3801ICT – Problems for Computational Errors

1. The exponential function can be computed using:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$

   Add terms one at a time to estimate $e^{0.5}$ (= 1.648721..) until the approximate error estimate falls below an error criterion conforming to 3 significant figures.

2. Implement a program to solve Problem 1 above and graph the results.

3. Create a floating point number-set for a computer using 7 bit words (1st for sign, 3 for signed exponent and 3 for mantissa). Determine the smallest and largest possible numbers and identify some of the numbers that can be represented.

4. Using a hypothetical decimal computer with a 4 digit mantissa and a 1 digit exponent investigate errors in addition, subtraction, multiplication and division (use truncation rather than rounding).

5. Investigate the effect of round-off errors on large number of independent calculations by summing a number 100,000 times. Sum the number 1 in single precision and 0.00001 in single and double precision.

6. Compute the values of a quadratic equation with a = 1, b = 3000.001 and c = 3 (true roots are -0.001 and -3000).

7. The infinite series $f(n) = \sum_{i=1}^{n} \frac{1}{i^4}$ converges onto $\pi^4/90$ as n approaches infinity. Write a program that calculates *f(n)* for *n* = 10,000 in both increasing *i* and decreasing *i*. Compute the true percent relative error and explain the results.

8. Evaluate e^{-5} using two approaches and compare the results. The correct value is 6.737947×10^{-3}.

9. Determine the number of terms necessary to approximate $\cos(x)$ to 8 significant figures using the Maclaurin series approximation

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

   Calculate the approximation using a value of $x = 0.3\pi$.