Study Unit 9

Activity 9-6

1. *Express the following sentence symbolically and then determine whether or not it is a tautology*:
If demand has remained constant and prices have been increased, then turnover must have decreased.

*Use **p** to represent the sentence 'demand has remained constant', let **q** represent 'prices have been increased' and let **r** represent 'turnover must have decreased',* then this sentence as a whole is represented by $(p \wedge q) \rightarrow r$.

To determine whether this is a tautology, one can compile a truth table. Sometimes, as in this case, there is also a faster way: one works backward from a truth value of F for the whole sentence and determines whether truth values for the sentences can be found to support it.

From our knowledge of the truth table of '$\rightarrow$', we know that
if $(p \wedge q) \rightarrow r$ is F then $p \wedge q$ is T and r is F, ie p is T, q is T and r is F.

So the sentence $(p \wedge q) \rightarrow r$ is not a tautology, because allocating the values mentioned above to p, q, and r would make the sentence as a whole false.

2. Refer to the truth tables in Activity 9-5. Determine whether each of the statements is a tautology, a contradiction or neither of the two.

(a) $[(\neg q) \rightarrow (\neg p)] \rightarrow (p \rightarrow q)]$ is a tautology. For all possible combinations of the truth values p and q,
$[(\neg q) \rightarrow (\neg p)] \rightarrow (p \rightarrow q)$ is true as can be seen from the final column of the table.
(b) $[\neg p \rightarrow (q \wedge (\neg q))] \rightarrow p$ is a tautology. An interesting observation can be made from the fourth column where all the truth values are F: The statement $q \wedge (\neg q)$ is a contradiction.

(c) $p \vee (\neg p)$ is a tautology. Feel free to practice your truth table technique on this one. However, here is a second method: For $p \vee (\neg p)$ to be F, both p and $\neg p$ have to be F, which is impossible, so $p \vee (\neg p)$ is always T.

(d) $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology.
In order for the statement as a whole to assume the value F, q must be F while
$p \wedge (p \rightarrow q)$ is T.

For $p \wedge (p \rightarrow q)$ to be T, p must be T and $p \rightarrow q$ must be T.
But since q is F, $p \rightarrow q$ can only be T if p is F, whereas we know p is T.
So it is impossible for $[p \wedge (p \rightarrow q)] \rightarrow q$ to be F.

(a) From the truth table it is clear that $(p \vee q) \wedge (\neg p \vee \neg q)$ is neither a tautology nor a contradiction.

Let p be T and q be F, then $(p \vee q) \wedge (\neg p \vee \neg q)$ is T, whereas if p is T and q is also T, then $(p \vee q) \wedge (\neg p \vee \neg q)$ is F.

(b) This is neither a tautology nor a contradiction. There are F and T truth values in the final column.

(g) The final column tells us that this is not a tautology. We also know that two statements p and q are **logically equivalent** iff the statement $p \leftrightarrow q$ is a tautology.

The two given statements are not logically equivalent because the eighth and ninth columns are not identical, ie $(p \rightarrow [\, q \wedge r \,])$ is not logically equivalent to $([\, p \rightarrow q \,] \vee [\, p \rightarrow r \,])$.

The left-hand side (of the biconditional, $\leftrightarrow$) does not have exactly the same truth value as the right-hand side.