AE5535

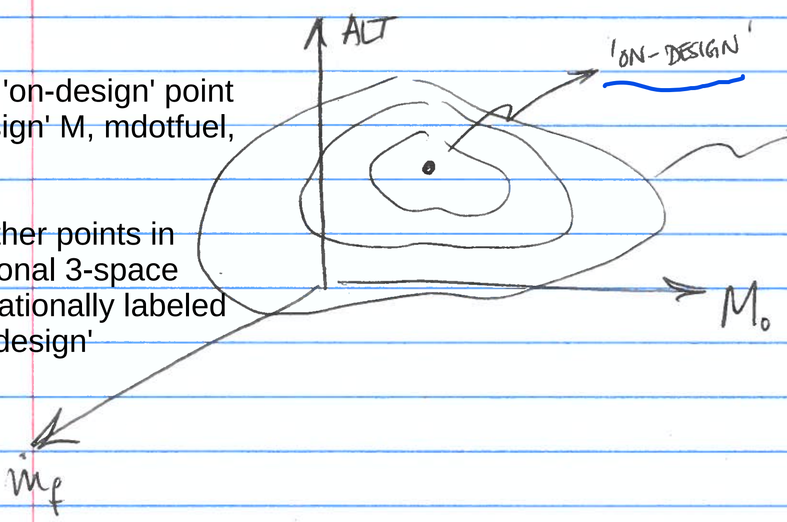
ENGINE OFF-DESIGN PERFORMANCE & COMPONENT MATCHING

TO DATE, (IDEAL AND NON-IDEAL CYCLE ANALYSIS), WE HAVE BEEN ANALYSING FAMILIES OF ENGINES, (i.e. π_c FIXED AS YOU VARY M_0; ANALYZE $F/\dot{m}$, S, ...). OBVIOUSLY, IN A REAL GIVEN ENGINE, π_c WOULD VARY AS YOU VARY CONDITIONS SUCH AS M_0.

SPECIFICALLY, WE now WANT TO ASSESS THE PERFORMANCE Thrust, mdot(air) captured, RPM and $(\frac{F}{\dot{m}}, S)$ OF A GIVEN ENGINE ACROSS THE OPERATIONAL '3-SPACE' OF FLIGHT MACH NUMBER, M_0, FUEL THROTTLE SETTING, $\dot{m}_f$, AND ALTITUDE, T_0, P_0, ρ_0. [CONTOUR MAP, 3D]

engine 'on-design' point (at 'design' M, mdotfuel, ALT)

... all other points in operational 3-space are notationally labeled as 'off-design'



i.e., we want to generate contours of THRUST, MASS CAPTURE, RPM, SPECIFIC FUEL CONSUMPTION

'mass capture' is mdot air inducted into engine (captured streamtube mdot)

☆ THE ENGINE IS 'BOSS' OF $\dot{m}_{air}$; IT TAKES THE $\dot{m}$ IT WANTS!
(based on mdotfuel throttled, flight Mach M and altitude (ALT) it is at!

* TO DO ALL OF THIS, WE NEED INFORMATION ON HOW (FOR INSTANCE) τ_c FOR A GIVEN ENGINE VARIES WITH M_0, [$M_0 \Rightarrow \tau_r, \pi_r$ UNIQUELY DETERMINED] HOW IT VARIES WITH ALTITUDE (T0, P0), AND/OR, FUEL THROTTLE SETTING (mdotfuel).

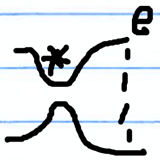
* WE WILL MAKE SOME PHYSICALLY PLAUSIBLE APPROXIMATIONS OR ASSUMPTIONS INITIALLY.
* NOTE NOW THAT T_{t4} (HENCE τ_λ at given altitude (T0)) IS NOT PURELY A MEASURE OF THE MAX DESIGN (ALLOWABLE) TEMP AT COMBUSTOR EXIT FOR OFF-DESIGN. IT IS RATHER A CONVINIENT MEASURE OF THE THROTTLE SETTING (HOW MUCH BURNING IN COMBUSTOR) FOR GIVEN ALTITUDE AND M_0.
BUT NOTE THAT $\dot{m}_f$ DEPENDS ALSO ON T_{t3} (not just Tt4)

since assuming Cp const and f<<1 - $\dot{m}_f h = \dot{m}_{air} C_p (T_{t4} - T_{t3}) \eta_b$ (burner efficiency) TURBOJET

