AE/ME 5830 Spring 2021 Homework I, Due: Thursday, February 11 by 5:00 pm

1. The non-dimensional displacement of an airplane wing tip ($f(t)$) obtained from an aeroelastic flutter study is represented by

$$f(t) = 2Sin(t^2) - 3tSin(t^2) + t^2Sin(t^2)$$

where t is the non-dimensional time.

   (a) For three different starting points $t_0 = 2.2$, $t_0 = 2.3$, and $t_0 = 2.4$, find the roots of $f(t)$ (non-dimensional times at which the displacement is equal to zero) using Newton's Method. For each case, tabulate t_n, $f(t_n)$, $\Delta_n t = t_{n+1} - t_n$, and t_{n+1} at each iteration level n. Use a convergence criterion of $(\epsilon_n)_r < 10^{-8}$ where $(\epsilon_n)_r = \frac{|t_{n+1}-t_n|}{|t_n|}$.

   (b) Use the method of linear interpolation (Regula-Falsi) to find the root of $f(t) = 0$ for $3.2 \leq t \leq 3.8$ with the same convergence criterion given in part (a). Tabulate t_{n+1}, $f(t_{n+1})$, and $(\epsilon_n)_r$ at each iteration level n.

2. The free vibrations of a cantilever beam satisfy the equation

$$cosh(\alpha) \times cos(\alpha) = -1$$

where the dimensionless parameter α is proportional to the square root of the natural frequency. Using Secant Method, find the three smallest positive values of α satisfying this equation (pick the appropriate initial guesses for finding each root). For each root: (1) use a convergence criterion of $(\epsilon_n)_r = \frac{|\alpha_{n+1}-\alpha_n|}{|\alpha_n|} < 10^{-6}$, (2) tabulate the values of α_n, $f(\alpha_n)$, $\Delta_n\alpha = \alpha_{n+1} - \alpha_n$, and α_{n+1} at each iteration.

3. For $-1.0 \leq x \leq 0.0$, find the root of the function

$$f(x) = e^{6x} + 3(ln2)^2e^{2x} - e^{4x}ln8 - (ln2)^3 = 0$$

   (a) Use Newton's Method with a starting point of $x_0 = -1.0$.

   (b) Use Modified Newton's Method with a starting point of $x_0 = -1.0$.

For each method: (1) approximate the root with a convergence criterion of $(\epsilon_n)_r = \frac{|x_{n+1}-x_n|}{|x_n|} < 10^{-5}$, (2) tabulate the values of x_n, $f(x_n)$, $\Delta_n x = x_{n+1} - x_n$, and x_{n+1} at each iteration, and (3) plot $|f(x_n)|/|f(x_0)|$ vs. iteration number on a semi-log plot. Comment on the convergence of each method.

4. (Modified from problem 8.35 in the textbook) Mechanical and Aerospace engineers use thermodynamics extensively in their work. The following polynomial (curve-fit) can be used to relate the specific heat at constant pressure of dry air, c_p (kJ/kgK) to temperature T for a range of values:

$$\begin{aligned} c_p = 0.99403 + 1.671 \times 10^{-4}T + 9.7215 \times 10^{-8}T^2 \\ -9.5838 \times 10^{-11}T^3 + 1.9520 \times 10^{-14}T^4 \end{aligned}$$

Use a fixed point iteration scheme to determine the temperature that corresponds to specific heat of 1.2 (kJ/kgK) with a convergence criterion of $(\epsilon_n)_r = \frac{|T_{n+1}-T_n|}{|T_n|} < 10^{-5}$. Show the derivation of the fixed-point function $g(T)$ used in your scheme. Solve the problem with two different initial guesses, $T_0 = 500\ K$ and $T_0 = 2000\ K$. For each case, tabulate the values of T_n, T_{n+1}, and $(\epsilon_n)_r$ at each iteration level n and limit the maximum number of iterations to $N_{max} = 100$.