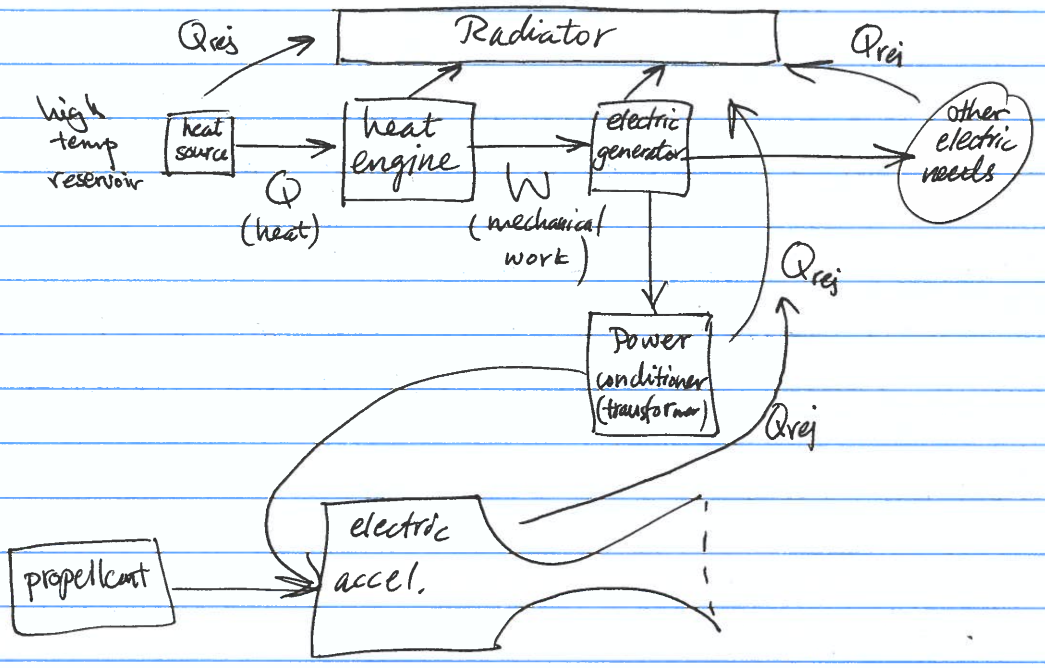
AE5535

Power Generation in Space

Consider a 'typical' electric rocket schematic: ~~rocket~~



Note: 'Direct' conversion of heat into electricity (i.e., 'bypass' the heat engine/generator) is not practical at this time for large scale power needs.

The heat engine works on a traditional thermodynamic fluid cyclic system using a liquid vapor or a gas as the working fluid.

A power plant system in space has a heat source represented by the reactor while the heat sink (radiator) is environmental space. Waste heat is transferred into space by radiation alone (i.e., no convection).

Terrestrial power plants usually use air or H_2O (waste heat rejected mainly by convection).

So, the radiator is really the only unconventional portion (item) in a space power plant other than the accel., the area of the radiator required needs to be minimized to greatest extent possible.

There are 3 thermodynamic cycles that we will consider in this course for use in generating power in space:

1) Carnot cycle → ideal cycle.
(most efficient possible) but not practically feasible.
↪ gives an upper limit or 'best possible'.

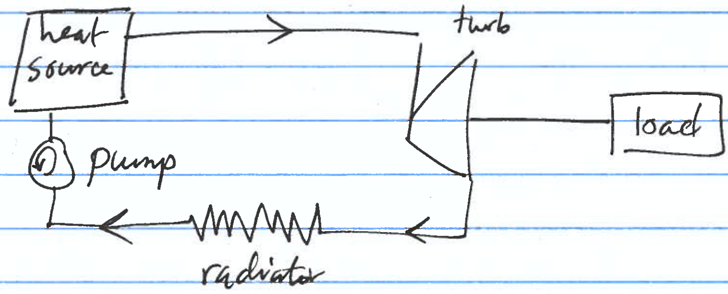
2) Rankine cycle:
vapor or 2-phase cycle.