AND, T_{t3} IS DEPENDANT ON ALTITUDE (T_0) AND M_0 τ_r (Tt0) as well as τ_c (compressor)

WE WILL APPROACH OFF-DESIGN IN TWO WAYS: THE FIRST IS ANALYTICAL (SIMPLIFIED, WITH HEAVY DUTY, YET GOOD ASSUMPTIONS), AND THE SECOND IS 'COMPONENT MATCHING', INVOLVING ACTUAL specific COMPONENT, PERFORMANCE MAPS. (obtained from hardware testing)

ANALYTICAL METHOD FOR OFF-DESIGN



REVIEW $\dot{m}$ RELATIONSHIPS

$$\dot{m} = \rho u A = \left(\frac{P}{RT}\right) u A = \frac{P_t A}{\sqrt{T_t}} \sqrt{\frac{\gamma}{R}}\, M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{(1+\gamma)}{2(\gamma-1)}}$$

ANY LOCATION IN ENGINE

so if Pte, Tte, Ae, gamma, R,mdot known, can find Me (then Ue, Pe, Te, etc.)

AND AT AN AERODYNAMIC THROAT $(M = 1.0)$ AT A^*:

$$\dot{m} = \frac{P_t^* A^*}{C^*} \quad : \quad C^* = \text{CHARACTERISTIC VELOCITY} = \sqrt{\frac{R T_t^*}{\gamma}} \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

OR

$$\dot{m}_{throat} = \frac{\Gamma A^* P_t^*}{\sqrt{R T_t^*}} \quad : \quad \Gamma = \sqrt{\gamma} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

AE5535

ANALYTICAL TURBOJET OFF-DESIGN

- ASSUME THAT THE FLOW IS CHOKED AT STATION 4 AND STATION 8, THAT APPROXIMATION IS VALID FOR A VERY WIDE OPERATING RANGE OF A TURBOJET; TRUE ACROSS THE 3-SPACE, of M0, mdotfuel, ALT, i.e. true for on-design and most off-design points in 3 space
POSSIBLE afterburner AB. BETWEEN 5 & 8.

- FOR STEADY FLOW with stations 4 and 8 choked:
$$\dot{m}_4 = \frac{\Gamma_t A_4 P_{t4}}{\sqrt{RT_{t4}}} = \dot{m}_8 = \frac{\Gamma_t A_8 P_{t8}}{\sqrt{RT_{t8}}}$$

SOLVE FOR:
$$\frac{A_8}{A_4} = \sqrt{\frac{T_{t8}}{T_{t4}}}\,\frac{P_{t4}}{P_{t8}}$$

BUT $\dfrac{P_{t8}}{P_{t4}} = \pi_t \pi_{AB}$; $\pi_t = \dfrac{P_{t5}}{P_{t4}}$, $\pi_{AB} = \dfrac{P_{t8}}{P_{t5}}$

AND $\dfrac{T_{t8}}{T_{t4}} = \tau_t \tau_{AB}$; $\tau_t = \dfrac{T_{t5}}{T_{t4}}$, $\tau_{AB} = \dfrac{T_{t8}}{T_{t5}}$

SO
$$\boxed{\frac{\tau_t^{1/2}}{\pi_t} = \frac{A_8}{A_4}\frac{\pi_{AB}}{\sqrt{\tau_{AB}}}}$$

Note: this holds in entire 3-space of mdotfuel, ALT, M0 for the given engine! (of course assuming choked flow at stations 4 and 8)

NOW ASSUME NO AB: $\pi_{AB} = 1.0$, $\tau_{AB} = 1.0$

$$\boxed{\frac{\tau_t^{1/2}}{\pi_t} = \frac{A_8}{A_4}} \quad *$$

π_t IS RELATED TO τ_t THROUGH η_t

$$\left\{\begin{aligned} \eta_t &= \frac{1-\tau_t}{1-\pi_t^{\frac{\gamma_t-1}{\gamma_t}}} \\ \pi_t &= \pi_t(\eta_t, \tau_t) \end{aligned}\right.$$

η_t IS approx. constant across most operating points

* IF η_t IS APPROXIMATELY CONSTANT ACROSS OPERATING RANGE OF ENGINE (3-SPACE), THEN $\left[\frac{\tau_t^{1/2}}{\pi_t} = \frac{A_8}{A_4}\right]$ UNIQUELY DETERMINES τ_t IN TERMS OF A_8/A_4

$\therefore$ τ_t IS 'ALMOST' CONSTANT ACROSS (3-SPACE) of mdotfuel, ALT, M0 FOR FIXED $\frac{A_8}{A_4}$!

