1. For part a, Newton's Method was used to determine the roots of the given equation:

$$f(t) = 2sin(t^2) - 3tsin(t^2) + t^2\sin(t^2)$$

which represents for the non-dimensional displacement of an airplane wing tip. Three different starting values were used to illustrate the significance of the initial value. Notice that Newton's Method convergence to a different root of the function for each starting value.

Problem 1a					
t_0=2.2	*n*	t_n	$f(t_n)$	Δt_n	t_{n+1}
	1	2.200000E+00	-2.380485E-01	-1.897974E-01	2.010203E+00
	2	2.010203E+00	-8.069191E-03	-9.784865E-03	2.000418E+00
	3	2.000418E+00	-3.167566E-04	-4.169946E-04	2.000001E+00
	4	2.000001E+00	-5.858235E-07	-7.740718E-07	2.000000E+00
	5	2.000000E+00	-2.019718E-12	-2.668754E-12	2.000000E+00

t_0=2.3	1	2.300000E+00	-3.267301E-01	-9.054107E-01	1.394589E+00
	2	1.394589E+00	-2.223678E-01	4.707378E+00	6.101968E+00
	3	6.101968E+00	-9.386574E+00	4.187691E-02	6.143844E+00
	4	6.143844E+00	1.016639E+00	-3.879403E-03	6.139965E+00
	5	6.139965E+00	1.266620E-03	-4.847219E-06	6.139960E+00
	6	6.139960E+00	3.175883E-09	-1.215383E-11	6.139960E+00

t_0=2.4	1	2.400000E+00	-2.797995E-01	1.957904E-01	2.595790E+00
	2	2.595790E+00	4.177726E-01	-7.741000E-02	2.518380E+00
	3	2.518380E+00	4.645478E-02	-1.139232E-02	2.506988E+00
	4	2.506988E+00	1.378283E-03	-3.594493E-04	2.506629E+00
	5	2.506629E+00	1.401831E-06	-3.663359E-07	2.506628E+00
	6	2.506628E+00	1.456721E-12	-3.805845E-13	2.506628E+00

For part b, the Regula-Falsi method was used to find the root located 3.2 < t < 3.8.

Problem 1b			
n	t_{n+1}	$f(t_{n+1})$	e_n
1	3.371259E+00	-3.031785E+00	1.128266E-01
2	3.537005E+00	-2.181314E-01	6.920932E-02
3	3.548412E+00	9.808767E-02	3.225279E-03
4	3.544874E+00	-9.438935E-04	9.972298E-04
5	3.544908E+00	-3.800625E-06	9.877250E-04
6	3.544908E+00	-1.529859E-08	9.876867E-04
7	3.544908E+00	-6.157993E-11	9.876865E-04
8	3.544908E+00	-2.463663E-13	9.876865E-04
9	3.544908E+00	5.058057E-15	2.505505E-15

2. The Secant method was used to determine the three smallest positive values of the natural frequency, α that satisfy the given equation for the free vibration of a cantilever beam:

$$cosh(\alpha)\ x\ cos(\alpha) = -1$$

To obtain the three smallest possible values, three different sets of initial guesses were selected.

Problem 2					
	n	α_n	$f(\alpha_n)$	$\Delta\alpha_n$	α_{n+1}
α_0=1	1	3.000000E+00	-8.966910E+00	-1.660440E+00	1.339560E+00
α_1=3	2	1.339560E+00	1.467453E+00	2.335186E-01	1.573078E+00
	3	1.573078E+00	9.942625E-01	4.906669E-01	2.063745E+00
	4	2.063745E+00	-8.934669E-01	-2.322338E-01	1.831511E+00
	5	1.831511E+00	1.746824E-01	3.797893E-02	1.869490E+00
	6	1.869490E+00	2.313557E-02	5.797970E-03	1.875288E+00
	7	1.875288E+00	-7.617044E-04	-1.848052E-04	1.875103E+00
	8	1.875103E+00	3.147030E-06	7.603925E-07	1.875104E+00

α_0=5	1	6.000000E+00	1.946814E+02	-1.127732E+00	4.872268E+00
α_1=6	2	4.872268E+00	1.139759E+01	-7.012861E-02	4.802139E+00
	3	4.802139E+00	6.457488E+00	-9.166901E-02	4.710470E+00
	4	4.710470E+00	8.933837E-01	-1.471856E-02	4.695751E+00
	5	4.695751E+00	8.921431E-02	-1.632873E-03	4.694118E+00
	6	4.694118E+00	1.467719E-03	-2.731271E-05	4.694091E+00
	7	4.694091E+00	2.478843E-06	-4.620672E-08	4.694091E+00

α_0=7	1	8.000000E+00	-2.158648E+02	-3.425108E-01	7.657489E+00
α_1=8	2	7.657489E+00	2.075965E+02	1.679116E-01	7.825401E+00
	3	7.825401E+00	3.676965E+01	3.614214E-02	7.861543E+00
	4	7.861543E+00	-8.812643E+00	-6.987534E-03	7.854555E+00
	5	7.854555E+00	2.605987E-01	2.006937E-04	7.854756E+00
	6	7.854756E+00	1.759544E-03	1.364281E-06	7.854757E+00

