Study Unit 6, sections 6.1 - 6-3

Activity 6-12

Determine whether P is a partition of X in each of the following cases. If so, describe the corresponding equivalence relation.

(a) $X = \{1, 2, 3\}$ and $P = \{ \emptyset, \{1\}, \{2, 3\} \}$:
A partition of X must consist of nonempty subsets of X.
So P is not a partition of X because $\emptyset \in P$.

(b) $X = \{1, 2, 3\}$ and $P = \{ \{1\}, \{2\}, \{1, 3\} \}$:
P is not a partition of X since $\{1\} \cap \{1, 3\} = \{1\} \neq \emptyset$.

(c) $X = \{1, 2, 3\}$ and $P = \{ \{1,3\}, \{2\} \}$:
P satisfies all the requirements to be a partition of X:
P is a collection of nonempty subsets of X,
and for each $x \in X$ there is some $Y \in P$ such that $x \in Y$,
and for all $Y, W \in P$, if $Y \neq W$ then $Y \cap W = \emptyset$.
The equivalence classes of the corresponding equivalence relation (that we call R) are:
$[2] = \{2\}$, so $(2, 2) \in R$, and
$[1] = [3] = \{1, 3\}$, so (1, 1), (3, 3), (1, 3) and (3, 1) must all be in R.
Therefore $R = \{ (1, 1), (2, 2), (3, 3), (1, 3), (3, 1) \}$.

(d) $X = \{1, 2, 3\}$ and $P = \{ \{1\}, \{2\} \}$:
P is not a partition of X because there is no $Y \in P$ such that $3 \in Y$.

(e) $X = \mathbb{Z}$ and $P = \{\{0\}, \mathbb{Z}^+, Neg\}$ where $Neg = \{x \mid x \in \mathbb{Z} \text{ and } x < 0\}$:
P is a partition of $\mathbb{Z}$ with the equivalence classes $\{0\}$, $\mathbb{Z}^+$ and Neg.
The corresponding equivalence relation is:
$\{(x, y) \mid (x = 0 \text{ and } y = 0) \text{ or } (x \in \mathbb{Z}^+ \text{ and } y \in \mathbb{Z}^+) \text{ or } (x \in Neg \text{ and } y \in Neg)\}$.

(f) $X = \mathbb{Z}$ and $P = \{ [0], [1], [2], [3], [4] \}$ where
$[0] = \{x \mid x - 0$ is divisible by 5 with zero remainder$\}$
$[1] = \{x \mid x - 1$ is divisible by 5 with zero remainder$\}$
$[2] = \{x \mid x - 2$ is divisible by 5 with zero remainder$\}$
$[3] = \{x \mid x - 3$ is divisible by 5 with zero remainder$\}$
$[4] = \{x \mid x - 4$ is divisible by 5 with zero remainder$\}$.
P is a partition of $\mathbb{Z}$. The reasons are:

- Every element of P is a nonempty subset of $\mathbb{Z}$. Each of them contains at least the representative given between square brackets.
- For all $Y, W \in P$, if $Y \neq W$, then $Y \cap W = \emptyset$, ie different classes do not have any elements in common.

No integer can be in two different sets Y, W $\in$ P, because no integer gives two different remainders on integer division by 5. (*Note*: If, say, x - 3 is divisible by 5 with zero remainder, then it means x itself leaves 3 as remainder when divided by 5.)

- For each x $\in \mathbb{Z}$, there is some Y $\in$ P such that x $\in$ Y, because, after all, any integer x will, when divided by 5, give a remainder of 0, 1, 2, 3 or 4. Subtracting this remainder from x results in a value which is divisible by 5 with zero remainder.

The corresponding equivalence relation is: $\{(x, y) \mid x - y = 5k$, or some integer $k\}$.