# 2790 Project

Gari Pahayo

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The Duke boys were at the last stoplight in town. Bo was telling Luke how General Lee could accelerate from 0 to 90 m/s in 15 seconds with a constant acceleration of 7 $m/s^2$. Luke was astonished. Bo began driving. They get pulled over by Roscoe for speeding. They were told that they were going 91 in a 60 m/s zone. They will go to jail. They get a person to prove they were not travelling faster than 89 m/s to avoid going to jail. There fine will be much smaller if they can prove they were going less than 85 m/s. Bo was accelerating less than 7 $m/s^2$, 6.158 $m/s^2$. Bo was 1.544 m in front of the stop line and started to rollback with a velocity of 3.4 m/s. They use a Kalman filter to estimate the velocity.

Step 1 is to make the state transition matrix. The state Matrix use kinematic equations,

**The Kinematic Equations**

$$d = v_i * t + \frac{1}{2} * a * t^2 \qquad v_f^2 = v_i^2 + 2 * a * d$$

$$v_f = v_i + a * t \qquad d = \frac{v_i + v_f}{2} * t$$

.

$$\frac{dv}{dt} = a$$

Multiply to dt to both sides, getting dv = adt. Integrate getting v = at + $v_o$. dx = (at + $v_o$)dt, resulting in x = $x_o$+ $at^2$ + $v_o$t

The code in matlab looks like: `"F = [1 dt 1/2*dt^2;0 1 dt;0 0 1];".` The order is x, x', x''. It is a 3x3 matrix. The true matrix, $x_k$, is given, allowing the data to be plotted over time to find the true position and velocity at 15 seconds.

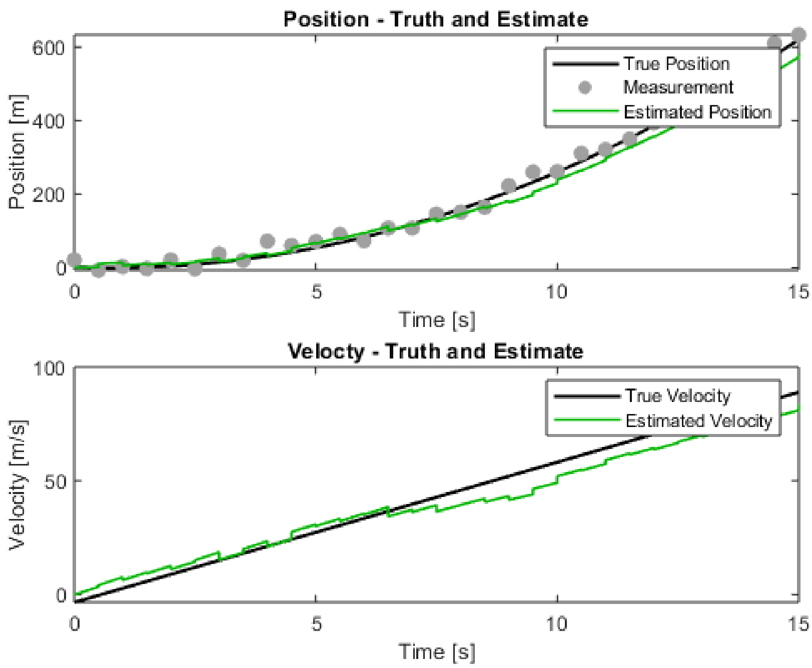


*Figure 1*

The true velocity at 15 seconds is 88.97 m/s, however they don't know this
The judge however does not know much about measurement noise and now wants a proper fit line a truth.

