Gari Pahayo

Homework 2

Computational Methods

Question 1

(50 points) Develop a computer routine to solve a set of linear equations ($Ax = b$) using the Gaussian elimination scheme with partial pivoting. Your routine should take A matrix and the right hand side vector b as inputs and return the solution vector x as output.

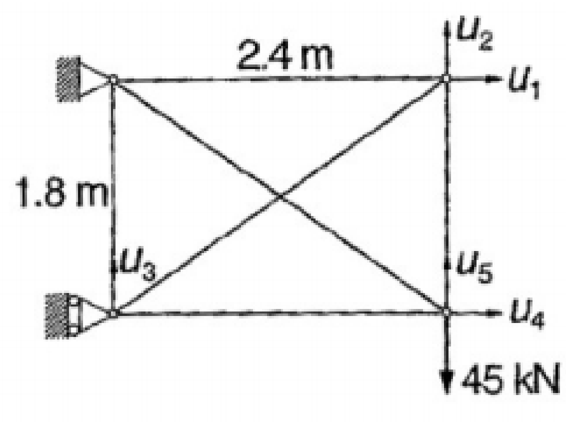


Figure 1: **Truss System**

(a) The displacement formulation for a plane truss system shown in Figure 1 is similar to that of a mass-spring system. The differences are: (1) the stiffnesses of the members are $k_i = (EA/L)_i$ where E is the modulus of elasticity, A represents the cross-sectional area and L is the length of the member; (2) there are two components of displacement at each joint. For the statically indeterminate truss shown, the displacement formulation yields the symmetric equation **Ku=p** where

$$\mathbf{K} = \begin{bmatrix} 27.58 & 7.004 & -7.004 & 0.0000 & 0.0000 \\ 7.004 & 29.57 & -5.253 & 0.0000 & -24.32 \\ -7.004 & -5.253 & 29.57 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 27.58 & -7.004 \\ 0.0000 & -24.32 & 0.0000 & -7.004 & 29.57 \end{bmatrix} \text{MN/m}$$

$$\mathbf{p} = \begin{bmatrix} 0 & 0 & 0 & 0 & -45 \end{bmatrix}^T \text{kN}$$

Using the program you have developed, determine the displacements u_i $(i = 1, ..., 5)$ of the joints.

(b) Use your program to solve $Ax = b$ where A is a $n \times n$ Hilbert matrix defined as

$$a_{ij} = \frac{1}{i + j - 1} \qquad (i, j = 1, 2, ..., n)$$

and the vector b is given by $b_i = 1.0$ $(i = 1, 2, ..., n)$. Obtain the solution vector for $n = 5$ and $n = 10$.

Results

Part a)

$$\vec{u} = \begin{bmatrix} 1.44043701280587 \\ -6.48248565744001 \\ -0.810405015922897 \\ -1.8518167281592 \\ -7.29199105691471 \end{bmatrix} m$$

Part b)

n = 5

$$\vec{x} = \begin{bmatrix} 4.99999999999443 \\ -119.999999999913 \\ 629.999999999665 \\ -1119.99999999954 \\ 629.999999999788 \end{bmatrix}$$

n = 10

$\vec{x} =$

-9.99765861488567

989.797482562022

-23755.6835643676

240200.742162159

-1261072.69524723

3783265.01075107

-6725874.97493985

7000463.3406193

-3937791.25466355

923685.713419866

Method

1) Rearrange the matrix so that A(k,k) from k = 1 to k=n-1 is not zero (partial pivoting)
2) Make the A matrix into an upper triangular matrix (elimination)
3) Apply back substitution.
    a. $$x_i = \frac{b_i^{(i-1)} - \sum_{j=i+1}^{n} a_{ij}^{(i-1)} x_j}{a_{ii}^{(i-1)}} \qquad \text{for } i = n-1, n-2, \ldots, 1$$

This applies to both parts.

