Study unit 2

Activity 2-8:

1. *Define the words "even" and "odd" for positive integers.*

Definitions:

- An integer n is *even* if n is a multiple of 2.

(We can say a positive integer n is even if n = 2k for some positive integer k. You can think of even positive integers as numbers n of the form n = 2k, where k is some positive integer.)

- An integer n is *odd* if n is not even.

(Using the general form of an even positive integer, we can now say that n is odd if n = 2k + 1 for some positive integer k. You can think of odd positive integers as numbers n of the form n = 2k + 1, where k is some positive integer.)

2. *Is it the case that m + (n·k) = (m + n)(m + k) for all positive integers m, n and k?*

Substitute a few values and see whether the idea is plausible.
Take m = 1, n = 2, and k = 3, then the left-hand side is
$m + (n \cdot k) = 1 + (2 \cdot 3) = 7$
while the right-hand side becomes
$(m + n)(m + k) = (1 + 2) \cdot (1 + 3) = 3 \cdot 4 = 12.$
This is a counterexample to show that it is not the case that m + (n·k) = (m + n) (m + k) for all positive integers m, n and k.

3. *Are there any even prime numbers besides 2?*

No. Any even prime number other than 2 would have three factors: 1, because 1 is a factor of every number; 2, because the number we are talking about is supposedly even; and the number itself, because a number is always a factor of itself. But primes cannot have so many factors, which means that 2 is the only even prime number.

4. *If m and n are even positive integers, is m + n even?*

If m and n are even positive integers, then each is a multiple of 2, in other words

$m = 2k$ for some $k \in \mathbb{Z}^+$.
and $n = 2j$ for some $j \in \mathbb{Z}^+$.
So $m + n = 2k + 2j$
$= 2(k + j)$
which means m + n is also even.

5. *If m and n are odd positive integers, is m·n odd?*

If m and n are odd positive integers, then both m and n can be written in the following general form:

$m = 2k + 1$ for some k in $\mathbb{Z}^+$.
and $n = 2j + 1$ for some j in $\mathbb{Z}^+$.
So $m \cdot n = (2k + 1)(2j + 1)$
$= 4kj + 2k + 2j + 1$
$= 2(2kj + k + j) + 1$

which means that $m \cdot n$ is odd .

An additional exercise:

If m and n are prime, is $m + n$ and $m - n$ prime?

No, not usually.

It can occasionally happen that $m + n$ is also prime: take $m = 3$ and $n = 2$ then $m + n = 5$.

But for other values $m + n$ may not be prime: take $m = 3$ and $n = 7$, for instance. $3 + 7 = 10$ which is not a prime number.

What about the difference between two prime numbers? The difference $m - n$ will sometimes be prime and sometimes not. E.g. $5 - 3 = 2$, which is a prime number, but $23 - 3 = 20$, which is not prime.