2021

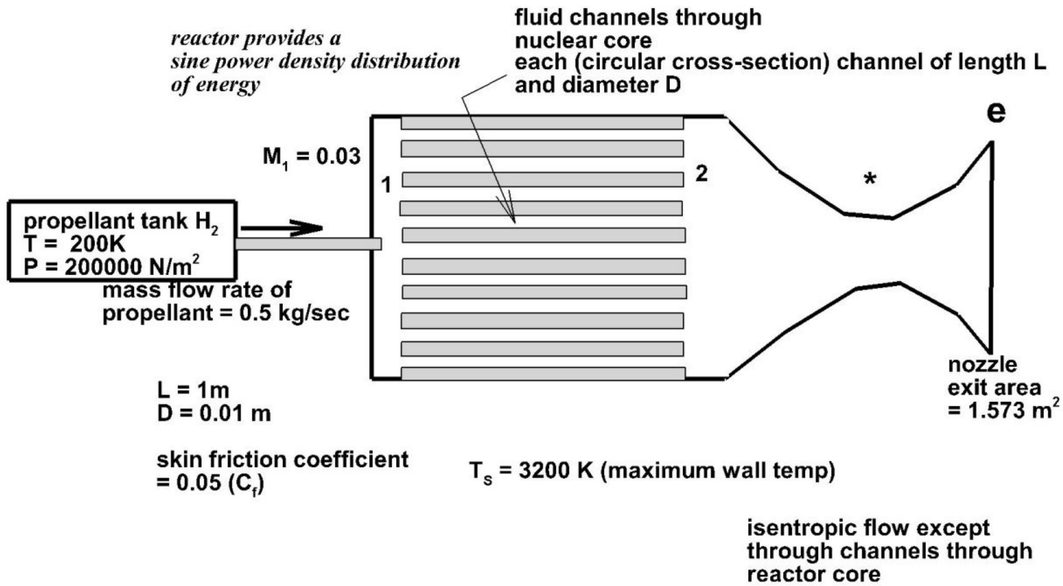
AE_5335 Homework 6

MATT PAHAYO
AE_5335 -> DR. RIGGINS

Homework 6 **AE 5335** **Assigned: 3/19/2021**

Due: 3/31/2021

Consider a conventional nuclear thermal rocket as shown in the sketch below:



Let the skin friction coefficient = 0.05, L (length of tubes through core) =1 m, D (diameter of each tube) = 0.01 m.

Maximum temperature of wall in tubes = 3200K

Mach number at entrance of each tube (station 1) = 0.03

Total temperature and total pressure of H_2 in propellant tank = 200K and 200000 N/m^2

Mass flow rate of propellant is 0.5 kg/sec

Area of nozzle exit = 1.573 m^2

Assume an axial sine power density distribution for the nuclear reactor in this nuclear rocket. Use ratio of specific heats γ= 1.4 and R (gas constant) = 4125 J/kgK for the hydrogen propellant.

Calculate and plot both the wall temperature and the total temperature of the propellant from tube entrance (station 1) to tube exit (station 2).

Find the axial location of the maximum wall temperature.

