Question 1:

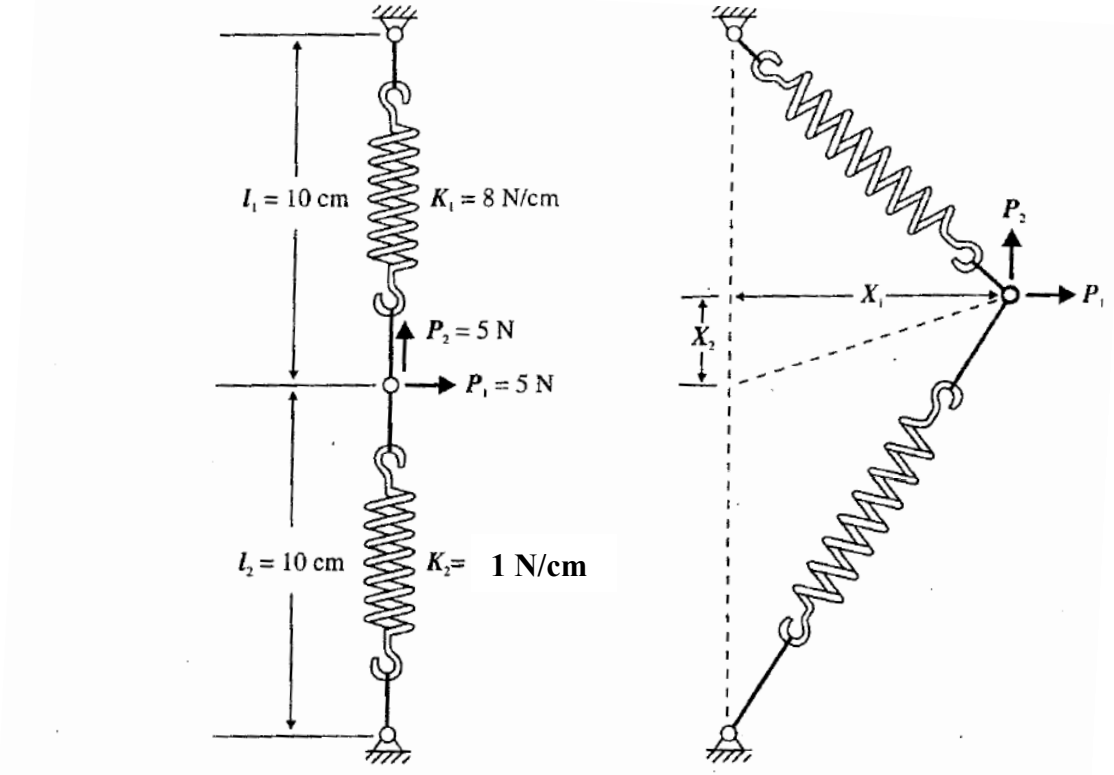
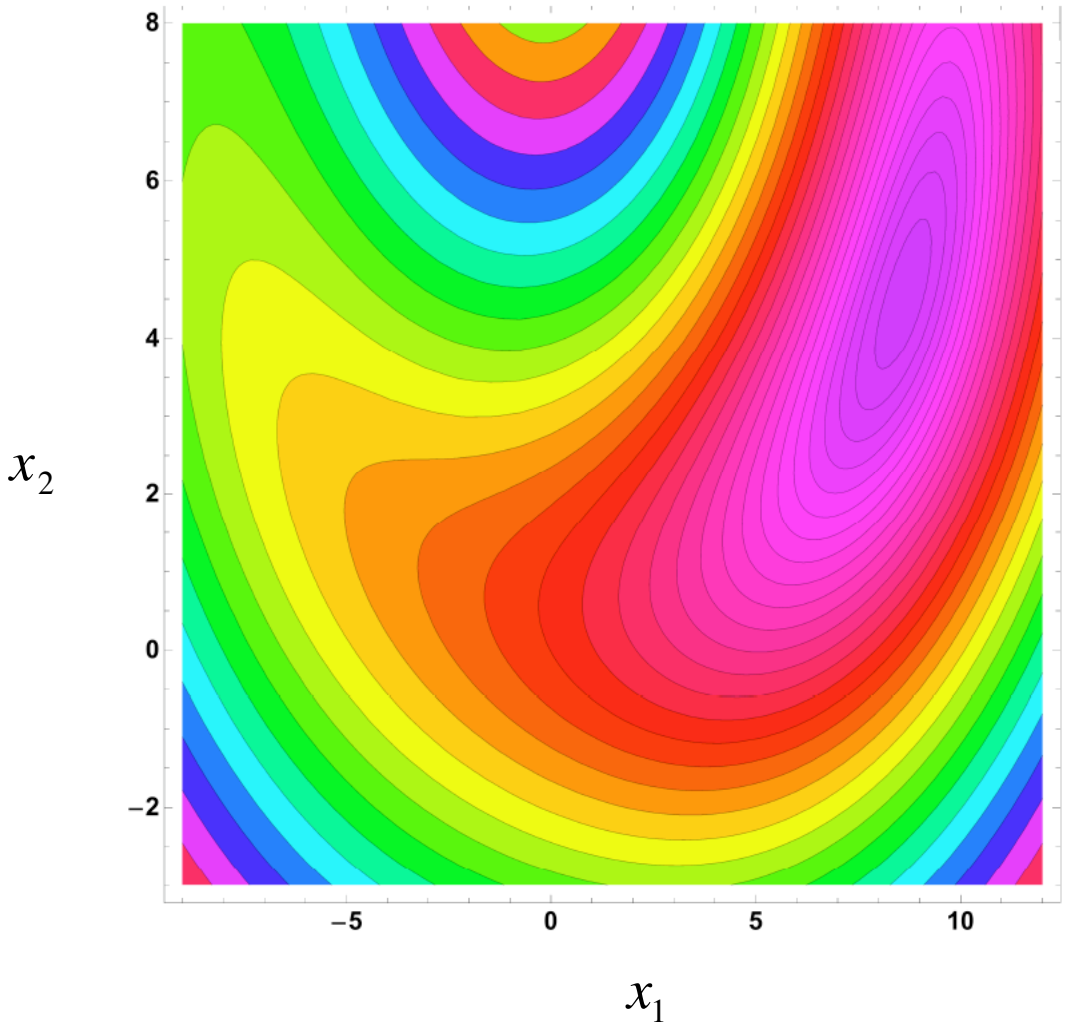


