$$\boxed{\vec{f}(\vec{x}) = 0} \Rightarrow \begin{Bmatrix} f_1(\vec{x}) \\ f_2(\vec{x}) \\ \vdots \\ f_n(\vec{x}) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix}$$

$$\vec{x} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$$

<u>Example:</u> $f_1(x_1, x_2)$; $f_2(x_1, x_2)$ $\qquad \vec{f} = \begin{Bmatrix} f_1(\vec{x}) \\ f_2(\vec{x}) \end{Bmatrix}$; $\vec{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$

$$\boxed{\vec{f} = 0}$$

Write Taylor Series Expansion at $\vec{x}^{\,k} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}^k$ $\quad$ k = iteration number

$$\overset{\approx 0}{f_1(x_1^k + \Delta x_1^k,\ x_2^k + \Delta x_2^k)} = f_1(x_1^k, x_2^k) + \left.\frac{\partial f_1}{\partial x_1}\right|_{\vec{x}_k} \Delta x_1^k + \left.\frac{\partial f_1}{\partial x_2}\right|_{\vec{x}_k} \Delta x_2^k$$

$$+ \frac{1}{2!}\left[\left.\frac{\partial^2 f_1}{\partial x_1^2}\right|_{\vec{x}^k} (\Delta x_1^k)^2 + 2 \left.\frac{\partial^2 f_1}{\partial x_1 \partial x_2}\right|_{\vec{x}^k} \Delta x_1^k \Delta x_2^k + \left.\frac{\partial^2 f_1}{\partial x_2^2}\right|_{\vec{x}^k} (\Delta x_2^k)^2 \right] + \cdots$$

$$\overset{\approx 0}{f_2(x_1^k + \Delta x_1^k,\ x_2 + \Delta x_2^k)} = f_2(x_1^k, x_2^k) + \left.\frac{\partial f_2}{\partial x_1}\right|_{\vec{x}^k} \Delta x_1^k + \left.\frac{\partial f_2}{\partial x_2}\right|_{\vec{x}^k} \Delta x_2^k$$

$$+ \frac{1}{2!}\left[\left.\frac{\partial^2 f_2}{\partial x_1^2}\right|_{\vec{x}^k} (\Delta x_1^k)^2 + 2 \left.\frac{\partial^2 f_2}{\partial x_1 \partial x_2}\right|_{\vec{x}^k} \Delta x_1^k \Delta x_2^k + \left.\frac{\partial^2 f_2}{\partial x_2^2}\right|_{\vec{x}^k} (\Delta x_2^k)^2 \right] + \cdots$$

$$-f_1(x_1^k, x_2^k) = \left.\frac{\partial f_1}{\partial x_1}\right|_{\vec{x}^k} \Delta x_1^k + \left.\frac{\partial f_1}{\partial x_2}\right|_{\vec{x}^k} \Delta x_2^k$$

$$-f_2(x_1^k, x_2^k) = \left.\frac{\partial f_2}{\partial x_1}\right|_{\vec{x}^k} \Delta x_1^k + \left.\frac{\partial f_2}{\partial x_2}\right|_{\vec{x}^k} \Delta x_2^k$$

$$\vec{x}^k = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}^k$$

$$\vec{x}^{k+1} = \vec{x}^k + \Delta x^k$$

$$\Delta x^k = \begin{Bmatrix} \Delta x_1 \\ \Delta x_2 \end{Bmatrix}^k$$

$$\vec{x}^0 = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}^0$$

$$-\begin{Bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{Bmatrix}^{(k)} = \underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}^{(k)}}_{\text{Jacobian Matrix} = J} \begin{Bmatrix} \Delta x_1 \\ \Delta x_2 \end{Bmatrix}^{(k)}$$

$$\boxed{J = \left[\frac{\partial f_i}{\partial x_j}\right]} \quad \begin{array}{l} i = 1, \dots, n \\ j = 1, \dots, n \end{array}$$

$$\boxed{\vec{f}(\vec{x}) = 0} \qquad \vec{x}^0$$

$$\left(\frac{\partial f_1}{\partial x_2}\right)^k \approx \frac{f_1(x_1^k, x_2^k + h, x_3^k) - f_1(x_1^k, x_2^k, x_3^k)}{} +$$

$h \rightsquigarrow$ step size (small)