AE/ME 5830 Spring 2021, Homework V, Due Sunday, April 11 by 11:59 PM CST

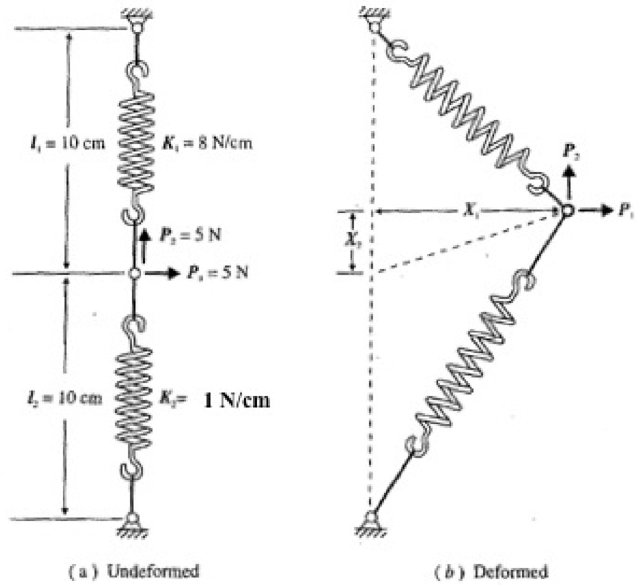

Figure 1: Equilibrium of a spring-force system

1. Consider a simple two-spring system shown in the undeformed position in Figure 1a and after deformation in Figure 1b. The springs are linearly elastic with constants $K_1 = 8\ N/cm$ and $K_2 = 1 N/cm$. The loads P_1 and P_2 are both constant and equal to 5 N. In terms of the original lengths of the springs $l_1 = l_2 = 10\ cm$ and the displacements x_1 and x_2, the total potential energy of the system is given as

$$PE(x_1, x_2) = \frac{1}{2}K_1\left[\sqrt{x_1^2 + (l_1 - x_2)^2} - l_1\right]^2 +$$

$$\frac{1}{2}K_2\left[\sqrt{x_1^2 + (l_2 + x_2)^2} - l_2\right]^2 - P_1x_1 - P_2x_2.$$

This equation can be solved for the equilibrium displacements as an unconstrained minimization problem where $\vec{x} = (x_1, x_2)$ is the independent design variable vector and $PE(x_1, x_2)$ is the objective function. Calculate the minimum objective function value and the corresponding design variables by using **(i)** Steepest Descent, and **(ii)** the BFGS Variable Metric (Quasi-Newton) Methods. For each method, use the initial design point $\vec{x}^0 = \{-4, 5\}^T$ and limit the maximum number of iterations to 200. Use the convergence criterion $\left|PE(\vec{x}^k) - PE(\vec{x}^{k-1})\right| \leq 10^{-10}$ and make sure to satisfy this at 4 successive iterations. For each method, output the objective function and design variable vector values at the minimum along with the number of iterations to converge. Plot the convergence history of the objective function ($PE(\vec{x}^k)$ versus k, the iteration number) for each method. For one dimensional search at each iteration, you may use the routine you have developed in Homework 4 with $n = 15$ or 20 for golden section search. (Note that at the minimum, $PE = -41.8082$, $x_1 = 8.6321$, and $x_2 = 4.5319$.)

2. It can be shown that solution to a non-linear set of equations ($f_i(\vec{x}) = 0$, $i = 1, 2, ..., n$ and $\vec{x} = \{x_1, x_2, ..., x_n\}$) can be obtained by formulating the problem as a non-linear optimization problem with the objective function

$$F(\vec{x}) = f_1^2(\vec{x}) + f_2^2(\vec{x}) + ... + f_n^2(\vec{x}) \tag{1}$$

By implementing the above strategy, use Steepest Descent and the BFGS Variable Metric (Quasi-Newton) methods to find the solution vector to the following system of equations:

$$\begin{aligned} 4x_1 - x_2 + x_3 &= x_1 x_4 \\ -x_1 + 3x_2 - 2x_3 &= x_2 x_4 \\ x_1 - 2x_2 + 3x_3 &= x_3 x_4 \\ x_1^2 + x_2^2 + x_3^2 &= 1 \end{aligned}$$

For each method, obtain the solution using two starting vectors: $x^0 = \{1, 1, 1, 1\}^T$ and $x^0 = \{3, 3, 3, 3\}^T$ and use the same stopping criteria given in Question 1 with the objective function you have defined for this question. Comment on the the convergence of two methods.