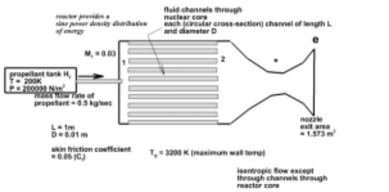
Hw 6

Saturday, April 3, 2021 4:04 AM

Homework 6 **AE 5335** **Assigned: 3/19/2021**
Due: 3/31/2021

Consider a conventional nuclear thermal rocket as shown in the sketch below:



Let the skin friction coefficient = 0.05, L (length of tubes through core) =1 m, D (diameter of each tube) = 0.01 m.
Maximum temperature of wall in tubes = 3200K
Mach number at entrance of each tube (station 1) = 0.03
Total temperature and total pressure of H_2 in propellant tank = 200K and 200000 N/m^2
Mass flow rate of propellant is 0.5 kg/sec
Area of nozzle exit = 1.573 m^2

Assume an axial sine power density distribution for the nuclear reactor in this nuclear rocket. Use ratio of specific heats γ= 1.4 and R (gas constant) = 4125 J/kgK for the hydrogen propellant.

Calculate and plot both the wall temperature and the total temperature of the propellant from tube entrance (station 1) to tube exit (station 2).
Find the axial location of the maximum wall temperature.
Find the total heat rate generated by the reactor for this rocket.
Find the thrust and the specific impulse of this rocket.

$C_f = .05$ $\quad T_S = 3200K$

$L = 1m$

$D = .01m$

$M_1 = .3$

$T_{t1} = 200K \quad P_{t1} = 20000 \text{ pa}$

$\dot{m} = 0.5 kg/s$

$\gamma = \frac{c_p}{c_p - R} \Rightarrow \frac{\gamma R}{\gamma - 1}$

$T_S = T_{t1} + \left[\frac{2C_f L}{\pi D}\right] \Delta T_m \left\{1 + \sqrt{1 + \left(\frac{\pi D}{2C_f L}\right)^2}\right\}$

solve for ΔT_m

$\frac{T_s - T_{t1}}{[\ldots]} = \Delta T_m$

$\frac{3200 - 200}{\left[\frac{2(.05)}{\pi(.01)}\right]\left\{1 + \sqrt{1 + \left(\frac{\pi(.01)}{2(.05)}\right)^2}\right\}} = 460.152K$

$T_t(x) = 200 + 1464.7\,(1 + \cos(\pi x))$

$T_w(x) = T_t(x) + 460.1 \sin \pi x$

$T_{t2} = T_{t1} + \frac{4C_f L}{\pi D} \Delta T_m = 200 + \frac{4(.05)}{\pi(.01)}(460.152) = 3129.42 \text{ K}$

get $T_w(x)$ and $T_t(x)$

$T_w(x) = T_s \Rightarrow$ get x at max wall temp: $\left(\frac{x}{L}\right)_{T_w = T_s} = 1 + \frac{1}{\pi}\arctan\left(\frac{-\pi D}{2C_f L}\right) = .903$; $T_w(.903) = T_S$ ✓

$\delta q = 2c_f\, c_p\, [T_w(x) - T_t(x)]\, \frac{dx}{D}$ or $\dot{Q} = \dot{m} c_p (T_{t2} - T_{t1}) = 2114675.048 \text{ J/s}$

$q = \int_0^1 \delta q\, dx \Rightarrow q\,\dot{m} = Q/s$

$T_{te} = T_{t2} \quad P_{te} = P_{t2}$

$\sqrt{T_{t2}/T_{t1}} = \frac{M_2}{M_1}[\ldots][\ldots]^{1/2} \Rightarrow M_2 = .138$ from Newton

$\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2 \left(1 - \frac{C_f L}{D}\right)}{1 + \gamma M_2^2 \left(1 - \frac{C_f L}{D}\right)} \Rightarrow \frac{1 + 1.4(.03)^2(1-5)}{1 + 1.4(.138)^2 6} = .8575$

$\frac{P_{t2}}{P_{t1}} = \frac{P_2}{P_1}\left\{\frac{\left(1 + \frac{\gamma-1}{2}M_2^2\right)}{1 + \frac{\gamma-1}{2}M_1^2}\right\}^{1/2} \Rightarrow P_{te}$ $\quad P_{te} = 173690$

$\dot{m} = \frac{P_{te} A_e}{\sqrt{T_{te}}} \sqrt{\frac{\gamma}{R}}\, M_e \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}}$ $\quad M_e = 7$

$\frac{T_{te}}{T_e} = 1 + \frac{\gamma-1}{2} M_e^2 \Rightarrow T_e$, get P_e $\quad T_e = \frac{T_{te}}{1 + .2M_e^2} = 289.763 K \quad P_e = 41.958 \text{ pa}$

$u_e = M_e \sqrt{\gamma R T_e}$ $\quad u_e = 9055 m/s$

$F = \dot{m}u + P_e A_e$

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$u_e = 7\sqrt{1.4 \cdot 412 \cdot 5} = 9055\ m/s$

$\text{Thrust} = \frac{1}{2}u_e + P_e(1.573) = 4.543\ kN$

$I_{sp} = \text{Thrust}/\text{mdot}/g_0 = 937.46\ s$