Tutorial letter 102/0/2026

Linear Algebra

MAT1503

Year Module

Department of Mathematical Sciences

TUTORIAL RESOURCE FOR MAT1503

IMPORTANT INFORMATION:
This tutorial letter contains the tutorial activities for the module MAT1503

BAR CODE

Contents

TABLES

FIGURES

1. Contacts of academic staff involved in the module

Lecturers	Email	Telephone
Dr ZI Ali	alizi@unisa.ac.za	0116709163
Dr Katlego Sebogodi	sebogk@unisa.ac.za	0116709151
Prof Talat Nazir	talatn@unisa.ac.za	0116709287
Dr PP Ghosh	Ghoshpp@unisa.ac.za	
Prof A Adem	ademar@unisa.ac.za	

2. Contacts of learner support structures in Unisa

	Email	Telephone
Career Guidance		
Counselling		
Academic Literacy		

3. Contacts and physical locations of Unisa' learning centres

LEARNING CENTRES: PHYSICAL ADDRESS	CONTACTS: TUTORIAL SERVICES
UNISA POLOKWANE, Tutorial Services Office 23A Landros Mare' Street, Polokwane, 0742	☎ (015) 290 3417 ☎ (015) 290-3443
UNISA NELSPRUIT, Tutorial Services Office Standard Bank Centre**: 1st Floor**, 31 Brown Street, Nelspruit, 1201	☎ (013) 755 2476
UNISA PRETORIA HUB (THUTONG) Tutorial Service Centre Bld 14, Sunnyside Campus, Cnr Walker & Joubert Str., Pretoria 0003	☎ (012) 441 5868 / 5766
UNISA JOHANNESBURG Tutorial Services Office Old JSE Annekes Building 1 Kerk Street, Johannesburg, 2000	☎ (011) 630-4506
UNISA FLORIDA, Tutorial Services Office, F-Block, Cnr Christiaan de Wet/Pioneer Ave., Florida 0001	☎ (011) 471-3978
UNISA BENONI, 90 General Building, Elston Ave, Benoni, 1501	☎ (011) 421-6592 ☎ (011) 421-6593
UNISA DURBAN Tutorial Services Office, Kwazulu Natal 230 Stalwart Simelane Street, Durban, 4001	☎(031) 332-2202 ☎(031) 335-1751/49
UNISA PIETERMARITZBURG, Tutorial Services Office 1 Langalibalele Str, Pietermaritzburg, 3201	☎ (033) 355-1713
UNISA NEWCASTLE, Tutorial Services Office Cnr Sutherland and Harding Str, Newcastle, 2940	☎ (033) 355-1713
UNISA PAROW, Tutorial Services Office 15 Jean Simonis Street, Parow, 7499	☎ (021) 936-4146

UNISA MTHATHA, Tutorial Services Office 32 Cnr Victoria & York Rd Str, Economic Affairs Building, Umtata, 5100	☎ (047) 531-5002/6
UNISA EAST LONDON Tutorial Services Office, 10 St Lukes Road, Southernwood, East London, 5201	☎ (043) 743 9246
LEARNING CENTRES: PHYSICAL ADDRESS	**CONTACTS: TUTORIAL SERVICES**
UNISA PORT ELIZABETH Tutorial Services Office Cnr Hurd Street & 76 5th Avenue Newton Park; Port Elizabeth, 6045	☎ (041) 365 6650/6645
UNISA MAFIKENG Tutorial Services Office 29 Main Street Opposite ABSA Bank, Mafikeng Mafikeng, 2745	☎ (018) 381-6617/7318
UNISA RUSTENBURG Tutorial Services Office Forum Building (1st Floor) Cnr. OR Tambo & Steen Street Rustenburg, 0300	☎ (014) 594 8800/8856
UNISA POTCHEFSTROOM Tutorial Services Office, 20 Auret Street, Potchefstroom, 2531	☎ (018) 294 3362/41
UNISA KIMBERLEY Tutorial Services Office Shop 3 – Liberty Life Building, Chapel St, Kimberley, 8301	☎ (053) 832 6391
UNISA BLOEMFONTEIN Tutorial Services Office NRE House, 161 Zastron Street Bloemfontein, 9301	☎ (051) 411-0459/411 0440 ☎ (051) 430-4353/411 0452
UNISA KROONSTAD Tutorial Services Office NFS Building 1st floor, 36 Brand Street, Kroonstad, 9500	☎ (056) 213-2053/4

4. Word of Welcome

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Welcome to tutorials for **MAT1503.**

Unisa recognizes the importance of tutorials as one way we can provide you with quality teaching to increase your chances of succeeding in your studies. In order to make it easier for you to take advantage of the tutorials, you have the choice of attending tutorials in face-face classes at the learning center, or participating in e-tutoring on myUnisa.

Before you start your tutorials, please read carefully through this Tutorial Resource and through **Tutorial Letter 101** for **MAT1503**. The Tutorial Resource contains the

week-by-week activities which you will go through in the tutorials. The activities are designed to pace your studies in order to help you to complete your assignments, and to prepare you for the final examination.

Remember, this tutorial resource does not replace your study guide for module **MAT1503,** but complements it by providing interactive activities around the more difficult concepts in the module. Under the guidance of your tutor, you will go through some of the activities on your own, and some in collaboration with your fellow students. Please use the resource with close reference to other study materials in your study pack. It is important for you to understand the rules and your responsibilities with regard to the tutorials and to your assessment.

We wish you the best in your studies, and hope that the tutorials will enrich your learning experience.

Signed…… ……

Dr ZI. Ali
Lecturer for MAT1503

5. Word of caution

Your choice to study by distance is commendable because of the many advantages that studying by distance offers to you. However, although studying through distance learning should motivate you to learn to study independently, independent study may not be easy for you. Studying alone can at times be indeed a lonely experience, which may affect your success. This is because in the distance education environment, minimal, yet vital interaction takes place between yourself and your lecturers and between yourself and other students in the same module. This is the main reason why we consider tutorials as a vital element of your learning. Tutorials give you the opportunity to interact with others to exchange knowledge, with guidance by a mentor who understands the subject. You must therefore make every effort to participate in the tutorials.

Please note also that tutors mark your assignments, discuss the feedback with you, and use the formative assessment to help you to prepare for the final examination. Therefore, given also that the assignments contribute 20% to your final mark, it is extremely important that you participate in the tutorials and seize every opportunity to get assistance from the tutor.

Before and during each tutorial, ask yourself the following questions;

- *Have I prepared adequately for the tutorial by reading around the subject?*
- *Have I thought critically about my question or comment before I post it on myUnisa or before I pose it to others in a face-face class?*
- *Will I or have I completed (at the prescribed times) all the activities that are linked to the tutorial?*
- *Am I willing to share information, and not shy to engage in conversation with others?*
- *Am I using the correct language to communicate by writing or speaking formally, and not using shorthand or slang?*
- *Will I be known by others in the tutorial group for being a critical thinker?*
- *Will I be known by others to be respectful and considerate of their opinions?*

6. The choice between face-face and e-tutoring

In this module, we have given you the choice to attend face-face tutorials which are delivered by a tutor at a learning centre, or to participate in **e-tutorials** on myUnisa. You can opt for face-face or e-tutoring, or both. However, **we strongly recommend that you opt for e-tutoring** for the following reasons;

- Face-face tutors are not always available for the module at all centers, particularly at the small centers.
- There may be very few students to constitute a meaningful tutorial group, particularly in the small learning centers.
- Online tutorials have large groups with more interactive opportunities and access to online resources.
- Online tutorials give you flexible access times.
- It is easier, quicker and more secure to complete and submit your assignment on the online platform.
- Upon registration, you are automatically allocated to an e-tutorial group, which gives you more time to be active in the tutorials. On the other hand, you have to book for face-face tutorials by completing the tutorial booking form (**Annexure 1**) and submitting at the Unisa learning centre nearest to you, or book online on **http/www.f2ftutorialbooking…**

7. Time Frames for tutorials

One of the reasons why **MAT1503** is offered over a year is to create ample time for you to access tutorials. Tutorials are delivered over a period of 30-40 weeks during the year, depending on the date on which you register for the module (for online tutorials) and on the date on which you enrol for tutorials at a learning centre. Once a tutorial site is created on myUnisa, e-tutorials are open all the time, although the tutor is only live on specific days and times, depending on availability.

Depending on the subject content of the tutorial, a tutorial session typically takes approximately 2-3 weeks in an e-tutorial. You can be on the tutorial site myUNISA at any time. However, you are encouraged to maintain a regular presence on myUnisa (even if sometimes you do not do much more than browse through announcements and, or other students' conversations). The ideal is to spend an average of 2-3 hours per week on myUnisa

8. Navigating through the tutorial resource

Figure 1 shows the structure of a tutorial session.

Figure 1: Structure of a tutorial

Different icons are used to guide you through the different activities in a tutorial (**Figure 2**). The purpose of each segment of the tutorial is also explained. It is important that you understand the importance and purpose of each activity in the tutorial so that you can plan to make the best of each experience. Once you are used to the tutorial routine, you will find it easy to follow the tutorial schedule, which is important for you to prepare for the tutorials according to the different activities.