3) Brayton Cycle:
gas turbine engines.

A thermodynamic power generator is one of two categories:

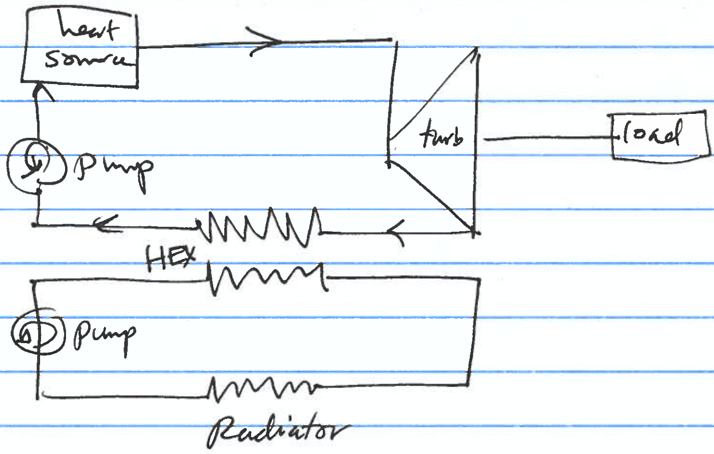
i) Direct cycle: heat transmitted from the working fluid to the radiator directly.

ii) Indirect cycle: employs an intermediate (or secondary) fluid, heat is transmitted from working medium to secondary fluid through a heat exchange (HEX). The second fluid then passes through radiator.
Second fluid recirculates in a closed loop indefinitely and its function is to act simply as a heat carrier.



Direct

(Base direct cycle)

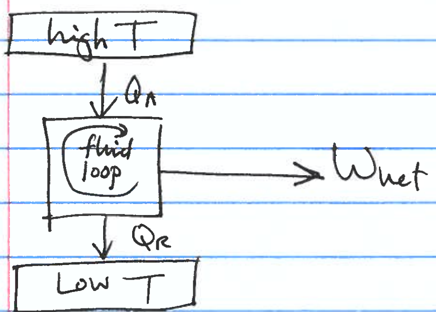


Indirect

Although the use of the indirect cycle schematically is more complex, using a heat exchanger w/ secondary fluid maybe beneficial (i.e., w/better rad/HEX performance).

Basic cycle considerations:

Consider thermal efficiency, η, of the heat engine



$$\boxed{\eta = \frac{\text{output}}{\text{input}} = \frac{W_{net}}{Q_A} = \frac{W_{net}}{Q_R + W_{net}}}$$

where W_{net} = net work output [J]

Q_R = heat rejected [J]

Q_A = heat added [J]

So ... $$\frac{1}{\frac{Q_R}{W_{net}} + 1} = \eta \quad \text{or} \quad \frac{Q_R}{W_{net}} = \frac{1}{\eta} - 1$$

i.e., the heat in joules rejected per joules of net work out is:

$$\frac{1}{\eta} - 1 = Q_R' = \frac{Q_R}{W_{net}} \qquad \text{Eq. (1)}$$

or for British Imperial Systems:

$$Q_R' = 3413 \left(\frac{1}{\eta} - 1 \right) \left[\frac{Btu}{kW\text{-}hr} \right]$$

[1 BTU = 1055 J]

Power Generation In Space cont.

Eq. (1) $Q_R' = \frac{1}{\eta} - 1$ from cycle considerations

The heat rejected from the power cycle must be radiated into space. Then the heat trasferred by radiation in J per J of work output is:

Eq. (2) $Q_R' = \varepsilon \sigma A T_R^4$

A is in $\frac{m^2}{\text{Watt of power output}}$

σ = Stefan Boltzmann constant of radiation $= 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$

T_R = radiator temp. ('heat rejection' T)

ε = emissivity of radiator surface (≤ 1.0)

Equate ① & ②, obtain:

$$\boxed{A = \frac{1}{\varepsilon \sigma T_R^4}\left(\frac{1}{\eta} - 1\right)}$$

$\left[\frac{m^2}{W}\right]$ watts of mechanical power produced

— Valid for any cycle.

