Study Unit 4

Activity 4-11

Prove the given sets equal.

1. $\{y \in \mathbb{Z}^+ \mid y \text{ is an even prime number}\} = \{u \in \mathbb{Z}^+ \mid u^2 = 4\}$

Proof:
If $x \in \{y \in \mathbb{Z}^+ \mid y$ is an even prime number $\}$
then $x \in \mathbb{Z}^+$ and x is an even prime number
ie $x \in \mathbb{Z}^+$ and $x = 2$ (since 2 is the only even prime number)
ie $x \in \{u \in \mathbb{Z}^+ \mid u^2 = 4\}$.

Conversely, if $x \in \{u \in \mathbb{Z}^+ \mid u^2 = 4\}$
then $x \in \mathbb{Z}^+$ and $x = 2$ (since $2 \in \mathbb{Z}^+$) (this excludes -2)
ie $x \in \mathbb{Z}^+$ and x is an even prime
ie $x \in \{y \in \mathbb{Z}^+ \mid y$ is an even prime$\}$.

2. $\mathcal{P}(\{0,1\}) = \{\emptyset\} \cup \{\{0\}\} \cup \{\{1\}\} \cup \{\{0,1\}\}$

Proof:
$S \in \mathcal{P}(\{0,1\})$
iff $S \in \{\emptyset, \{0\}, \{1\}, \{0,1\}\}$
iff $S = \emptyset$ or $S = \{0\}$ or $S = \{1\}$ or $S = \{0,1\}$
iff $S \in \{\emptyset\}$ or $S \in \{\{0\}\}$ or $S \in \{\{1\}\}$ or $S \in \{\{0,1\}\}$
iff $S \in \{\emptyset\} \cup \{\{0\}\} \cup \{\{1\}\} \cup \{\{0,1\}\}$.

Note: The following rules have to be applied when doing exercises 3 – 5.

Let *a* and *b* be two factors, then consider the following options:

(i) If $ab < 0$, i.e ab is a negative number,
then *a* is a negative number and *b* is a positive number (since a minus times a plus gives a minus)
OR
a is a positive number and *b* is a negative number (since a plus times a minus gives a minus)

(ii) If $ab > 0$, i.e ab is a positive number,
then *a* is a negative number and *b* is a negative number (since a minus times a minus gives a plus)
OR

a is a positive number and *b* is a positive number (since a plus times a plus gives a plus)

3. $\{x \in \mathbb{R} \mid x^2 + 6x + 5 < 0\} = \{x \in \mathbb{R} \mid -5 < x < -1\}$

Proof:

$y \in \{x \in \mathbb{R}\colon\ x^2 + 6x + 5 < 0\}$
iff $y \in \mathbb{R}$ and $y^2 + 6y + 5 < 0$
iff $y \in \mathbb{R}$ and $(y + 1)(y + 5) < 0$
iff $y \in \mathbb{R}$ and either $(y + 1 > 0$ and $y + 5 < 0)$ or $(y + 1 < 0$ and $y + 5 > 0)$
iff $y \in \mathbb{R}$ and either $(y > -1$ and $y < -5)$ or $(y < -1$ and $y > -5)$
iff $y \in \mathbb{R}$ and $-5 < y < -1$ (since there is no real number simultaneously less than -5 and greater than -1)
iff $y \in \{x \in \mathbb{R} \mid -5 < x < -1\}$

4. $\{x \in \mathbb{Z} \mid x^2 - 5x + 4 < 0\} = \{x \in \mathbb{Z}^+ \mid x \text{ is a prime factor of } 6\}$

Proof:

$y \in \{x \in \mathbb{Z} \mid x^2 - 5x + 4 < 0\}$
iff $y \in \mathbb{Z}$ and $y^2 - 5y + 4 < 0$
iff $y \in \mathbb{Z}$ and $(y - 1)(y - 4) < 0$
iff $y \in \mathbb{Z}$ and either $(y - 1 < 0$ and $y - 4 > 0)$ or $(y - 1 > 0$ and $y - 4 < 0)$
iff $y \in \mathbb{Z}$ and either $(y < 1$ and $y > 4)$ or $(y > 1$ and $y < 4)$
iff $y \in \mathbb{Z}$ and $1 < y < 4$
iff $y \in \mathbb{Z}$ and $y \in \{2, 3\}$
iff $y \in \mathbb{Z}^+$ and $y \in \{2, 3\}$ (since 2 and 3 are positive integers.)
iff $y \in \{x \in \mathbb{Z}^+ \mid x \text{ is a prime factor of } 6\}$

5. $\{x \in \mathbb{R} \mid x^2 + x - 2 > 0\} = \{x \in \mathbb{R} \mid x < -2 \text{ or } x > 1\}$

Note: There is a mistake in the exercise given in the study guide.

Proof:

$y \in \{x \in \mathbb{R} \mid x^2 + x - 2 > 0\}$
iff $y \in \mathbb{R}$ and $y^2 + y - 2 > 0$
iff $y \in \mathbb{R}$ and $(y - 1)(y + 2) > 0$
iff $y \in \mathbb{R}$ and either $(y - 1 < 0$ and $y + 2 < 0)$ or $(y - 1 > 0$ and $y + 2 > 0)$
iff $y \in \mathbb{R}$ and either $(y < 1$ and $y < -2)$ or $(y > 1$ and $y > -2)$
iff $y \in \mathbb{R}$ either $y < -2$ or $y > 1$
iff $y \in \{x \in \mathbb{R} \mid x < -2 \text{ or } x > 1\}$