Code written for the question will be given in the appendix (q1a.m and q1b.m)

Question 2

(50 points) Develop a computer program to obtain the LU decomposition of a matrix using Crout's Method. Use this program (by adding forward and backward substitution routines) to solve the linear set of equations described in part (b) of question 1. Your results should include the original A matrix, L and U matrices obtained after the decomposition, and the solution vector for each case.

-

Results

n = 5

A

$$= \begin{bmatrix} 1 & 0.5 & 0.333333333333333 & 0.25 & 0.2 \\ 0.5 & 0.333333333333333 & 0.25 & 0.2 & 0.1666666667 \\ 0.333333333333333 & 0.25 & 0.2 & 0.166666666666667 & 0.1428 \\ 0.25 & 0.2 & 0.166666666666667 & 0.1428 & 0.125 \\ 0.2 & 0.166666666666667 & 0.1428 & 0.125 & .11111 \end{bmatrix}$$

L

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ .5 & 0.0833333333333333 & 0 & 0 & 0 \\ .3333 & 1 & 0.00555555555555552 & 0 & 0 \\ .25 & .9 & 1.5 & 0.000357142857142872 & 0 \\ .2 & .8 & 1.7143 & 2 & 2.2676e-05 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & .5 & .3333 & .25 & .2 \\ 0 & 1 & .0833 & .075 & .0667 \\ 0 & 0 & 1 & .0083 & .0095 \\ 0 & 0 & 0 & 1 & 7.1429e-04 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} 4.99999999999443 \\ -119.999999999913 \\ 629.999999999665 \\ -1119.99999999954 \\ 629.999999999788 \end{bmatrix}$$

n =10

$$A =$$

$$\begin{bmatrix}
1 & 0.5000 & 0.3333 & 0.2500 & 0.2000 & 0.1667 & 0.1429 & 0.1250 & 0.1111 & 0.1000 \\
0.5000 & 0.3333 & 0.2500 & 0.2000 & 0.1667 & 0.1429 & 0.1250 & 0.1111 & 0.1000 & 0.0909 \\
0.3333 & 0.2500 & 0.2000 & 0.1667 & 0.1429 & 0.1250 & 0.1111 & 0.1000 & 0.0909 & 0.0833 \\
0.2500 & 0.2000 & 0.1667 & 0.1429 & 0.1250 & 0.1111 & 0.1000 & 0.0909 & 0.0833 & 0.0769 \\
0.2000 & 0.1667 & 0.1429 & 0.1250 & 0.1111 & 0.1000 & 0.0909 & 0.0833 & 0.0769 & 0.0714 \\
0.1667 & 0.1429 & 0.1250 & 0.1111 & 0.1000 & 0.0909 & 0.0833 & 0.0769 & 0.0714 & 0.0667 \\
0.1429 & 0.1250 & 0.1111 & 0.1000 & 0.0909 & 0.0833 & 0.0769 & 0.0714 & 0.0667 & 0.0625 \\
0.1250 & 0.1111 & 0.1000 & 0.0909 & 0.0833 & 0.0769 & 0.0714 & 0.0667 & 0.0625 & 0.0588 \\
0.1111 & 0.1000 & 0.0909 & 0.0833 & 0.0769 & 0.0714 & 0.0667 & 0.0625 & 0.0588 & 0.0556 \\
0.1000 & 0.0909 & 0.0833 & 0.0769 & 0.0714 & 0.0667 & 0.0625 & 0.0588 & 0.0556 & 0.0526
\end{bmatrix}$$

