Navigation and the Kalman Filter

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AREUS Lab Seminar Series Week 2

February 18, 2019

Missouri S&T | Mechanical and Aerospace Engineering

Introduction

Guidance

Introduction

Guidance

Navigation

Introduction

Guidance

Navigation

Control

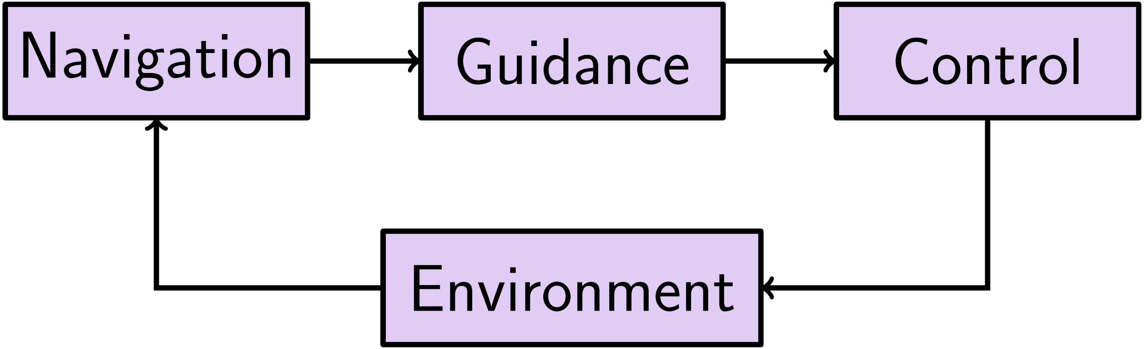
Introduction

Navigation

Guidance

Control

Introduction



Navigation I/O

Time t_k

Navigation

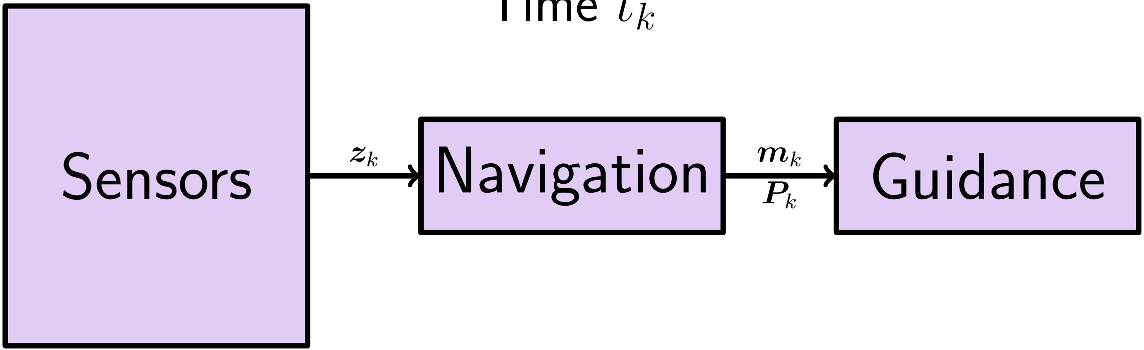
Navigation I/O

Time t_k

$$\boxed{\text{Navigation}} \xrightarrow[\boldsymbol{P}_k]{\boldsymbol{m}_k} \boxed{\text{Guidance}}$$

