Study Unit 10

Activity 10-3

1. *Write down the English equivalent of each of the following statements. Give an opinion on whether or not the statement is true.*

(a) $\exists\ y \in \mathbb{Q}, y = \sqrt{2}$

There exists some rational number y which is equal to $\sqrt{2}$.
This is not true, since $\sqrt{2}$ is not a rational number.

(b) $\forall\ x \in \mathbb{R}, 2x < x^2$

For all real numbers it holds that $2x < x^2$.
We give a counterexample to show that this statement does not hold.
Choose x = 0. $2 \cdot 0 = 0$ and $0^2 = 0$. In this case it does not hold that $2x < x^2$.

(c) $\forall\ x \in \mathbb{Z}, x > 0$

For all integers x, it holds that x > 0. The set $\mathbb{Z}$ includes all integers, so if we choose x = 0 or x equal any negative integer, the statement does not hold.

(d) $\exists\ x \in \mathbb{Z}^+, x = 0$

There exists a positive integer which is equal to 0.
The set $\mathbb{Z}^+$= {1, 2, 3, …}. The value 0 does not belong to $\mathbb{Z}^+$, so the statement is not true.