AE_5535 Project 1

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θ-β-M relation

$$\tan\theta = 2\cot\beta\left[\frac{M^2\sin^2\beta-1}{M^2(\gamma+\cos 2\beta)+2}\right] \quad (1)$$

Solve for β → Either program Numerical method or use Virginia Tech CAC.

I will use V.T. CAC

$\theta_A = 11°$ $M = 10$ → $\beta_A = 15.492°$ $M_{2,A} = 6.357$

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left((M_1\sin\beta)^2-1\right) \quad (2)$$

$$M_{2n}^2 = \frac{1+[(\gamma-1)/2]M_{1n}^2}{\gamma M_{1n}^2-(\gamma-1)/2} \quad (3)$$

$$\left(\frac{P_2}{P_1}\right)_A = 8.157$$

Use eqn. 1, 2 and 3 on surface C, E

$\theta_C = 4°$ → $\beta_C = 8.653°$, $\left(\frac{P_2}{P_1}\right)_C = 2.474$, $M_{2_C} = 8.623$

$\theta_E = 9°$ → $\beta_E = 13.384°$, $\left(\frac{P_2}{P_1}\right)_E = 6.084$, $M_{2E} = 9.967$

To find pressure at surface

$$P_{2,i} = \left(\frac{P_2}{P_1}\right)_i * P_0 \quad (4)$$

$P_{2,A} = \left(\frac{P_2}{P_1}\right)_A P_0 = 8.157 * 1185.5 = 9670.124$ Pa

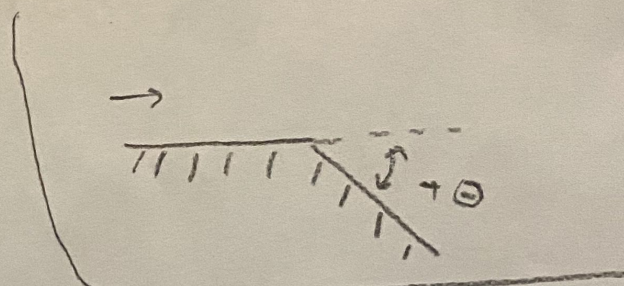
$P_{2,C} = 2932.927$ Pa

$P_{2,E} = 7217.582$ Pa

Prandtl – Meyer Expansion (Isentropic) → $P_{0,1} = P_{0,2}$

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1} \quad (5)$$

$$\theta = \nu(M_2) - \nu(M_1) \quad (6)$$



$\nu(M_2) = \theta + \nu(M_1)$

↳ solve for M_2

$$\frac{P_0}{P} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{(\gamma/\gamma-1)} \quad (7)$$

from Eqn 5.

$\nu(M_1) = 102.316$

$\nu_D(M_2) = 102.813 \rightarrow M_{2,D} = 10.186$, $\frac{P_2}{P_{o_2}} = .00002083$

$$\frac{P_2}{P_{o2}} \frac{P_{o1}}{P_1} P_1 = P_2 \quad (8)$$

$T_1 = \frac{T_1}{T_0} T_0$

$\frac{T_2}{T_1} T_1 = T_2$

$M = \frac{|U|}{\sqrt{\gamma R T}}$

$T\left(1 + \frac{\gamma - 1}{2} M^2\right) = T_{t0}$

$= 4856.04 K$

$\left(\frac{T_2}{T_1}\right)_A (T_1) = 2.31(231.24) = 534.16 K$

$|V|_A = M\sqrt{\gamma R T_A} = 6.357\sqrt{1.4(287)(534.16)} = 2945.063$ m/s

$u = |V|_A \cos 11^\circ = 2890.32$ m/s $= 554.6903$ m/s

Eqn. 7

↳ $\frac{P_{01}}{P_1} = 42439.234$

Eqn. 6

↳ $(\nu(M_2))_F = 9 + 102.316 = 111.316$

↳ $M_{2,F} = 14,846$

Eqn. 7

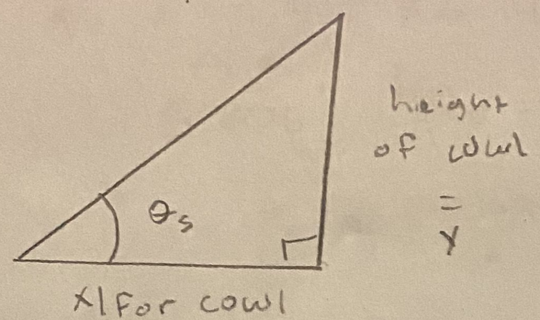
↳ $529371.1115 \rightarrow \left(\frac{P_2}{P_{02}}\right)_F =$