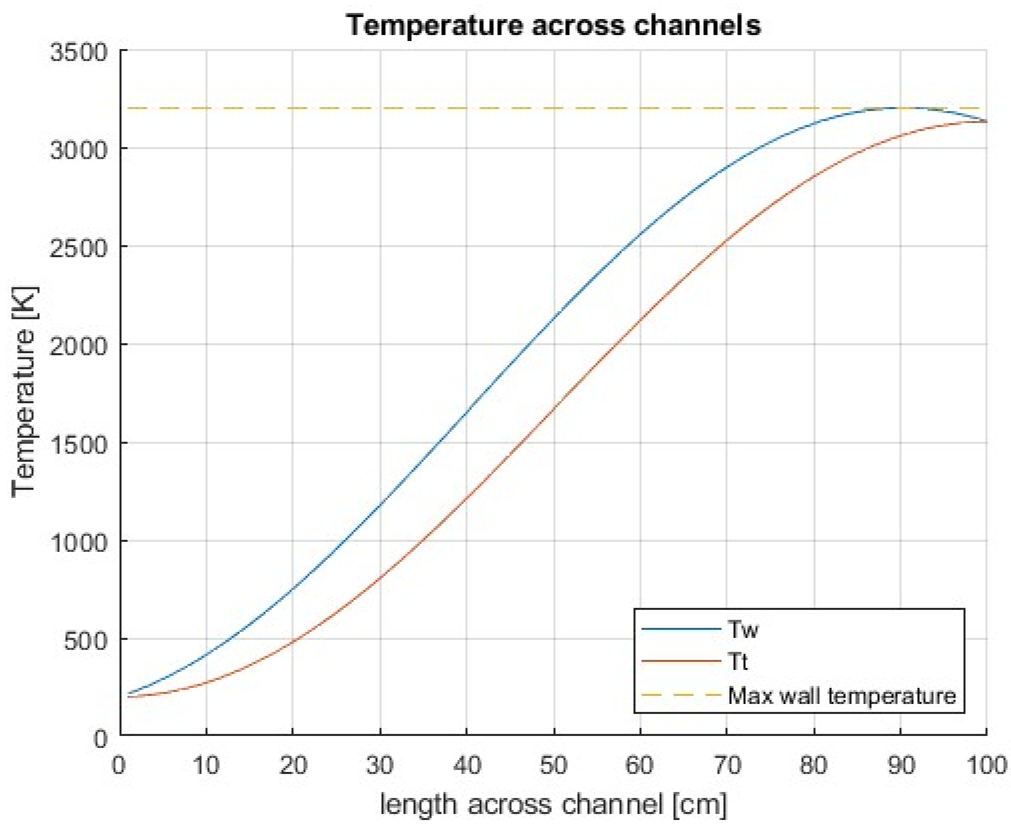
Find the total heat rate generated by the reactor for this rocket.

Find the thrust and the specific impulse of this rocket.

Results

Calculate and plot both the wall temperature and the total temperature of the propellant from tube entrance (station 1) to tube exit (station 2).

Station	0	2
T_w [K]	200	3129.42
T_t [K]	200	3129.42



Find the axial location of the maximum wall temperature.

$$\left(\frac{\mathrm{x}}{\mathrm{L}}\right)_{Tw=Ts} = 0.903107$$

Find the total heat rate generated by the reactor for this rocket.

$$\dot{Q} = 21146750.47962\, J/s$$

Find the thrust and the specific impulse of this rocket.

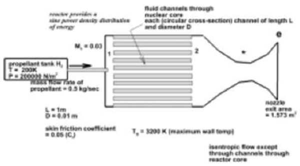
$$thrust = 4.593\ kN$$

$$ISP = 937.461\ sec$$

Scratch work and code

Homework 6 AE 5335 Assigned: 3/19/2021 Due: 3/31/2021

Consider a conventional nuclear thermal rocket as shown in the sketch below:



Let the skin friction coefficient = 0.05, L (length of tubes through core) =1 m, D (diameter of each tube) = 0.01 m.
Maximum temperature of wall in tubes = 3200K
Mach number at entrance of each tube (station 1) = 0.03
Total temperature and total pressure of H_2 in propellant tank = 200K and 200000 N/m²
Mass flow rate of propellant is 0.5 kg/sec
Area of nozzle exit = 1.573 m²

Assume an axial sine power density distribution for the nuclear reactor in this nuclear rocket. Use ratio of specific heats γ= 1.4 and R (gas constant) = 4125 J/kgK for the hydrogen propellant.