	***Ice breaker*** *This icon draws your attention to a social activity which is intended to loosen you and warm you up to the discussion. This part of the tutorial is a lighter moment of the tutorial which helps you to feel comfortable in interacting with others in the session.*
	***Outcomes and Assessment Criteria*** *This icon draws your attention to the learning outcomes of each tutorial unit. This part of the tutorial assists you to focus on the main concepts of the tutorial.*
	***Foundation Content*** *This icon draws your attention to the foundational reading resources which support the stated foundation outcomes. This part of the tutorial helps you to focus on the basic concepts which you need to master before you get to the more complex concepts.*

	***Common mistakes and misconceptions*** *This icon draws your attention to some of the most common mistakes around a concept which may be applicable to you. This part of the tutorial assists you to avoid the common mistakes and misconceptions which may slow your learning and may embarrass you as you engage with others*
	***Frequently asked questions*** *This icon draws your attention to some of the most frequently asked questions around the subject. This part of the tutorial assists you to focus on the subject and stimulates your thinking by looking at the subject from different angles.*
	***Glossary*** *This icon draws your attention to important terms and definitions in the subject. This part of the tutorial clarifies the key terms and definitions around the concept.*
	***Discussions*** *This icon draws your attention to an open discussion on the main concept of the tutorial. This part of the tutorial is the focal or main activity of the tutorial, where we exchange information and construct new knowledge together.*
	***Learning activity*** *This icon draws your attention to what you need to do practically, independently or with others in order to understand the concept better?* It *supports the learning of the concept from its foundations, and its application in relation to other, more complex concepts.*
	***Worksheets and Assignments*** *This icon draws your attention to your worksheets so that you can practice for assignments.. This part of the tutorial helps you to see whether or not you are achieving or showing potential to do well in your assignments.*
	***Additional Resources*** *This icon draws your attention to additional reading resources relevant to the tutorial. This part of the tutorial encourages you to read wider around the tutorial content to broaden your understanding of concepts.*
	***Reflection*** *This icon draws your attention to a checklist of what you have covered in the tutorials. This part of the tutorial helps you take stock of what you have learnt in the tutorial.*
	***Whats Next ?*** *This icon draws your attention to the content of the next tutorial. This part of the tutorial gives you opportunity to prepare for the next tutorial, and also provides continuity in the learning process.*

Figure 2: Signposts used for navigation through the tutorials

9. Guidelines for online participation and netiquette

In order for the tutorials to be effective, exciting and meaningful, we all need to follow certain ground rules. Before starting on tutorials, the tutor will ask you to discuss and

agree on a set of rules to be followed by everybody in the group. This is important for the tutorials to be a pleasant and useful experience for everyone. We refer to good behavior in tutorials as **netiquette.** You must therefore sincerely to commit to the netiquette, and respect it at all times! **Misconduct during tutorials can lead to expulsion from the tutorial group**.

Commitment by the student

During tutorials, I undertake to do my best to;

Share information with others.

Prepare to engage in meaningful discussions with others by prior study around the concept.

Complete all the recommended learning activities within the prescribed times.

Attend 75 % of the face-face tutorials or maintain a regular presence on myUnisa, an average of 3-4
hours per week

Pose at least three relevant questions in each face-face class or post at least one question or comment in appropriate sites every time I visit the tutorial site on myUnisa

Respect the university policy on communication which, (among other things) encourages the use of appropriate language, courtesy and respect for others.

Commitment by the Tutor

During Tutorials, I undertake to do my best to;

Nurture your independent and group learning skills

Create and maintain a friendly learning environment

Respond to your questions within 48 hours

Respect the confidentiality of any personal information

Conduct assessment without fear or favour

Respect the university policy on communication

In addition, for our mutual benefit, each of us needs to participate actively and make meaningful contributions during the tutorials. If you prefer online tutorials, your participation is considered satisfactory only if you are active in 75% of all discussions and complete the same percentage of the compulsory related learning activities and tasks. A minimum of 3 postings per discussion, plus responses to a minimum of 3 other students are required for a student to be recorded as active in a discussion.

If you opt to attend face-face tutorials, your participation is considered satisfactory only if you attend with participation in a minimum of 30 (75%) of the 40 hours allocated for tutorial in the module. You also need to complete the same percentage of the learning activities and tasks as students in the e-tutorials.

During the tutorials, always reflect on whether you are meeting your obligations as a member of a learning community. **Table 1** provides a summary of these obligations; your tutor will explain more on what you need to do during the tutorials.

Table 1: Active Participation in tutorials

Focus of the tutorial	Actions that describe meaningful participation in the tutorial
Foundation content	Foundation content is meant to refresh the basic knowledge you may need to understand higher concepts. Do not assume you know, read though all the recommended resources and test yourself using the self-assessment exercises. ***Remember, it is important to read and understand additional resources for you to score high in the journal of tutorial reflections***.
Common mistakes misconceptions	Identifying the common mistakes and misconceptions around a concept helps to ensure that everybody in the tutorial group starts off at the same level of understanding. Clearing misconceptions and avoiding common mistakes helps us to narrow discussions to key issues, and thereby accelerate learning. Therefore, before each tutorial, clear your mistakes and any misconceptions using the list provided. Thereafter, be prepared to make other mistakes or expose other misconceptions. Others may in fact share the same, and you can help each other in correcting them.
Frequently asked questions (FAQ)	Identifying the FAQ around a concept similarly helps to ensure the tutorial group starts off at the same level of understanding. Clearing your question against the list of FAQ also saves tutorial time. Before you ask a question, clear it by looking at the FAQ. We encourage you to ask questions in the public domain in the hope that it may help others too. However, questions you may not feel comfortable to ask in an open discussion such as in a face-face class or in the discussions forums on myUnisa can be asked in private consultations with a face-face tutor, or by posting in the Q & A tool on myUnisa.
Discussions	Read around before engaging in a discussion so that you can engage in meaningful conversations with others. In the discussions on myUnisa, posting your own topics **[only when invited to open a discussion topic]** is a good way to direct the discussion in your favour. Expressing your comments or opinions allows the tutor to identify gaps in your knowledge. The tutor does not always give you a straight answer, but will ask you probing or prompting questions to help you to think through the question. Stay on a discussion until it's closed by the tutor. Be known as a critical thinker. ***Remember, it is important for you to make the required minimum number of contributions in each discussion for you to score high in the journal of tutorial reflections.***

Assignments & feedback	As a rule, except if they are group activities, assignments are not discussed before the due date, but after. Once the discussion on an assignment is open, look critically at the feedback to your assignments regardless of the mark you get, and ask more questions if necessary. The tutor will always probe your mastery of concepts through more questions.
Self-assessment/ Learning activities	Do as many of the learning activities provided in the tutorials as possible. Do them as well as if you are doing an assignment. This helps the tutor to identify knowledge gaps and to suggest appropriate additional learning activities tailor-made to address your specific needs. ***Remember, it is important to do the activities diligently for you to score high in the journal of tutorial reflections.***
Reflection on the tutorial session	Do not go out of a tutorial session with unanswered questions. Use the checklist, and further test your knowledge in the reflective discussions with your peers.
Additional (including online) learning resources	Look at all recommended learning resources and take notes. It is important to read and understand all the resources recommended for further reading for you to score high in the journal of tutorial reflections

We believe the tutorials are most meaningful if members of the group all believe in the power of interactive or collaborative learning;

[1]The Power of Interactive Learning

What I hear, I forget.

What I hear and see, I remember a little.

What I hear, see, and ask questions about or discuss with someone else I begin to

understand.

What I hear, see, discuss, and do, I acquire knowledge and skill.

What I teach to another, I master."

[1] Silberman, M. 1996. Active learning: 101 Strategies to Teach any Subject. Boston: Allyn and Bacon.

10. Tutorial Sessions

This section onwards contains your tutorials. The resource contains an orientation session, [inset number] subject-based tutorial sessions, and an exam preparation session. Please note that the module **MAT1503** is very important and practical. The module is an introduction of linear algebra which is an indispensable tool in many areas of science. To make it a real-life learning experience and therefore make easier for you to master the practical skills, the tutorials are designed to assist you to apply the knowledge on practical level. The tutorial resource is therefore structured around questions from a recommended book.