Eqn. 8

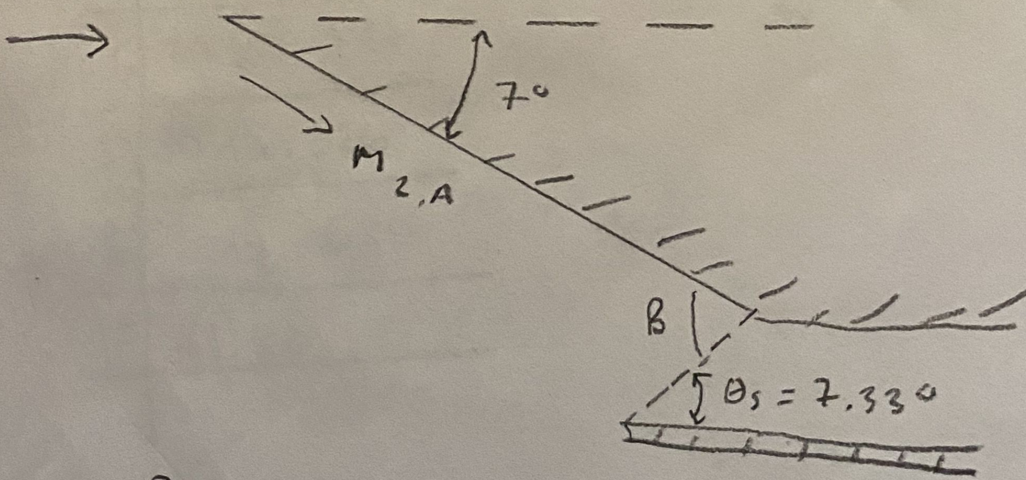
$(P_2)_D = (.00002083)(42493.234)(1186.5)$