3. For part a, Newton's method was used in order to determine the root of the equation:

$$f(x) = e^{6x} + 3(ln2)^2 e^{2x} - e^{4x} ln8 - (ln2)^3 = 0$$

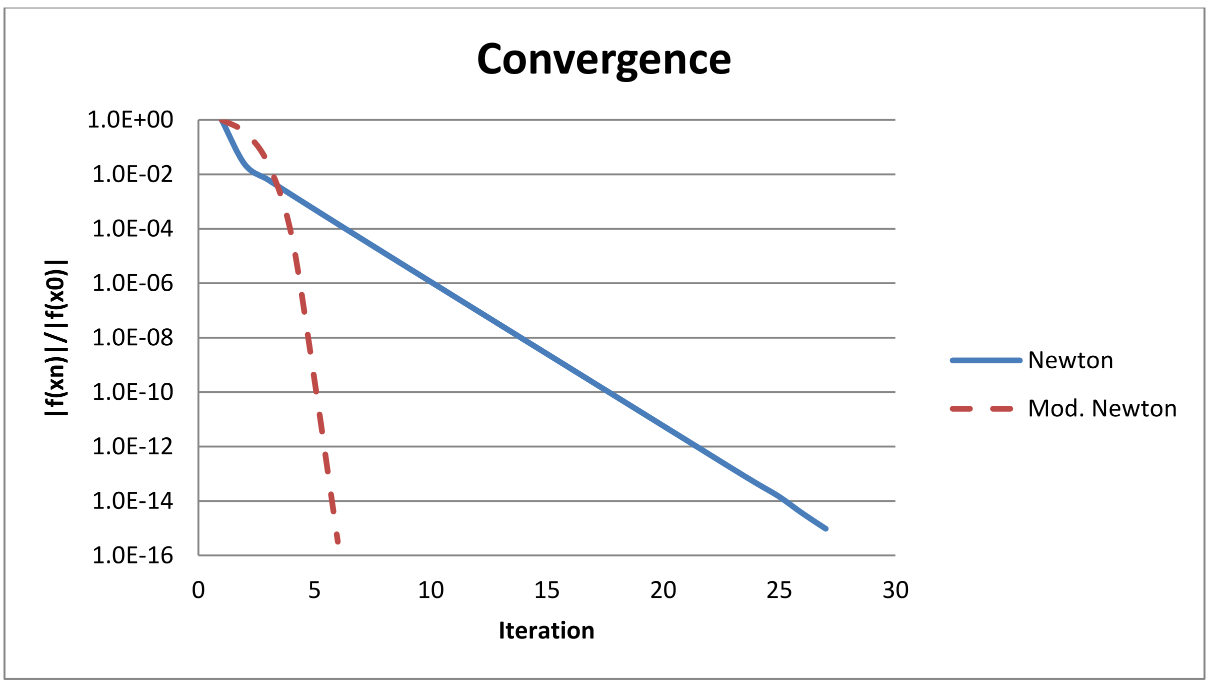
located between -1 < x < 0.

Problem 3a				
n	x_n	$f(x_n)$	Δx_n	x_{n+1}
1	-1.000000E+00	-1.735655E-01	6.869506E-01	-3.130494E-01
2	-3.130494E-01	-3.979888E-03	4.939887E-02	-2.636506E-01
3	-2.636506E-01	-1.091184E-03	2.907270E-02	-2.345779E-01
4	-2.345779E-01	-3.091491E-04	1.801592E-02	-2.165619E-01
5	-2.165619E-01	-8.911714E-05	1.147993E-02	-2.050820E-01
6	-2.050820E-01	-2.594953E-05	7.436306E-03	-1.976457E-01
7	-1.976457E-01	-7.602881E-06	4.866100E-03	-1.927796E-01
8	-1.927796E-01	-2.236249E-06	3.204806E-03	-1.895748E-01
9	-1.895748E-01	-6.594055E-07	2.119478E-03	-1.874553E-01
10	-1.874553E-01	-1.947580E-07	1.405515E-03	-1.860498E-01
11	-1.860498E-01	-5.758439E-08	9.337225E-04	-1.851161E-01
12	-1.851161E-01	-1.703814E-08	6.210301E-04	-1.844951E-01
13	-1.844951E-01	-5.043635E-09	4.133777E-04	-1.840817E-01
14	-1.840817E-01	-1.493484E-09	2.753004E-04	-1.838064E-01
15	-1.838064E-01	-4.423311E-10	1.834074E-04	-1.836230E-01
16	-1.836230E-01	-1.310249E-10	1.222155E-04	-1.835008E-01
17	-1.835008E-01	-3.881490E-11	8.145168E-05	-1.834193E-01
18	-1.834193E-01	-1.149952E-11	5.429080E-05	-1.833650E-01
19	-1.833650E-01	-3.406997E-12	3.618924E-05	-1.833288E-01
20	-1.833288E-01	-1.009359E-12	2.412277E-05	-1.833047E-01
21	-1.833047E-01	-2.990386E-13	1.607938E-05	-1.832886E-01
22	-1.832886E-01	-8.854029E-14	1.071029E-05	-1.832779E-01
23	-1.832779E-01	-2.625677E-14	7.138957E-06	-1.832708E-01
24	-1.832708E-01	-7.938095E-15	4.847957E-06	-1.832659E-01
25	-1.832659E-01	-2.498002E-15	3.488396E-06	-1.832624E-01
26	-1.832624E-01	-6.106227E-16	2.138460E-06	-1.832603E-01
27	-1.832603E-01	-1.665335E-16	1.413974E-06	-1.832589E-01

