Final Exam

Tuesday, May 11, 2021 7:47 PM

$I_{sp} = 3500$ sec $\qquad \frac{\phi_A - \phi_C}{d} = 10^5$ V/cm

$q/m = 2500 \frac{C}{kg}$

$I_{sp} = \frac{1}{g_0}\sqrt{2\frac{q}{m}(\phi_A - \phi_C)} \Rightarrow \frac{(I_{sp} \cdot g_0)^2}{2}\left[\frac{q}{m}\right]^{-1} = \phi_A - \phi_C$

$= \frac{(3500 \cdot 9.81)^2}{2 \cdot 2500} =$ 235778.445 V

$\frac{\phi_A - \phi_C}{d} = C \Rightarrow \frac{\phi_A - \phi_C}{C} = \frac{235778.445\ V}{10^5} =$ 2.35778 cm

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$c = I_{sp} g_0 = 3500 \cdot 9.81 =$ 34335 m/s

$\Delta V = 70000$ m/s

$m_p = m_0\left(1 - e^{\frac{-\Delta V}{c}}\right) \qquad k = \frac{\Delta V}{C} =$ 2.038736

$m_p = 80000\left(1 - e^{-k}\right) =$ 69584.54615 kg

$m_0 = m_e + m_L + m_p \Rightarrow m_0 - m_p - m_L = m_e =$ 5415.45385 kg

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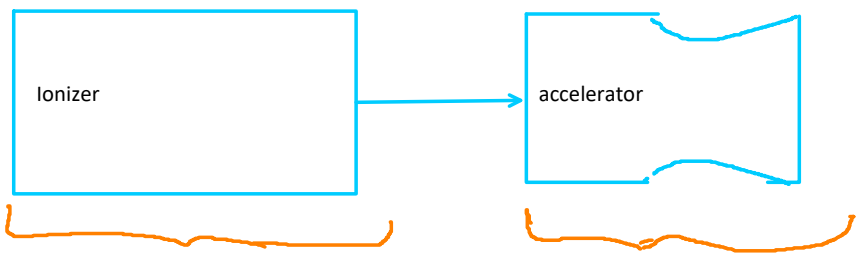
$\alpha' = 600$ W/kg

$\dot{W}_e = \dot{m}_e \alpha' = 5415.45385 \cdot 600 =$ 3249272.31 W = 3249.27231 kW



.35 W e = 1137.245309 kW .65 W e = 2112.027002 kW

$$j = .4444\,\varepsilon_0\sqrt{2\left(\frac{q}{m}\right)}\,\frac{(\phi_A-\phi_C)^{3/2}}{d^2} = .4444\,\varepsilon_0\sqrt{2\cdot 2500}\,\frac{(235778.445)^{3/2}}{.02^2}$$

j = 55.327101 Amp/m^2

$$Power_{AC} = V\,j\,A \Rightarrow \frac{Power_{AC}}{V\,j} = A = \frac{2112.027002\ kW}{235778.445\ V \times 55.327101\ Amp/m^2} = 0.161904\ m^2$$

Diameter of beam = (4(0.161904 m^2)/pi)^(1/2) = 0.4540291 m

$$\frac{Thrust}{Area} = \left(\frac{q}{m}\right)^{-1} j\sqrt{2\,q/M}\,(\phi_A-\phi_C)^{1/2} \quad \frac{1}{2500}*55.327101\sqrt{2*2500(235778.445)} = 759.8624116$$

$$Thrust = CA = 759.8624116 * 0.161904 = 123.0247639\ N$$

$$\dot{m} = \frac{Thrust}{V_e} = \frac{123.0247639}{34335} = 0.0035830716\,\frac{kg}{s} = 3.5830716*10^{-3}\,\frac{g}{s}$$

$T_3 = 3000\ K$
$\eta_c = \eta_t = 0.9$
$\gamma = 1.4$
R = 296.8 J/kg K
Cp = 1038.8 J/kg K

From code: Pi_c ranges from 1 to 6 and T_a ranges from 500 K to 1700 K

$\pi_c = 2.64$
$T_a = 1472\ K$
$T_2 = 1994.80286196426\ K$
$\eta = 0.130519423193357$
$A_{rad} = 1.05785661271208e\text{-}05$ m2/watt

$$\pi_t = \frac{1}{\pi_c} = 1/2.64 = 0.3787878$$

$$\tau_T = 1-\eta_t\left(1-\pi_t^{\frac{\gamma-1}{\gamma}}\right) = 1-0.9(1-0.3787878^{((\Upsilon-1)/\Upsilon)} = 0.781999795794567$$

$$T_4 = T_3\cdot\tau_t = 3000*0.781999795794567 = 2345.999387383701\ K$$

$$Q_R' = \frac{1}{\eta} - 1 = \frac{1}{0.130519423193357} - 1 = 6.66169490742047$$

$$\dot{m}_{gas}' = \frac{Q_R'}{c_p(T_4-T_a)} = \frac{6.66169490742047}{1038.8(2345.999387383701-1472)} = 7.33739e-6\,\frac{\frac{kg}{sec}}{watt}$$

Net power out required by ion drive = Wdot_e = 3249272.31 W = 3249.27231 kW
kg

mdot_gas = 3249272.31 * $7.33739e-6 = 23.84117856 \frac{kg}{s}$

Total power Supplied by turbine = mdot_gas*Cp*(T3-T4) = $23.84117856 * 1038.8 * (3000 - 2345.999387383701)$ = 16197120.63 watts =16197.12063 kW

Power required by the compressor = mdot_gas*Cp*(T2-Ta) = 11709537.94 W =11709.53794 kW

Heat rate required = $\eta * net\ power = 0.130519423193357 * 3249.27231 = 24894.9331103493\ kW$

Area of radiator = $A_{rad} * net\ power = 1.05785661271208e\text{-}05 * 3249272.31 = 34.3726419963576\ m^2$

$$\eta_A = \frac{\dot{m}_p \frac{c^2}{2}}{\dot{W}_e} = \frac{0.0035830716 * \frac{34335^2}{2}}{3249272.31} = .65$$

$$\tau = \frac{m_p}{\dot{m}_p}$$

$$\tau_A \Rightarrow \left[1 - e^{-\frac{\Delta V_A}{c}}\right] m_0 = \left[1 - e^{\frac{35000}{34335}}\right] * 80000 = 51134.16711\ kg = m_{p,A}$$

The rest of the propellant is for the deceleration burn => 18450.54614 kg

$m_{p,A} + m_{p,D} = m_p = 51134.16711\ kg$ +18450.54614 kg = 69584.54614 kg

$$\tau_A = \frac{m_{p,D}}{\dot{m}_p} = \frac{51134.16711}{0.0035830716} = 14271042.51\ sec = 3964.1785\ days$$

$$\tau_D = \frac{m_{p,A}}{\dot{m}_p} = \frac{18450.54614}{0.0035830716} = 5149365.74\ sec = 1430.3794\ days$$