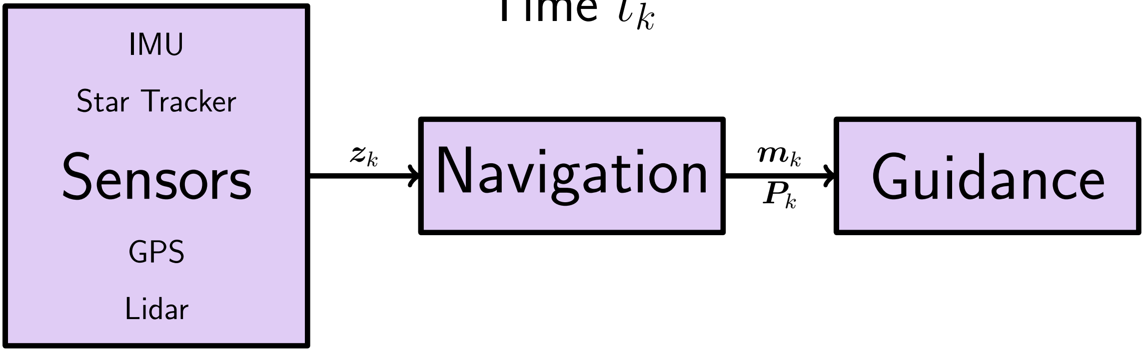
Navigation I/O

Time t_k

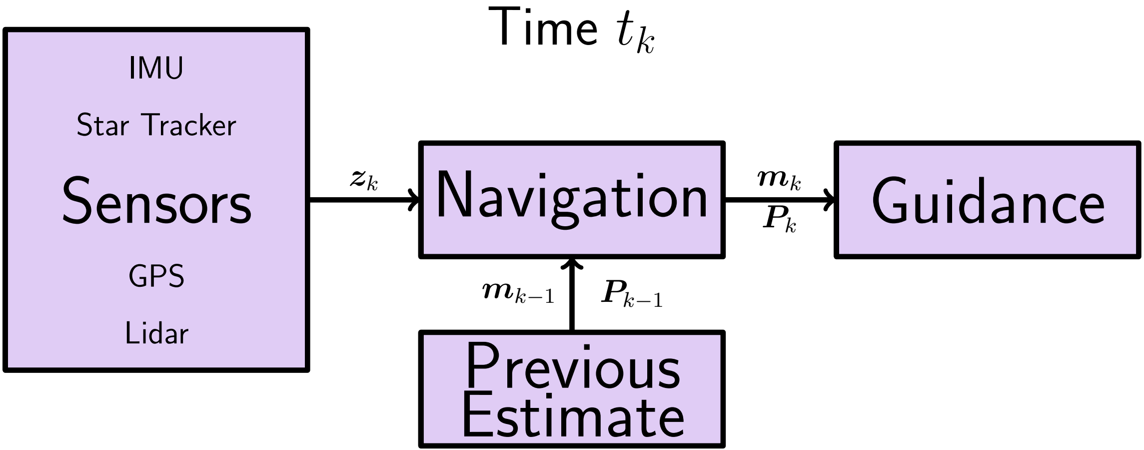


Navigation I/O

Time t_k



Navigation I/O



Kalman Filter

Kalman Filter

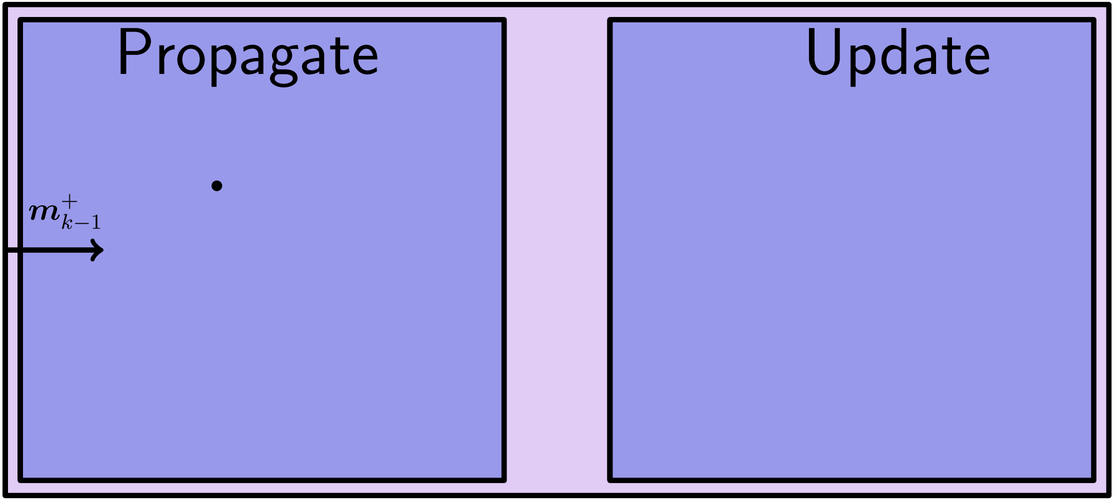
Propagate

Kalman Filter

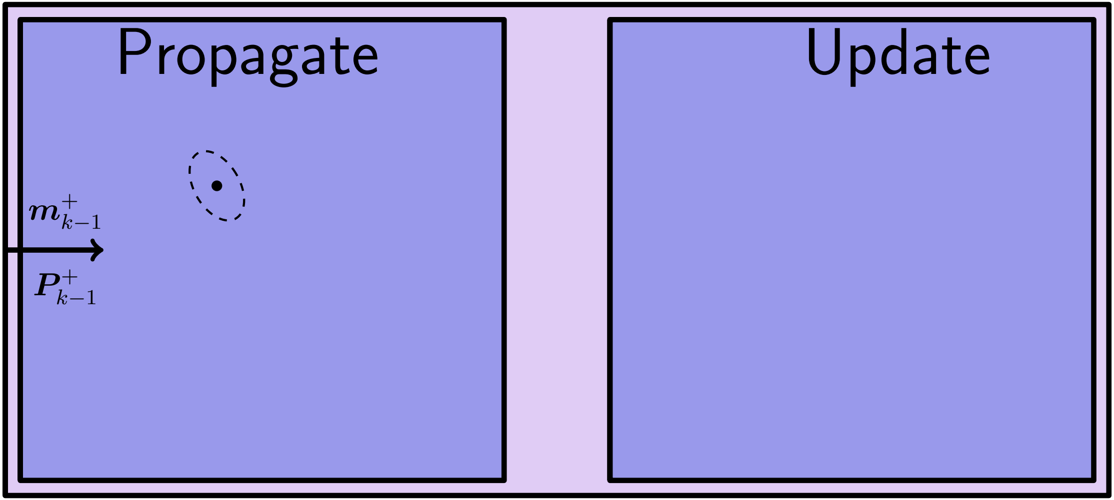