* For minimum area, we like high η and high T_R.

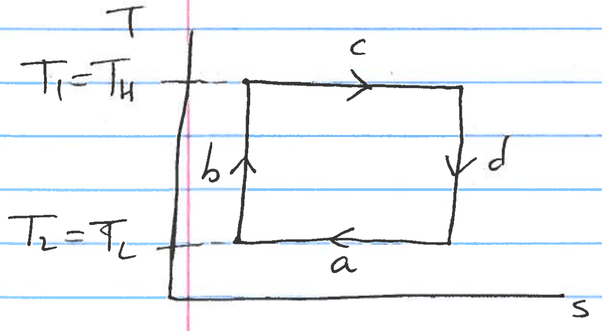
So the area of the radiator surface is a function of the thermal efficiency of the cycle.

2nd law of thermo implies there is no existing cycle with higher efficiency than the Carnot cycle

* Carnot Cycle $\longrightarrow$ Ideal Cycle

so Carnot cycle gives

– The highest η (thermal efficiency) possible $\longrightarrow$ smallest area possible (and smallest possible radiator area)



in 'a' entropy decreases

a : const. temp heat rejection at T2 or TLow

b : isentropic compression to T1 or Thigh

c : const. temp heat addition at T1 entropy increases

d : isentropic expansion to T2

Then repeat a, b, c, d (cyclic)

Work is the area inclosed.

$\xrightarrow{\text{only}}$ Carnot $\quad Q_{Added} = T_1 \Delta S \quad , \quad Q_{rejected} = T_2 \Delta S$

$W_{Carnot} = \Delta S \left(T_H - T_L \right)$ (1) (2)

– The lower the T_L is the greater W_{net}

$W_{net} \longrightarrow Q_A$ as $T_L \longrightarrow 0$

$\eta_{carnot} = \dfrac{T_1 - T_2}{T_1}$, η goes up as T_2 goes down

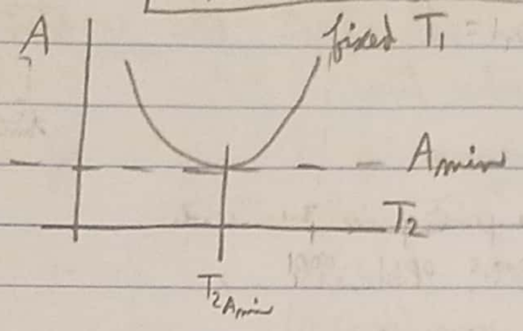
assuming $T_2 = T_R$

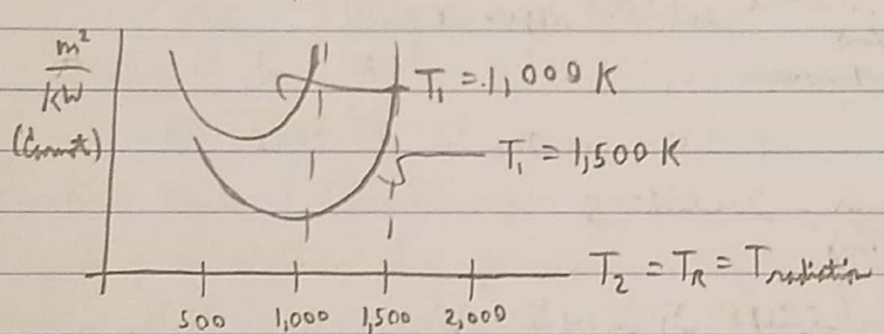
Recall $A = \frac{1}{\varepsilon \sigma T_R^4}\left(\frac{1}{\eta} - 1\right)$

Assume $T_2 \sim T_R$

Carnot Cycle

$$\boxed{A_{Carnot} = \frac{1000}{\varepsilon \sigma T_2^3}\left(\frac{1}{T_1 - T_2}\right) \text{ in } \frac{m^2}{kW \text{ of power generated}}}$$