Figure 3-1 Equilibrium of a spring-force system.

$$F(x_1,x_2)=\frac{1}{2}K_1\left\{\sqrt{x_1^2+(l_1-x_2)^2}-l_1\right\}^2+\frac{1}{2}K_2\left\{\sqrt{x_1^2+(l_2+x_2)^2}-l_2\right\}^2-P_1x_1-P_2x_2$$

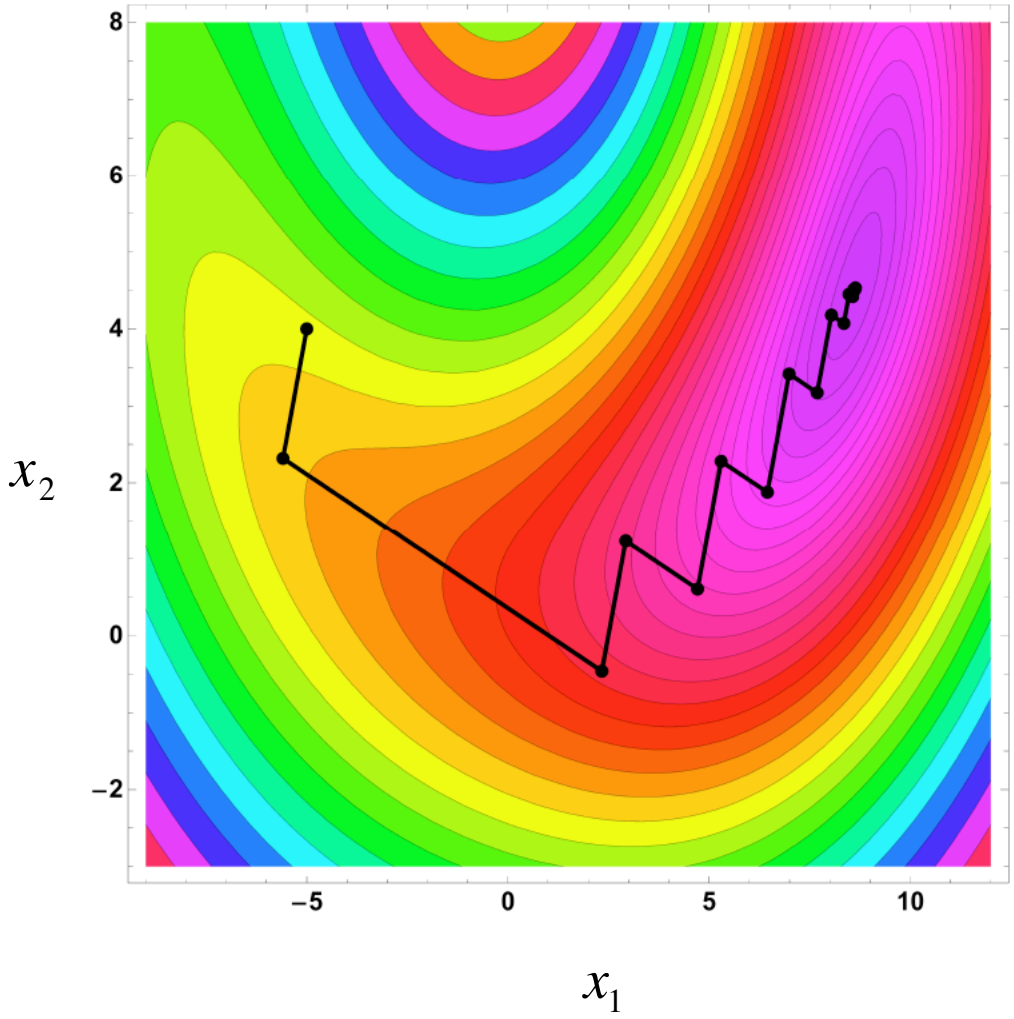


At the optimum

$x_1 = 8.631$ cm $x_2 = 4.533$ cm

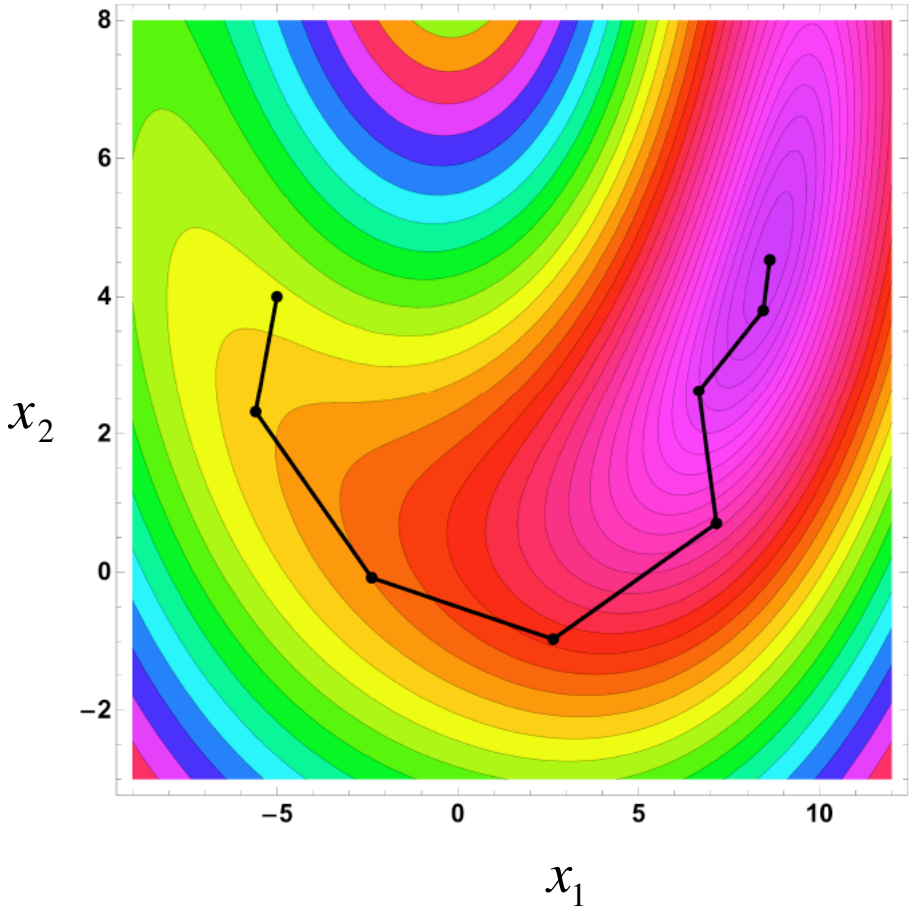
$F(x_1,x_2) = -41.81$ N - cm

Steepest Descent (number of iterations $= 20$)