Propagate

Update

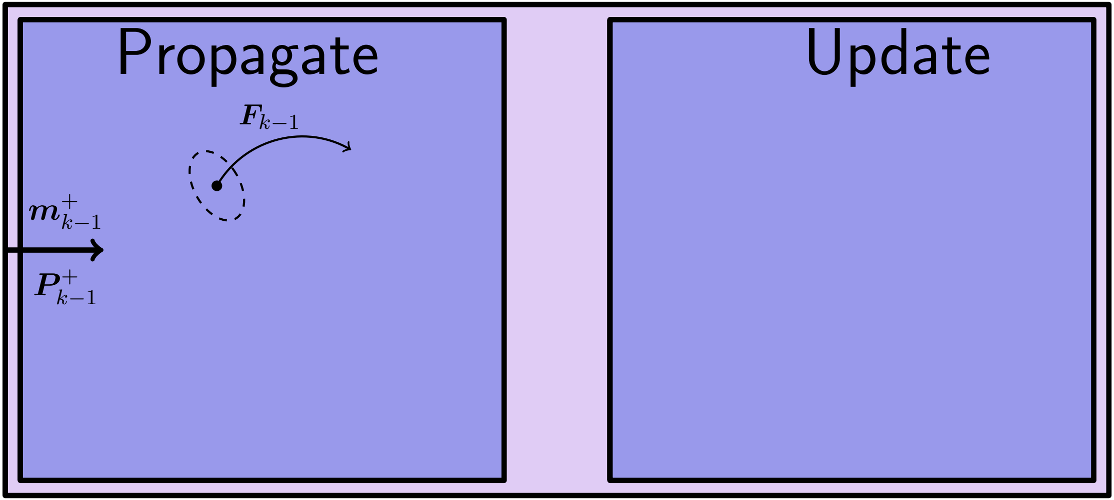
Kalman Filter



Kalman Filter

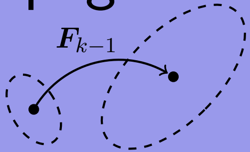


Kalman Filter



Kalman Filter

Propagate



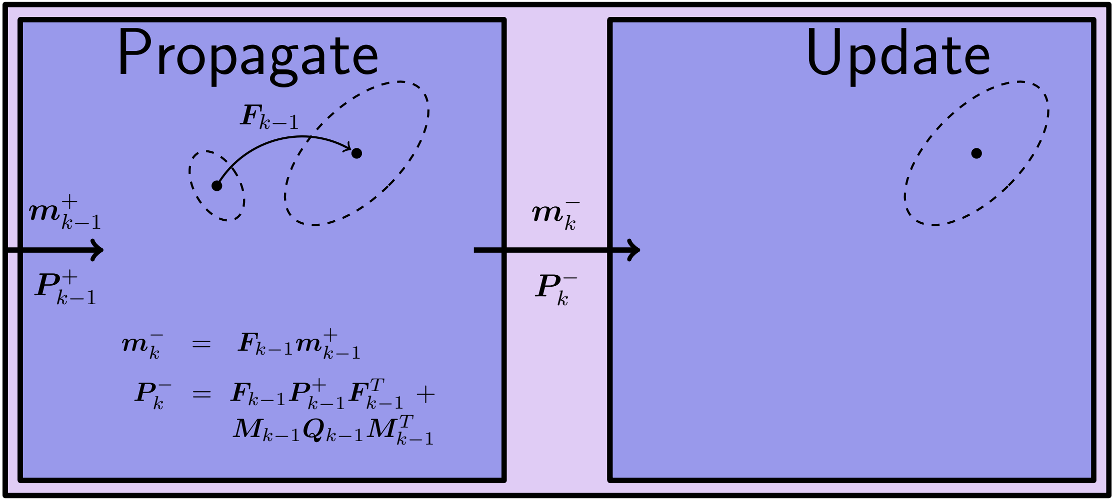
$\boldsymbol{m}_{k-1}^{+}$

$\boldsymbol{P}_{k-1}^{+}$

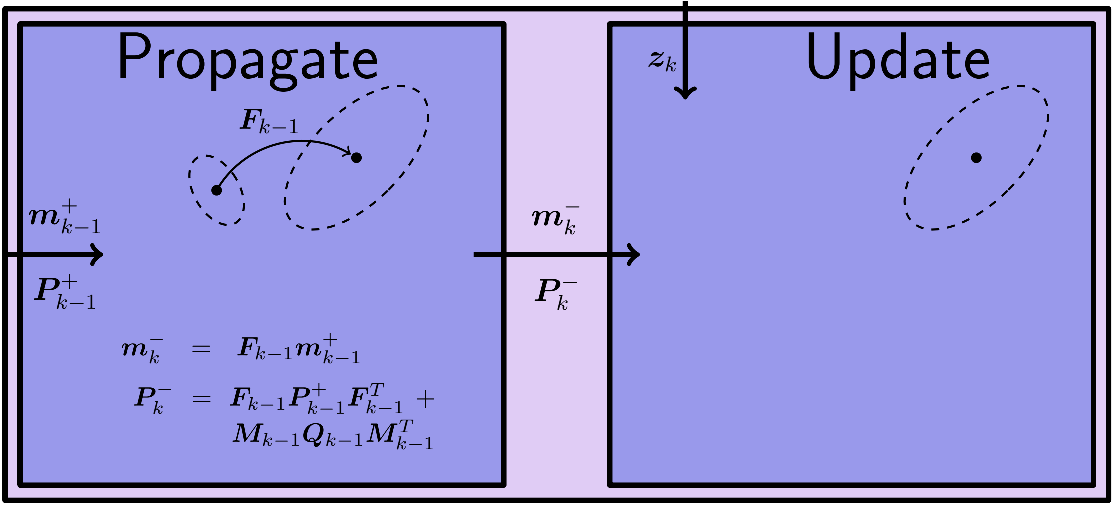
$$
\begin{aligned}
\boldsymbol{m}_k^- &= \boldsymbol{F}_{k-1}\boldsymbol{m}_{k-1}^+ \\
\boldsymbol{P}_k^- &= \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1}^+\boldsymbol{F}_{k-1}^T + \\
&\quad \boldsymbol{M}_{k-1}\boldsymbol{Q}_{k-1}\boldsymbol{M}_{k-1}^T
\end{aligned}
$$