The same root was determined in part b using the modified Newton's Method in order to illustrate the difference in convergence rate between the two methods.

Problem 3b				
n	x_n	$f(x_n)$	Δx_n	x_{n+1}
1	-1.000000E+00	-1.735655E-01	4.023762E-01	-5.976238E-01
2	-5.976238E-01	-5.955575E-02	2.816993E-01	-3.159245E-01
3	-3.159245E-01	-4.215352E-03	1.165259E-01	-1.993986E-01
4	-1.993986E-01	-1.067757E-05	1.588432E-02	-1.835142E-01
5	-1.835142E-01	-4.560480E-11	2.577224E-04	-1.832565E-01
6	-1.832565E-01	5.551115E-17	-3.186202E-08	-1.832566E-01

Method Convergence Comment: Note that the standard Newton's method takes about four times more iterations to converge. The Modified Newton's method uses more information about the behavior of the function (i.e. its derivative), and thus convergence is faster. The figure below shows that the convergence of the modified method is not linear like the regular method, but rather quadratic.



4. Using the Fixed point iteration method, the goal was to determine the Temperature the satisfies the given equation for a polynomial curve fit of the specific heat at constant pressure of dry air at a given specific heat of 1.2 (kJ/kgK). Two different starting values were used, namely T=500 K and T=2000K.

Derivation of Fixed Point Equation:

$$f(T) = -1.2 + 0.99403 \ + 1.671x10^{-4}\,T + 9.7215x10^{-8}\,T^2 - 9.5838x10^{-11}\,T^3 + 1.9520x10^{-14}\,T^4$$

$$g(T) = T + cf(T)$$

$$g(T) = T + c(-1.2 + 0.99403 \ + 1.671x10^{-4}\,T + 9.7215x10^{-8}\,T^2 - 9.5838x10^{-11}\,T^3 + 1.9520x10^{-14}\,T^4)$$

In order to determine the value of ‘c’, a guess and check method was employed. Recall that divergence is observed for the fixed point iteration method when $|g'(x)| > 1.0$. For small values of ‘c’ the solution moved only slightly from the initial guess. For extremely large values of the ‘c’, the method diverged. It was determined that a value for ‘c’ of 5000 produced the desired solution for the problem, however multiple values are acceptable as long as convergence is achieved in less than 100 iterations.

Problem 4				
T_0=500	n	T_n	T_{n+1}	e_n
	1	5.000000E+02	1.044380E+03	1.088760E+00
	2	1.044380E+03	1.101223E+03	5.442745E-02
	3	1.101223E+03	1.117941E+03	1.518113E-02
	4	1.117941E+03	1.123331E+03	4.821191E-03
	5	1.123331E+03	1.125115E+03	1.588116E-03
	6	1.125115E+03	1.125710E+03	5.292517E-04
	7	1.125710E+03	1.125909E+03	1.770547E-04
	8	1.125909E+03	1.125976E+03	5.930726E-05
	9	1.125976E+03	1.125998E+03	1.987439E-05
	10	1.125998E+03	1.126006E+03	6.661039E-06

Problem 4 (cont.)				
T_0=2000	n	T_n	T_{n+1}	e_n
	1	2.000000E+03	1.686470E+03	1.567650E-01
	2	1.686470E+03	1.433762E+03	1.498443E-01
	3	1.433762E+03	1.266394E+03	1.167336E-01
	4	1.266394E+03	1.180825E+03	6.756921E-02
	5	1.180825E+03	1.145561E+03	2.986394E-02
	6	1.145561E+03	1.132712E+03	1.121581E-02
	7	1.132712E+03	1.128274E+03	3.918395E-03
	8	1.128274E+03	1.126771E+03	1.332311E-03
	9	1.126771E+03	1.126265E+03	4.487360E-04
	10	1.126265E+03	1.126095E+03	1.506534E-04
	11	1.126095E+03	1.126038E+03	5.052380E-05
	12	1.126038E+03	1.126019E+03	1.693773E-05
	13	1.126019E+03	1.126013E+03	5.677557E-06