# Individual Project Help

AE 2790: Intro to Spacecraft Design

Due Friday, April 19, 2019 11:59pm on Canvas

## 1 How to Use the Code

In the following explanation $\boldsymbol{x}_k$ is not the same as $x_k$ because engineers like to use the same variables for different things. The bolding will always imply some sort of multidimensional value. If a capital letter is bolded, it is a matrix, if a lower case letter is bolded it is a vector (either row or column), and if a lower case letter is not bolded, it is a scalar.

*Any code that is given does not need to be changed.* When you are completing your project you will only change given code twice (Problems 10 and 11 where specified.)

The code will not run when you first get it. I suggest completing `kf_setup.m` by including `F` and `Hk` and then using the command `break` just after the Polynomial fit section. This is a bit of a cheat to make sure that only code above the code runs, this way you can complete the first four problems and run the code.

Here are some word, variable relations

- Truth $\boldsymbol{x}$
- The estimated mean (a.k.a Kalman filter estimate) $\boldsymbol{m}$
- Uncertainty will come from $\boldsymbol{P}$
- Measurement $\boldsymbol{z}$

### 1.1 Question 2

It seems most people are over-thinking question 2. I am asking for a plot with Time `T` in the $x$-axis, and the true velocity `xk(2,:)` in the $y$-axis. Both of these variables are given. In addition I just want you to pull the very last value from the true velocity.

### 1.2 Question 3

The biggest hurdle with question 3 is that you need to fit a line to the *measurements* `zk` (given), not the truth. So when you're calculating the coefficients, it should look something like `a = (V*V')\V'*zk'`. Note that the multiplication doesn't have a period, the last term is `zk`, and that `zk` is transposed. I am also not looking for a specific velocity at the end of this problem. I just want you to look at the fitted lines, and make a judgment call about if a polynomial fit could be used to imply the velocity was less than the truth.

### 1.3 Question 4

From lecture yesterday, I told you that if velocity was being measured, `Hk = [0,1,0]`. When you multiply this by the state you get

$$\begin{aligned}\hat{\boldsymbol{z}}_k &= \boldsymbol{H}_k \boldsymbol{x}_k \\ &= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ v_k \\ a_k \end{bmatrix} \\ &= v_k \end{aligned}$$

So `Hk` is effectively "selecting" `v_k` out of the state vector. Note in the above equations, $\boldsymbol{x}_k$ is the state (position, velocity, and acceleration) and $x_k$ is just the position.
You need to change this to make sure the correct element of the state is being selected.

### 1.4 Question 5

If you go to slide 24 of my Kalman filter lecture, all of the equations you need are given. Starting with `kf_prop.m` the input/output variables have the following mathematical notation

$$\texttt{mkm1} = \boldsymbol{m}_{k-1}^{+} \qquad \texttt{Pkm1} = \boldsymbol{P}_{k-1}^{+} \qquad \texttt{F} = \boldsymbol{F}_{k-1} \qquad \texttt{Q} = \boldsymbol{Q}_{k-1} \qquad \texttt{M} = \boldsymbol{M}_{k-1}$$
$$\texttt{mkm} = \boldsymbol{m}_{k}^{-} \qquad \texttt{Pkm} = \boldsymbol{P}_{k}^{-}$$

And similarly for `kf_update.m`

$$\texttt{mkm} = \boldsymbol{m}_{k}^{+} \qquad \texttt{Pkm} = \boldsymbol{P}_{k}^{+} \qquad \texttt{zk} = \boldsymbol{z}_{k} \qquad \texttt{Hk} = \boldsymbol{H}_{k} \qquad \texttt{Rk} = \boldsymbol{R}_{k} \qquad \texttt{Lk} = \boldsymbol{L}_{k}$$
$$\texttt{mkp} = \boldsymbol{m}_{k}^{+} \qquad \texttt{Pkp} = \boldsymbol{P}_{k}^{+}$$

At no point in these two functions should you be using the `.*` multiplication that you might have used in the polynomial fit. Every multiplication will just be `*`

### 1.5 Question 6

Until questions 10 and 11, all of your code can go in `driver.m` below where `kf_plotting` is called. All through the code you have been after $\boldsymbol{m}$ and $\boldsymbol{P}$, these are the mean and covariance respectively. The mean is on "average" what we think the position, velocity, and acceleration are. The covariance is our confidence in that average. The covariance matrix at each time step is a $[3 \times 3]$ matrix

$$\boldsymbol{P} = \begin{bmatrix} \sigma_x^2 & \sigma_{xv} & \sigma_{xa} \\ \sigma_{xv} & \sigma_v^2 & \sigma_{va} \\ \sigma_{xa} & \sigma_{va} & \sigma_a^2 \end{bmatrix}$$

What we care about most is the diagonal terms $\sigma_x^2$, $\sigma_y^2$, and $\sigma_a^2$. These tell us the confidence we have in position, velocity, and acceleration respectively. What I have done is pulled each of those diagonal elements and put them in the matrix `Pk` so it looks like

$$\texttt{Pk} = \begin{bmatrix} \sigma^2_{x,k=0^-} & \sigma^2_{x,k=0^+} & \sigma^2_{x,k=1^-} & \sigma^2_{x,k=1^+} & \cdots & \sigma^2_{x,k=15^+} \\ \sigma^2_{v,k=0^-} & \sigma^2_{v,k=0^+} & \sigma^2_{v,k=1^-} & \sigma^2_{v,k=1^+} & \cdots & \sigma^2_{v,k=15^+} \\ \sigma^2_{a,k=0^-} & \sigma^2_{a,k=0^+} & \sigma^2_{a,k=1^-} & \sigma^2_{a,k=1^+} & \cdots & \sigma^2_{a,k=15^+} \end{bmatrix}$$

The reason there are two $\sigma^2$ for each time step is because the one with the minus sign comes comes from `Pkm` and the one with the plus sign comes from `Pkp`. So when I ask you to plot the mean $\pm 3\sigma$, you will add/subtract values from `Pk` to the mean `mk`. Just make sure that you take note whether you are using the standard deviation ($\sigma$), or the variance ($\sigma^2$).

## 1.6 Question 7

Take a look at the mean plus/minus $3\sigma$ and see if at the last time step you can see where Roscoe's claim comes from. Also discuss why he is wrong, start with describing what the $3\sigma$ means.

## 1.7 Question 8

Look at the mean velocity at the end and see if it helps.

## 1.8 Question 9

Plot the things that are listed and make a decision of which method gives the lowest velocity.

## 1.9 Questions 10 and 11

Change `Rk` and nothing else. Rerun `driver.m` to get the black and gray plots again.

## 1.10 Question 12

Talk about the plots are different. Also discuss which of the different values for `Rk` make the plots look the best (hint: think about how often the mean should be "inside" the standard deviation lines. )