Update

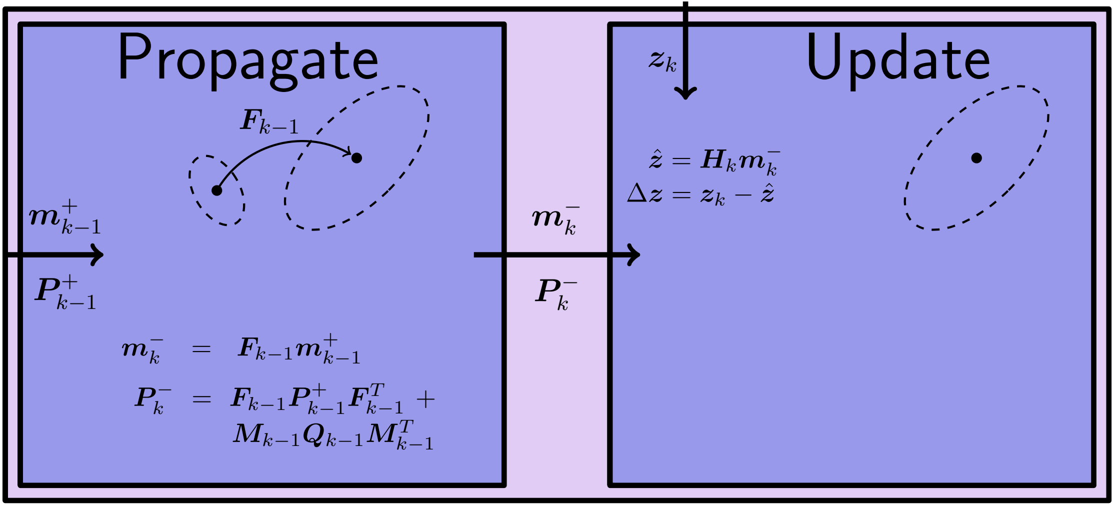
Kalman Filter



Kalman Filter



Kalman Filter



Kalman Filter

Propagate

F_{k-1}

m_{k-1}^+

P_{k-1}^+

$$m_k^- = F_{k-1} m_{k-1}^+$$

$$P_k^- = F_{k-1} P_{k-1}^+ F_{k-1}^T + M_{k-1} Q_{k-1} M_{k-1}^T$$

m_k^-

P_k^-

Update

z_k

$$\hat{z} = H_k m_k^-$$

$$\Delta z = z_k - \hat{z}$$

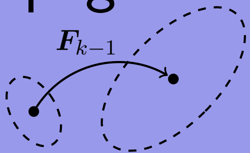
$$W_k = H_k P_k^- H_k^T + L_k R_k L_k^T$$

$$C_k = P_k^- H_k^T$$

$$K_k = P_k^- H_k^T W_k^{-1}$$

Kalman Filter

Propagate



$\boldsymbol{m}_{k-1}^{+}$, $\boldsymbol{P}_{k-1}^{+}$

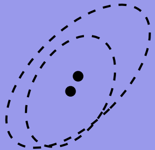
$$\boldsymbol{m}_k^- = \boldsymbol{F}_{k-1}\boldsymbol{m}_{k-1}^+$$

$$\boldsymbol{P}_k^- = \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1}^+\boldsymbol{F}_{k-1}^T + \boldsymbol{M}_{k-1}\boldsymbol{Q}_{k-1}\boldsymbol{M}_{k-1}^T$$

$\boldsymbol{m}_k^-$, $\boldsymbol{P}_k^-$

Update

$\boldsymbol{z}_k$



$$\hat{\boldsymbol{z}} = \boldsymbol{H}_k\boldsymbol{m}_k^-$$

$$\Delta\boldsymbol{z} = \boldsymbol{z}_k - \hat{\boldsymbol{z}}$$

$$\boldsymbol{W}_k = \boldsymbol{H}_k\boldsymbol{P}_k^-\boldsymbol{H}_k^T + \boldsymbol{L}_k\boldsymbol{R}_k\boldsymbol{L}_k^T$$

$$\boldsymbol{C}_k = \boldsymbol{P}_k^-\boldsymbol{H}_k^T$$

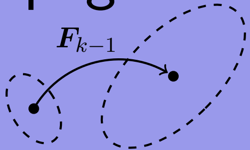
$$\boldsymbol{K}_k = \boldsymbol{P}_k^-\boldsymbol{H}_k^T\boldsymbol{W}_k^{-1}$$

$$\boldsymbol{m}_k^+ = \boldsymbol{m}_k^- + \boldsymbol{K}_k\Delta\boldsymbol{z}$$

$$\boldsymbol{P}_k^+ = \boldsymbol{P}_k^- - \boldsymbol{C}_k\boldsymbol{K}_k^T - \boldsymbol{K}_k\boldsymbol{C}_k^T + \boldsymbol{K}_k\boldsymbol{W}_k\boldsymbol{K}_k^T$$

Kalman Filter

Propagate



m_{k-1}^+, P_{k-1}^+ →

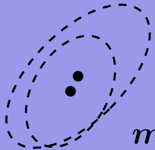
$$m_k^- = F_{k-1} m_{k-1}^+$$

$$P_k^- = F_{k-1} P_{k-1}^+ F_{k-1}^T + M_{k-1} Q_{k-1} M_{k-1}^T$$

→ m_k^-, P_k^- →

Update

z_k ↓



$$\hat{z} = H_k m_k^-$$

$$\Delta z = z_k - \hat{z}$$

$$W_k = H_k P_k^- H_k^T + L_k R_k L_k^T$$

$$C_k = P_k^- H_k^T$$

$$K_k = P_k^- H_k^T W_k^{-1}$$

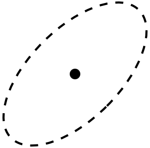
$$m_k^+ = m_k^- + K_k \Delta z$$

$$P_k^+ = P_k^- - C_k K_k^T - K_k C_k^T + K_k W_k K_k^T$$

→ m_k^+, P_k^+

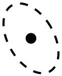
The Kalman Gain

$$\boldsymbol{K}_k = \boldsymbol{P}_k^- \boldsymbol{H}_k^T \boldsymbol{W}_k^{-1}$$



Large Kalman Gain — Small Kalman Gain

Large Residual — Small Residual



$$\delta \boldsymbol{z} = \boldsymbol{z}_k - \boldsymbol{H}_k \boldsymbol{m}_k^-$$