Code is written in matlab to plot the first through fifth order polynomial.

```
x = T';
y = zk';
figure
ax = axes;
scatter(x,y)
hold on
for k = 1:5
    for n=1:length(x)
        V(n,1)  = 1;
        V(n,k+1)  =  (x(n))^k;
    end
    a = (inv(V'*V))*(V')*y;
    if k<5
        a(k+2:6)=0;
    end
    p = 0;
    for j = 0:5
        p = p + a(j+1)*x.^(j);
    end
    plot(x,p)

end
title('Position vs Time');
xlabel('Time (s)');
ylabel('Position (m)');
```

```
legend ('points','1st','2nd','3rd','4th','5th','Location', 'NorthEast');
hold off
```

When running the code, it outputs:

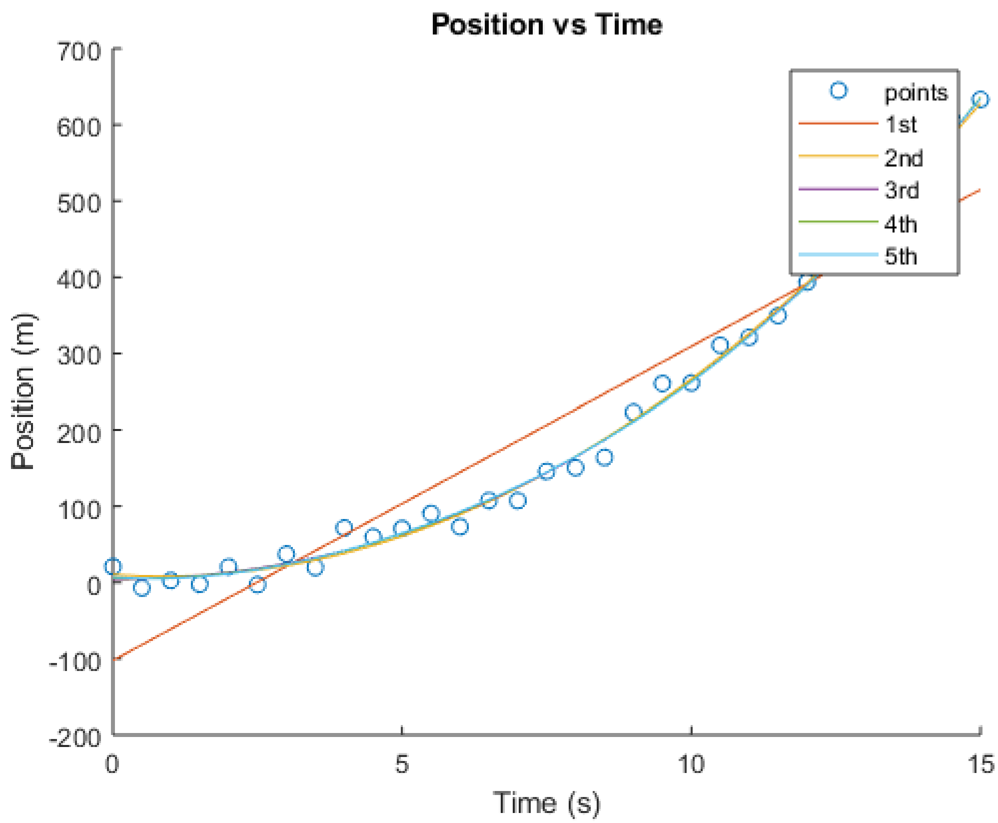


*Figure 2*

Similar code gives an enhanced view of times in between 14 and 15 seconds.

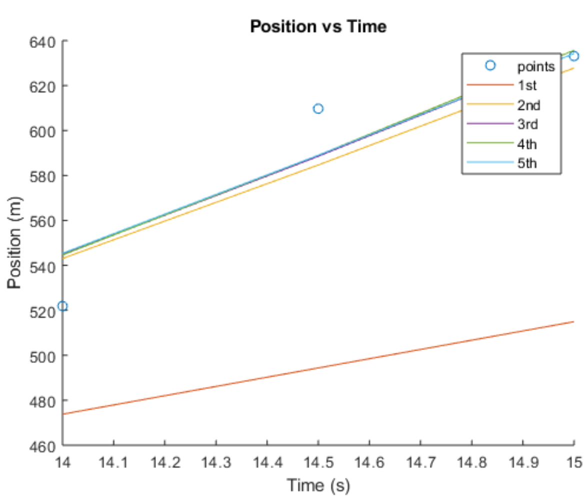


*Figure 3*

Fitting polynomials to Roscoe's data do not show that the Bo's speed was less than the truth. They decide to run a Kalman Filter. A Kalman filter will help predict the General Lee's velocity at the relevant time. It also gives an estimation how correct a value is (residual). The measurement mapping matrix is $H_k$ = [1 0 0]. To construct the Kalman filter eight equations arep needed $m_k^-$, $P_k^-$, $z_k$, $W_k$, $C_k$, $K_k$, $m_k^+$, and $P_k^+$.

