Study unit 1

Activity 1-11:

1. *Factorising:* If we need to factorise an expression of the form $x^2 + ax + b$ or ($x^2 – ax – b$ or $x^2 + ax – b$ or $x^2 – ax + b$) we need to find some c and some d such that $a = c + d$ and $b = (c)(d)$ and $(x + c)(x + d) = x^2 + ax + b$.

(a) $x^2 + 6x + 9 = x \cdot x + (3x + 3x) + (3)(3)$
$= (x + 3)(x + 3)$

(b) $x^2 – x – 2 = x \cdot x + (x – 2x) + (1)(-2)$
$= (x + 1)(x – 2)$

(c) $x^2 – 5x + 6 = x \cdot x – 2x – 3x + (-2)(-3)$
$= (x – 2)(x – 3)$

(d) $x^2 + 4x – 12 = x \cdot x – 2x + 6x + (-2)(6)$
$= (x – 2)(x + 6)$

2. *Solve $x^2 – 4x + 4 = 0$ by factorising.*

Well, at school we learned that:
the '+' in front of the last term means that our factorised version
$x^2 – 4x + 4$ will be either of the form $(x + ?)(x + ?)$ or of the form $(x – ?)(x – ?)$,
and that the '–' in front of the middle term tells us that our factorised version must be of the form
$(x – ?)(x – ?)$.

Experimenting with numbers that, when multiplied give us 4, we soon find that our factorised version has to be $(x – 2)(x – 2)$, or $(x – 2)^2$ if you prefer.
Our equation $x^2 – 4x + 4 = 0$ may therefore be rewritten as $(x – 2)^2 = 0$.
By Property 9, at least one of the factors on the left-hand side must be zero, and both factors are
$(x - 2)$, so we get that $x – 2 = 0$
ie that $x = 2$.

3. *Complete the square to solve $x^2 – 4x = 12$.*

If	$x^2 – 4x$	=	12	
then	$x^2 – 4x + 4 – 4$	=	12	(by Property 8, since $4 – 4 = 0$)
ie	$x^2 – 4x + 4$	=	$12 + 4$	(by Property 6 with $k = 4$)
ie	$(x – 2)^2$	=	16	(factorise)
ie	$x – 2 = 4$ or $x – 2$	=	–4	(taking square roots)
ie	$x = 6$ or x	=	–2	(using Property 6 again, with $k = 2$).

4. *Is 21 a prime number?*

No. Refer to the definition of prime numbers on p 16.
The numbers 3 and 7 are factors of 21 ($3 \times 7 = 21$).

5. *What is the value of 5! (5 factorial)?*
$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.