The Kalman Gain

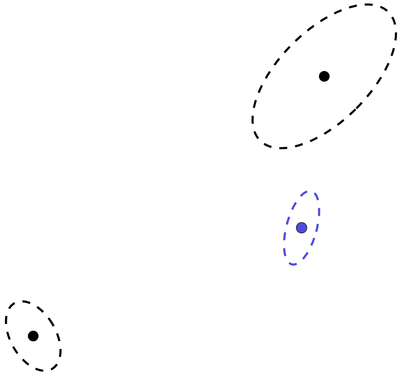
$$\boldsymbol{K}_k = \boldsymbol{P}_k^- \boldsymbol{H}_k^T \boldsymbol{W}_k^{-1}$$

Large Kalman Gain

Large Residual

Small Kalman Gain

Small Residual

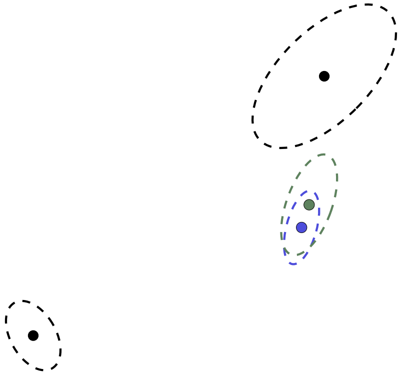


$$\delta \boldsymbol{z} = \boldsymbol{z}_k - \boldsymbol{H}_k \boldsymbol{m}_k^-$$

The Kalman Gain

$$\boldsymbol{K}_k = \boldsymbol{P}_k^- \boldsymbol{H}_k^T \boldsymbol{W}_k^{-1}$$

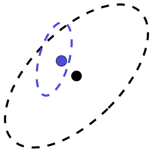
Large Kalman Gain	Small Kalman Gain
Large Residual	Small Residual



$$\delta \boldsymbol{z} = \boldsymbol{z}_k - \boldsymbol{H}_k \boldsymbol{m}_k^-$$

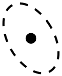
The Kalman Gain

$$\boldsymbol{K}_k = \boldsymbol{P}_k^{-} \boldsymbol{H}_k^T \boldsymbol{W}_k^{-1}$$



Large Kalman Gain | Small Kalman Gain

Large Residual | Small Residual

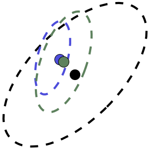


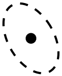
$$\delta \boldsymbol{z} = \boldsymbol{z}_k - \boldsymbol{H}_k \boldsymbol{m}_k^{-}$$

The Kalman Gain

$$\boldsymbol{K}_k = \boldsymbol{P}_k^- \boldsymbol{H}_k^T \boldsymbol{W}_k^{-1}$$

Large Kalman Gain	Small Kalman Gain
Large Residual	Small Residual





$$\delta \boldsymbol{z} = \boldsymbol{z}_k - \boldsymbol{H}_k \boldsymbol{m}_k^-$$

The Kalman Gain

$$\boldsymbol{K}_k = \boldsymbol{P}_k^- \boldsymbol{H}_k^T \boldsymbol{W}_k^{-1}$$

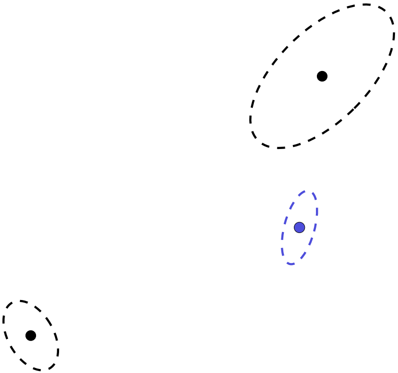
Large Kalman Gain

Small Kalman Gain

Large Residual

Small Residual

$$\delta \boldsymbol{z} = \boldsymbol{z}_k - \boldsymbol{H}_k \boldsymbol{m}_k^-$$



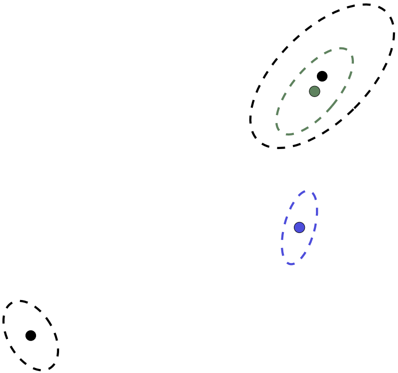
The Kalman Gain

$$\boldsymbol{K}_k = \boldsymbol{P}_k^- \boldsymbol{H}_k^T \boldsymbol{W}_k^{-1}$$