This is unlike cycle analysis where taut varies as M0, ALT, mdotfuel are changed (say at fixed compressure pressure ratio piec) - this due to TCPB turbine-compressor-power balance at that fixed piec!

WE KNOW TURBINE-COMPRESSOR POWER BALANCE IS VALID WHATEVER OPERATING CONDITION IS. whether on or off design

f<<1 and constant Cp

$$\tau_c = \frac{T_{t3}}{T_{t2}} \qquad \dot{m} C_p (T_{t3} - T_{t2}) = \eta_m \dot{m} C_p (T_{t4} - T_{t5})$$

SOLVE FOR $\tau_c \Rightarrow \boxed{\tau_c = \frac{T_{t3}}{T_{t2}} = 1 + \eta_m \left(\frac{\tau_\lambda}{\tau_r}\right)(1 - \tau_t)}$

TCPB

WHILE ASSUMING C_p, γ CONSTANTS

SO, $\tau_c = f^n(M_0, \tau_\lambda)$ = f(M0, Tt4, T0)

$$\tau_\lambda = \frac{T_{t4}}{T_0}$$

THEN $\pi_c = f^n(M_0, \tau_\lambda, \eta_c)$ $\quad \because \eta_c = \frac{\pi_c^{\frac{\gamma_c - 1}{\gamma_c}} - 1}{\tau_c - 1}$

$$\tau_r = 1 + \frac{\gamma - 1}{2} M_0^2$$

unlike eta(turbine), eta (compressor) changes signficantly across the 3 space of mdotfuel, ALT, M0

so as turbojet moves off-design in 3-space of mdotfuel, M0, ALT,we know taut stays fixed (for A8/A4 constant). hence piec (π_c) and tauc (τ_c) at off-design can be found from TCPB (and will of course vary in general)

quick
'Recap'

* WE WANT TO BRIDGE PERFORMANCE ANALYSIS ACROSS OPERATIONAL RANGE.

* ASSUMPTIONS: SIMPLE SINGE-SPOOL TURBOJET; $f \ll 1$, NO AB., γ, c_p CONSTANT ACROSS ENGINE.

DEFINED: $\boxed{\dfrac{\tau_t^{1/2}}{\pi_t} = \dfrac{A_8}{A_4}}$ CHOCKED FLOW AT ④ & ⑧

ALSO, $\pi_t = \pi_t(\eta_t, \tau_t)$: $\eta_t = \dfrac{1-\tau_t}{1-\pi_t^{\frac{\gamma_t-1}{\gamma_t}}}$

SO, FOR SOME GIVEN $\frac{A_8}{A_4}$ AND η_t NOT CHANGING, τ_t IS UNIQUELY DETERMINED AND DOESN'T CHANGE 'ANYWHERE' IN THE OFF-DESIGN 3-SPACE.

THEN, BASED ON τ_t, WE CAN FIND $\tau_c = \frac{T_{t3}}{T_{t2}}$ FROM TURBINE/COMPRESSOR POWER BALANCE.

$$\tau_c = 1 + \eta_m \left(\frac{\tau_\lambda}{\tau_r}\right)(1-\tau_t)$$

SO, $\tau_c = \tau_c(M_0, \tau_\lambda)$: η_m IS KNOWN, ALSO η_t, $\frac{A_8}{A_4}$

THEN, π_c IS FOUND FROM: $\eta_c = \dfrac{\pi_c^{\frac{\gamma_c-1}{\gamma_c}} - 1}{\tau_c - 1}$

