Linear System $\rightarrow$ $A\vec{x} = \vec{b}$, with LU decomposition $\boxed{A = LU}$

$L \cdot \underbrace{U \cdot \vec{x}}_{} = \vec{b}$ $\longrightarrow$ (1) $L \cdot \vec{x}^{*} = \vec{b}$ } obtain $\vec{x}^{*}$ with forward subst $\sim O(n^2)$

Define $U\vec{x} = \vec{x}^{*}$

(2) $\underbrace{U \vec{x} = \vec{x}^{*}}_{}$

obtain $\vec{x}$ with backward subst $\sim O(n^2)$

Example: $A_{3\times 3} \rightarrow \boxed{L \cdot U = A}$

$$\underbrace{\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}}_{U} = \underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{A}$$

For Crout's

$u_{ii} = 1.0$

Doolittle's

$l_{ii} = 1.0$

Doolittle's method for LU decomposition:

$$\boxed{L \cdot U = A}$$

L & U ← the results of Gauss Elimination $\quad l_{ii} = 1.0$

↳ $l_{ik} = m_{ik} = \dfrac{a_{ik}^{(k)}}{a_{kk}^{(k)}}$ $\quad k$: elimination step

$k = 1, \ldots, (n-1)$

$i = k+1, \ldots n$

$$L = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ \dfrac{a_{21}^{(1)}}{a_{11}^{(1)}} & 1 & \cdots & \cdots & 0 \\ \dfrac{a_{31}^{(1)}}{a_{11}^{(1)}} & \dfrac{a_{32}^{(2)}}{a_{22}^{(2)}} & 1 & \cdots & 0 \\ \vdots & & & & \\ \dfrac{a_{n1}^{(1)}}{a_{11}^{(1)}} & \dfrac{a_{n2}^{(2)}}{a_{22}^{(2)}} & \cdots & \cdots & 1 \end{bmatrix} \qquad U = \begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & \cdots & a_{1n}^{(1)} \\ & a_{22}^{(2)} & \cdots & a_{2n}^{(2)} \\ & 0 & \ddots & \vdots \\ & & & a_{nn}^{(n)} \end{bmatrix}$$

Coeff. matrix obtained as the result of Gauss Elimination