Large Kalman Gain | Small Kalman Gain

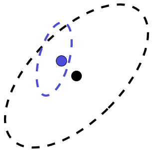
Large Residual | Small Residual

$$\delta \boldsymbol{z} = \boldsymbol{z}_k - \boldsymbol{H}_k \boldsymbol{m}_k^-$$

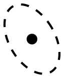


The Kalman Gain

$$\boldsymbol{K}_k = \boldsymbol{P}_k^- \boldsymbol{H}_k^T \boldsymbol{W}_k^{-1}$$



Large Kalman Gain	Small Kalman Gain
Large Residual	Small Residual



$$\delta \boldsymbol{z} = \boldsymbol{z}_k - \boldsymbol{H}_k \boldsymbol{m}_k^-$$

The Kalman Gain

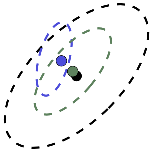
$$\boldsymbol{K}_k = \boldsymbol{P}_k^- \boldsymbol{H}_k^T \boldsymbol{W}_k^{-1}$$

Large Kalman Gain

Small Kalman Gain

Large Residual

Small Residual



$$\delta \boldsymbol{z} = \boldsymbol{z}_k - \boldsymbol{H}_k \boldsymbol{m}_k^-$$

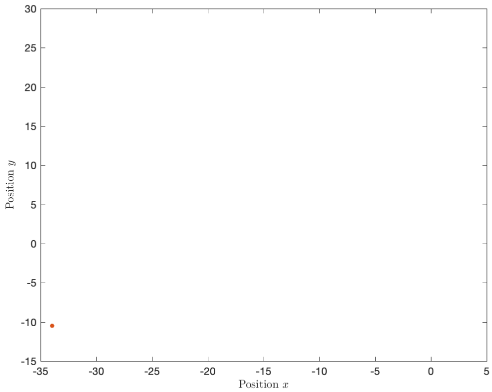
- Truth

$$\Delta x = \dot{x}_{k-1}\Delta t \qquad \Delta \dot{x} = 0$$
$$\Delta y = \dot{y}_{k-1}\Delta t \qquad \Delta \dot{y} = 0$$

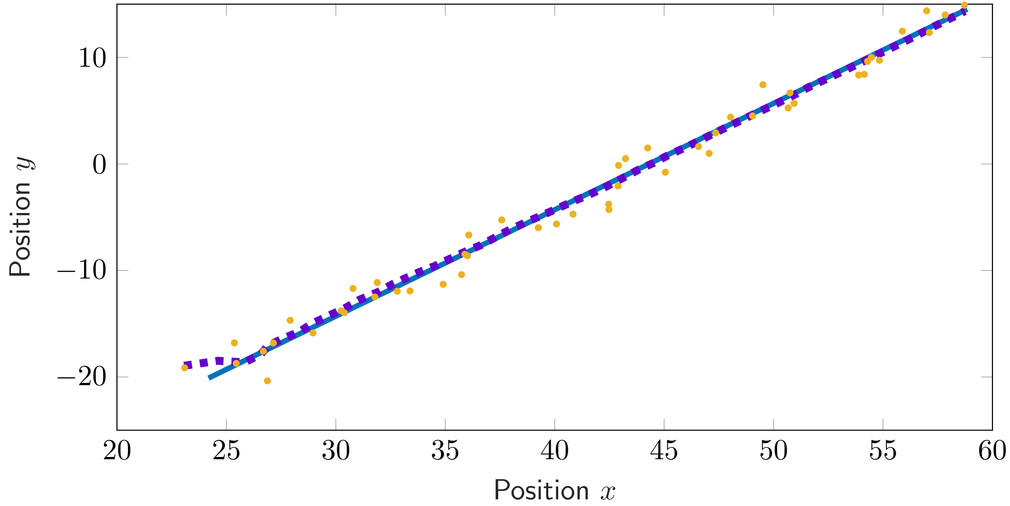
- Initial Estimate

$$\boldsymbol{m}_0 = \begin{bmatrix} 0 \\ 0 \\ 100\cos(\pi/4) \\ 100\sin(\pi/4) \end{bmatrix} \quad \boldsymbol{P}_0 = \text{diag}\begin{bmatrix} 800 \\ 500 \\ 0.05 \\ 0.05 \end{bmatrix}$$