10.1 Orientation

10.1.1 Tutorial 0: Orientation – Linear Algebra [1 = weeks)] [2 = hours)

Ice breaker

Introduce yourself, and tell us the following things about yourself:

1. *Who you are*
2. *Name / Province/town/village*
3. *Why you have enrolled for this module and what you know about linear algebra?*
4. *Share your contact details (phone, social media addresses) of you are willing to be part of a study group.*

Frequently asked questions

1. *How much time would we spend each unit?*
   Each unit is compiled to include a lot of discussion and interesting information on the relevant topic, with two or three worksheet to complete per unit.
2. *How many units are there?*
   There are 13 study units and an Exam preparation spread out throughout the year.
3. *What do we cover?*
   *The units correspond with the **MAT1503** Study Guide for ease of reference, and cover the whole module in week-by-week steps.*

Competency Test [if in the tutorial plan]

10.2 Study Guide Unit 1: Introduction to Systems of Linear Equations

10.2.1 Tutorial 1: Introduction to Systems of Linear Equations [1 = week] [2 = hours]

Ice breaker :

Visit the following link and choose the video that best describe the systems of linear equations:

- *http://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring2010/video-lectures/*

Outcomes and Assessment Criteria:

1. Outcomes:

You should be able to

- *Identify and analyse a linear equation*
- *Find the augmented matrix of a system of linear equations*
- *Determine whether a given sequence of numbers or a given element is a solution of a linear equation (or system of linear equations)*
- *Determine algebraically or geometrically whether a system of linear equations in 2 unknowns has no solution, exactly one solution or infinitely many solutions*
- *State the three elementary row operations*
- *Find the solution of a linear system of equations using elementary row operations*

2. Assessment Criteria:

- *A linear equation is identified*
- *The augmented matrix is obtained from a given system of linear equations*
- *The augmented matrix is solved using elementary row operations*

Foundation Content:

Basic concepts of system of linear equations in the following websites:

- *http://www.analyzemath.com/Tutorial-System-Equations/TutorialSystem-Equations.html*
- *http://www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/int_alg_tut19_systwo.htm*
- *http://www.algebra.com/algebra/homework/coordinate/Linearsystems.faq*

Common mistakes and misconceptions:

- *Adding up the variables in the equations as they are, instead of making them cancel out*
- *Not ensuring that one of the variables cancel out when multiplying*

- *Making arithmetical mistake when solving the equation (Hint: substitute your answers back into the system of equations, they should satisfy both equations otherwise the solution is incorrect)*
- *Re-arranging the equation and then substituting it back into itself. This will make everything cancel out.*

Frequently asked questions:

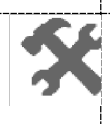
- *Is there a particular method I should use to solve the linear equations?*

Glossary:

- *A "system" of equations is a set or collection of equations that you deal with all together at once*
- *A linear equation is an equation of the form $Ax + By = C$, where $A \neq 0$ and $B \neq 0$. The graph of a linear equation is a straight line.*

Discussions:

1. *Generalize the case where there are three straight lines in the plane defined by three linear equations. What if there are n lines defined by n equations?*
2. *Consider a system composed of two linear equations in two variables. Can the sys-tem have exactly two solutions? Exactly three solutions? Exactly a finite number of solutions?*
3. *Suppose at least one of the equations in a system composed of two equations in two variables is nonlinear. Can the system have no solution? Exactly one solution? Exactly two solutions? Exactly a finite number of solutions? Infinitely many solutions? Illustrate each answer with a sketch.*



Learning activity:

Attempt the following problems: ***Anton****, Exercise Set 1.1, Questions 1, 2, 3, 7, 12, and 14. Pages 20-24 (from e-book, 10th Edition).*

Worksheets and Assignments

Test yourself by attempting tutorial- related questions given on the worksheets that you can find in the following webpages:

- *http://www.math-aids.com/Algebra/Pre-Algebra/Systems/*
- *http://edhelper.com/LinearEquations.htm*
- *http://www.mathworksheetsgo.com/sheets/algebra/systems-of-linear-equations/*
- *http://cdn.kutasoftware.com/Worksheets/Alg1/Systems%20of%20Equations%20 Word%20Problems.pdf*

The assessment related to this session is Assessment one. See the tutorial letter for details.

Additional Resources

- *Ayres, Frank: Schaum's Outline of Theory and Problems of Matrices, McGraw-Hill, New York, 1974*
- *Cullen, Charles G.: Matrices and Linear Transformations, Addison-Wesley, Reading, MASS., 1972*
- *Johnson, Lee W.: Introduction to Linear Algebra (2 nd or earlier editions), Addison-Wesley, Reading, MASS., 1989*
- *Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.*
- *Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.*
- *Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974*
- *Nicholson, W.K.: Linear Algebra with Applications (3-rd edition), PWS Publishing Company, Boston.*
- *Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co.,Belmont, CA., 1991.*
- *Anton, Howard and Rorres, Chris:Elementary Linear Algebra; Applications Version,(10th edition, 2011), John Wiley & Sons, Inc*

Reflection

This was the introduction to systems of linear equations. Specific methods of solutions will be explored in the next unit.

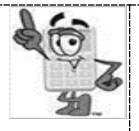
What's next?

The next tutorial is devoted to some techniques which will be used to solve systems of linear equations.

10.3 Study Guide Unit 2: Gaussian Elimination

1.1.1 Tutorial 22: Gaussian Elimination [2 = weeks] [2 = hours]

10.3.1



Ice breaker:

Visit the following links and choose the video that best describe the Gaussian elimination:

- *http://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring2010/video-lectures/*
- *https://www.youtube.com/watch?v=Ncu9Pks3AJQ*
- *https://www.youtube.com/watch?v=ybQbWYBEZzc*

Outcomes and Assessment Criteria:

1. Outcomes:

You should be able to

- *Identify matrices that are in row-echelon form, reduced row-echelon form, or generalized row-echelon form*
- *Solve a linear system by using Gauss-Jordan elimination (i.e. by reducing the augmented matrix to reduced row-echelon form)*
- *Solve a linear system by using Gaussian elimination (i.e. by reducing the augmented matrix to row-echelon form)*
- *Solve a linear system by reducing the augmented matrix to generalized row-echelon form*
- *Determine if/when a linear system has no solution, exactly one solution or infinitely many solutions*
- *Determine if/when a linear system of homogeneous equations has only the trivial solution (i.e. only one solution) or the trivial as well as nontrivial solutions (i.e. infinitely many solutions).*

2. Assessment Criteria:

- *Matrices in either row-echelon form, reduced row-echelon form or generalized row-echelon form are identified*
- *The augmented matrix of a system of linear equations is solved using the Gaussian and the Gauss-Jordan elimination methods*
- *A system of linear equations is analyzed to determine when it has no solution, exactly one solution, or infinitely many solutions*
- *A homogenous system of linear equation is analyzed to check whether it has only the trivial solution or infinitely many solutions in addition to the trivial solution*

Foundation Content:

The Gaussian elimination methods are used for solving systems of linear equations

Common mistakes and misconceptions:

- *Adding Rows after multiplying one of them by a real number*
- *Selecting the wrong pivot element.*
- *Pivoting on the right hand side*

Frequently asked questions:

- *What is a good way to implement Gaussian elimination when the operators are custom operators, rather than standard arithmetic ones?*
- *What is the difference between Gauss-Jordan elimination and Gaussian elimination?*

Glossary:

- *A matrix is a rectangular array of numbers or other mathematical objects, for which operations such as addition and multiplication are defined.*

Discussions:

- *Can the phrase "a nonzero constant multiple of itself" in a type-2 row operation be replaced by "a constant multiple of itself"? Explain*
- *Can a row of an augmented matrix be replaced by a row obtained by adding a constant to every element in that row without changing the solution of the system of linear equations? Explain.*

Learning activity:

*Attempt the following problems: **Anton**, Exercise Set 1.2, Questions 15, 19, 23, 27, 28, 29, and 30. Pages 76-77 (from e-book, 10th Edition).*

Worksheets and Assessments

Test yourself by attempting tutorial- related questions given on the worksheets that you can find in the following webpages:

- *http://www.math-aids.com/Algebra/Pre-Algebra/Systems/*
- *http://edhelper.com/LinearEquations.htm*
- *http://www.mathworksheetsgo.com/sheets/algebra/systems-of-linearequations/*
- *http://www.pkwy.k12.mo.us/west/teachers/mooney/Finite_Math/finitehw022712.pdf*
- *http://www.personal.soton.ac.uk/jav/soton/HELM/workbooks/workbook_8/8_3_gauss_elim.pdf*
- *http://lhsblogs.typepad.com/files/gauss-jordan-eliminationworksheet.pdf*
- *http://www.math.uiuc.edu/~wgreen4/Math124S07/Exam%203%20Worksheet.pdf*

The assessment related to this session is Assessment one. See the tutorial letter for details.

Additional Resources

- *Ayres, Frank: Schaum's Outline of Theory and Problems of Matrices, McGraw-Hill, New York, 1974*
- *Cullen, Charles G.: Matrices and Linear Transformations, Addison-Wesley, Reading, MASS., 1972*
- *Johnson, Lee W.: Introduction to Linear Algebra (2-nd or earlier editions), Addison-Wesley, Reading, MASS., 1989*
- *Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.*
- *Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.*
- *Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974*
- *Nicholson, W.K.: Linear Algebra with Applications (3 rd edition), PWS Publishing Company, Boston.*
- *Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co.,Belmont, CA., 1991.*
- *Anton, Howard and Rorres, Chris:Elementary Linear Algebra; Applications Version,(10th edition, 2011), John Wiley & Sons, Inc*

Reflection

This was the Gaussian elimination method.

What's next?

The next tutorial is devoted to the study of matrices as objects in their own right. Fundamental definitions will be given and see how matrices can be combined using the arithmetic operations of addition, subtraction and multiplication. See tutorial letter for the assessment related to this session.