For Carnot: set $\left.\frac{dA}{dT_2}\right)_{fixed\ T_1} = 0$ & find $\left.\frac{T_2}{T_1}\right)_{\substack{min\ A \\ Carnot}} = \frac{3}{4}$ or $\eta_{\substack{min\ area \\ (Carnot)}} = 0.25$

or $A_{\substack{min\ area \\ Carnot}} = \frac{9481.5}{\varepsilon \sigma T_1^4} \frac{m^2}{kW \text{ of net power output}}$

- Ex. heat is added in Carnot cycle at temperature of 811.1 K & rejected at 533.3 K; find
  - a) Area of radiator required per kW of power output
  - b) η

$T_1 = 811.1$ K $\quad T_2 = 533.3$ K $\quad \varepsilon = 1.0$

$\sigma = 5.669 \cdot 10^{-8} \frac{W}{m^2 \cdot K^4}$

$\eta = \frac{T_1 - T_2}{T_1} = 0.342 \qquad A = 0.4181\ m^2/kW$

if $\eta = 0.25$ (min A)

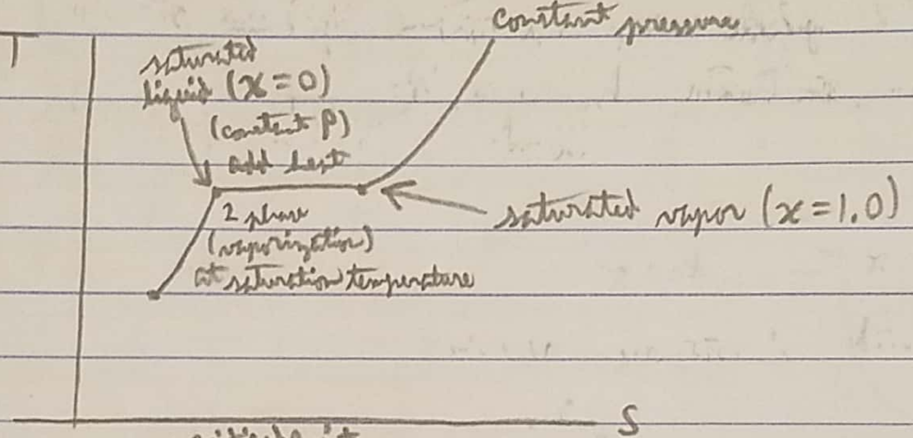
$T_1 = 811.1\ K \rightarrow T_2 = 608.3\ K$

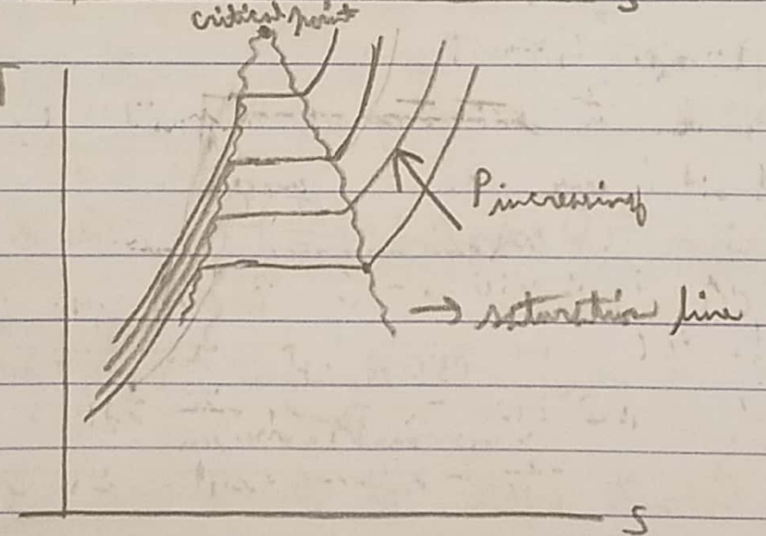
$A_{min} = 0.3865\ m^2/kW$

· Rankine cycle (2 phase) vapor-turbine

· Review of 2 phase thermodynamics

- Consider a liquid (like H_2O) where you add heat at constant pressure, you get a 'line' that looks like





1) hence for a given fluid in its 2 phase state ('saturated' state) → distinct 'saturation temp' for a given P and vice-versa

