Pitfalls of Elimination Methods

	Fix
① Division by zero (for $m_{ik} = \frac{a_{ik}}{a_{kk}}$)	← Partial pivoting
② Growth of round-off errors (especially for large "n")	← Use double precision; ← Partial pivoting

Ill-conditioned system: Two or more equations are nearly identical.

$$\underbrace{\begin{aligned} x_1 + 2x_2 &= 10.0 \\ 1.1x_1 + 2x_2 &= 10.4 \end{aligned}}_{x_1 = 4.0,\ x_2 = 3.0} \Longrightarrow \underbrace{\begin{aligned} x_1 + 2x_2 &= 10.0 \\ 1.05x_1 + 2x_2 &= 10.4 \end{aligned}}_{x_1 = 8.0,\ x_2 = 1.0}$$

Hilbert Matrix: Used as an example of an ill-conditioned system matrix

$$[A] \rightarrow a_{ij} = \frac{1}{i+j-1} \qquad \begin{array}{l} i=1,\ldots,n \\ j=1,\ldots,n \end{array}$$

③ **Singular Systems**: Two or more equations are linearly dependent or identical.

$$A\vec{x} = \vec{b}$$

$$\vec{x} = \underbrace{A^{-1}}_{\text{inverse of } A} \cdot \vec{b}$$

For a singular system, A^{-1} does not exist.

Example: Calculation of the inverse of a square matrix A

$$A \cdot A^{-1} = I \qquad I: \text{identity matrix}$$

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{A}
\underbrace{\begin{bmatrix} \overset{\vec{x}_1}{\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}_1} & \overset{\vec{x}_2}{\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}_2} & \overset{\vec{x}_3}{\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}_3} \end{bmatrix}}_{A^{-1}}
=
\underbrace{\begin{bmatrix} \overset{\vec{b}_1}{\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}} & \overset{\vec{b}_2}{\begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}} & \overset{\vec{b}_3}{\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}} \end{bmatrix}}_{I}$$

1st eqn: $A \vec{x}_1 = \vec{b}_1$

2nd eqn. $A \vec{x}_2 = \vec{b}_2$

3rd eqn $A \vec{x}_3 = \vec{b}_3$

Example for the solution of Multiple RHS with the same A