10.4 Study Guide Unit 3: Matrices and Matrix Operations

10.4.1 Tutorial 3: Matrices and Matrix Operations [1 = week] [2 = hours]

Ice breaker :

Visit the following links to self-learning the concepts of Matrices and its operations:

- *https://www.youtube.com/watch?v=Ncu9Pks3AJQ*
- *https://www.youtube.com/watch?v=XdY0TTknvmo*

Outcomes and Assessment Criteria:

1. Outcomes:

You should be able to

- *Explain what is meant by a matrix and the entries of a matrix*
- *Determine the size of a matrix*
- *Determine if/when two matrices are equal*
- *Find the sum and difference of two matrices*
- *Determine if matrix addition and subtraction are defined or undefined*
- *Multiply a matrix by a scalar*
- *Multiply two matrices*
- *Determine if the product of two matrices is defined or undefined*
- *Multiply two matrices where possible*
- *Write the product of certain types of matrices as a linear combination of row or column matrices*
- *Determine the transpose of a matrix*
- *Find the trace of a matrix*
- *Determine when two matrices are equal*

2. Assessment Criteria:

- *Matrix operations are understood*
- *Arithmetic operations of matrix addition, subtraction, scalar multiplication, and scalar multiplication are performed where possible*
- *Matrix operations including Gaussian reduction are performed*
- *The product of a matrix and a column vector is expressed as a linear combination of the columns of the matrix.*
- *The transpose of a matrix is defined and calculated*
- *The trace of a matrix is defined and calculated*
- *An understanding of when two matrices are equal is demonstrated*

Foundation Content:

Many practical problems are solved by using arithmetic operations on the data associated with the problems. By properly organizing the data into blocks of numbers, we can then carry out these arithmetic operations in an orderly and efficient manner. In particular, this systematic approach enables us to use the computer to full advantage.

Common mistakes and misconceptions:

- *Confusing vectors with scalars and performing arithmetic operations by treating them as numbers*
- *Incorrect reasoning about the projection of a vector*

Frequently asked questions:

- *Is a vector a matrix?*
- *Is a point a matrix?*

Glossary:

- *A matrix is a rectangular array of numbers arranged in rows and columns*

Discussions:

- *Define (a) a matrix, (b) the size of a matrix, (c) a row matrix, (d) a column matrix, and (e) a square matrix.*
- *When are two matrices equal? Give an example of two*

Learning activity:

Attempt the following problems: ***Anton****, Exercise Set 1.3, and Questions 1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29 and 30. Pages 63-72 (from e-book, 10th Edition).*

Worksheets and Assessments

Test yourself by attempting tutorial- related questions given on the worksheets that you can find in the following webpages:

- *http://www.mathworksheets4kids.com/matrices/*
- *http://cdn.kutasoftware.com/Worksheets/Alg2/Basic%20Matrix%20Operations.pdf*

The assessment related to this session is Assessment two and three. See the tutorial letter for details.

Additional Resources

- *Ayres, Frank: Schaum's Outline of Theory and Problems of Matrices, McGraw-Hill, New York, 1974*
- *Cullen, Charles G.: Matrices and Linear Transformations, Addison-Wesley, Reading, MASS., 1972*
- *Johnson, Lee W.: Introduction to Linear Algebra (2 nd or earlier editions), Addison-Wesley, Reading, MASS., 1989*
- *Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.*
- *Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.*
- *Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974*

- *Nicholson, W.K.: Linear Algebra with Applications (3-rd edition), PWS Publishing Company, Boston.*
- *Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co.,Belmont, CA., 1991.*
- *Anton, Howard and Rorres, Chris:Elementary Linear Algebra; Applications Version,(10th edition, 2011), John Wiley & Sons, Inc*

Reflection

This was the Matrices and Matrix operations

Whats next?

In the next tutorial we discuss some properties of the arithmetic operations on matrices. We see that most of the basic rules of arithmetic for real numbers also hold for matrices, but that there are some exceptions when it comes to matrix multiplication. We define the inverse of a matrix and give some properties of inverses and transposes of matrices. We give the formula for the inverse of a 2× 2 invertible matrix.

10.5 Study Guide Unit 4: Inverses; Rules of Matrix Arithmetic

10.5.1 Tutorial 4: Inverses; Rules of Matrix Arithmetic [2 = weeks] [2 = hours]

Ice breaker :

Visit the following link to self-learning the concepts of inverses of matrices and its rules:

- *https://www.youtube.com/watch?v=QNpj-gOXW9M*

Outcomes and Assessment Criteria:

1. Outcomes:

You should be able to

- *Apply the various properties of matrix arithmetic*
- *State which basic rules of arithmetic for real numbers do not hold for matrices*
- *Explain what is meant by an invertible (nonsingular) matrix*
- *Find the inverse of a 2 × 2 invertible matrix*
- *Prove and apply the properties of inverse matrices and laws of matrix exponents*
- *Apply the properties of the transpose of a matrix.*

2. Assessment Criteria:

- *The arithmetic rules for matrix combination are applied*
- *The understanding of the basic arithmetic rules of real numbers which do not apply to matrices is demonstrated*
- *The inverse matrix of a 2×2 matrix is calculated*
- *The properties of inverse matrices are proved and applied The relationship between the properties of the matrix transpose and its relationship with invertible matrices is demonstrated*
- *Inverse matrices are calculated*

Foundation Content:

Square matrices

Common mistakes and misconceptions:

- *Considering the inverse of matrix as the inverse of each of the entries*

Frequently asked questions:

- *Is the inverse of a matrix one over the matrix?*
- *Explain how you would find the inverse of a nonsingular matrix.*

Glossary:

- *An n by n matrix, B, is called an inverse of A if AB = I and BA = I.*

Discussions:

- *In defining the inverse of a matrix A, why is it necessary to require that A be a square matrix?*
- *Suppose A is a square matrix with the property that one of its rows is a nonzero constant multiple of another row. What can you say about the existence or nonexistence of A^{-1} ? Explain your answer.*

Learning activity:

Attempt the following problems: ***Anton****, Exercise Set 1.4, All odd numbered questions. Pages 90-97 (from e-book, 10th Edition).*

Worksheets and Assessments

Test yourself by attempting tutorial- related questions given on the worksheets that

you can find in the following webpages:

- *http://www.mathworksheets4kids.com/matrices/inverse.html*
- *http://edhelper.com/Matrices.htm*

The assessment related to this session is Assessment four. See the tutorial letter for details.

Additional Resources

- *Ayres, Frank: Schaum's Outline of Theory and Problems of Matrices, McGraw-Hill, New York, 1974*
- *Cullen, Charles G.: Matrices and Linear Transformations, Addison-Wesley, Reading, MASS., 1972*
- *Johnson, Lee W.: Introduction to Linear Algebra (2-nd or earlier editions), Addison-Wesley, Reading, MASS., 1989*
- *Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.*
- *Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.*
- *Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974*
- *Nicholson, W.K.: Linear Algebra with Applications (3-rd edition), PWS Publishing Company, Boston.*
- *Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co., Belmont, CA., 1991.*
- *Anton, Howard and Rorres, Chris: Elementary Linear Algebra; Applications Version,(10th edition, 2011), John Wiley & Sons, Inc*

Reflection

These were Matrices and Rules of matrix arithmetic

What's next?

In the next tutorial we develop an algorithm for finding the inverse of an invertible matrix. We also discuss some of the basic properties of invertible matrices.

10.6 Study Guide Unit 5: Elementary Matrices and a Method for Finding the Inverse of A

10.6.1 Tutorial 5: Elementary Matrices and a Method for Finding the Inverse of A

[1 weeks] [2 = hours]

Ice breaker :

Visit the following links to self-learning the concepts of elementary matrices and related techniques:

- *https://www.youtube.com/watch?v=b5JXb8fmjb4*
- *https://www.youtube.com/watch?v=BYK0F8YlvF0*

Outcomes and Assessment Criteria:

1. Outcomes:

You should be able to

- *Explain what is meant by an elementary matrix*
- *Find for any invertible matrix A, a sequence $E_1, E_2, \cdots, E_k$ of elementary matrices such that*

$$E_k \cdots E_2 E_1 A = I_n$$

- *Determine the inverse A^{-1} of any invertible matrix A by using the matrix inversion method (which we call the matrix inverse algorithm), **Anton**.*

2. Assessment Criteria:

- *An elementary matrix is described*
- *A given square matrix is classified either as an elementary matrix or not*
- *An invertible matrix is expressed as a product of elementary matrices*
- *The inversion algorithm is applied to obtain the inverse of an invertible matrix*
- *Elementary matrices are calculated*

Foundation Content:

The elementary matrices are based on identity matrices

Common mistakes and misconceptions:

- *Trying to get elementary matrices from non-square matrices*
- *Forgetting the starting point of performing an elementary row operation on a matrix*

Frequently asked questions:

- *Can we perform an elementary row operation on a matrix without starting with an identity matrix?*
- *Is it possible to get an elementary matrix from a non-invertible matrix?*

Glossary:

- *Elementary matrix is a square matrix that can be obtained from the identity matrix by performing elementary row operations*

Discussions:

- *We have 3 kinds of elementary matrices, which are invertible. Any invertible matrix can be written as a product of some elementary matrices (Try to show this)*
- *Three different types of elementary matrices*
- *Suppose that A is some unknown invertible matrix, but you know of a sequence of elementary row operations that produces the identity matrix when applied in succession to A. Explain how you can use the known information to find A.*
- *Indicate whether the statement is always true or sometimes false. Justify your answer with a logical argument or a counterexample*

