Individual Project: Intro to the Kalman Filter

AE 2790: Intro to Spacecraft Design

Due Friday, April 19, 2019 11:59pm on Canvas
350 points

1 Instructions

The submission for this project should be typed up as a technical document. 20 points are dedicated entirely to proper technical writing and grammar. In addition to a written report, please submit *all* code that you use. This project should be completed individually. You may use classmates for help debugging, but all work should be your own. This is intended to be a technical document with full descriptions of all steps taken and equations used. All plots must include axis labels and legends where applicable. All figures and tables in your document must have captions. When referencing figures/equations/tables you must use its caption number not it's relative position in the document (e.g. "as seen in fig (3)", not "as in the figure above"). Do not crop any figures unless explicitly told to do so.

Your written report should be at least 5 pages, 11 point font, with 1.5 spacing. It should include an abstract, introduction, technical overview (including explanations and equations of measurement noise, polynomial fits, and the Kalman filter), results (all plots and discussion from the deliverables), and conclusion. The introduction and the technical overview should cover the same material. The difference is that the introduction should go over the topic's importance and use, and the technical overview should have all the equations and implementation discussion. Remember, when writing technical documents to:

- Introductory - Tell them what you're gonna tell them
- Technical discussion - Tell them
- Conclusion - Tell them what you told them.

A good abstract should be a few sentences, somewhere around 4-8. Think how you wrote your mission statements. A sentence describing the problem. A few sentences how you are going to solve the problem. A few sentences with results. (I generally write abstracts last.)

Hint: If your results tell indicate that you don't have to do the entire project (e.g. the part indicates they were going faster than Roscoe says.), you have made a mistake.

If you happen to find some sort of mistake on my part, bring it to me. If you're the first one to bring it in I'll give you 5 bonus points.

2 Problem Description

The incident started when the Duke boys were at the last stoplight in the town with the fastest speed limits on the planet. Bo was bragging to Luke that the General Lee could go from 0 m/s to 90 m/s in 15 seconds with a constant acceleration of 7 m/s^2. Luke told Bo he was crazy, so when the light turned green, Bo began to drive.

Unfortunately, Roscoe P. Coltrane is at the end of the road and he pulls them Duke boys over for speeding. He says they were going over 91 m/s in a 60 m/s zone. Unfortunately, the law in the state where they are speeding says they're going to jail. The Duke boys know you're an expert in state estimation so they ask you to help.

If you can prove to the judge that Bo was going no faster than 89 m/s, then they won't go to jail. They would be slapped with a hefty fine, but they won't have to go to the slammer. If you can prove that they were going less than 85 m/s, they get a much smaller fine and they'll let you drive the General Lee since they won't have to pay for court costs.

This particular day, Roscoe left his radar gun at home, and instead was taking position measurements. He then used a real-time Kalman filter to estimate the General Lee's velocity. Everybody knows that ole Roscoe P. Coltrane has it out for them Duke boys so it's safe to say he's going to pull every trick in the book to ensure they go to jail.

The Duke boys had just installed a new accelerometer that continuously measures and stores the acceleration of the General Lee. In truth, Bo was accelerating much slower than 7 m/s^2 and was instead accelerating at 6.158 m/s^2. He also confides to you that he had started driving before the light turned green. The General was actually 1.544 m in front of the stop line but Luke had thrown a fit so he had started rolling backwards at a velocity of -3.400 m/s.

Your first step is to propagate the initial state forward in time from 0 to 15 seconds at half second intervals. If it turns out they were going even faster than Roscoe says, then you decide to not pursue the case, just to make sure you don't accidentally get them in even more trouble.

1. (15 points) What does the state transition matrix look like for a simple constant acceleration system? Like question 11 from Homework #4 This goes in line 13 of `kf_setup.m`. Begin by writing down the kinematic equations for a constant acceleration system. You don't have to derive these yourself, but you must cite your source if you get them elsewhere. The state $\boldsymbol{x}$ is in the order $[x,\ \dot{x},\ \ddot{x}]^T$.

You are looking for $\boldsymbol{T}$ in the three-dimensional system $\boldsymbol{x}_{k+1} = \boldsymbol{T}\boldsymbol{x}_k$. We know there's a possibility there's some error in our dynamics model, so we set the process noise to diag$[1,\ 0.1,\ 0.01]$.

2. (10 points) Plot the true velocity over time. What is the Duke boy's true velocity at 15 seconds?

If you decide to take the case, you are given access to Roscoe's raw position measurements. Roscoe also left his binoculars at home, so he assumed that the Bo started at the stop line, from rest, and started accelerating at the ideal 7 m/s^2 as soon as the light turned green. He knew that he might be wrong, so he assigned the following standard deviations to each of the states: 14 m, 5 m/s, and 1 m/s^2. He also tells you his position measurements have a variance of 200 m^2.

Now that you have all the required information you decide to fit a line to the measurements. The judge doesn't know much about measurement noise and would accept a proper line fit as truth.

3. (30 points) Plot the first through fifth order polynomials (without cropping) to the data without using `polyfit`. Additionally, crop the plot so only the last two data points are seen (`xlim([14,15])`). In both plots, include the true position and the measurements. Can you use a polynomial fit to lessen the Duke boys' penalties? Note that for this to work, the polynomial must reasonably fit the data, and the data must suggest they were going slower than the charges state.