The mean propagation is equal to the state transition matrix multiplied by the mean from the last time step. It represents the mean at the current time step. The covariance of propagation is covariance from current time step.

The measurement estimate is a measurement of the true state. The innovation covariance is the covariance of the new dimension. The cross covariance is a measure of the similarity of two signals. The Kalman gain is a matrix o gain from each measurement. The update mean is the mean at the current time step. The update covariance is the covariance at the current time step.

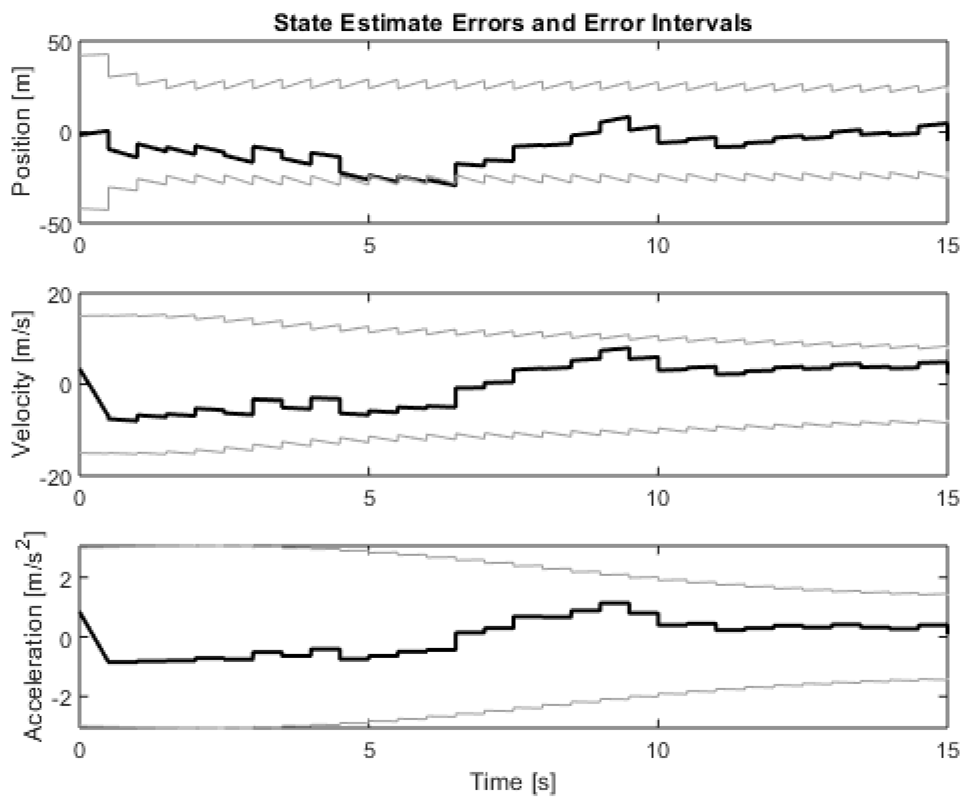


*Figure 4*

The fastest possible speed Roscoe can be at least the tiniest sure Bo was driving is 70.35 m/s. Agreeing with the mean velocity (81.02 m/s) helps the Duke boys.