1. *Every square matrix can be expressed as a product of elementary matrices.*
2. *The product of two elementary matrices is an elementary matrix.*
3. *If A is invertible and a multiple of the first row of A is added to the second row, then the resulting matrix is invertible.*
4. *If A is invertible and AB=0, then it must be true that B=0*

Learning activity:

Attempt the following problems: ***Anton****, Exercise Set 1.5. Questions 1, 2, 3, 4, 5(c), 6(a), 7(b), 8(c), 10, 13 and 19. Pages 107-112 (from e-book, 10th Edition).*

Worksheets and Assessments

Test yourself by attempting tutorial- related questions given on the worksheets that you can find in the following webpages:

- http://math.kennesaw.edu/~plaval/math3260/elemmat.pdf
- https://math.uc.edu/~halpern/Matrix.methods/Homatrixmethods/elementaryMatri ces.pdf
- http://www.d.umn.edu/~mhampton/m3280s14/m3280s14w15.pdf
- http://www.d.umn.edu/~mhampton/m3280s10/m3280s10w9.pdf
- http://math.berkeley.edu/~jtener/pdf/110fa12/ws8sol.pdf

The assessment related to this session is Assessment five. See the tutorial letter for details

Additional Resources

- *Ayres, Frank: Schaum's Outline of Theory and Problems of Matrices, McGraw-Hill, New York, 1974*
- *Cullen, Charles G.: Matrices and Linear Transformations, Addison-Wesley, Reading, MASS., 1972*

- *Johnson, Lee W.: Introduction to Linear Algebra (2-nd or earlier editions), Addison-Wesley, Reading, MASS., 1989*
- *Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.*
- *Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.*
- *Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974*
- *Nicholson, W.K.: Linear Algebra with Applications (3-rd edition), PWS Publishing Company, Boston.*
- *Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co.,Belmont, CA., 1991.*
- *Anton, Howard and Rorres, Chris:Elementary Linear Algebra; Applications Version,(10th edition, 2011), John Wiley & Sons, Inc*

Reflection

These were elementary matrices and a method for finding the inverse of a matrix.

Whats next?

In the next tutorial we establish further results about systems of linear equations and invertibility of matrices. We also give another method for solving n linear equations in n unknowns

10.7 Study Guide Unit 6: Further Results on Systems of Equations and Invertibility

10.7.1 Tutorial 6: Further Results on Systems of Equations and Invertibility [1 weeks] [2 = hours]

Ice breaker:

Visit the following links to self-learning concepts of systems of equations and invertibility techniques:

- *http://axiomagick.wordpress.com/2012/04/10/1-6-further-results-onsystems-of-equations-and-invertibility/*
- *https://www.youtube.com/watch?v=MJE4EPbQMD0*
- *http://mhhe.com/math/devmath/streeter/ia/graphics/streeter5ia/ch05/others/strI_5.3.pdf*

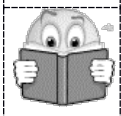
Outcomes and Assessment Criteria:

1. Outcomes:

- *Solve linear systems by using the inverse of its coefficient matrix*
- *Solve multiple linear systems with the same coefficient matrix simultaneously*
- *Determine the consistency of a linear system by elimination*
- *Apply different equivalent statements for the fact that a matrix is invertible, in various problems*

2. Assessment Criteria:

- *Systems of linear equations are solved by inverting its coefficient matrix*
- *Multiple linear systems with the same coefficient matrix are solved simultaneously*
- *Understanding of the conditions required for a linear system to be consistent is demonstrated*
- *Understanding of the equivalent statements which define invertibility is demonstrated*



Foundation Content:

Invertibility of matrices

Common mistakes and misconceptions:

- *Adding up the variables in the equations as they are, instead of making them cancel out*
- *Not ensuring that one of the variables cancel out when multiplying*
- *Making a mistake when solving the equation (Hint: put your answers back into the equations, they should make both equations true when you plug them in)*
- *Re-arranging the equation and then substituting it back into itself. This will make everything cancel out.*

Frequently asked questions:

- *Can we solve a system of linear equations without the invertibility condition?*

Glossary:

- *A "system" of equations is a set or collection of equations that you solve simultaneously*
- *Linear equation is an equation of the form $Ax + By = C$, where $A \neq 0$ and $B \neq 0$. The graph of a linear equation is a straight line.*

Discussions:

- *Generalize the case where there are three straight lines in the plane defined by three linear equations. What if there are n lines defined by n equations?*
- *Consider a system composed of two linear equations in two variables. Can the sys-tem have exactly two solutions? Exactly three solutions? Exactly a finite number of solutions?*
- *Suppose at least one of the equations in a system composed of two equations in two variables is nonlinear. Can the system have no solution? Exactly one solution? Exactly two solutions? Exactly a finite number of solutions? Infinitely many solutions? Illustrate each answer with a sketch*

Learning activity:

Attempt the following problems: ***Anton****, Exercise Set 1.6, Questions 1, 4, 6, 9, 10, 11, 15, 28, 30 and using theorem 1.6.2 solve 13, 14, 15, 16, 18, 23. Pages 123-125 (from e-book, 10th Edition).*

Worksheets and Assessments

Test yourself by attempting tutorial- related questions given on the worksheets that you can find in the following webpages:

- *http://pc30sfall2010.pbworks.com/f/Solving+Systems+of+Linear+Equations+in+Th ree+Variables+Worksheet.pdf*
- *http://www.mathworksheets4kids.com/equations/systems.html*
- *http://www.math-only-math.com/worksheet-on-simultaneous-linear-equations.html*
- *http://cdn.kutasoftware.com/Worksheets/Alg2/Basic%20Matrix%20Operations.pdf*

The assessment related to this session is Assessment five. See the tutorial letter for details.

Additional Resources

- *Ayres, Frank: Schaum's Outline of Theory and Problems of Matrices, McGraw-Hill, New York, 1974*
- *Cullen, Charles G.: Matrices and Linear Transformations, AddisonWesley, Reading, MASS., 1972*
- *Johnson, Lee W.: Introduction to Linear Algebra (2-nd or earlier editions), Addison-Wesley, Reading, MASS., 1989*
- *Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.*
- *Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.*
- *Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974*

- *Nicholson, W.K.: Linear Algebra with Applications (3-rd edition), PWS Publishing Company, Boston.*
- *Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co.,Belmont, CA., 1991.*
- *Anton, Howard and Rorres, Chris: Elementary Linear Algebra; Applications Version,(10th edition, 2011), John Wiley & Sons, Inc*

Reflection

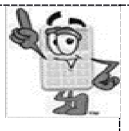
These were Further Results on Systems of Equations and Invertibility

What's next?

In the next tutorial section we define certain classes of square matrices that have special forms. These are amongst some of the most important matrices encountered in linear algebra.

10.8 Study Guide Unit 7: Diagonal, Triangular and Symmetric Matrices

10.8.1 Tutorial 7: Diagonal, Triangular and Symmetric Matrices [1 weeks] [2 = hours]



Ice breaker :

Visit the following links to self-learning concepts of diagonal, triangular and symmetric matrices:

- *https://www.youtube.com/watch?v=7NUnMtLVfvQ*
- *https://www.youtube.com/watch?v=YJG1U3NrUYI*
- *http://www.utdallas.edu/dept/abp/PDF_Files/LinearAlgebra_Folder/Special_Matrices.pdf*

Outcomes and Assessment Criteria:

1. Outcomes:

- *Identify a matrix, a diagonal matrix, a triangular matrix and a symmetric matrix*
- *Write down the inverse of an invertible diagonal matrix by inspection*
- *Understand the effect of the transpose operation on diagonal and triangular matrices*
- *Understand the effect of inversion on diagonal and triangular matrices*

2. Assessment Criteria:

- *A given matrix is analyzed to check whether it is diagonal, triangular or symmetric matrix*
- *The inverse of an invertible diagonal matrix is obtained by inspection*
- *Understanding of the effect of the transpose operation on diagonal and triangular matrices is demonstrated*
- *Understanding of the effect of inversion on diagonal and triangular matrices is demonstrated*

Foundation Content:

Matrices

Common mistakes and misconceptions:

- *Confusing rows and columns for symmetries*

Frequently asked questions:

- *Does the product of symmetric matrices commute?*
- *Is the product of symmetric matrices is symmetric?*
- *Is an invertible symmetric matrix symmetric?*

Glossary:

- *A square matrix is symmetric if it is equaled to its transpose*
- *A diagonal matrix is a square matrix in which all the entries off the main diagonal are zero*
- *A lower triangular matrix is a square matrix in which all the entries above the main diagonal are zero*
- *An upper triangular matrix is a square matrix in which all the entries below the main diagonal are zero*
- *A triangular matrix is a matrix that is either upper triangular or lower triangular*