$= 1049.326$ Pa

$(P_2)_F = 81.886$ Pa



$\tan\theta_s = \frac{y}{x} \implies \theta_s = 7.33°$



$B = 14.33°$

Eqn 1

↳ $M_{2,A} = 6.357$ $\beta = 14.33°$

$M_{2,B} =$

↳ 5.306 , $\left(\frac{P_2}{P_1}\right)_B = 1.817$

$P_{1,B} = P_{2,A} = 9670.124$ Pa

$\left(\frac{P_2}{P_1}\right)_B (P_{1,B}) = 26302.737$ Pa

	Pressure Pa
A	9670.124
B	26302.737
C	2932.927
D	1049.326
E	7217.582
F	81.886

$R_A = A_A \cdot P_A$

$A_A = 5 \cos 7° = 5.0375$

$R_A = 48713.72$ Pa

$F_{Ax} = R_A \cos 11° = 47818.72$ Pa

$F_{Ay} = R_A \sin 11° = 9295.017$ Pa

$R_B = A_B P_B = 31911.38$ Pa

$F_{Bx} = R_B \cos 4° = 31833$ Pa

$F_{By} = R_B \sin 4° = -2226$ Pa

$R_C = A_C P_C$

$A_C = A_{forcowl} + A_{comb} + A_{noz} = 4.71324 \, m^2$

$R_C = 13823.6$ Pa

$F_{Cx} = -13789.92$ Pa

$F_{Cy} = 964.29$ Pa

$R_D = A_D * P_D$

$A_D = 5/\cos 3.50298 = 5.00935\ m^2$

$R_D = 5256.45\ Pa$

$F_{Dx} = 5256.25$

$F_{Dy} = .45.6$

$R_E = A_E P_E = 32154.939$

$\hookrightarrow 14.5\ m^2$

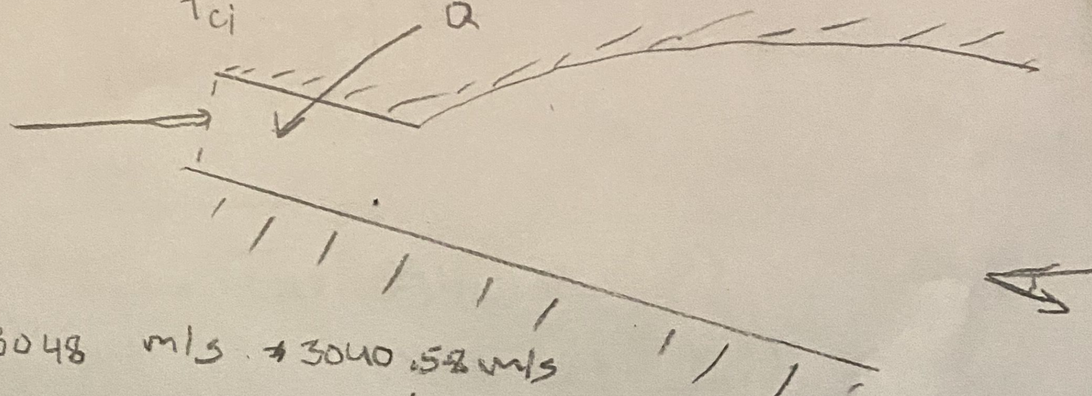
$F_{Ex} = -31759.058$

$F_{Ey} = 5030.14$

$R_F = 1187.35$

$F_{Fx} = 1172.73$

$F_{Fy} = -185.74$



$|V_i| = 3048$ m/s $\ast$ 3040.58 m/s

$u_i = 3048 \cos 40° =$

$v_i = 212.62$ m/s

$M_{ci} = M_B$

$T_{ci} = T_B = \frac{T_B}{T_A} T_A = 1.61 \cdot 231.24 = 372.3\,K$

M

$T_A = \frac{T_A}{T_0} T_0 = 2.31 \cdot 231.24 = 534.8\,K$

$\frac{T_B}{T_A} T_A = 1.369(534.8) = 732.14$

$P_{ci} = P_B = 26302.737$ Pa

$T_{tci} = T_{t0} = T_0\left(1 + \frac{\gamma-1}{2} M_0^2\right) = 4856.04\,K$

$P_{tci} = P_{ci} \ast \left(1 + \frac{\gamma-1}{2} M_{ci}\right)^{\frac{\gamma}{\gamma-1}}$

$= 19745357$ Pa

Solve For M_{ce}

$$f(M^2) = \left[\frac{1 + \frac{\gamma-1}{2}M^2}{(1+\gamma(M^2))^2}\right] M^2 \quad (9)$$

$f_{ci} = .114$

$$M = \sqrt{\frac{2f}{1-\gamma f \pm [1-(2(\gamma+1)f]^{1/2}}} \quad (10)$$

$$G(M) = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\left(\frac{\gamma+1}{2(\gamma+1)}\right)} \quad (11)$$

$$\frac{A_2}{A_1} = \frac{G(M_2)}{G(M_1)} \quad (12)$$

$\Sigma F_x = 0 \rightarrow$ F_x comb-Noz $= 85691.298\,N$

Numerically solve for $T_{t\,ce}$

Shuch that $\Sigma F_x = 0$

Ce — NOZ → Isent.

$P_{t\,ce} = P_{t\,NOZ}$

$T_{t\,ce} = T_{t\,NOZ}$

$\left(\frac{A^*}{A_e}\right)^{-1} = \frac{1.2403}{.156} \rightarrow M_e = 3.71$

$G(M_e) = .477$

$G(M_e)\frac{A_1}{A_2}$

Vehicle mass = $\frac{\text{excess Lift}}{g}$

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Results Summary (Please fill out and put this in your submission)

M_i (Flight Mach number) = 10

$\dot{Q}$ required to cruise =

$\dot{m}$ (air flow rate captured by vehicle and processed in engine) = 56.631 Kg/sec

Mass of vehicle required for cruise = excess lift / g

Aerodynamic forces (summary):

Component	F_x (N)	F_y(N)
Inlet*	-15985	-11521.017
Combustor-nozzle (ci to e)	85691.268	915.257
Wing top	1172.73	-186.74
Wing bottom	-31759.058	5030.14
Wing total	-30586.27	4843.4
Bottom of vehicle (cowl bottom)	-13789.92	964.29
Top of vehicle	5256.25	-45.6
Overall vehicle (total)	0	0

* actual force associated **with captured stream** tube on inlet surfaces from i to ci!