Calculate and plot both the wall temperature and the total temperature of the propellant from tube entrance (station 1) to tube exit (station 2).
Find the axial location of the maximum wall temperature.
Find the total heat rate generated by the reactor for this rocket.
Find the thrust and the specific impulse of this rocket.

$C_p = .05$

$L = 1m$

$D = .01m$

$M_1 = .3$

$T_{t1} = 200K \quad P_{t1} = 20000$ pa

$\dot{m} = 0.5 kg/s$

$T_s = 3200K$

$\gamma = \frac{C_p}{C_p - R} \Rightarrow \frac{\gamma R}{\gamma - 1}$

$$T_s = T_{t1} + \left[\frac{2C_fL}{\pi D}\right] \Delta T_m \left\{1 + \sqrt{1 + \left(\frac{\pi D}{2C_fL}\right)^2}\right\}$$

solve for ΔT_m

$$\frac{T_s - T_{t1}}{[\ldots]} = \Delta T_m$$

$$\frac{3200 - 200}{\left[\frac{2(.05)}{\pi(.01)}\right]\left\{1 + \sqrt{1 + \left(\frac{\pi(.01)}{2(.05)}\right)^2}\right\}} = 460.152K$$

$T_t(x) = 200 + 1464.7(1 + \cos(\pi x))$

$T_w(x) = T_t(x) + 460.1 \sin \pi x$

$$T_{t2} = T_{t1} + \frac{4C_fL}{\pi D}\Delta T_m = 200 + \frac{4(.05)}{\pi(.01)}(460.152) = 3129.42 \text{ K}$$

get $T_w(x)$ and $T_t(x)$

$T_w(x) = T_s \Rightarrow$ get x at max wall temp: $\left(\frac{x}{L}\right)_{T_w = T_s} = 1 + \frac{1}{\pi}\arctan\left(\frac{-\pi D}{2C_fL}\right) = .903$; $T_w(.903) = T_s$ ✓

$\delta q = 2c_f c_p [T_w(x) - T_t(x)] \frac{dx}{D}$ or $\dot{Q} = \dot{m} c_p (T_{t2} - T_{t1}) = 2114675.048$ J/s

$q = \int_0^1 \delta q \, dx \Rightarrow q\dot{m} = Q/s$

$T_{te} = T_{t2} \quad P_{te} = P_{t2}$

$\sqrt{T_{t2}/T_{t1}} = \frac{M_2}{M_1}[\ldots][\ldots]^{1/2} \Rightarrow M_2 = .138$ from Newton

$$\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2 (1 - \frac{C_fL}{D})}{1 + \gamma M_2^2 (1 - \frac{C_fL}{D})} \Rightarrow \frac{1 + 1.4(.03)^2(1-5)}{1 + 1.4(.138)^2 6} = .8575$$

$$\frac{P_{t2}}{P_{t1}} = \frac{P_2}{P_1}\left\{\frac{(1 + \frac{\gamma-1}{2}M_2^2)}{1 + \frac{\gamma-1}{2}M_2^2}\right\}^{1/2} \Rightarrow P_{te}$$

$P_{te} = 173690$

$$\dot{m} = \frac{P_{te} A_e}{\sqrt{T_{te}}}\sqrt{\frac{\gamma}{R}} M_e \left(1 + \frac{\gamma-1}{2}M_e^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}}$$

