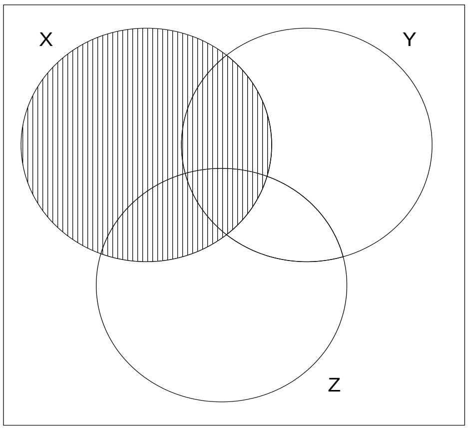
Study Unit 4

Activity 4-8

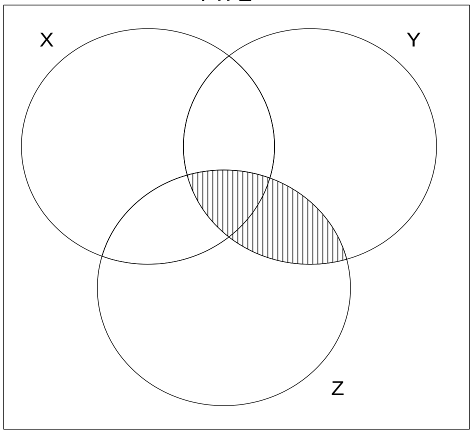
1. *Is it the case that for all X, Y, Z $\subseteq$ U, X + (Y $\cap$ Z) = (X + Y) $\cap$ (X + Z)?*

Determine the Venn diagram for X + (Y $\cap$ Z):

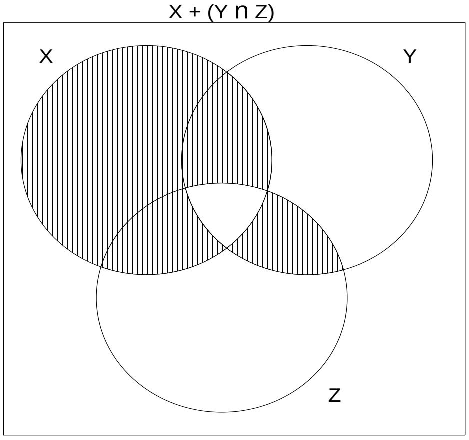
X



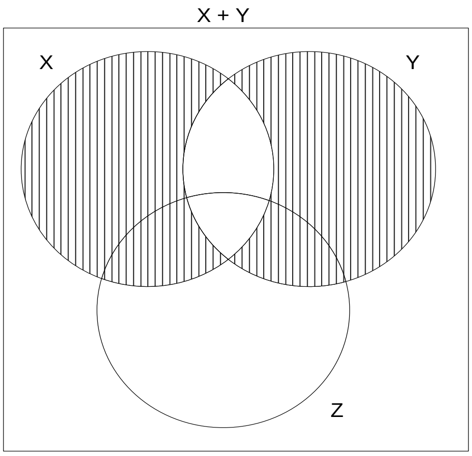
Y n Z

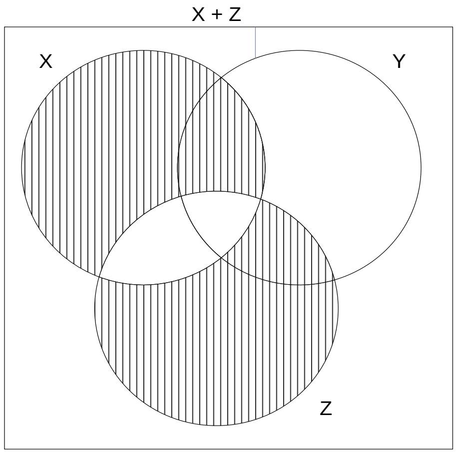


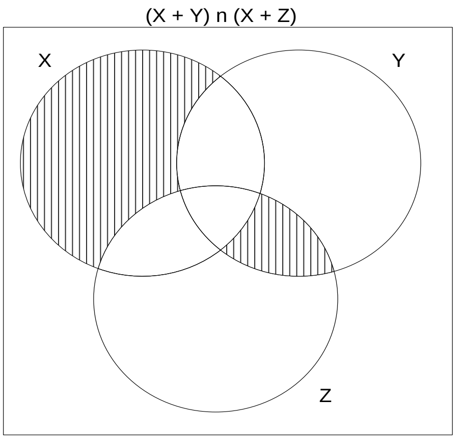
X + (Y n Z)



Determine the Venn diagram for (X + Y) $\cap$ (X + Z):







From the Venn diagrams it is clear that it is *not* the case that for all X, Y, Z $\subseteq$ U,
X + (Y $\cap$ Z) = (X + Y) $\cap$ (X + Z).

