AE/ME 5830 Spring 2021, Homework VII, Due Friday May 7 by 9 am CST

1. Using composite Trapezoidal and Simpson's 1/3 rule, approximate

$$\int_0^{\pi} xSin(x)dx.$$

   Obtain your results for $n = 2, 4, 6, ..., 40$ panels. For each case, calculate the error in the approximation using the exact value of the integral. For each n (number of panels), tabulate h (mesh size), result obtained with the trapezoidal method, error of the trapezoidal method, result obtained with Simpson's method, and the error of the Simpson's method. On the same log-log plot, show the change of the error with the mesh size for each method. Comment on your results.

2. Use 3 and 4 point Gaussian Quadrature to approximate the integral given in question 1.

3. To numerically solve

$$\int_0^{0.5} \int_0^{0.5} xye^{(y-x)}dydx$$

   (a) Use Simpson's 1/3 method with two panels in each direction

   (b) Use 3 point Gaussian quadrature for each direction

   Evaluate the error for each method using the exact value of the integral.

4.
$$\frac{dy}{dt} = te^{3t} - 2y, \qquad 0 \leq t \leq 1, \qquad y(0) = 0$$

   Approximate the solution to the above initial value problem using

   (a) Modified-Euler Method

   (b) Midpoint Method

   (c) Heun's Method

   (d) 4-stage Runge-Kutta Method

with a time step of $h = 0.1$. For each method, tabulate the approximate solution and the error at each time step using the exact solution to this problem

$$y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}$$

Plot the approximate solutions and the exact function for comparison. In another graph, plot the error distributions. Note that you may need to give a separate plot for the error of 4-stage Runge-Kutta scheme to see the trend in a larger scale.

5.

$$\frac{dy}{dt} = \frac{y}{t} - \left(\frac{y}{t}\right)^2, \qquad 1 \le t \le 2, \qquad y(1) = 1$$

Approximate the solution to the above initial value problem using

(a) 2-step Adams-Bashfort Method

(b) 3-step Adams-Bashfort Method

with a time step of $h = 0.1$. In each case use starting values obtained from 4-stage Runge-Kutta method. For each method, tabulate the approximate solution and the error at each time step using the exact solution to this problem

$$y(t) = \frac{t}{1 + ln(t)}$$

Plot the approximate solutions and the exact function for comparison. In another graph, plot the error distributions.