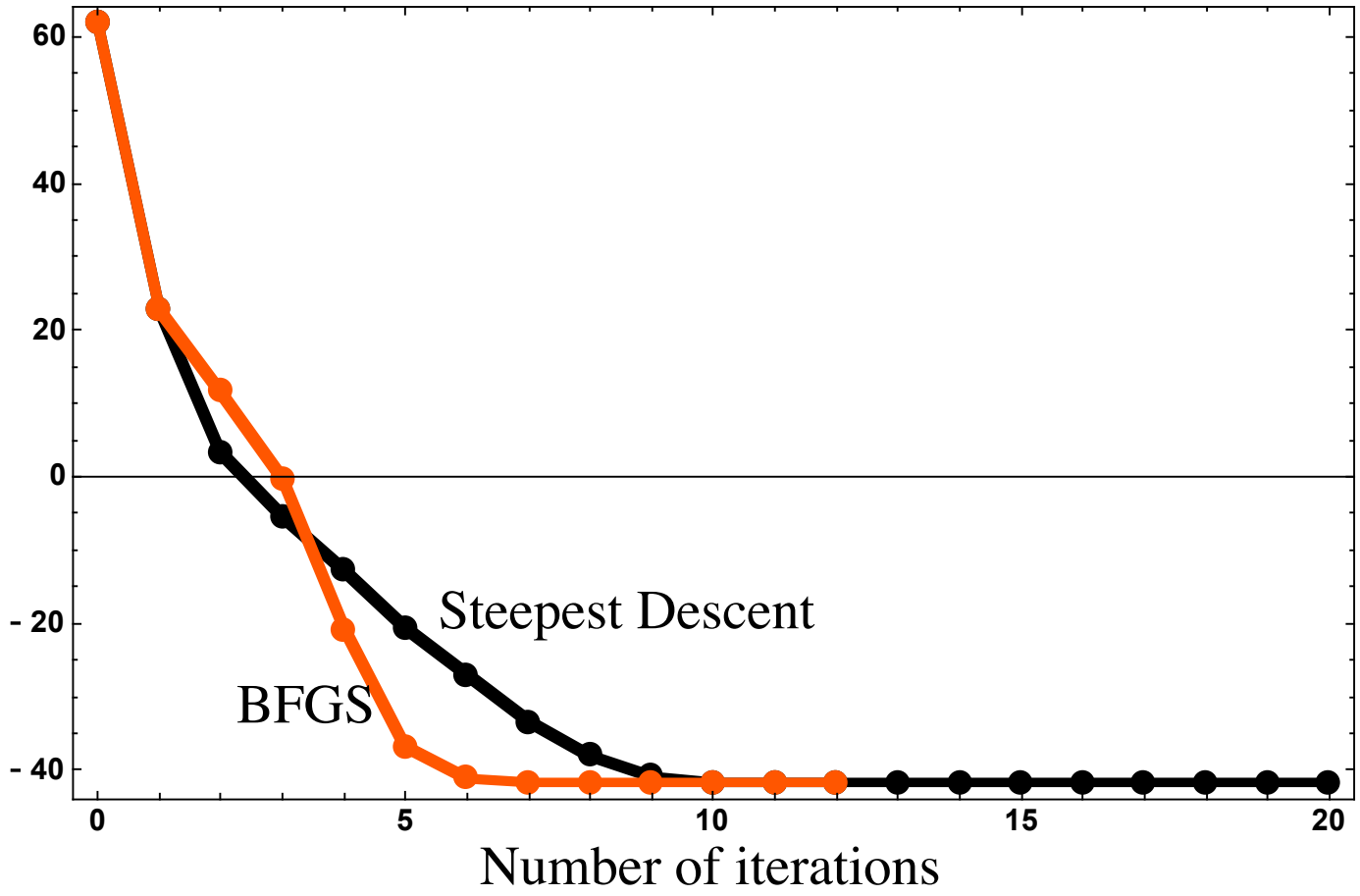
Quasi - Newton (BFGS)

(number of iterations $=12$)



Note that the number of itereations to converge may slightly change depending on the 1-D minimization results at each iteration and precision of the calculations

$F(x_1, x_2)$



Question 2 :

