AE/ME 5830, Spring 2021, Homework III, Due Monday March 15 by 11:59 pm

1. Develop a computer routine to solve a set of linear equations ($Ax = b$) using the Gauss-Seidel scheme with over/under relaxation. Your routine should take A (the coefficient matrix), b (the right hand side vector), ω (the relaxation factor), $NMAX$ (maximum number of iterations allowed), and p (tolerance to stop the iteration process) as inputs. Your output should include the solution vector x. In your program, define the residual vector $r^{(k)} = b - Ax^{(k)}$ where k indicates the iteration number. Stop your iteration process if

$$\frac{||r^k||_2}{||r^0||_2} \leq 10^{-p} \qquad or \qquad k \geq NMAX$$

   Use your program to solve $Ax = b$ where A is a $n \times n$ Hilbert matrix and the vector b is given by $b_i = 1.0$ ($i = 1, 2, ..., n$). Obtain the solution vector for $n = 2$ and $n = 3$ with a precision goal of $p = 8$. Set $NMAX$ to 5000 iterations at each case. For $n = 2$, start your iteration with $x^0 = \{1, 1\}^T$ and use $x^0 = \{1, 1, 1\}^T$ for $n = 3$. For each case, make a plot of the number of iterations to achieve the precision goal (or the maximum number of iterations) versus the relaxation factor by changing ω between 0.1 and 1.9 with at least 0.1 increments.

2. Develop a computer program to solve n non-linear equations with n unknowns using the Newton's Method. Use your program to find the solution vector of the following system of equations:

$$\begin{aligned} 4x_1 - x_2 + x_3 &= x_1x_4 \\ -x_1 + 3x_2 - 2x_3 &= x_2x_4 \\ x_1 - 2x_2 + 3x_3 &= x_3x_4 \\ x_1^2 + x_2^2 + x_3^2 &= 1 \end{aligned}$$

   Obtain the solution using three different starting vectors:
   $x^0 = \{1, 1, 1, 1\}^T$, $x^0 = \{3, 3, 3, 3\}^T$, and $x^0 = \{6, 6, 6, 6\}^T$. For each case, use the following stopping criteria

$$\frac{||f^k||_2}{||f^0||_2} \leq 10^{-16} \qquad or \qquad k \geq NMAX$$

where $NMAX = 100$ and f^k is the function vector evaluated at k^{th} iteration. Note that for this question you can use the methods (direct or indirect) you have developed or build-in functions (or commands) in Matlab for solving the linear set of equations at each iteration.