GP

AE 5535 **Homework 3** **Assigned: 2/22/2021**
Due: 3/01/2021

Variable Area Turbojet Problem

Consider the performance of an ideal non-afterburning turbojet with flow at station 4 (turbine entrance) and station 8 (nozzle throat) choked. A4 is fixed and A8 is varied in order to maintain constant compressor total pressure ratio (π_C).

On-design conditions are as follows: $\pi_C = 15$ $M_0 = 2.0$ $\tau_\lambda = 7.0$

Find the required ratio of nozzle throat area (off-design) to nozzle throat area (on-design) for the engine operating at the same flight Mach number (2.0) but at the off-design condition such that τ_λ = 6.0.

1) $A_4 = A_{4R}$

$T_{tC} = T_{tCR}$

$M_o = M_{oR}$

$\tau = \pi^{\frac{\gamma-1}{\gamma}}$

$\gamma = 1.4$

$$\tau_{tR} = (\tau_c - 1)\frac{\tau_{tR}}{\tau_{\lambda R}} + 1$$

$\tau_c = \tau_{cR}$

$$\Rightarrow \tau_t = 1 - (1 - \tau_{tR})\frac{\tau_{\lambda R}}{\tau_\lambda}$$

$A_4 = A_{4R}$

$$\left(\frac{\tau_t}{\pi_t}\right) / \left(\frac{\tau_{tR}}{\pi_{tR}}\right) = \frac{\left(\frac{A_8}{A_4}\right)}{\left(\frac{A_{8R}}{A_{4R}}\right)} = \frac{A_8}{A_{8R}}$$

- On design

$$\tau_r = T_{tC}^{\frac{\gamma-1}{\gamma}}$$

$$= 15^{\frac{1.4}{1.4}} = 2.168 \rightarrow \tau_{8R} = 1.8 = \tau_r$$

$$\tau_{tR} = 1 + (2.168 - 1)\frac{1 + \frac{\gamma-1}{2}(2^2)}{7} = .6997$$

$$\frac{\gamma}{\gamma-1}$$

$$T_{tR} = \tau_{tR} = .6997^{(1.4/.4)} = .2865$$

$$\frac{A_{8R}}{A_{4R}} = \frac{.6997^{1/2}}{.2865} = 2.919$$

- Off - design

$$\tau_t = 1-(1-\tau_{tR})\frac{\tau_{xR}}{\tau_x} = 1-(1-.6997)\frac{7}{6} = 0.6496$$

$$\tau_{t+} = \tau_t^{\frac{k}{k-1}} = .6496^{1.4/.4} = .221$$

$$\frac{A_8}{A_4} = \frac{.6496}{.221}^{1/2} = 3.647$$

- $$\frac{A_8}{A_{8R}} = \frac{3.647}{2.919} = \boxed{1.249}$$