2. *Find examples of sets A and B such that $\mathcal{P}$(A $\cup$ B) is not a subset of $\mathcal{P}$(A) $\cup$ $\mathcal{P}$(B).*

This means we need to find a counterexample to show that it is not the case that $\mathcal{P}$(A $\cup$ B) is a subset of

$\mathcal{P}$(A) $\cup$ $\mathcal{P}$(B) for all sets A and B of U.

Let a universal set be U = {1, 2 } and let A = {1} and B = {2}.

$\mathcal{P}$(A) = **{** $\emptyset$, {1} **}**

$\mathcal{P}$(B) = **{** $\emptyset$, {2} **}**

$\mathcal{P}$(A) $\cup$ $\mathcal{P}$(B) = **{** $\emptyset$, {1}, {2}**}**

$\mathcal{P}$(A $\cup$ B) = $\mathcal{P}$({1, 2})
= { $\emptyset$, {1}, {2}, {1, 2}**}**

3. *Is it the case that, for all X, Y $\subseteq$ U, $\mathcal{P}$(X) $\cap$ $\mathcal{P}$(Y) = $\mathcal{P}$(X $\cap$ Y)? Justify your answer*.

S $\in$ $\mathcal{P}$(X) $\cap$ $\mathcal{P}$(Y)

iff S $\in$ $\mathcal{P}$(X) and S $\in$ $\mathcal{P}$(Y)

iff S $\subseteq$ X and S $\subseteq$ Y
iff the elements of S all belong to X and all belong to Y
iff the elements of S all belong to X $\cap$ Y
iff S $\subseteq$ X $\cap$ Y

iff S $\in$ $\mathcal{P}$(X $\cap$ Y).

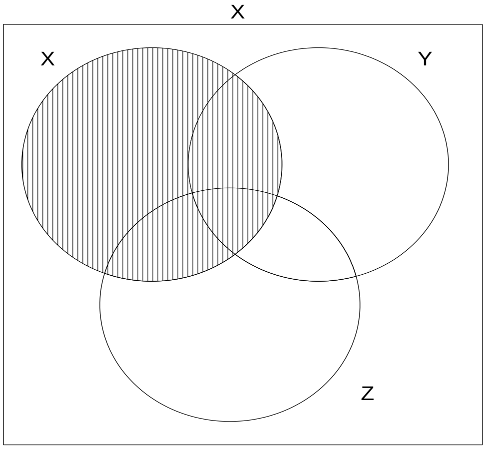
This proves that $\mathcal{P}$(X) $\cap$ $\mathcal{P}$(Y) = $\mathcal{P}$(X $\cap$ Y).

4. *Use Venn diagrams to investigate whether or not, for all sets X, Y, Z $\subseteq$ U, X − (Y − Z) = (X − Y) − Z. If the statement appears to hold, give a proof; if not, give a counterexample.*

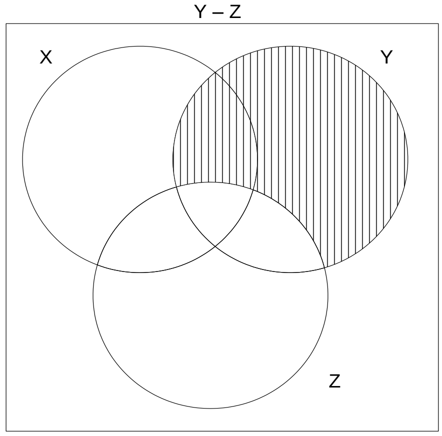
We draw the Venn diagrams as follows:

Left-hand side:

