \cup W)$
iff $x \in X$ and ($x \in Y$ or $x \in W$)
iff ($x \in X$ and $x \in Y$) or ($x \in X$ and $x \in W$)
iff ($x \in X \cap Y$) or ($x \in X \cap W$)
iff $x \in (X \cap Y) \cup (X \cap W)$.
Thus $X \cap (Y \cup W) = (X \cap Y) \cup (X \cap W)$.