2) x = 'dryness' fraction ('quality') of liquid-vapor

$x=0$ all saturated liquid

$x=1$ all saturated vapor

3) h (enthalpy) and entropy (s) increase as heat is added on a constant pressure line (even in saturation region)

$$h_{2\text{-phase}} = \underset{\substack{\text{liquid} \\ (x=0)}}{h_{sat}} + x h_{fg}$$

h_{fg} = latent heat of vaporization (specific to fluid)

$$h_{\substack{sat \\ vapor \\ (x=1.0)}} = h_{\substack{sat \\ liquid \\ (x=0)}} + h_{fg}$$

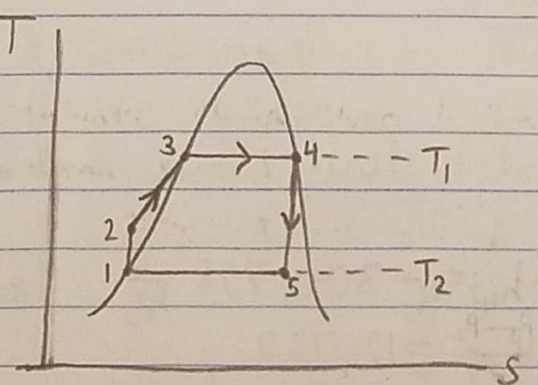
$$s_{2\,phase} = s_{\substack{sat \\ liquid \\ (x=0)}} + x\, s_{fg}$$

4) for a given 2 phase fluid (saturated $H_2O \rightarrow$ steam)
saturation temperature, $h_{\substack{sat\ liquid \\ (x=0)}}$, $h_{\substack{sat\ vapor \\ (x=1.0)}}$

h_{fg}, $s_{\substack{sat\ liquid \\ (x=0)}}$, $s_{\substack{sat\ vapor \\ (x=1.0)}}$, s_{fg}

are tabulated at various pressures

- Rankine cycle (saturated vapor cycle)
- Liquid pressurization by pump $(1 \rightarrow 2)$
- Heating fluid with vaporization to saturation vapor point $(2 \rightarrow 4)$
  - 3 at plateau between liquid & vapor on T-s diagram
- Turbine expansion of vapor (extracts work) $\rightarrow$ run pump + external work $(4 \rightarrow 5)$
- Condensor (radiator) $(5 \rightarrow 1)$
- Rankine cycle has lower temperatures than Brayton cycle
  - Advantage is lower temperatures and pressures, otherwise not as good as other cycles



How do we compute work, heat for this cycle?

Recall energy equation:

$$\underset{A \rightarrow B}{\Delta h_{total}} = \underset{heat/mass}{q_{A \rightarrow B}} + \underset{work/mass}{w_{A \rightarrow B}}$$

$h_t = h + \frac{u^2}{2}$, for power generation systems $\frac{u^2}{2} << h$ so
$h_B - h_A = q_{A \rightarrow B} + w_{A \rightarrow B}$
so in Rankine cycle, what is turbine work (assume turbine is adiabatic)?
$h_5 - h_4 = w_{turbine}$

heat added?

$h_4 - h_2 = q_{2\to4}$

pump power required? (liquid)

$\frac{dP}{\rho} + u\,du = \delta w_{pump}$ (per mass) (for fluids, not specifically a liquid or specifically a gas)

assume $u\,du$ term is negligible

$\frac{dP}{\rho} = \delta w_{pump}$ ρ = constant (liquid)

$\frac{P_2 - P_1}{\rho} = \delta w_{pump}$

· Ex. Saturated vapor cycle (H_2O), isentropic turbine

$T_H = T_3 = T_4 = 617\ K = 343°C$

$P_L = 300$ psiA $= 2.07$ MPa $= P_5 = P_1$

look up P_H for $T_H = 343°C = 14.8$ MPa

look up T_L for $P_L = 2.07$ MPa $= 487\ K = 214°C$

$h_4 = (h_g$ at $343°C) = 2{,}610.5$ kJ/kg

$s_4 = s_5$ ($= s_g$ at $343°C$) $= 5.3098\ \frac{kJ}{kg \cdot K}$

$s_1 = 2.455\ \frac{kJ}{kg \cdot K}$ ($= s_f$ at $214°C$), $h_1 = 915\ \frac{kJ}{kg}$