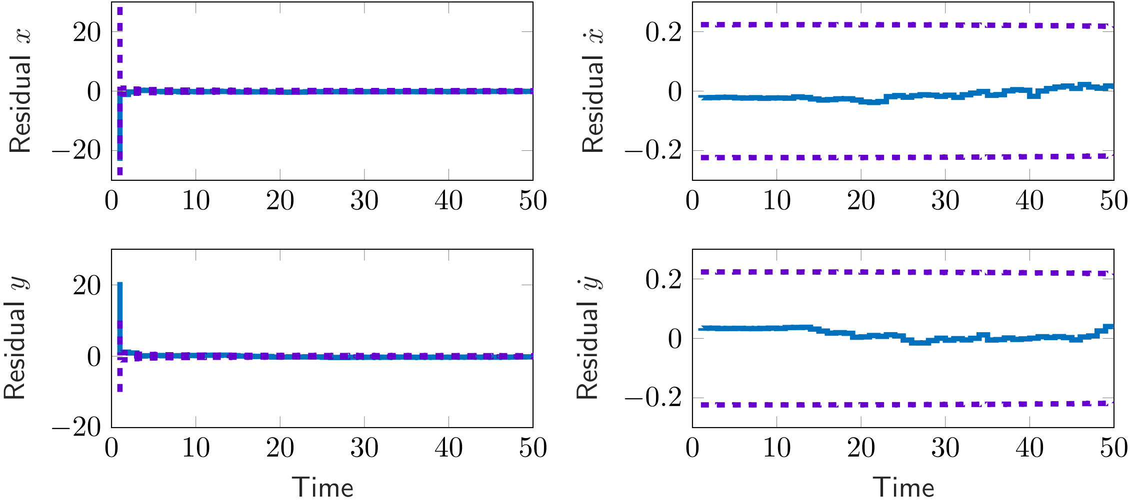
- No process error
- Directly measure x and y
- Measurement error covariance $\boldsymbol{R} = \boldsymbol{I}_2$



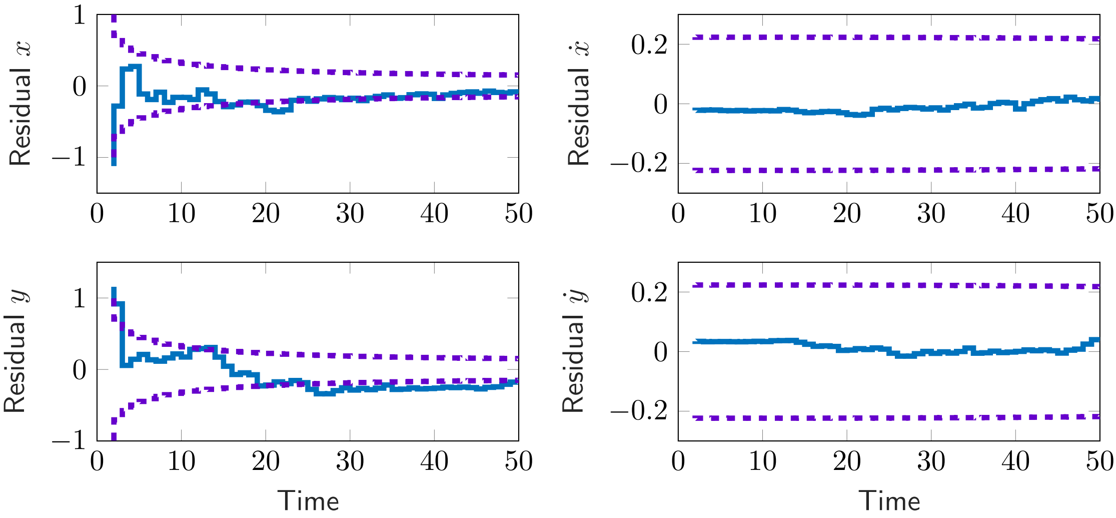
Raw Results



Residuals


Residual x
Residual $\dot{x}$
Residual y
Residual $\dot{y}$
20
0
−20
0.2
0
−0.2
0
10
20
30
40
50
Time
Time

Trimmed Residuals



Questions

x_{k-1}

$$x_k = F_k x_{k-1} + B_k u_k$$

x_k

$$z_k = H_k x_k + v_k$$

$$r_k = z_k - y_k$$

$$y_k = H_k m_k^-$$

m_{k-1}^+

P_{k-1}^+

$$m_k^- = F_k m_{k-1} + B_k u_k$$
$$P_k^- = F_k P_{k-1}^+ F^T + Q_k$$

$$W_k = H P_k^- H_k^T + R_k$$
$$K_k = P_k^- H_k^T W_k^{-1}$$

$$m_k^+ = m_k^- + K_k r_k$$
$$P_k^+ = \left(I_n - K_k H_k\right) P_k^-$$

m_k^+

P_k^+