Study Unit 9

Activity 9-9

1. *Rewrite $p \leftrightarrow q$ as a statement built up using only $\neg$, $\wedge$ and $\vee$:*

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$
$$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$$

2. *Refer to study unit 6 for the definition of equivalence relations. This kind of relation is reflexive, symmetric and transitive. Show that $\equiv$ is an equivalence relation*.

Let's think about the meaning of the symbol '$\equiv$'. In the context of this question, it states that $p \equiv q$ means that p is equivalent to q, where p and q are two statements.
It must be the case that $p \equiv p$ (p is equivalent to itself), so the relation is reflexive.
If $p \equiv q$, it is also the case that $q \equiv p$ (p and q are equivalent statements), so the relation is symmetric.
If $p \equiv q$ and $q \equiv r$, it must be the case that $p \equiv r$, so the relation is transitive.
This means that $\equiv$ is an equivalence relation on statements.

3. *Truth table for the exclusive OR (XOR)*:

p	**q**	**$p \oplus q$**
T	T	F
T	F	T
F	T	T
F	F	F

4.

p	**q**	**$\neg p$**	**$\neg q$**	**$p \vee \neg q$**	**$\neg (p \vee \neg q)$**	**$\neg p \wedge q$**
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	F	F

5. *Use the property of double negation and De Morgan's laws to rewrite the following statements so that the not symbol ($\neg$) does not appear outside parentheses:*

(a) $\neg [(p \vee q \vee \neg q) \wedge (q \wedge \neg p)]$

$\equiv \neg (p \vee q \vee \neg q) \vee \neg (q \wedge \neg p)$ *De Morgan's law*

$\equiv (\neg p \wedge \neg q \wedge \neg \neg q) \vee (\neg q \vee \neg \neg p)$ *De Morgan's law*

$\equiv (\neg p \wedge \neg q \wedge q) \vee (\neg q \vee p)$ *Double negation*

(a) $\neg [(p \vee (p \rightarrow q)) \vee (p \wedge q)]$

$\equiv \neg [(p \vee (\neg p \vee q)) \vee (p \wedge q)]$ *Refer to Study Guide, comment p 147, and second table p 148*.

$\equiv \neg (p \vee (\neg p \vee q)) \wedge \neg (p \wedge q)$

$\equiv (\neg p \wedge \neg (\neg p \vee q)) \wedge (\neg p \vee \neg q)$ De Morgan's law

$\equiv (\neg p \wedge (\neg\neg p \wedge \neg q)) \wedge (\neg p \vee \neg q$ De Morgan's law

$\equiv (\neg p \wedge (p \wedge \neg q)) \wedge (\neg p \vee \neg q)$ Double negation

6. *Determine whether or not the following statements are equivalent:* $\neg p \wedge (\neg p \wedge \neg q)$ *and* $\neg (p \vee (p \rightarrow q))$.

$\neg (p \vee (p \rightarrow q))$

$\equiv \neg (p \vee (\neg p \vee q))$ *Refer to Study Guide, comment p 147, and second table p 148*.

$\equiv \neg p \wedge \neg (\neg p \vee q)$ *De Morgan's law*

$\equiv \neg p \wedge (\neg\neg p \wedge \neg q)$ *De Morgan's law*

$\equiv \neg p \wedge (p \wedge \neg q)$ *Double negation*

Clearly the two given expressions are equivalent.