$$L =$$

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.5000 & 0.0833 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.3333 & 1.0000 & 0.0056 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.2500 & 0.9000 & 1.5000 & 3.5714e-04 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.2000 & 0.8000 & 1.7143 & 2.0000 & 2.2676e-05 & 0 & 0 & 0 & 0 & 0 \\
0.1667 & 0.7143 & 1.7857 & 2.7778 & 2.5000 & 1.4315e-06 & 0 & 0 & 0 & 0 \\
0.1429 & 0.6429 & 1.7857 & 3.3333 & 4.0909 & 3.0000 & 9.0097e-08 & 0 & 0 & 0 \\
0.1250 & 0.5833 & 1.7500 & 3.7121 & 5.5682 & 5.6538 & 3.5000 & 5.6600e-09 & 0 & 0 \\
0.1111 & 0.5333 & 1.6970 & 3.9596 & 6.8531 & 8.6154 & 7.4667 & 4.0000 & 3.5514e-10 & 0 \\
0.1000 & 0.4909 & 1.6364 & 4.1119 & 7.9301 & 11.6308 & 12.6000 & 9.5294 & 4.5000 & 2.2268e-11
\end{bmatrix}$$

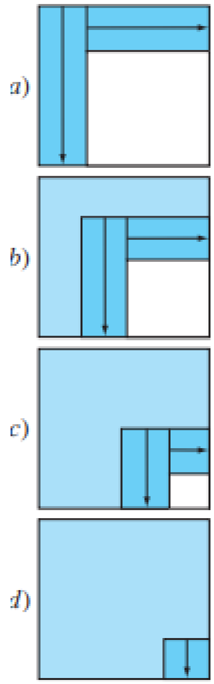
$$U =$$

$$\begin{bmatrix}
1 & 0.5000 & 0.3333 & 0.2500 & 0.2000 & 0.1667 & 0.1429 & 0.1250 & 0.1111 & 0.1000 \\
0 & 1 & 0.0833 & 0.0750 & 0.0667 & 0.0595 & 0.0536 & 0.0486 & 0.0444 & 0.0409 \\
0 & 0 & 1 & 0.0083 & 0.0095 & 0.0099 & 0.0099 & 0.0097 & 0.0094 & 0.0091 \\
0 & 0 & 0 & 1 & 7.1429e-04 & 9.9206e-04 & 0.0012 & 0.0013 & 0.0014 & 0.0015 \\
0 & 0 & 0 & 0 & 1 & 5.6689e-05 & 9.2764e-05 & 1.2626e-04 & 1.5540e-04 & 1.7982e-04 \\
0 & 0 & 0 & 0 & 0 & 1 & 4.2946e-06 & 8.0938e-06 & 1.2333e-05 & 1.6650e-05 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 3.1534e-07 & 6.7273e-07 & 1.1352e-06 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2.2640e-08 & 5.3936e-08 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1.5981e-09 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

$$\vec{x} = [-9.997 \quad 989.77 \quad -23755.13 \quad 240195.71 \quad -1261048.59 \quad 3783198.50 \quad -6725765.4 \quad 7000357.23 \quad -3937735.417 \quad 923673.408]^T$$

Method

1) Decompose A into L and U

    a. 

2) Forward substitute to get {D}

    a. L{D}={B}

    b. $$d_i = b_i - \sum_{j=1}^{i-1} a_{ij} d_j \qquad \text{for } i = 2, 3, \ldots, n$$

3) Backward substitution

    a. U{X}={D}

    $$x_n = d_n / a_{nn}$$

    $$x_i = \frac{d_i - \sum_{j=i+1}^{n} a_{ij} x_j}{a_{ii}} \qquad \text{for } i = n-1, n-2, \ldots, 1$$

    b.

This applies to both parts.

Code written for the question will be given in the appendix (q2.m)

