Study unit 3

Activity 3-6

1. Let $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$. Determine the required sets.

(a) $A \cup B = \{1, 2, 3, 4, 5\} = B \cup A$ (1, 2, 3, 4 and 5 are elements of A or B or both.)

(b) $A \cap B = \{3\} = B \cap A$ (3 is an element of both A and B)

(c) $A - B = \{1, 2\}$ (1 and 2 are elements of A but not of B)
$B - A = \{4, 5\}$ (4 and 5 are elements of B but not of A)

(d) $A + B = \{1, 2, 4, 5\} = B + A$ (1, 2, 4 and 5 are elements that belong either to A or to B but not to both)

2. *Let $U = \{a, e, i, o, u\}$, $A = \{i, o, u\}$ and $B = \{a, e, o, u\}$. Determine the following sets:*

(a) $A' = \{a, e\}$ (a and e are elements of U but not of A)
$(A')' = \{i, o, u\}$
$= A$

(b) $B' = \{i\}$
$(B')' = \{a, e, o, u\}$

(c) $A \cup B = \{a, e, i, o, u\}$
$(A \cup B)' = \emptyset$

(d) $A' \cap B' = \{a, e\} \cap \{i\}$
$= \emptyset$ (We can also refer to this set merely as $A' \cap B'$.)

(e) $A \cap B = \{o, u\}$
$(A \cap B)' = \{a, e, i\}$

(f) $A' \cup B' = \{a, e\} \cup \{i\}$
$= \{a, e, i\}$ (Can also be called $A' \cup B'$.)

(g) $A - B = \{i\}$
$B - A = \{a, e\}$

(h) $A \cap B' = \{i, o, u\} \cap \{i\}$
$= \{i\}$ (or $A \cap B'$)
$B \cap A' = \{a, e, o, u\} \cap \{a, e\}$
$= \{a,e\}$ (or $B \cap A'$)

(i) $A + B = \{i, o, u\} + \{a, e, o, u\}$

$= \{a, e, i\}$

$B + A = \{a, e, i\}$

3. *Let U = {1, 2, 3, 4, 5}, A = {3} and B = {{3}, 4, 5}. Determine $\mathcal{P}$(A) and $\mathcal{P}$(B).*

Note*: If the cardinality of some finite set C is n (ie |C| = n ≥ 0), then a total of 2^n subsets of C can be formed, so |$\mathcal{P}$(C)| = 2^n. In the case of B = {{3}, 4, 5}, B has 3 elements namely {3}, 4 and 5, so the power set of B, namely $\mathcal{P}$(B) has 2^3 = 8 elements.*

$\mathcal{P}$(A) = **{** ∅, {3} **}** (All the subsets of A = {3} are elements of $\mathcal{P}$(A).)

$\mathcal{P}$(B) = **{** ∅, {{3}}, {4}, {5}, {{3}, 4}, {{3}, 5}, {4, 5}, {{3}, 4, 5} **}**

Subsets of **B** are *members* of **$\mathcal{P}$(B).** We determine two *members* of **$\mathcal{P}$(B)**:

{3}, 4 and 5 are *members* of **B** = {{3}, 4, 5}.

We form subsets of B:

Keep the outside brackets of B then throw away the members 4 and 5 of B then we are left with the *subset* **{**{3}**}** of B which is then a *member* of **$\mathcal{P}$(B)**. (Note that {3} is **not** a member of $\mathcal{P}$(B).)

Keep the outside brackets of B then throw away the members {3} and 4 of B then we are left with the *subset* **{**5**}** of B which is then a member of $\mathcal{P}$(B). All the subsets of B are the members of $\mathcal{P}$(B).

4. *Let U = {a, e, i, o, u}, A = {i, o, u} and B = {a, e, o, u}. Determine the following sets:*

(a) $\mathcal{P}$(A) = **{** ∅, {i}, {o}, {u}, {i, o}, {i, u}, {o, u}, {i, o, u} **}**

$\mathcal{P}$(B) = **{** ∅, {a}, {e}, {o}, {u}, {a, e}, {a, o}, {a, u},
{e, o}, {e, u}, {o, u}, {a, e, o}, {a, o, u},
{a, e, u}, {e, o, u}, {a, e, o, u} **}**

(b) $\mathcal{P}$(A ∩ B) = $\mathcal{P}$({o, u}) = **{** ∅, {o}, {u}, {o, u} **}**

$\mathcal{P}$(A) ∩ $\mathcal{P}$(B) = **{** ∅, {o}, {u}, {o, u} **}**

(c) $\mathcal{P}$(A′) = $\mathcal{P}$({a, e}) = **{** ∅, {a}, {e}, {a, e} **}**

In order to be able to determine ($\mathcal{P}$(A))′, we need to determine $\mathcal{P}$(U) first.

$\mathcal{P}$(U) = **{** ∅, {a}, {e}, {i}, {o}, {u},
{a, e}, {a, i}, {a, o}, {a, u}, {e, i},
{e, o}, {e, u}, {i, o}, {i, u}, {o, u},
{a, e, i}, {a, i, o}, {a, o, u}, {a, i, u},
{a, e, o}, {a, e, u}, {e, i, o}, {e, o, u},
{e, i, u}, {i, o, u}, {a, e, i, o}, {a, e, i, u},
{a, e, o, u}, {a, i, o, u}, {e, i, o, u},
{a, e, i, o, u} **}**

($\mathcal{P}$(A))′ = **{** {a}, {e}, {a, e}, {a, i}, {a, o}, {a, u},
{e, i}, {e, o}, {e, u}, {a, e, i}, {a, i, o},
{a, o, u}, {a, i, u}, {a, e, o}, {a, e, u},
{e, i, o}, {e, o, u}, {e, i, u}, {a, e, i, o},
{a, e, i, u}, {a, e, o, u}, {a, i, o, u},
{e, i, o, u}, {a, e, i, o, u} **}**

(d) $\mathcal{P}$(A) ∪ $\mathcal{P}$(B)
= **{** ∅, {i}, {o}, {u}, {a}, {e}, {i, o}, {i, u}, {o, u}, {a, e}, {a, o}, {a, u},
{e, o}, {e, u}, {i, o, u}, {a, e, o}, {a, o, u}, {a, e, u}, {e, o, u}, {a, e, o, u} **}**

$\mathcal{P}$(A ∪ B) = $\mathcal{P}$({a, e, i, o, u})
= $\mathcal{P}$(U)