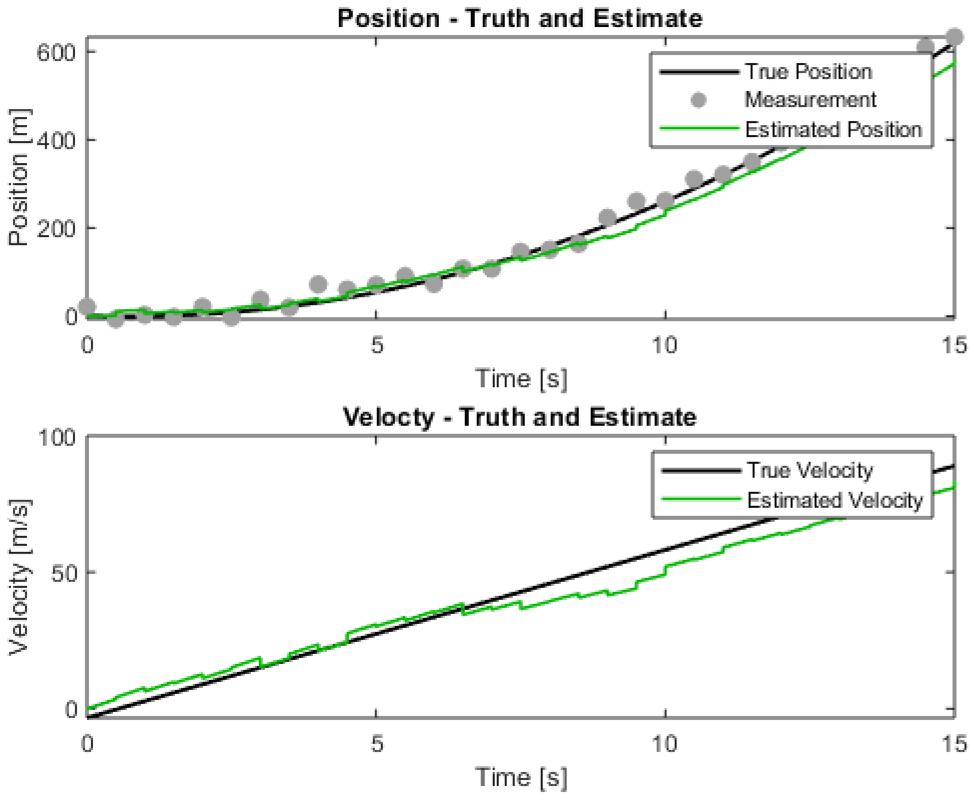
Position - Truth and Estimate
600
400
200
0
Position [m]
0
5
10
15
Time [s]
True Position
Measurement
Estimated Position
Velocty - Truth and Estimate
100
50
0
Velocity [m/s]
0
5
10
15
Time [s]
True Velocity
Estimated Velocity

*Figure 5*

Roscoe now panics and says the variance, $R_k$, was actually 20 $m^2$, but it does not help his case neither does changing the variance to 2000.