$M_e = 7$

$\frac{T_{te}}{T_e} = 1 + \frac{\gamma - 1}{2}M_e^2 \Rightarrow T_e$, get P_e

$T_e = \frac{T_{te}}{1 + .2M_e^2} = 289.763 K \quad P_e = 41.958$ pa

$u_e = M_e\sqrt{\gamma R T_e}$

$F = \dot{m}u + P_e A_e$

$u_e = \sqrt{\ldots} = 9055$ m/s

Saturday, April 3, 2021 4:04 AM

$u_e = M_e\sqrt{\gamma R T_e}$

$F = \dot{m}u + P_e A_e$

$u_e = 7\sqrt{14.4125} = 9055\ m/s$

$Thrust = \frac{1}{2}u_e + Pe(1.573) = 4.543\ KN$

$Isp = Thrust/mdot/\ g_0 = 93746\ s$

```matlab
classdef rootFind
    %rootFind is a class of functions that find the root of a function /
    %data set
%-------------------------------------------------------------------------%
    methods (Static)
        function x = Bisect(f,a,b,tol)
            %Bisect uses the bisection algoritm using the interval
            iter = 0;
            while (b-a)/2 >= tol
                c = (a+b)/2;
                if f(c) > 0
                    b = c;
                end
                if f(c) < 0
                    a = c;
                end
                iter = iter + 1;
            end
            x = (a+b)/2
        end
%-------------------------------------------------------------------------%
        function x = newRap(f,x0)
            %newRap is a function that utilizes the Newton-Raphson
            %algorithm to find the roots of the function
            %x0 is the initial guess
            fp = diff(f);

            x=x0;
            nmax=25;
            eps=1;
            n=0;

            while eps>=1e-5&&n<=nmax
                y=x-double(f(x))/double(fp(x));
                eps=abs(y-x);
                x=y;
                n=n+1;
            end
        end
%-------------------------------------------------------------------------%

    end
end
```

```
% matthew Pahayo
% main.m
clc
clear all
close all

format longg
syms x M2 Me

cf = .05;
L = 1;
D = .01;
M1 = .03;
Tt1 = 200;
Pt1 = 200000;
mdot = .5;
Ts = 3200;
gam = 1.4;
R = 4125;
cp = gam*R/(gam-1);
Ae = 1.573;

delTm = (Ts-Tt1)/(2*cf*L/pi/D)/(1+sqrt((1+((pi*D)/(2*cf*L))^2)))
Tt2 = Tt1+4*cf*L/pi/.01*delTm
Tt = symfun(2*cf/D*delTm*L/pi*(1-cos(pi*x/L))+Tt1,x)
Tw = Tt + delTm*sin(pi*x/L)
xbyL = 1 + 1/pi*atan(-pi*D/2/cf/L)
Qdot = int(2*cf*cp*(Tw-Tt)/D,0,1)*mdot

f = symfun(-sqrt(Tt2/Tt1)+(M2/M1)*((1+gam*M1^2*(1-
cf*L/D))/(1+gam*M2^2*(1+cf*L/D)))*...
    ((1+(gam-1)/2*M2^2)/(1+(gam-1)/2*M1^2))^(1/2),M2)
M2 = rootFind.newRap(f,0)

Pr = ((1+gam*M1^2*(1-cf*L/D))/(1+gam*M2^2*(1+cf*L/D)))
Pte = Pr*((1+(gam-1)/2*M2^2)/(1+(gam-1)/2*M1^2))^(gam/(gam-1))*Pt1
g = symfun(Pte*Ae/sqrt(Tt2)*sqrt(gam/R)*Me*(1+(gam-1)/2*Me^2)^(-
(gam+1)/2/(gam-1))-mdot,Me)
Me = rootFind.newRap(g,3)
Te = Tt2/(1+(gam-1)/2*Me^2)
Pe = Pte/((1+(gam-1)/2*Me^2))^(gam/(gam-1))
ue = Me*sqrt(gam*R*Te)
thrust = mdot*ue + Pe*Ae
ISP = thrust/mdot/9.8

for i = 1:100
    Twall(i) = double(Tw(i/100));
    Ttotal(i) = double(Tt(i/100));
    Tes(i) = Ts;
end
i = 1:100;
hold on
plot(i,Twall)
plot(i,Ttotal)
plot(i,Tes,'--')
legend('Tw','Tt','Max wall temperature','Location','southeast')
```

```
ylabel('Temperature [K]')
xlabel('length across channel [cm]')
title('Temperature across channels')
grid on
hold off
```