Appendix

```
%{
 2/28/2020
 Matthew Pahayo
 file Name: q1a.m
%}
clc
clear all
close all

format longg

a = [27.58 7.004 -7.004 0 0;...
     7.004 29.57 -5.253 0 -24.32;...
     -7.004 -5.253 29.57 0 0;...
     0 0 0 27.58 -7.004;...
     30 -24.32 0 -7.004 29.57];

b = transpose([0 0 0 0 -45]);
n = length(a);

k = 1 ;
p = k ;
big = abs(a(k,k));

%**************************************************
% pivoting portion
%**************************************************
for ii=k+1:n
    dummy = abs(a(ii,k));
    if dummy > big
       big = dummy;
       p = ii ;
    end
end
if p ~= k
    for jj = k:n
        dummy = a(p,jj);
        a(p,jj) = a(k,jj);
        a(k,jj) = dummy;
    end
    dummy = b(p);
    b(p)=b(k);
    b(k) = dummy;
end

%**************************************************
% elimination step
%**************************************************
for k=1:(n-1)
    for i=k+1:n
        factor = a(i,k)/a(k,k);
       for j=k+1:n
           a(i,j) = a(i,j) - factor*a(k,j);
       end
```

```
        b(i) = b(i) - factor*b(k);
    end
end

%****************************************************
% back substitution
%****************************************************
x(n,1) = b(n)/a(n,n);
for i = n-1:-1:1
    sum = b(i);
   for j = i + 1:n
        sum = sum - a(i,j)*x(j,1);
   end
   x(i,1) = sum/a(i,i);
end
%}




%{
 2/28/2020
 Matthew Pahayo
 file Name: q1b.m
%}
clc
clear all
close all

format longg

n = 10;

for i = 1:n
    for j = 1:n
        a(i,j) = 1/(i+j-1);
    end
end

for i=1:n
    b(i,1) = 1;
end

k = 1 ;
p = k ;
big = abs(a(k,k));

%****************************************************
% pivoting portion
%****************************************************
for ii=k+1:n
    dummy = abs(a(ii,k));
    if dummy > big
       big = dummy;
       p = ii ;
```

```
    end
end
if p ~= k
    for jj = k:n
        dummy = a(p,jj);
        a(p,jj) = a(k,jj);
        a(k,jj) = dummy;
    end
    dummy = b(p);
    b(p)=b(k);
    b(k) = dummy;
end

%****************************************************
% elimination step
%****************************************************
for k=1:(n-1)
    for i=k+1:n
        factor = a(i,k)/a(k,k);
       for j=k+1:n
           a(i,j) = a(i,j) - factor*a(k,j);
       end
       b(i) = b(i) - factor*b(k);
    end
end

%****************************************************
% back substitution
%****************************************************
x(n,1) = b(n)/a(n,n);
for i = n-1:-1:1
    sum = b(i);
   for j = i + 1:n
       sum = sum - a(i,j)*x(j,1);
   end
   x(i,1) = sum/a(i,i);
end
%}




%{
 2/28/2020
 Matthew Pahayo
 file Name: q2.m
%}
clc
clear all
close all

format longg

n = 10;

for i = 1:n
```

```matlab
    for j = 1:n
        a(i,j) = 1/(i+j-1);
    end
end
A = a

for i=1:n
    b(i,1) = 1;
end
%****************************************************
% modified A
%****************************************************
for i = 1:n
    o(i) = i;
end
for k = 1:n-1
    for i = 1+k:n
        factor = a(o(i),k)/a(o(k),k);
        a(o(i),k) = factor ;
        for j = k+1:n
            a(o(i),j) = a(o(i),j) - factor*a(o(k),j);
        end
    end
end



%****************************************************
% get L and U
%****************************************************
for i = 1:n
    L(i,1) = a(i,1);
    U(i,i) = 1;
end
for k = 1:n
    for j = k+1:n
        U(k,j) = a(k,j);
    end
end
for i = 1:n
    for j = 1:i
        L(i,j) = a(i,j);
    end
end
(L)
(U)

%****************************************************
% forward substitution
%****************************************************
for i = 2:n
    sum = b(i);
    for j = 1:i-1
        sum = sum - a(i,j)*b(j);
    end
    b(i) = sum;
end
```

```
D = b

%**************************************************
% back substitution
%**************************************************
x(n,1) = b(n)/a(n,n);
for i = n-1:-1:1
    sum = b(i);
   for j = i + 1:n
       sum = sum - a(i,j)*x(j,1);
   end
   x(i,1) = sum/a(i,i);
end
x
%}
```