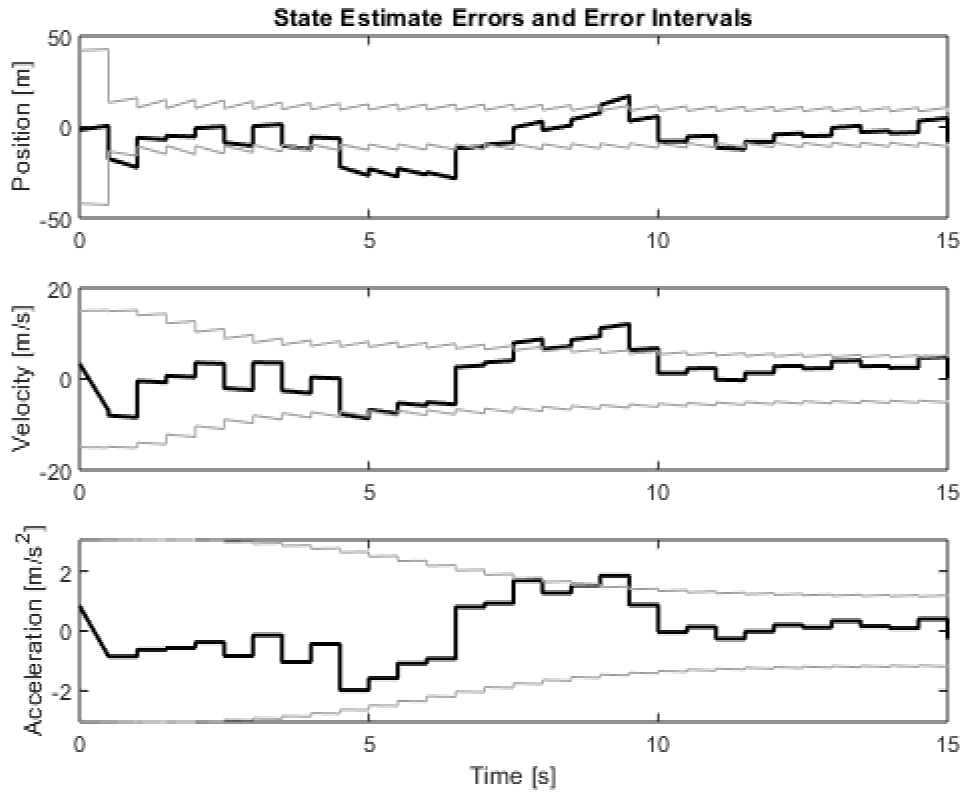


*Figure 6 - 20 m^2*

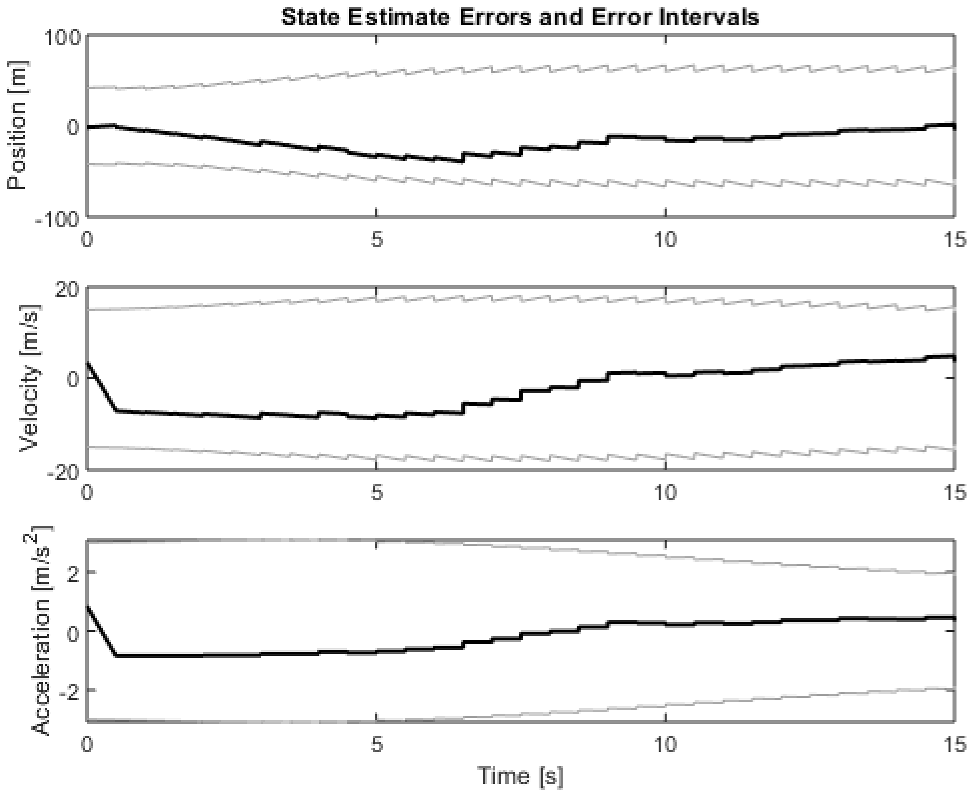


*Figure 7 - 2000 m^2*