

HEP

AE 5535

Homework 3

Assigned: 2/22/2021

Due: 2/26/2021

Variable Area Turbojet Problem

Consider the performance of an ideal non-afterburning turbojet with flow at station 4 (turbine entrance) and station 8 (nozzle throat) choked. A_4 is fixed and A_8 is varied in order to maintain constant compressor total pressure ratio (π_c).

On-design conditions are as follows: $\pi_c = 15$ $M_0 = 2.0$ $\tau_\lambda = 7.0$

Find the required ratio of nozzle throat area (off-design) to nozzle throat area (on-design) for the engine operating at the same flight Mach number (2.0) but at the off-design condition such that $\tau_\lambda = 6.0$.

Homework 3 VAT jet ($\pi_c = \text{constant}$)

$$\pi_c = \pi_{CR} \text{ so } \tau_c = \tau_{CR} \quad (CR = \text{on-design})$$

$$\tau_{CR} = \tau_c = 1 + \eta_m \left(\frac{\tau_\lambda}{\tau_r} \right) (1 - \tau_t)$$

$$\text{solve for } \tau_t : \tau_t = 1 - (\tau_{CR} - 1) \frac{\tau_r}{\tau_\lambda}$$

$$\pi_t = \tau_t^{\frac{\gamma}{\gamma-1}}$$

$$\text{also know that } \frac{\tau_t^{\frac{1}{2}}}{\pi_t} = \frac{A_0}{A_t} \quad \begin{cases} \text{fixed } A_t = A_{tR} \\ (A_0 \neq A_{0R}) \end{cases}$$

$$\text{so } \boxed{\frac{A_0}{A_{0R}} = \frac{\tau_t^{1/2}}{\tau_{tR}^{1/2}} \cdot \frac{\pi_{tR}}{\pi_t}} \Rightarrow \text{maintains } \pi_c = \pi_{CR}$$

$$\text{have } \begin{cases} \tau_{CR} = \tau_c^{\frac{\gamma-1}{\gamma}} ; \tau_{tR} = 1 - (\tau_{CR} - 1) \frac{\tau_{rR}}{\tau_{\lambda R}} ; \pi_{tR} = \tau_{tR}^{\frac{\gamma}{\gamma-1}} \\ \tau_t = 1 - (\tau_{CR} - 1) \frac{\tau_r}{\tau_\lambda} , \pi_t = \tau_t^{\frac{\gamma}{\gamma-1}} \end{cases}$$

$$\text{for this problem: } \pi_c = 15, \gamma = 1.4, M_{0R} = 2.0, \tau_{\lambda R} = ?$$

$$\{ M_0 = 2.0, \tau_\lambda = 6.0 \} \text{ off-design ; } \boxed{\text{find } \frac{A_0}{A_{0R}}} \quad (\text{fixed } A_t)$$

$$\tau_{rR} = 1.8 = \tau_r$$

$$\tau_{CR} = \pi_c^{\frac{\gamma-1}{\gamma}} = 2.168$$

$$\tau_{tR} = 0.7 ; \pi_{tR} = .2865 ; \tau_t = .6496 ; \pi_t = .20$$

$$\boxed{\frac{A_0}{A_{0R}} = 1.25}$$