Analyze the effect of the step (size grid size) $\boldsymbol{h}$ for the following finite difference formulas to calculate the first derivative of

$$f(x) = Sin(e^x - 2) \quad at\ x_i = 0.9$$

$$\left(\frac{df}{dx}\right)_i = \frac{1}{12h}\left(-f_{i+2} + 8f_{i+1} - 8f_{i-1} + f_{i-2}\right) + O(h^4)$$

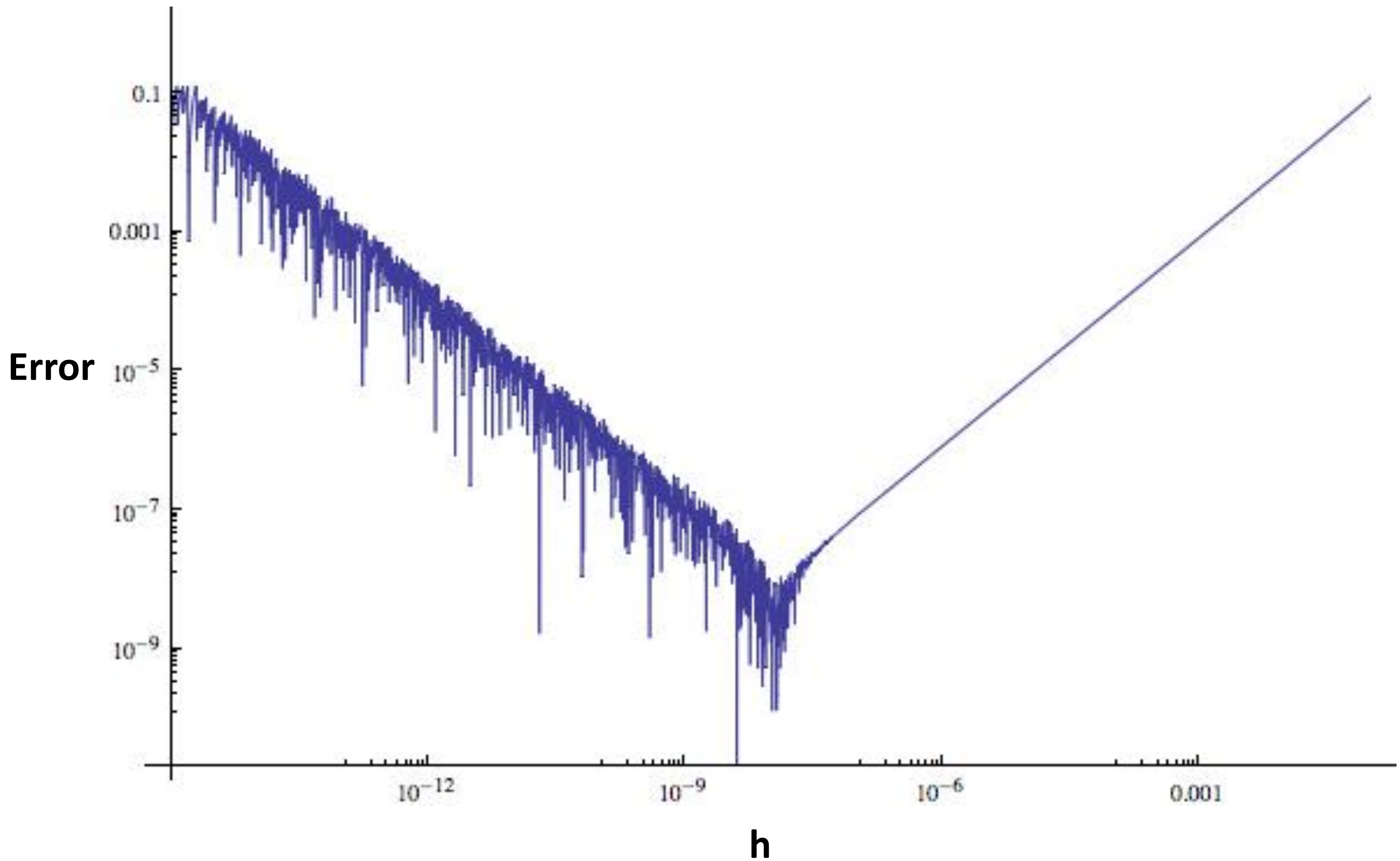
$$\left(\frac{df}{dx}\right)_i = \frac{1}{h}\left(f_{i+1} - f_i\right) + O(h)$$

Error Table

h	Error (1st Order)	Error (4^{th} Order)
10^{-1}	8.67541×10^{-2}	5.74867×10^{-5}
10^{-2}	8.26649×10^{-3}	5.58038×10^{-9}
10^{-3}	8.22587×10^{-4}	5.57554×10^{-13}
10^{-4}	8.22181×10^{-5}	3.35509×10^{-13}
10^{-5}	8.22139×10^{-6}	4.10538×10^{-12}
10^{-6}	8.22042×10^{-7}	6.99095×10^{-11}
10^{-7}	8.07461×10^{-8}	3.00165×10^{-10}
10^{-8}	9.18195×10^{-9}	9.18195×10^{-9}
10^{-9}	1.20204×10^{-7}	4.61894×10^{-8}
10^{-10}	1.23043×10^{-6}	2.4987×10^{-7}

Comment: As expected, the reduction in error is consistent with the order of each method. The round-off error becomes important at small mesh sizes, so the error starts to increase after a certain grid size for each method. The optimal grid size for the first order approximation is approximately 10^{-8} whereas the optimal grid size for the fourth order is around 10^{-3} (see also the plots in the next two pages).

Error vs. Step Size (1^{st} Order Approximation)



Error vs. Step Size (4^{th} Order Approximation)