$x = 0.737$ based on $x = \frac{s_5 - s_1}{s_{fg}}$

$= \frac{s_5 - s_1}{s_{sat\ vapor\ 214°C} - s_1}$

$h_5 = h_1 + x h_{fg\,(214°C)} = 2{,}304\ \frac{kJ}{kg}$

work of turbine per mass $= h_5 - h_4 = -306{,}778\ \frac{J}{kg}$ (out)

pump work per mass $= \frac{P_H - P_L}{\rho_{H_2O}} = \frac{P_2 - P_1}{\rho} = 12{,}730$

q_{added} (per mass) $= h_4 - h_2$ but $h_2 = h_1 +$ pump work so

$q_{added} = 1{,}682{,}770\ \frac{J}{kg}$

$\eta = \frac{\overbrace{|W_{turbine}| - |W_{pump}|}^{\to W_{net}}}{q_{added}} = 0.175$

Scanned with CamScanner

$|W_{net\ (out)}| = 294{,}048$ J/kg

$q_{rejected} = q_{added} - W_{net\ out} = 1{,}388{,}722$ J/kg

Now for a 1,000 kW power requirement (say need to power an ion drive)

$$1{,}000{,}000 \ \frac{J}{s} = \dot{m}_{H_2O} \cdot W_{net\ out} \Longrightarrow \dot{m}_{H_2O} = 3.4 \text{ kg/s}$$

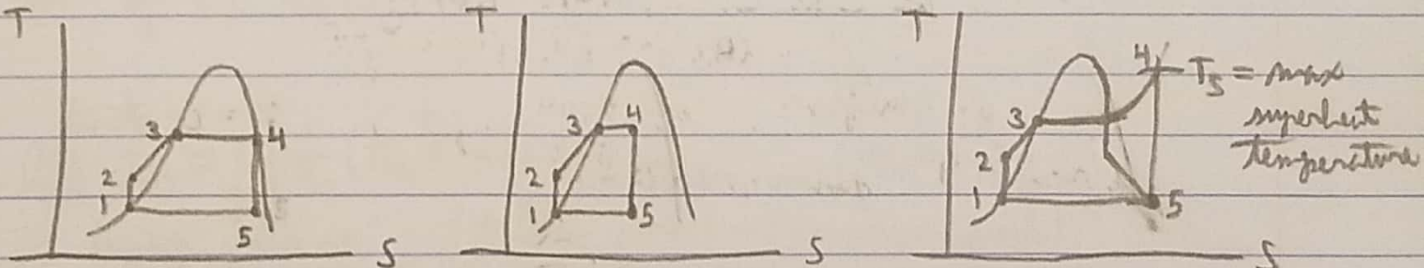
$(\varepsilon = 1)$

Required area of radiation in m^2/kW power output

$$= \frac{1000}{\varepsilon \sigma T_L^4}\left(\frac{1}{\eta} - 1\right) = 1.48 \ m^2/kW$$

so total area of radiator $= 1{,}480 \ m^2$ $(15{,}907 \ ft^2)$

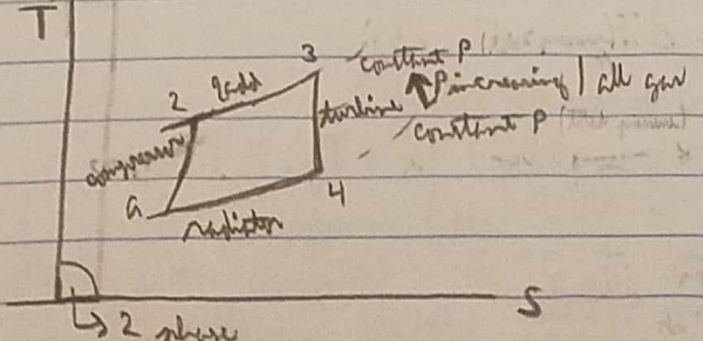
3 possibilities for Rankine cycle



Saturated vapor | Wet vapor | Superheated vapor cycle (avoids erosion, used most often)

