Study unit 7

Activity 7-4

1. Write down the possible surjective functions from X to Y:

(b) X = {a, b} and Y = {c, d}:
To obtain a surjective function from X to Y, we must try to fill in, using all the members of Y, the template { (a,), (b,) }.
This can be done in two ways:
g_1 = { (a, c), (b, d) }
and g_2 = { (a, d), (b, c) }.

(c) X = {a, b} and Y = {c, d, e}:
The template
{ (a,), (b,) }
cannot be completed in a way that uses all the elements of Y, because Y has 3 members and there are only two gaps to be filled, so there are no surjective functions from X to Y in this case.

2. *Let f: $\mathbb{Z} \to \mathbb{Z}$ be defined by f(x) = x + 1.*

(a) *Determine f[$\mathbb{Z}$] (= ran(f)) . (Do not give specific examples.)*

$$\begin{aligned} f[\mathbb{Z}] &= \{y \mid \text{for some } x \in \mathbb{Z}, (x, y) \in f\} \\ &= \{y \mid \text{for some } x \in \mathbb{Z}, y = x + 1\} \\ &= \{y \mid \text{for some } x \in \mathbb{Z}, x = y - 1\} \\ &= \{y \mid y - 1 \text{ is an integer}\} \\ &= \mathbb{Z} \end{aligned}$$

(b) *Is f surjective? If f is not surjective, provide a counterexample to show why it is not surjective.*
f is surjective because the range of f is equal to the codomain of f: f[$\mathbb{Z}$] = $\mathbb{Z}$.

3. *Let g: $\mathbb{Z} \to \mathbb{Z}$ be defined by g(x) = 4x + 8.*
(a) *Determine g[$\mathbb{Z}$] (= ran(g)) . (Do not give specific examples.)*

$$\begin{aligned} g[\mathbb{Z}] &= \{y \mid \text{for some } x \in \mathbb{Z}, (x, y) \in g\} \\ &= \{y \mid \text{for some } x \in \mathbb{Z}, y = 4x + 8\} \\ &= \{y \mid \text{for some } x \in \mathbb{Z}, x = (y - 8)/4\} \\ &= \{y \mid (y - 8)/4 \text{ is an integer}\} \\ &\neq Z \end{aligned}$$

(b) *Is g surjective? If g is not surjective, provide a counterexample to show why it is not surjective.*

We give a counterexample: Let $y = 9$ ($9 \in \mathbb{Z}$), then $x = (y - 8)/4 = (9 - 8)/4 = ¼ \notin \mathbb{Z}$, so if $y = 9$, no

integer x can be found such that $(x, 9) \in g$. Thus $9 \notin g[\mathbb{Z}]$.

This means that $\mathbb{Z}$ is not a subset of $g[\mathbb{Z}]$ because each element of $\mathbb{Z}$ is not an element of $g[\mathbb{Z}]$, so $g[\mathbb{Z}] \neq \mathbb{Z}$. Thus g is not surjective because the range of g is not equal to the codomain.