Study unit 7

Activity 7-10

Determine $f \circ f$, $g \circ g$, $g \circ f$, and $f \circ g$ in the following cases:

(a) $f: Z \to Z$ is defined by the rule $f(x) = x + 1$ and $g: Z \to Z$ is defined by the rule $g(x) = x - 1$:

$f \circ f: Z \to Z$ is defined by $(f \circ f)(x)$:

$(f \circ f)(x)$

$= f(f(x))$
$= f(\mathbf{x + 1})$ (replace $f(x)$ by $\mathbf{x + 1}$)
$= (\mathbf{x + 1}) + 1$ ($f(\mathbf{x}) = \mathbf{x} + 1$, so $f(\mathbf{x + 1}) = (\mathbf{x + 1}) + 1$)
$= x + 2$.

Note: If you want to express what this means in words: if you feed $f \circ f$ an element x, it spits out the same thing you get if you feed x to f and then feed the result to f again.
First you feed **x** to f to get **x** + 1. Now feeding **x + 1** to f gives you (**x + 1**) + 1 = x + 2, because f takes anything you feed it and adds 1 to it.

$g \circ g: Z \to Z$ is defined by $(g \circ g)(x) = g(g(x))$

$= g(x - 1)$
$= (x - 1) - 1$
$= x - 2$.

$g \circ f: Z \to Z$ is defined by $(g \circ f)(x) = g(f(x))$

$= g(\mathbf{x + 1})$ (replace $f(x)$ by $\mathbf{x + 1}$)
$= (\mathbf{x + 1}) - 1$ ($g(\mathbf{x}) = \mathbf{x} - 1$, so $g(\mathbf{x + 1}) = (\mathbf{x + 1}) - 1$)
$= x$.

$f \circ g: Z \to Z$ is defined by $(f \circ g)(x) = f(g(x))$

$= f(x - 1)$
$= (x - 1) + 1$
$= x$.

(b) $f: R \to R$ is defined by $f(x) = 3x - 2$, and
$g: R \to R$ is defined by $g(x) = x^2 + x$:

$f \circ f: R \to R$ is defined by $(f \circ f)(x) = f(f(x))$

$= f(\mathbf{3x - 2})$ (replace $f(x)$ by $\mathbf{3x - 2}$)
$= 3(\mathbf{3x - 2}) - 2$ ($f(\mathbf{x}) = 3\mathbf{x} - 2$, so $f(\mathbf{3x - 2}) = 3(\mathbf{3x - 2}) - 2$)
$= 9x - 8$.

$g \circ g: R \to R$ is defined by $(g \circ g)(x) = g(g(x))$

$= g(x^2 + x)$
$= (x^2 + x)^2 + (x^2 + x)$
$= (x^2 + x)(x^2 + x) + (x^2 + x)$
$= x^4 + 2x^3 + x^2 + x^2 + x$
$= x^4 + 2x^3 + 2x^2 + x.$

$g \circ f: R \to R$ is defined by $(g \circ f)(x) = g(f(x))$

$= g(3x - 2)$
$= (3x - 2)^2 + (3x - 2)$
$= 9x^2 - 12x + 4 + 3x - 2$
$= 9x^2 - 9x + 2.$

$f \circ g: R \to R$ is defined by

$(f \circ g)(x) \quad = f(g(x))$
$\quad = f(x^2 + x)$
$\quad = 3(x^2 + x) - 2$
$\quad = 3x^2 + 3x - 2.$

(c) $f: \mathbb{Z}^{\geq} \to \mathbb{Z}^{\geq}$ is defined by $f(x) = 113$, and
$g: \mathbb{Z}^{\geq} \to \mathbb{Z}^{\geq}$ is defined by $g(x) = x + 1$:

$f \circ f: \mathbb{Z}^{\geq} \to \mathbb{Z}^{\geq}$ is defined by $(f \circ f)(x) = f(f(x))$

$= f(113) = 113.$

$g \circ g: \mathbb{Z}^{\geq} \to \mathbb{Z}^{\geq}$ is defined by $(g \circ g)(x) = g(g(x))$

$= g(x + 1)$
$= (x + 1) + 1$
$= x + 2.$

$g \circ f: \mathbb{Z}^{\geq} \to \mathbb{Z}^{\geq}$ is defined by $(g \circ f)(x) = g(f(x))$

$= g(113)$
$= 113 + 1$
$= 114$

$f \circ g: \mathbb{Z}^{\geq} \to \mathbb{Z}^{\geq}$ is defined by $(f \circ g)(x) = f(g(x))$

$= f(x + 1)$
$=113$. (because f does not care what you feed it is constantly going to spit out 113 nothing else).