Gas turbine direct power plant (closed Brayton cycle)

all gas



T_3 = max temperature in cycle

$P_4 = P_a$

$P_2 = P_3$

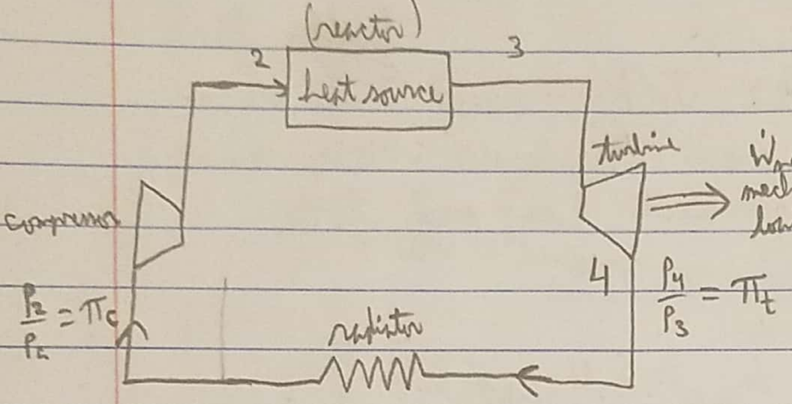
$\frac{u^2}{2}$ are negligible

→ good for large power generation

→ high temperature materials

→ compressor heavy

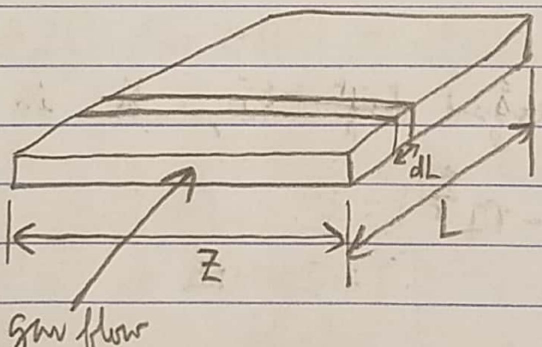
→ single phase (simpler)



$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{add}}$$

Total heat in Watts transferred per Watt of net power output is $Q_R' = \frac{1}{\eta} - 1$

Consider a flat section of radiator; gas flowing through



from 4th power radiation:

Z = constant for one side of panel

$$* \, dQ_R' = \varepsilon \sigma T_R^4 \underbrace{(Z\,dL)}_{dA}$$

$Z\,dL$ is the differential radiator area required per Watt power output

Energy equation:

$$** \, dQ_R' = -\dot{m}_{gas}' C_{p_{gas}} dT_{gas}$$

$\dot{m}_{gas}'$ is kg/s/Watts power output

steady operation $* = **$ $\longrightarrow$ also $T_R = T_{gas} = T$

so

$$\frac{-\dot{m}_{gas}' C_{p_{gas}} dT}{\varepsilon \sigma T^4} = Z\,dL$$

integrate between 4 & 1

Eq. ① $\dfrac{\dot m_{gen}' C_{p_{gen}}}{3\varepsilon\sigma}\left[\dfrac{1}{T_a^3}-\dfrac{1}{T_4^3}\right]=ZL$

(**) $dQ_R' = -\dot m_{gen}' C_{p_{gen}}\, dT_{gen}$

$Q_R' = \dot m_{gen}' C_{p_{gen}} (T_4 - T_a)$

We know $Q_R' = \frac{1}{\eta} - 1$ so

$$\dot m_{gen} C_{p_{gen}} = \frac{\left(\frac{1}{\eta}-1\right)}{(T_4 - T_a)}$$ sub into eq. ① & get

Brayton cycle: $A_{radiator} = \dfrac{\left(\frac{1}{\eta}-1\right)}{3(T_4-T_a)\sigma\varepsilon}\left(\dfrac{1}{T_a^3}-\dfrac{1}{T_4^3}\right)\ \dfrac{m^2}{W}$ (Eq. ②)