If fitting a polynomial to the data won't show their speed was less than the truth, then you decide to run a Kalman filter.

4. What is the measurement mapping matrix? Use problem 16 from homework #5. Put this in line `kf_setup.m` line 15. The state transition matrix and the measurement mapping matrix must be right for the rest of the project to work. As soon as you get these, you may send them to Christy to check. Don't worry about coming to me with a wrong answer, only your final submission will be graded.

5. (50 points) Complete the Kalman filter outline provided. Open `kf_prop.m` and `kf_update.m` and complete these files with the appropriate equations. Include the error plot that is automatically generated (the 3×1 black and grey plot). In addition, your writeup should include the 8 equations used in the Kalman filter propagation and update steps and a description of what each of these equations represent. These are:
   - Propagate: mean $\boldsymbol{m}_k^-$
   - Propagate: covariance $\boldsymbol{P}_k^-$
   - Update: Measurement estimate $\hat{\boldsymbol{z}}_k$
   - Update: Innovation covariance $\boldsymbol{W}_k$
   - Update: Cross covariance $\boldsymbol{C}_k$
   - Update: Kalman gain $\boldsymbol{K}_k$
   - Update: mean $\boldsymbol{m}_k^+$
   - Update: covariance $\boldsymbol{P}_k^+$
6. (15 points) What is the fastest possible speed Roscoe can be at least the tiniest sure Bo was driving? If you round, does this explain where Roscoe got his speed from? Hint: Plot the mean as well as the mean $\pm 3\sigma$. Remember the variance (σ^2) comes from the diagonal elements of the covariance (P).
7. (13 points) What is Roscoe's error with claiming 99% uncertainty that their speed is 91 m/s? (Hint: What do the sigma intervals mean?)
8. (12 points) If you argue that the General Lee was going mean velocity, does this help the Duke boys?
9. (15 points) If the the judge will accept the mean estimate as their actual speed, which speed will you chose to go with? Roscoe's speed, their true speed, or the estimated mean? Present two plots, one for the position and one for the velocity. In the position plot include truth, measurements, Kalman filter estimate, and what you have decided is the "best" polynomial fit. In the velocity plot include the truth and the estimate.
10. (10 points) If your argument lets the boys off easier, Roscoe begins to panic, and now says that his measurement variance (R_k) in `kf_setup.m` was actually 20 m^2. What does this do to his argument, can he argue they're going faster? Include the error plots (The 3×1 black and gray plots).
11. (10 points) What if he had claimed R_k was 2000 m^2? Include the error plots (The 3×1 black and gray plots).
12. (20 points) Based on your knowledge of the Kalman filter and standard deviation intervals, what do the state error/covariance plots tell you about using an incorrect measurement noise model?

3 Grading Rubric

Category	Points	100% – 90%	89% – 80%	79% – 70%	< 69%
Formatting	**20**				
11 point font	2	Yes	-	-	No
1.5 spacing	2	Yes	-	-	No
Length	5	$\geq$ 5 pages	< 5 pages	< 4 pages	< 3 pages
Grammar	2	No mistakes	-	-	Significant mistakes
Spelling	2	No mistakes	-	-	Significant mistakes
Figures	**15**				
Image resolution	5	Clear			Unreadable
Figure number	5	Always			Never
Figure caption	5	Always			Never
Axis labels	5	Always			Never
Legend	5	Always			Never
Abstract	**15**				
KF Importance	5	Motivating			Missing
KF Description	5	Clear			Missing
Results	5	Clear			Missing
Introduction	**20**				
Measurement noise	10	Clear overview			Missing
Polynomial fit	5	Clear overview			Missing
Kalman filter	5	Clear overview			Missing
Literature Review	**10**				
Cite (No web pages)	5	> 2	2	1	none
Technical Overview	**30**				
Polynomial fit	5	Clear development			Missing
Equations	5	Correct			Missing
Kalman filter	10	Clear development			Missing
Equations	10	Correct			Missing
Conclusions	**15**				
Restate problem	5	Clear recap			Missing
Individual findings	5	Clear recap			Missing
Closing statement	5	Convincing			Missing
Total	**125**				

Category	Points	100% – 90%	89% – 80%	79% – 70%	< 69%
STM	**10**				
Size	5	Correct			Incorrect
Values	5	Correct			Incorrect
Truth Propagation	**20**				
Code	10	Correct			Doesn't Run
Plot	10	Correct			Missing
Polynomial Fit	**35**				
Code	15	Correct			Doesn't Run
Plot	5	Correct			Missing
Discussion	15	Rational			Missing
Measurement Map	**10**				
Size	5	Correct			Incorrect
Values	5	Correct			Incorrect
Kalman Filter	**40**				
Code	15	Correct			Doesn't Run
Plot	5	Correct			Missing
Discussion	20	Rational			Missing
Roscoe's Claim	**20**				
Plot	10	Correct			Missing
Discussion	10	Rational			Missing
Roscoe's Wrong	**20**				
Discussion	20	Rational			Missing
Chosen Speed	**15**				
Answer	5	Correct			Missing
Discussion	10	Rational			Missing
Less Noise	**20**				
Answer	5	Correct			Missing
Plot	5	Correct			Missing
Justification	10	Logical			Missing
More Noise	**20**				
Answer	5	Correct			Missing
Plot	5	Correct			Missing
Justification	10	Logical			Missing
1 Noise Discussion	**15**	Rational			Missing
Total	**225**				