	***Discussions:*** • *What is the maximum number of distinct entries that an n by n symmetric matrix can have?* • *Propose and prove a theorem that describes how to multiply two diagonal matrices.* • *Suppose that A is a square matrix and D is a diagonal matrix such that AD= I. What can you say about the matrix A? Explain your reasoning.*
	***Learning activity:*** *Attempt the following problems:* ***Anton****, Exercise Set 1.7, Questions 1, 4, 6, 13, 14, 15, 17, 19, 23, and 24. Pages 133-136 (from e-book, 10th Edition).*
	***Worksheets and Assessments*** *Test yourself by attempting tutorial- related questions given on the worksheets that you can find in the following webpages:* • *http://educ.jmu.edu/~ohmx/Fall2011/worksheet2.pdf* • *http://www.pstcc.edu/facstaff/jahrens/math2000/matrices_worksheet.pdf* • *http://www.maths.manchester.ac.uk/~mprest/matrices-2012.pdf* *The assessment related to this session is Assessment five. See the tutorial letter for details.*

- *Matrices, McGraw-Hill, New York, 1974*
- *Cullen, Charles G.: Matrices and Linear Transformations, AddisonWesley, Reading, MASS., 1972*
- *Johnson, Lee W.: Introduction to Linear Algebra (2-nd or earlier editions), Addison-Wesley, Reading, MASS., 1989*
- *Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.*
- *Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.*
- *Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974*
- *Nicholson, W.K.: Linear Algebra with Applications (3-rd edition), PWS Publishing Company, Boston.*
- *Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co.,Belmont, CA., 1991.*
- *Anton, Howard and Rorres, Chris: Elementary Linear Algebra; Applications Version,(10th edition, 2011), John Wiley & Sons, Inc*

Reflection

These were Diagonal, Triangular and Symmetric Matrices

What's next?

In the next tutorial, we define the determinant function in terms of cofactor expansion along the first row of a square matrix. We also obtain a formula for the inverse of an invertible matrix, as well as a formula (known as Cramer's rule) for the solution to certain systems of linear equations in terms of determinants.

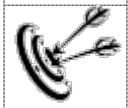
10.9 Study Guide Unit 8: Determinants by Cofactor Expansion

10.9.1 Tutorial 8: Determinants by Cofactor Expansion [1 weeks] [2 = hours]

Ice breaker :

Visit the following links to self-learning concepts of determinants by cofactor technique:

- *https://www.youtube.com/watch?v=9bzr8qUnmb0*
- *https://www.youtube.com/watch?v=vJLGWpob0T8*
- *https://www.youtube.com/watch?v=bqJFGtnoUXg*



Outcomes and Assessment Criteria:

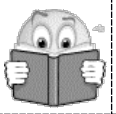
1. Outcomes:

You should be able to:

- *Find the minors and cofactors of a square matrix*
- *Evaluate the determinant of a matrix by cofactor expansion along any row or column of the matrix*
- *Determine the inverse of an invertible matrix by using its adjoint*

Additional Resources

Ayres, Frank: Schaum's Outline of Theory and Problems of

	• *Determine the inverse of a 2×2 matrix by using the determinant* • *Solve systems of linear equations by using Cramer's rule.* *2. Assessment Criteria:* • *The minors and cofactors of a square matrix are calculated* • *The determinant is calculated by row or column cofactor expansion* • *The inverse of a 2×2 matrix is obtained by using the determinant*
	***Foundation Content:*** *Minors, cofactors and adjoint matrix*
	***Common mistakes and misconceptions:*** • *Confusing rows and columns when using Cramer's rule.*
	***Frequently asked questions:*** • *Is Gaussian elimination far more efficient than Cramer's rule?* • *What are the common points between Gaussian elimination method and Cramer's rule?*
	***Glossary:*** • *A determinant is a real number associated with every square matrix.* • *A minor for any element is the determinant that results when the row and column that element is in are deleted.* • *A cofactor for any element is either the minor or the opposite of the minor, depending on where the element is in the original determinant.*
	***Discussions:*** • *What is the maximum number of zeros that a 4 by 4 matrix can have without having a zero determinant? Explain your reasoning.* • *Indicate whether the statement is always true or sometimes false. Justify your answer by giving* • *A logical argument or a counterexample.* *- In theory, Cramer's rule can be used to solve any system of linear equations, although the amount of computation may be enormous. - If A has a row of zeros, then so does adj(A)*
	***Learning activity:*** *Attempt the following problems:* ***Anton****, Exercise Set 2.1. Questions 1, 3(c), (f), 5, 7, 9, 10, 15, 16, 18, 21, 22, 23, 33 and 35. Pages 180-185 (from e-.book, 10th Edition). Questions 24-32 (Pages 215-216, from e-book, 10th Edition).*

Worksheets and Assessments

Test yourself by attempting tutorial- related questions given on the worksheets that you can find in the following webpages:

- *http://math.berkeley.edu/~valby/Summer2012/Worksheet_7_Determinants.pdf*
- *http://math.mit.edu/~dyatlov/54summer10/ws10.pdf*
- *http://web.clark.edu/jmitchell/spring%2009/math%20215/215%20Worksheet%202%20Solutions.pdf*

The assessment related to this session is Assessment six. See the tutorial letter for details.

- *Matrices, McGraw-Hill, New York, 1974*
- *Cullen, Charles G.: Matrices and Linear Transformations, AddisonWesley, Reading, MASS., 1972*
- *Johnson, Lee W.: Introduction to Linear Algebra (2-nd or earlier editions), Addison-Wesley, Reading, MASS., 1989*
- *Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.*
- *Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.*
- *Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974*
- *Nicholson, W.K.: Linear Algebra with Applications (3-rd edition), PWS Publishing Company, Boston.*
- *Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co.,Belmont, CA., 1991.*
- *Anton, Howard and Rorres, Chris: Elementary Linear Algebra; Applications Version,(10th edition, 2011), John Wiley & Sons, Inc*

Reflection

These were Determinants by Cofactor Expansion

What's next?

In the next tutorial, we consider the effect that each type of elementary row operation has on the value of the determinant of the resulting matrix in terms of the determinant of the original matrix. We also show that the determinant of a square matrix can be evaluated by reducing the matrix to row-echelon form (or generalized row-echelon form). This method is important since it is the most computationally efficient way to evaluate the determinant of a general matrix.

10.10 Study Guide Unit 9: Evaluating Determinants by Row Reduction

10.10.1 Tutorial 9: Evaluating Determinants by Row Reduction [1 weeks] [2 = hours]

Ice breaker:

Visit the following links to self-learning concepts of determinants by row reduction technique:

Additional Resources

Ayres, Frank: Schaum's Outline of Theory and Problems of

- *https://www.youtube.com/watch?v=pgqyULjZgbU*
- *https://www.youtube.com/watch?v=MMuPDjKWFaA*

Outcomes and Assessment Criteria:

1. Outcomes:

You should be able to:

- *State the effect that each elementary row operation has on the value of the determinant of the resulting matrix*
- *Determine, by inspection, the determinants of elementary matrices*

	• *Evaluate the determinant of a matrix by using elementary row operations to reduce the given matrix to an upper triangular matrix* • *Evaluate a determinant by using a combination of row or column operations and cofactor expansion* • *Evaluate a determinant in terms of a related determinant.* *2. Assessment Criteria:* • *Understanding of the effect of the elementary operations on the value of the determinant is demonstrated* • *The determinant of an elementary matrix is evaluated by inspection* • *The determinant of a square matrix is evaluated using a combination of elementary row or column operations and the cofactor expansion* • *The determinant is evaluated by row reduction*
	***Foundation Content:*** *Minors, cofactors and adjoint matrix*
	***Common mistakes and misconceptions:*** • *Misusing rows reduction method*
	***Frequently asked questions:*** • *What is the advantage of using the row reduction method to evaluate the determinant of a matrix?*
	***Glossary:*** • *A determinant is a real number associated with every square matrix.* • *A minor for any element is the determinant that results when the row and column that element is in are deleted.* • *A cofactor for any element is either the minor or the opposite of the minor, depending on where the element is in the original determinant.*

Discussions:

- *How many arithmetic operations are needed, in general, to find det (A) by row reduction? By cofactor expansion?*

Learning activity:

Attempt the following problems: ***Anton****, Exercise Set 2.2, Questions 2, 3, 4, 6, 8, 11, 12, 13, 15, 17, 29, 30 and 31. Pages 193-197 (from e-book, 10th Edition).*

Worksheets and Assessments

Test yourself by attempting tutorial- related questions given on the worksheets that you can find in the following webpages:

- *http://www.math.pitt.edu/~annav/0290H/row_reduction.pdf*
- *http://math.berkeley.edu/~gwyneth/ws209soln.pdf*

The assessment related to this session is Assessment six. See the tutorial letter for details.

- *Matrices, McGraw-Hill, New York, 1974*
- *Cullen, Charles G.: Matrices and Linear Transformations, Addison-Wesley, Reading, MASS., 1972*
- *Johnson, Lee W.: Introduction to Linear Algebra (2-nd or earlier editions), Addison-Wesley, Reading, MASS., 1989*
- *Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.*
- *Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.*
- *Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974*
- *Nicholson, W.K.: Linear Algebra with Applications (3rd edition), PWS Publishing Company, Boston.*
- *Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co.,Belmont, CA., 1991.*
- *Anton, Howard and Rorres, Chris: Elementary Linear Algebra; Applications Version,(10th edition, 2011), John Wiley & Sons, Inc*

Reflection

These were Evaluating Determinants by Row Reduction

Whats next?