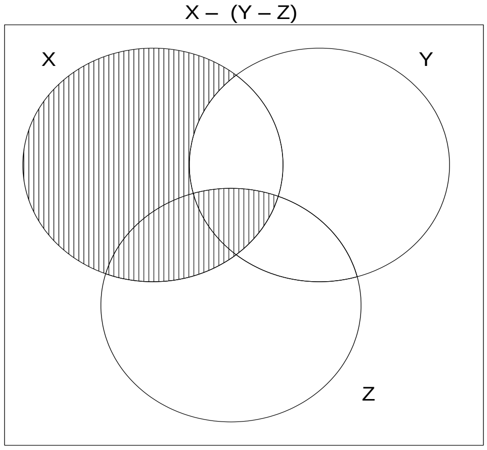
X



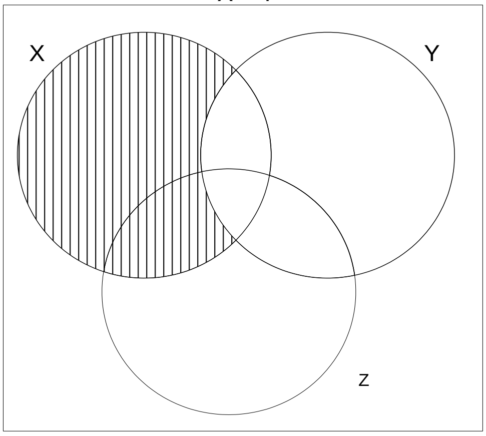
Y – Z

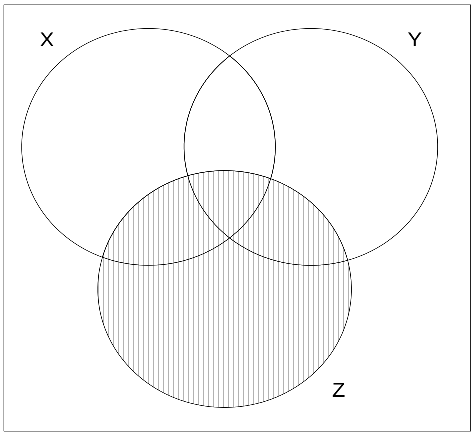


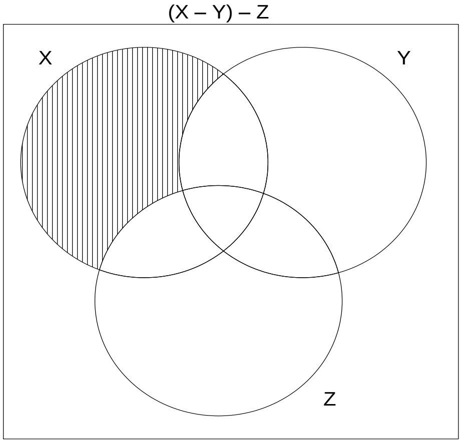
X – (Y – Z)



Right-hand side:







The shaded areas representing X − (Y − Z) and (X − Y) − Z differ, and therefore it seems as if the claim that

X − (Y − Z) = (X − Y) − Z is not always true. We have to give a counterexample with specific values for X, Y,

and Z. The two final Venn diagrams differ in the region $X \cap Z$, so we choose, for example, $1 \in X$ and $1 \in Z$.

Let X = {1, 2}, Y = {2, 3}, and Z = {1, 3},

then X − (Y − Z) = {1, 2} - {2} = {1}.

On the other hand, (X - Y) - Z = {1} - {1,3} = { }.

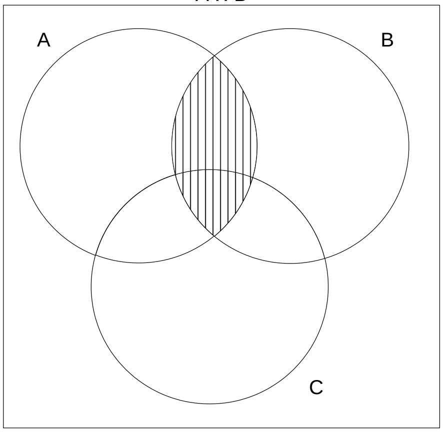
So, in this case, $X - (Y-Z) \neq (X-Y) - Z$.

5. *Use Venn diagrams to investigate whether or not, for all subsets A, B and C of U, $(A \cap B) + (C \cap A) = (A \cap B') \cup (B - C)$. If the statement appears to hold, give a proof; if not, give a counterexample.*

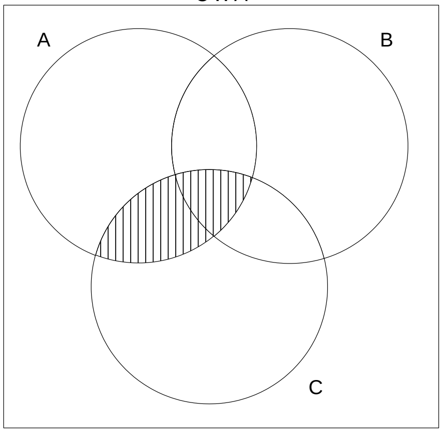
We draw the Venn diagrams as follows:

Left-hand side:

