Study Unit 8

Activity 8-10

Two matrices A and B can only be multiplied if the sizes of A and of B match up in the following way:

Schematically: $\underset{m\times n}{A} \cdot \underset{n\times k}{B} = \underset{m\times k}{C}$

(equal)

Determine the following:

1. $$\begin{bmatrix} 31 & -3 & 2 \\ 2 & 5 & 1 \\ 3 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 10 \\ 0 \end{bmatrix}$$

2. $$\begin{bmatrix} 9 & 3 \\ 1 & 5 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 4 \\ 5 & 1 \end{bmatrix} = ?$$

This multiplication cannot be performed because the sizes of the matrices do not match up as explained above. Both matrices are 3×2.

3. $$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 6 & 4 \\ 3 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & -1 & 3 \\ 1 & 1/3 & 1 \\ 1/2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 8 & 0 \\ 8 & 22 & 6 \\ 3/2 & 12 & 9 \end{bmatrix}$$

4. *Provide examples of matrices X and Y such that XY is a 3×3 matrix but YX is a 2×2 matrix.*

Let X = $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$

and Y = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ for example, then

XY = $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$ and YX = $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$.

5. *Provide examples of matrices X and Y such that both X and Y contain at least some nonzero entries, but XY is the 2×2 zero matrix,*

ie $XY = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Take X = $\begin{bmatrix} 3 & 0 \\ 5 & 0 \end{bmatrix}$ and Y = $\begin{bmatrix} 0 & 0 \\ 6 & 2 \end{bmatrix}$ for example.

6. *Prove that addition is a commutative operation on the set of 2×2 matrices and that there is a 2×2 matrix that acts as an identity element in respect of addition.*

Let X = $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$ and Y = $\begin{bmatrix} s & t \\ u & v \end{bmatrix}$.

$$\text{Then} \quad X + Y = \begin{bmatrix} x+s & y+t \\ z+u & w+v \end{bmatrix}$$
$$= \begin{bmatrix} s+x & t+y \\ u+z & v+w \end{bmatrix}$$
$$= Y + X$$

Notice we use the commutativity of ordinary addition inside the matrix.

Let O be the identity element: O = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

7. *Prove that multiplication is **not** a commutative operation on the set of 2×2 matrices, and that there is a*
2×2 matrix that acts as an identity element in respect of multiplication.

Let X = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and Y = $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

Then XY = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ but YX = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Let I be the identity element: I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.