In the next tutorial, we develop some of the fundamental properties of the determinant function. We investigate the relationship between a square matrix and its determinant and give the determinant test for the invertibility of a matrix.

10.11 Study Guide Unit 10: Properties of the Determinant Function

Additional Resources

Ayres, Frank: Schaum's Outline of Theory and Problems of

10.11.1 Tutorial 10: Properties of the Determinant Function [1 weeks] [2 = hours]

Ice breaker :

Visit the following links to self-learning the properties of the determinant functions:

- *https://www.youtube.com/watch?v=aKX5_DucNq8*
- *https://www.youtube.com/watch?v=srxexLishgY*

Outcomes and Assessment Criteria:

1. Outcomes:

You should be able to:

- *State and apply the basic properties of determinants*
- *Analyze how the determinant behaves with respect to basic arithmetic operations*
- *Use Cramer's rule to solve linear system of equations • Evaluate a determinant in terms of a related determinant.*

2. Assessment Criteria:

- *Understanding of the basic properties is demonstrated and applied to obtain the determinant in terms of a related determinant*
- *Cramer's rule is applied to solve a linear system of equations*

Foundation Content:

Minors, cofactors and adjoint matrix

Common mistakes and misconceptions:

- *Misusing rows reduction method*
- *det(A+B)=det(A) det(B)*
- *det (k A) = k det(A)*

Frequently asked questions:

- *What is the advantage of using the row reduction method to evaluate the determinant of a matrix?*

Glossary:

- *A determinant is a real number associated with every square matrix.*
- *A minor for any element is the determinant that results when the row and column that element is in are deleted.*
- *A cofactor for any element is either the minor or the opposite of the minor, depending on where the element is in the original determinant.*

Discussions:

- *Let A and B be n by n matrices. You know from earlier work that AB and BA need not be equal. Is the same true for det(AB) and det(BA)? Explain your reasoning.*

Learning activity:

*Attempt the following problems: **Anton**, Exercise Set 2.3, Questions 1, 5, 6, 8, 9, 12, 14(a), 15(a), 20, 24, 25, 30. Pages 212-215 (from e-book, 10th Edition).*

Worksheets and Assessments

Test yourself by attempting tutorial- related questions given on the worksheets that you can

find in the following webpages:

- *http://www.softschools.com/math/algebra/*
- *http://www.mathworksheets4kids.com/matrices/*

The assessment related to this session is Assessment five and seven. See the tutorial letter for details.

Additional Resources

- *Ayres, Frank: Schaum's Outline of Theory and Problems of Matrices, McGraw-Hill, New York, 1974*
- *Cullen, Charles G.: Matrices and Linear Transformations, AddisonWesley, Reading, MASS., 1972*
- *Johnson, Lee W.: Introduction to Linear Algebra (2-nd or earlier editions), Addison-Wesley, Reading, MASS., 1989*
- *Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.*
- *Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.*
- *Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974*
- *Nicholson, W.K.: Linear Algebra with Applications (3-rd edition), PWS Publishing Company, Boston.*
- *Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co.,Belmont, CA., 1991.*
- *Anton, Howard and Rorres, Chris: Elementary Linear Algebra; Applications Version,(10th edition, 2011), John Wiley & Sons, Inc*

Reflection

These were the properties of the determinant function

What's next?

In the next tutorial, we introduce vectors in 2-space and 3-space geometrically. We define arithmetic operations on vectors and give some basic properties of these operations. We also establish the basic rules of vector arithmetic and introduce the concept of the norm of a vector.

10.12 Study Guide Unit 11: Introduction to Vectors, and Norm of a Vector; Vector Arithmetic

10.12.1 Tutorial 11: Introduction to Vectors, and Norm of a Vector; Vector Arithmetic [1 weeks] [2 = hours]

Ice breaker :

Visit the following links to self-learning vectors and their properties and lengths:

- *https://www.youtube.com/watch?v=pimr9I92GZY*
- *https://www.youtube.com/watch?v=Iok2Gi2QEVA*
- *https://www.youtube.com/watch?v=oDJRsc265GQ*

Outcomes and Assessment Criteria:

1. Outcomes:

You should be able to:

- *Find the components of a vector given the initial point and terminal point of the vector*
- *Find the initial or terminal point of a vector given certain information about the vector*
- *Perform various arithmetic operations on vectors.*
- *Apply the properties of vector arithmetic in 2-space and 3-space*
- *Find the norm of a vector in 2-space and 3-space*
- *Find the distance between two points in 2-space and 3-space.*

2. Assessment Criteria:

- *Basic arithmetic operations of addition, subtraction, and scalar multiplication are performed on vectors*
- *The norm of a vector in 2-space and 3-space is evaluated*

Foundation Content:

Points in a plane

Common mistakes and misconceptions:

- *Confusing a vector to a point*
- *Plotting a vector as a point*

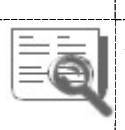
Frequently asked questions:

- *Is a vector a point or a set of points?*
- *Can a norm of a vector be expressed in terms of a distance between two different points?*

Glossary:

- *Vectors can be represented geometrically as directed line segments or arrows in 2-space or 3-space. The direction of the arrow specifies the direction of the vector, and the length of the arrow describes its magnitude. The tail of the arrow is called the initial point of the vector, and the tip of the arrow the terminal point.*

S

	• *The length of a vector u is often called the norm of u*
	***Discussions:*** • *Draw a picture that shows four nonzero vectors whose sum is zero* • *If you were given four nonzero vectors, how would you construct geometrically a fifth vector that is equal to the sum of the first four? Draw a picture to illustrate your method.* • *Is it possible to have || u + v||= || u|| + || v||? Explain your reasoning.* • *What does the inequality ||u||< 1tell you about the location of the point u in the plane?*
	***Learning activity:*** *Attempt the following problems:* ***Anton****, Exercise Set 3.1, Questions 1 (a selection), 5 (a selection), 7, 9, 11, 22, 23, 25, 27, 34; and Set 3.2, Questions 1, 11, 13, 15, 19, 21, 23, 25, 27, 29, 31, 33. Pages 243-250 and pages 273-278 (from e-book, 10th Edition).*
	***Worksheets and Assessments*** *Test yourself by attempting tutorial- related questions given on the worksheets that you can find in the following webpages:* • *http://www.colorado.edu/engineering/cas/courses.d/IFEM.d/IFEM.AppA.d/IFEM.AppA.pdf* • *http://mathsci.ucd.ie/courses/mst10030/tut4_sols.pdf* *The assessment related to this session is Assessment eight. See the tutorial letter for details.*

Additional Resources

- *Ayres, Frank: Schaum's Outline of Theory and Problems of Matrices, McGraw-Hill, New York, 1974*
- *Cullen, Charles G.: Matrices and Linear Transformations, AddisonWesley, Reading, MASS., 1972*
- *Johnson, Lee W.: Introduction to Linear Algebra (2-nd or earlier editions), Addison-Wesley, Reading, MASS., 1989*
- *Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.*
- *Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.*
- *Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974*
- *Nicholson, W.K.: Linear Algebra with Applications (3-rd edition), PWS Publishing Company, Boston.*
- *Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co.,Belmont, CA., 1991.*
- *Anton, Howard and Rorres, Chris: Elementary Linear Algebra; Applications Version,(10th edition, 2011), John Wiley & Sons, Inc*

Reflection

These were Introduction to vectors, Norm of a Vector; Vector Arithmetic

Whats next?

In the next tutorial, we introduce the concept of the dot product (an important type of product) of two vectors in 2 -space and 3 -space. We also discuss some geometric implications of this concept

10.13 Study Guide Unit 12: Dot Product; Projections, Cross Product, Lines and Planes in 3-Space, and Euclidean n-Space

10.13.1 Tutorial 12: Dot Product; Projections, Cross Product, Lines and Planes in 3space, and Euclidean *n*-Space [2 weeks] [2 = hours]

Ice breaker *:*

Visit the following links to self-learning concepts of dot and cross product, projections, lines and planes:

- *https://www.youtube.com/watch?v=KDHuWxy53uM*
- *https://www.youtube.com/watch?v=EK_TO7qnuts*
- *https://www.youtube.com/watch?v=Warl5ZxW7tA*

Outcomes and Assessment Criteria:

1. Outcomes:

You should be able to:

- *Determine the dot product of two vectors in 2-space or 3-space*
- *Use the dot product to find the angle (or cosine of the angle) between two vectors in 2-space or 3-space*
- *Determine if two vectors in 2-space or 3-space are orthogonal (perpendicular)*
- *Find the orthogonal projection of* ***u*** *on* ***a*** *(the vector component of* ***u*** *along* ***a****) and the vector component of* ***u*** *orthogonal to* ***a***
- *Find the distance between a point and a line in 2-space.*
- *Determine the cross product of two vectors in 3-space*
- *Use the cross product to find the area of a parallelogram in 3-space*
- *Use determinants to find the area of a parallelogram in 2-space and the volume of a parallelepiped in 3-space*
- *Find the equations of lines and planes in 3-space*
- *Find the distance between a point and a plane or between two parallel planes in 3-space*
- *Perform various arithmetic operations on vectors in n-space*
- *Apply the arithmetic properties of vectors in n-space*
- *Find the Euclidean norm (Euclidean length) of a vector in n-space*