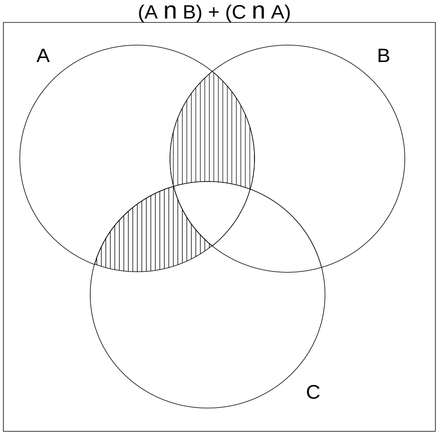
$A \cap B$



$C \cap A$

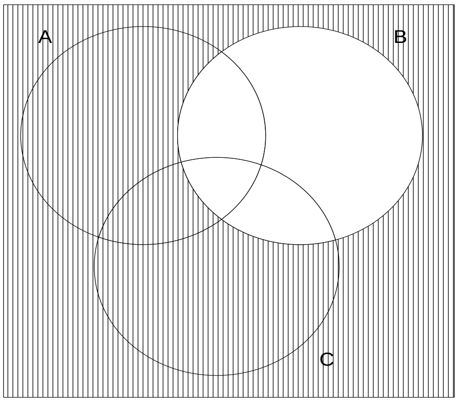


$(A \cap B) + (C \cap A)$

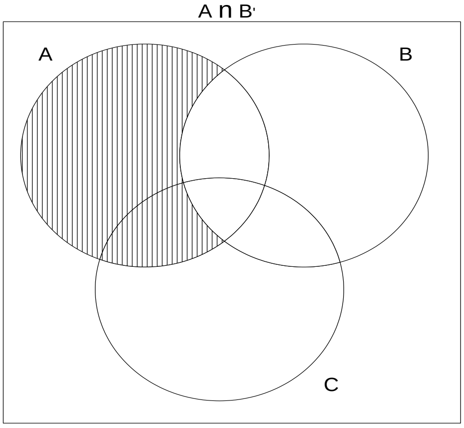


Right-hand side:

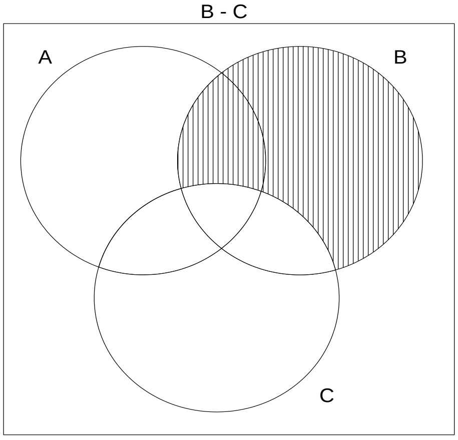
B'



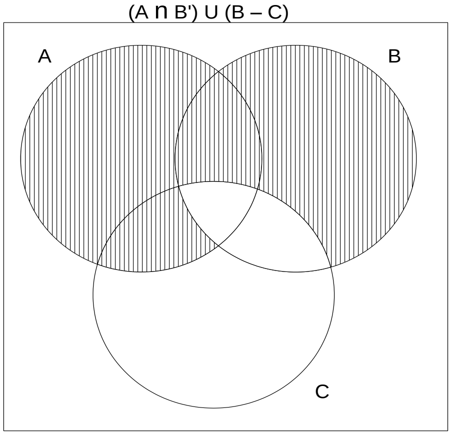
$A \cap B'$



$B - C$



$(A \cap B') \cup (B - C)$



It appears that the expression is not an identity, so we need a counterexample. That is, we want a concrete example of sets which shows that the left-hand side is different from the right-hand side.

Counterexample:

The final diagrams differ in some areas of B. We choose the element 2 that resides in set B only.

Let $A = \{1\}$, $B = \{1, 2\}$, $C = \{1, 3\}$ and $U = \{1, 2, 3\}$ with U as the universal set.

Determine B′:

$B' = U - B = \{3\}$

Now $(A \cap B) + (C \cap A)$ (Determine which members reside in either $A \cap B$ or $C \cap A$, but not in both $A \cap B$ and $C \cap A$.)

$= \{1\} + \{1\}$

$= \{\ \}$

while $(A \cap B') \cup (B - C)$ (Determine which members reside in $A \cap B'$ or $B - C$.)

$= \{\ \} \cup \{2\}$

$= \{2\}$.

In this example it is not the case that $(A \cap B) + (C \cap A) = (A \cap B') \cup (B - C)$.