Define :

$$f_1(x) = 4x_1 - x_2 + x_3 - x_1x_4$$

$$f_2(x) = -x_1 + 3x_2 - 2x_3 - x_2x_4$$

$$f_3(x) = x_1 - 2x_2 + 3x_3 - x_3x_4$$

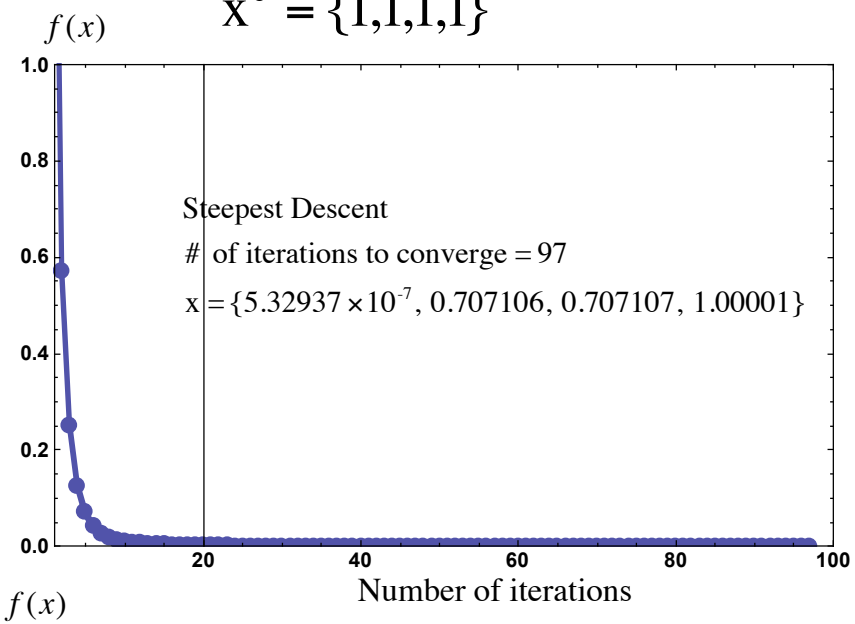
$$f_4(x) = x_1^{\,2} + x_2^{\,2} + x_3^{\,2} - 1.0$$

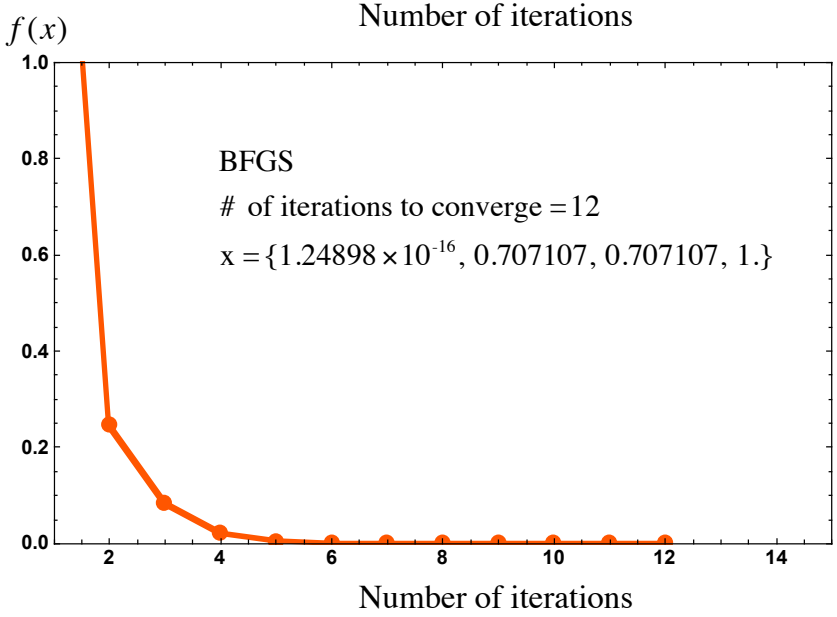
where $x = \{x_1, x_2, x_3, x_4\}$

Minimize $f(x) = f_1^2 + f_2^2 + f_3^2 + f_4^2$ by Steepest Descent and BFGS methods

As can be seen from next slide, for both starting vectors BFGC method exhibits a superior convergence rate over the steepest descent method.

$x^0 = \{1,1,1,1\}$





$x^0 = \{3,3,3,3\}$