	beta	theta	M	P	T
free-stream	x	x	10	1185.5	231.24
Region downstream of incident inlet shock	15.492	11	6.357	9670.124	534.1644
Region downstream of reflected inlet shock (ci)	14.33	4	5.306	26302.74	731.2711
Combustor exit (ce)	x	x			
Nozzle exit (e)	x	x	3.71		
Top surface of vehicle	x	-0.49702	10.186	1049.326	231.1475
Bottom of wing	13.384	9	3.967	7217.582	315.3025
Top of wing	x	9	14.846	81.886	107.4637
Cowl bottom (bottom of vehicle)	8.653	4	8.623	2932.927	305.9819

|V|	u	v	T_t	P_t
3048	3048	0	4856.041	50311712
2945.063	-2890.32	-554.69	4856.041	
2876.149	2869.061	-201.804	4856.041	
0				
0				
3104.225	3104.107	-27.0855	4856.041	
1411.987	-1394.4	222.167	4856.041	
3084.93	3046.504	-485.394	4856.041	
3023.509	3016.057	212.143	4856.041	

AE 5535 Hypersonic Air-Breathing Vehicle Analysis **Assigned: January 22, 2021**

Due: February 15, 2021

Consider the attached representation of a hypersonic air-breathing (scramjet-powered) vehicle (modeled after the X-43 or X-51). The vehicle is 'one meter width', where width is into the paper (i.e., this is analyzed as a '2-D' vehicle). The top figure shows the vehicle configuration and related nomenclature for sizing the vehicle, etc. The bottom sketch provides a view of the vehicle at angle of attack, in flight, with shock-on-lip condition at cowl lip (leading edge of the cowl) and reflected shock cancellation at combustor entrance (2-shock inlet, with contained shocks). For this vehicle, the cowl extends to the exit plane of the nozzle (trailing edge of vehicle) as shown. The combustor (from ci to ce) has constant cross-sectional area.

The vehicle to be analyzed has the following characteristics (referred to the vehicle configuration as shown on following page):

xlinlet = 5 m (inlet length from leading edge of vehicle, i, to combustor entrance, ci)

xlcomb = 0.5 m (combustor length from ci – combustor entrance - to ce – combustor exit)

xlnoz = 3 m (nozzle length from nozzle entrance at ce to exit plane of vehicle at e)

height of engine exit plane (at e) = 1.2903 m

wing setting angle = 5 degrees

total wing planform area = 14.5 m^2

combustor entrance height (Aci) = 0.156 m (also = Ace; constant area heat addition)

top surface angle = 3.50298 degrees

inlet angle = 7 degrees

xlforcowl (length of portion of cowl forward of combustor entrance) = 1.213243 m

Ai (height of captured air streamtube at vehicle leading edge at angle of attack = 4 degrees) = 1.0322 m

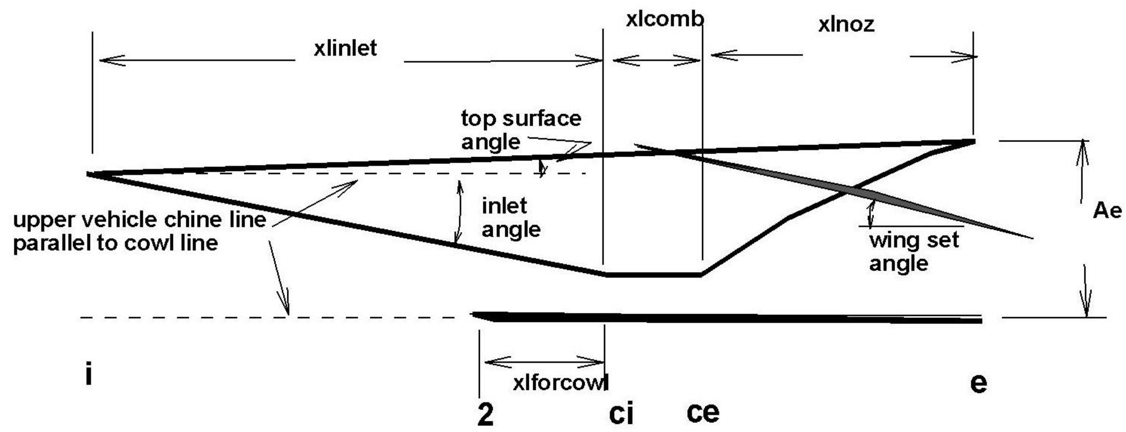
For this vehicle operating at 4 degrees angle of attack with a velocity of 3048.0 m/s at 30 km altitude (use ambient T = 231.24K and ambient P = 1185.5 N/m^2), find 1) the heat rate in the combustor and 2) the vehicle mass (use g = 9.81 m/s^2) ***required for cruise*** at these conditions.

