AE/ME-5830 Spring 2021 Homework II, due 11:59 PM CST Sunday February 28

1. **(50 points)** Develop a computer routine to solve a set of linear equations ($Ax = b$) using the Gaussian elimination scheme with partial pivoting. Your routine should take A matrix and the right hand side vector b as inputs and return the solution vector x as output.

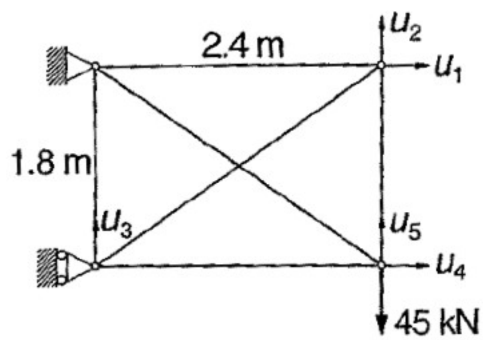

Figure 1: **Truss System**

(a) The displacement formulation for a plane truss system shown in Figure 1 is similar to that of a mass-spring system. The differences are: (1) the stiffnesses of the members are $k_i = (EA/L)_i$ where E is the modulus of elasticity, A represents the cross-sectional area and L is the length of the member; (2) there are two components of displacement at each joint. For the statically indeterminate truss shown, the displacement formulation yields the symmetric equation **Ku=p** where

$$\mathbf{K} = \begin{bmatrix} 27.58 & 7.004 & -7.004 & 0.0000 & 0.0000 \\ 7.004 & 29.57 & -5.253 & 0.0000 & -24.32 \\ -7.004 & -5.253 & 29.57 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 27.58 & -7.004 \\ 0.0000 & -24.32 & 0.0000 & -7.004 & 29.57 \end{bmatrix} \text{MN/m}$$

$$\mathbf{p} = \begin{bmatrix} 0 & 0 & 0 & 0 & -45 \end{bmatrix}^T \text{kN}$$

Using the program you have developed, determine the displacements u_i ($i = 1, ..., 5$) of the joints.

(b) Use your program to solve $Ax = b$ where A is a $n \times n$ Hilbert matrix defined as

$$a_{ij} = \frac{1}{i+j-1} \qquad (i, j = 1, 2, ..., n)$$

and the vector b is given by $b_i = 1.0$ $(i = 1, 2, ..., n)$. Obtain the solution vector for $n = 5$ and $n = 10$.

2. **(50 points)** Develop a computer program to obtain the LU decomposition of a matrix using Crout's Method. Use this program (by adding forward and backward substitution routines) to solve the linear set of equations described in part (b) of question 1. Your results should include the original A matrix, L and U matrices obtained after the decomposition, and the solution vector for each case.

3. **(Extra Credit: 15 points)** Consider a system of n linear equations represented by $Ax = b$

(a) Show that LU decomposition of the coefficient matrix A by Crout's method requires

$$\frac{1}{3}n^3 - \frac{1}{3}n \qquad \textit{multiplications/divisions}$$
$$\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n \qquad \textit{additions/subtractions}$$

(b) Show that solving $Ax = b$ by first decomposing A into $A = LU$ with Crout's method and then solving $Lx^* = b$ and $Ux = x^*$ requires the same number of operations as the Gaussian elimination algorithm.