	• *Find the Euclidean distance between two points in n-space* • *Determine if two vectors in n-space are orthogonal.* *2. Assessment Criteria:* • *The dot and cross product of a vector is evaluated for vectors in 2-space and 3-space* • *The dot product is used to evaluate the angle between two vectors in 2-space and 3-space* • *The orthogonal projection of a vector **u** along **a** is evaluated* • *The distance between a point and a plane is evaluated* • *The area of a parallelogram is evaluated using either the cross product or the absolute value of the determinant* • *The volume of a parallepiped is obtained using either the scalar triple product or the absolute value of the determinant* • *The equations of lines in R^2 and R^3 are expressed using either vector or parametric equations* • *The equations of planes in R^n are expressed using either vector or parametric equations* • *The equation of a line containing two given points in R^2 and R^3 is expressed using either vector or parametric equations*
	***Foundation Content:*** *vectors, points, lines and planes*
	***Common mistakes and misconceptions:*** • *Confusing a vector to a point*
	***Frequently asked questions:*** • *Is a vector a point or a set of points?* • *Difference between a line segment and a line*
	***Glossary:*** • *A dot product or scalar product is an algebraic operation that takes two equallength sequences of numbers and returns a single number.* • *A cross product or vector product is a binary operation on two vectors in threedimensional space. It results in a vector that is perpendicular to both and therefore normal to the plane containing them.* • *A plane is a flat, two-dimensional surface*
	***Discussions:*** • *Suppose that **u**, **v**, and w are mutually orthogonal nonzero vectors in 3-space, and suppose that you know the dot products of these vectors with a vector r in 3-space. Find an expression for **r** in terms of **u**, **v**, **w,** and the dot products.* • *Give some examples of algebraic rules that hold for multiplication of real numbers but not for the cross product of vectors.*

	• *Write parametric equations for two perpendicular lines through the point*

	***Learning activity:*** *Attempt the following problems:* ***Anton****, Exercise Set 3.3. Questions 1(a), (c), 3(a), (c), 4(a), (c), 5(a), (c), 6, 7, 8, 16, 17, 21, 23, 25, 27, 29, 31, and Exercise Set 3.4, Questions 1, 2, 5, 9, 13, 17, 21, 22, 25, 37, 38, Exercise Set 3.5, Questions 1(a), 2(a), 3, 4, 7, 9, 11, 13, 14, 15, 17, 21, 22, 25, 27, 29, 31, 37(b); and Exercise Set 4.1, Questions 1, 2, 3, 4, 5, 9, 11, 14, 15. Pages 289-291, page 303-304, pages 318-322, pages 337-338 (from e-book, 10th Edition).*
	***Worksheets and Assessments*** *Test yourself by attempting tutorial- related questions given on the worksheets that you can find in the following webpages:* • *http://www.math.armstrong.edu/faculty/hollis/classes/linear/linearProbSheet+solns.pdf* • *http://math.berkeley.edu/~preskill/math53/worksheet_crossplane_soln.pdf* *The assessment related to this session is Assessment nine-thirteen. See the tutorial letter for details.*

Additional Resources

- *Ayres, Frank: Schaum's Outline of Theory and Problems of Matrices, McGraw-Hill, New York, 1974*
- *Cullen, Charles G.: Matrices and Linear Transformations, AddisonWesley, Reading, MASS., 1972*
- *Johnson, Lee W.: Introduction to Linear Algebra (2-nd or earlier editions), Addison-Wesley, Reading, MASS., 1989*
- *Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.*
- *Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.*
- *Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974*
- *Nicholson, W.K.: Linear Algebra with Applications (3-rd edition), PWS Publishing Company, Boston.*
- *Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co.,Belmont, CA., 1991.*
- *Anton, Howard and Rorres, Chris: Elementary Linear Algebra; Applications Version,(10th edition, 2011), John Wiley & Sons, Inc*

Reflection

These were Dot Product; Projections; Cross Product; Lines and Planes in 3-Space; and
Euclidean n-Space

Whats next?

In the next tutorial, we focus on Complex Numbers

10.14 Study Guide Unit 13: Complex Numbers

10.14.1 Tutorial 13: Complex Numbers [2 weeks] [2 = hours]

Ice breaker:

Visit the following links to self-learning complex numbers and their properties:

- *https://www.youtube.com/watch?v=MDDZG-Y_AfY*
- *https://www.youtube.com/watch?v=2LgMw_LgTe4*
- *https://www.youtube.com/watch?v=sn3orkHWqUQ*

Outcomes and Assessment Criteria:

1. Outcomes:

You should be able to:

- *Find the modulus a complex number*
- *Find the argument of a complex*
- *Write a complex number in polar form*
- *Calculate the product and quotient of two complex numbers using the polar form of complex numbers*
- *State De Moivre's Theorem*
- *Use De Moivre's Theorem to calculate the roots and/or powers of a complex number*
-
- *Linearise trigonometric functions*

2. Assessment Criteria:

- *The modulus and argument are calculated.*
- *A complex number is converted from rectangular form to polar and exponential form and vice versa*
- *The product and quotient of complex numbers is evaluated using the polar form of the complex numbers*
- *DeMoivre's Theorem is applied to evaluate the power or roots of a complex number*

Foundation Content:

Points, modulus

Common mistakes and misconceptions:

- *Confusing real and imaginary parts*
- *Not knowing properties on trigonometric functions*

Frequently asked questions: •

Is a complex number a point?

• *How can we express distance between two different points in terms of complex numbers related those points?*

Glossary:

- *A complex number is an ordered pair of real numbers, denoted either by (a, b) or by a +i b, $i^2=-1$.*
- *A modulus of a complex number $z=a + ib$, denoted by $|z|$, is defined by $|z|= a^2+b^2$*
- *An argument of a complex number z is the angle between the vector with length $|z|$ and the horizontal x-axis.*

Discussions:

- *Express some properties defined in Vectors in terms of complex number*
- *Give the geometric interpretation of the module of any complex number*

Learning activity:

*Attempt the following problems: **Anton**, Exercise Set 10.1, Set 10.2 and Set 10.3*

Worksheets and Assessments

Test yourself by attempting tutorial- related questions given on the worksheets that you can find in the following webpages:

- *http://www.mathworksheetsland.com/topics/complex.html*
- *http://www.softschools.com/math/algebra/complex_numbers/*
- *http://www.vitutor.com/arithmetic/complex_numbers/exercises_complex.html*
- *http://www.lavc.edu/math/math125/Worksheets/complexM.pdf*
- *http://www.lavc.edu/math/math125/Worksheets/complexD.pdf*

The assessment related to this session is Assessment thirteen. See the tutorial letter for details.

Additional Resources

- *Ayres, Frank: Schaum's Outline of Theory and Problems of Matrices, McGraw-Hill, New York, 1974*
- *Cullen, Charles G.: Matrices and Linear Transformations, AddisonWesley, Reading, MASS., 1972*
- *Johnson, Lee W.: Introduction to Linear Algebra (2-nd or earlier editions), Addison-Wesley, Reading, MASS., 1989*
- *Knopp, Paul J.: Linear Algebra, an Introduction, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.*
- *Lipschutz, Seymour: Schaum's Outline of Theory and Problems of Linear Algebra, McGraw-Hill, New York, 1968.*
- *Nering, Evar D.: Elementary Linear Algebra, W.B. Saunders Publishing Co., Philadelphia, 1974*
- *Nicholson, W.K.: Linear Algebra with Applications (3-rd edition), PWS Publishing Company, Boston.*
- *Grossman, Stanley I.: Elementary Linear Algebra (any edition), Wadsworth Publishing Co.,Belmont, CA., 1991.*
- *Anton, Howard and Rorres, Chris: Elementary Linear Algebra; Applications Version,(10th edition, 2011), John Wiley & Sons, Inc*

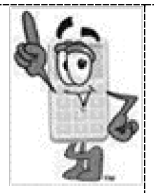
Reflection

These wereomplex numbers

What's next?

The next tutorial is devoted on the Exam preparation

10.15 Exam Preparation Session

	***Ice breaker:*** *Go through materials used in this course*
	***Exam Practice:*** *You must visit the following websites for your revision:* • *http://people.brandeis.edu/~levinea/spring12/la/practicefinalsolutions.pdf* • *https://www.math.dartmouth.edu/archive/m22s02/public_html/FXS.pdf* • *http://math.bard.edu/~mbelk/math601/LinearAlgebraSolutions.pdf* • *http://ocw.mit.edu/courses/mathematics/18-06-linear-algebraspring-2010/exams/MIT18_06S10_exam1_s10_sol.pdf*

11. Congratulations and best wishes

We congratulate you if you have successfully gone through the tutorials. We hope it was worthwhile!

We are confident you will be rewarded with a good pass in the module.

We extend the best wishes for you to pass this module, and hope the knowledge and skills you have mastered with help you in other modules in your program, and in future when you become a professional in **Linear Algebra**.

THE END