Also provide/tabulate Mach, pressure, temperature, total pressure, total temperature, and velocity magnitude at all relevant stations (i, 2, ci, ce, e) in the propulsive flow path, as well as all give the oblique shock angles and flow conditions behind all shocks and expansions on all relevant vehicle surfaces.

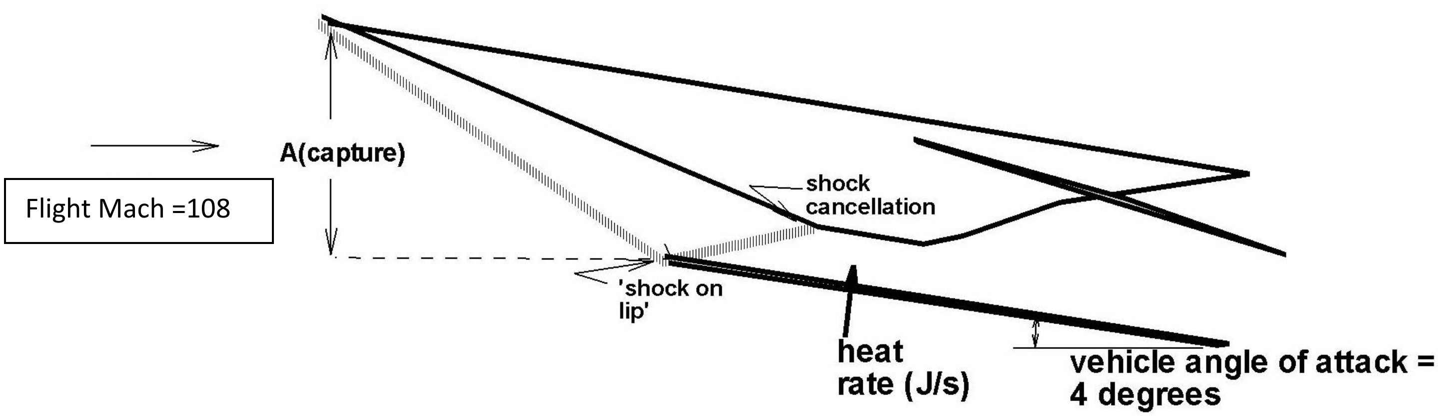
Provide a full and comprehensive break-down of all component forces (inlet, combustor-nozzle, top surface, wing top, wing bottom, cowl bottom) in terms of axial and vertical contributions to the overall axial and vertical aerodynamic forces experienced by the vehicle.

<u>Modeling assumptions:</u> Use constant ratio of specific heats $\gamma = 1.4$, R (gas constant) = 287 J/kg-K, and C_p = 1004.5 J/kg-K everywhere. Assume that ALL wetted surfaces of this vehicle (internal and external) are inviscid (i.e. no friction). Assume isentropic flow in the nozzle (from ce to ci). Use the equations for oblique shocks and Prandtl-Meyer expansion waves (as appropriate) in order to calculate the conditions necessary to do the force analysis; also use equations (**not tables or charts**) to calculate all necessary aspects of engine and airframe flow; all analysis in this project is done using basic compressible flow theory and techniques. Use one-dimensional inviscid heat addition ('Rayleigh') analysis in combustor.

vehicle configuration



vehicle in flight (angle of attack = 4 degrees)



Results Summary (Please fill out and put this in your submission)

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Mass of vehicle required for cruise =

Aerodynamic forces (summary):

Component	F_x (N)	F_y(N)
Inlet*		
Combustor-nozzle (ci to e)		
Wing top		
Wing bottom		
Wing total		
Bottom of vehicle (cowl bottom)		
Top of vehicle		
Overall vehicle (total)		

* actual force associated **with captured stream** tube on inlet surfaces from i to ci!

Fluids summary: β is shock angle, θ is turning angle (if not relevant to a station or region, enter X)

	β	θ	M	P	T	$\lvert\vec{V}\rvert$	u	v	T_t	P_t
Free-stream										
Region downstream of incident inlet shock										
Region downstream of reflected inlet shock (ci)										
Combustor exit (ce)										
Nozzle exit (e)										
Top surface of vehicle										
Bottom of wing										
Top of wing										
Cowl bottom (bottom of vehicle)										