Study Unit 4

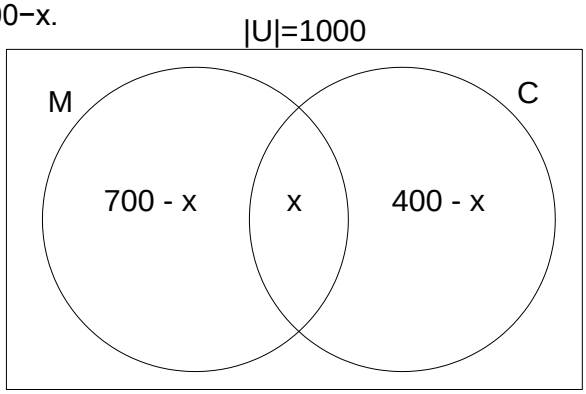
Activity 4-10

1. *Suppose that of 1000 first-year students, 700 take Mathematics, 400 take Computer Science and 800 take Mathematics or Computer Science.*

(a) *How many take Mathematics and Computer Science?*
Let U be the set of first-year students, M the set of those taking Mathematics, and C the set of those taking Computer Science. Then

$|U| = 1000$,
$|M| = 700$,
$|C| = 400$,
$|M \cup C| = 800$ and
$|M \cap C| = x$ (We do not know how many take Mathematics and Computer Science.)

By using this information, we can fill in the number of elements that reside in each region of the two sets, starting with the region in the middle that has x elements, and then $|M - C| = 700 - x$ and $|C - M| = 400 - x$.



We add the number of elements living in the three regions of the sets M and C, and since the total number of elements that reside in these regions is $|M \cup C| = 800$. We can determine x.

$|M - C| + x + |C - M| = 800$
ie $700 - x + x + 400 - x = 800$
ie $x = 300$, ie 300 students take Mathematics and Computer Science.

(b) *How many students take Mathematics but not Computer Science?*
$|M - C| = 700 - x = 700 - 300 = 400$.

(c) *How many students do not take any of the two subjects?*
There are 1000 students and 800 take Mathematics or Computer Science, so
$|(M \cup C)'| = |U| - |M \cup C| = 1000 - 800 = 200$ do not take any of the two subjects.

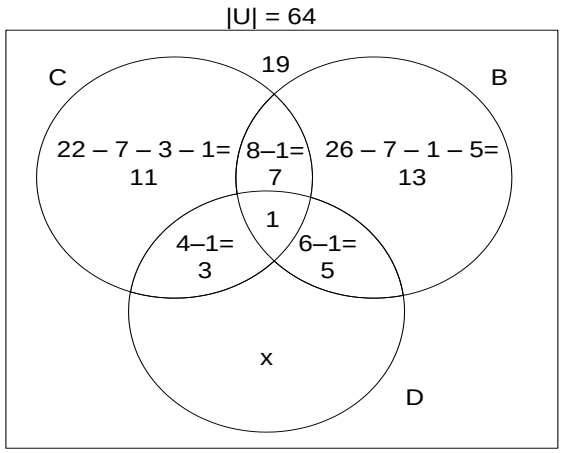
2. *A builder has a team of 64 construction workers. Of these, 45 are trained in the use of heavy machinery, ie cranes, bulldozers and backhoes. A total of 22 can operate cranes, 26 can operate backhoes, 4 can operate cranes and bulldozers, 6 can operate backhoes and*

bulldozers, 8 can operate cranes and backhoes, and 1 can operate all three kinds of machine. How many can operate bulldozers?

First we set out the available information neatly. Let U be the set of all workers in the team. Let C be the set of those who can operate cranes, B those who can operate backhoes and D those who can operate dozers. Then

$|U| = 64$, $|C| = 22$, $|B| = 26$,
$|D| = x$, the unknown we want to solve, ie those who operate bulldozers
$|C \cap D| = 4$,
$|B \cap D| = 6$,
$|C \cap B| = 8$,
$|C \cap D \cap B| = 1$,
$|C \cup D \cup B| = 45$ and
$|(C \cup D \cup B)'| = 64 - 45 = 19$.

Now we can fill in the various regions. We initially fill in x for the value of $|D - (B \cup C)|$.



$|C \cup B \cup D| = 45 = 11 + 7 + 1 + 3 + 5 + 13 + x$
ie $45 = 40 + x$
ie $x = 5$, ie 5 workers can operate dozers only.
Thus $3 + 1 + 5 + 5 = 14$ workers can operate bulldozers.

3. *A large software company employs 22 software engineers for the design of systems. Of these engineers, 17 are well versed in the secrets of a formal method (FM), 9 can use the Unified Modelling Language (UML), and 9 are familiar with the use of entity-relationship (ER) diagrams. If 5 engineers can use both an FM and UML, 4 both an FM and ER diagrams and 7 both UML and ER diagrams, answer the following:*

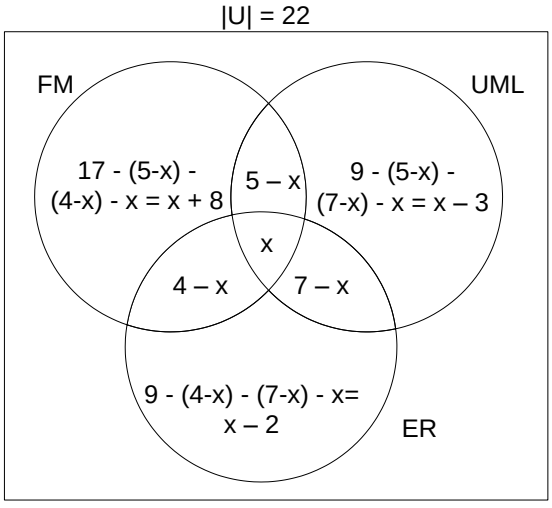
(a) How many engineers can use all 3 techniques, namely an FM, UML and ER diagrams?

Let U denote the number of software engineers. Let FM be the engineers who are well versed in the secrets of a formal method, UML those who can use the Unified Modelling Language and ER those who are familiar with the use of entity-relationship diagrams. Then

$|U| = 22$,
$|FM| = 17$,
$|UML| = 9$,
$|ER| = 9$,
$|FM \cap UML| = 5$,
$|FM \cap ER| = 4$,
$|UML \cap ER| = 7$ and
$|FM \cap UML \cap ER| = x$, ie the number of engineers who can use all 3 techniques.

Start by filling x into the intersection of the three circles. Then fill in the intersections of each pair of circles, e.g. 5 engineers can use both FM and UML, so insert $5 - x$ in the remaining overlap between FM and UML users, and so on ...

Now fill in the outstanding figures for each individual technique by subtracting the numbers already inside a particular circle from the total who uses that technique, e.g. in total, 9 use UML, so fill in: $9 - (5-x) - x - (7-x) = x-3$ in the open region of the UML circle, and so on …



Now let us solve for x:

$(x + 8) + (5 - x) + (x - 3) + x + (4 - x) + (7 - x) + (x - 2) = 22$

ie $x + 19 = 22$

ie $x = 3$

So 3 engineers can use all three techniques.

(b) *How many engineers can use UML only?*

$x - 3 = 3 - 3 = 0$, ie no engineer uses UML only.