(m^2/Watt power output)

Recall T_3 given (design) high temperature in cycle

We also know:

Eq. ③ $\dfrac{T_4}{T_3} = 1-\eta_t\left(1-\pi_t^{\frac{\gamma-1}{\gamma}}\right)=\tau_t$ $\left(\pi_t = \dfrac{P_4}{P_3}\right)$

Eq. ④ $\dfrac{T_2}{T_a} = 1+\dfrac{1}{\eta_c}\left(\pi_c^{\frac{\gamma-1}{\gamma}}-1\right)=\tau_c$ $\left(\pi_c = \dfrac{P_2}{P_a}\right)$

Eq. ⑤ $\eta = \dfrac{W_{net\ out}}{q_{add}} = \dfrac{|\text{turbine work}| - |\text{compressor work}|}{q_{added}}$

$$\eta = \frac{C_p(T_3-T_4) - C_p(T_2 - T_a)}{C_p(T_3-T_2)} = 1-\left[\frac{(T_4-T_a)}{(T_3-T_2)}\right]$$

Eq. ⑥ Since $P_3 = P_2$ & $P_4 = P_a \Longrightarrow \dfrac{P_4}{P_3} = \dfrac{P_a}{P_2}$ or $\boxed{\pi_t = \dfrac{1}{\pi_c}}$

So for given $T_3, C_p, \gamma, \eta_t, \eta_c$

use relationships ① through ⑥ to find absolute minimum area required (per Watt of net power output)

Along with associated T_a, π_c, η, etc.

Requires an iterative procedure!

Steps required (loops)

outer → (A) Choose (pick) a trial Π_c, determines Π_t from Eq. (6)

(B) Calculate τ_t & T_4 from Eq. (3)

inner → (C) Choose a trial T_a, calculate T_2 from (4)

(D) Calculate η from (5)

(E) Calculate A from (2)

(F) Iterate on T_a, Π_c until $A_{min\ absolute}$ (& η_{max}) found

· Ex. Brayton cycle

H_2 as gas $\gamma = 1.4$

$R = 4{,}125 \frac{J}{kg \cdot K}$

$C_p = R\left(\frac{\gamma}{\gamma-1}\right) = 14{,}438 \frac{J}{kg \cdot K}$

let $T_3 = 1{,}667$ K $\eta_c = \eta_t = 0.85$

look over range Π_c from 2 to 6

T_a from 500 to 900 K

choose $\Pi_c = 2.66$ $T_a = 711$ K

$\Pi_t = 0.376$

$\tau_t = 0.793$

$T_4 = 1{,}322$ K

$\tau_c = \ldots$ $T_2 = 980.7$ K

$\eta = 0.11$

use $\varepsilon = 1.0$ $A = 0.000182$ m^2/Watt net power output

→ This choice of T_a & Π_c are the optimal (minimum area)

Suppose 1,000 kW of net power output needed

area required $= 182\ m^2$

$\dot{Q}_{added} = 9{,}090.9$ kW

$\dot{m}_{gas} = 0.92$ kg/s

Turbine power $= \dot{m}_{gas} C_p (T_3 - T_4)$

compressor power $= \dot{m}_{gas} C_p (T_2 - T_a)$

net power = difference

Compare a Rankine/Brayton (1,000 kW)

net power requirement

Brayton H_2, $T_3 = 1{,}668$ K, $\eta_c = \eta_t = 0.85$

Rankine H_2O ($T_{high} = 617$ K, $P_{low} = 2$ MPa)

ideal turbine, pump

	Brayton	Rankine
net power output (kW)	1,000	1,000
Turbine power (kW)	4,577	1,043
Compressor/pump power (kW)	3,577	43
Heat rate required from reactor (kW)	9,091	5,722
Rejected heat rate (kW)	8,091	4,721
$\dot{m}$ working fluid (kg/s)	0.92	3.4
η	0.11	0.175
Area radiator (m^2)	182	1,478
max T in cycle (K)	1,668	617

[redacted]