$\therefore \pi_c = \pi_c(M_0, \tau_\lambda)$: η_c IS KNOWN

$\tau_t \longrightarrow$ FIXED ACROSS 3-SPACE

$\pi_c, \tau_c \longrightarrow$ VARY ACROSS 3-SPACE

GENERAL PERFORMANCE EQUATIONS for off-design flight/engine operation, i.e. in terms of performing Ttot, Ptot 'chains'

thrust, $\frac{F}{\dot m}$, S EQUATIONS ARE STILL VALID!, BUT THE MASS FLOW RATE OF AIR PROCESSED BY THE ENGINE VARY ACROSS 3-SPACE. ($\dot m$).. ENGINE WILL 'SPILL' OR 'INGEST' AIR TO GET ITS DEMANDED $\dot m$ (ENGINE IS BOSS).

RECALL $F = \dot m (u_9 - u_0) + (P9-P0)A9$

* SO, HOW DO WE COMPUTE THE $\dot m$ air DEMANDED BY THE ENGINE? IN TERMS OF THE (ALT, M_0, $\dot m_f$) in operational 3-space, recall τ_λ invariant

ASSUME $f \ll 1$, γ & C_p CONSTANT; ASSUME EFFICIENCIES $\pi_d, \pi_b, \eta_m, \eta_t, \eta_c$ KNOWN OR ESTIMATED. also A8, A4 known (we know what engine looks like!)

MASS FLOW RATE ANALYSIS FOR OFF-DESIGN:

flow at 4 choked!

$$\mathrm{I.}\quad \dot m_4 = \frac{\Gamma}{\sqrt{R}} A_4 \frac{P_{t4}}{\sqrt{T_{t4}}}, \text{ AND SINCE } P_{t4} = P_0 \pi_r \pi_d \pi_c \pi_b$$

$$\text{SO, } \pi_r = \pi_r(M_0) = \left(1+\frac{\gamma-1}{2}M_0^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\therefore\quad \dot m_4 = \frac{\Gamma}{\sqrt{R}} A_4 \left[P_0 \pi_r \pi_d \pi_c \pi_b\right] \frac{1}{\sqrt{\tau_\lambda T_0}}$$

ALT

(small f)

SO, FUNCTIONALLY, $\dot m_4$ (& HENCE $\dot m_{AIR}$) $= \dot m_4(P_0, T_0, M_0, \tau_\lambda)$

REMEMBER π_c IS FOUND FROM A 3-STEP PROCESS,

(1) $\frac{\tau_t^{1/2}}{\pi_t} = \frac{A_8}{A_4}$; $\eta_t = \frac{1-\tau_t}{1-\pi_t^{\frac{\gamma-1}{\gamma}}}$, $\frac{A_8}{A_4}$ KNOWN [τ_t UNIQUE]

(2) $\tau_c = 1 + \eta_m\left(\frac{\tau_\lambda}{\tau_r}\right)(1-\tau_t)$: τ_r, τ_λ KNOWN

(3) π_c FROM $\eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}}-1}{\tau_c - 1}$

SO τ_c, π_c FUNCTIONS OF $\left(M_0, \tau_\lambda, \frac{A_8}{A_4}, \text{EFFICIENCIES}\right)$

Recall $\tau_r = 1+\frac{\gamma-1}{2}M_0^2$

$\pi_r = \left(1+\frac{\gamma-1}{2}M_0^2\right)^{\frac{\gamma}{\gamma-1}}$

ALT $\quad M_0 \quad$ EFF. $\quad M_0, \tau_\lambda \quad$ EFF.

$$\text{So, } \dot m_4 \sim \boxed{\dot m = \frac{\Gamma}{\sqrt{R}} A_4 \left[P_0 \pi_r \pi_d \pi_c \pi_b \right] \frac{1}{\sqrt{\tau_\lambda T_0}}} \quad * \qquad T_0 \to \text{ALT}$$

$$\tau_\lambda = \frac{T_{t4}}{T_0}$$

$\dot m'$ air $= \dot m (\text{ALT}, M_0, \tau_\lambda)$: τ_λ IS A MAJOR PROBLEM

τ_λ IS NOT A DIRECT MEASURE OF $\dot m_{fuel}$ — Recall $\dot m_f$ depends on T_{t3} as well as T_{t4}

II. IN ORDER TO COMPUTE $\dot m$, in terms of $\dot m_f$, ALT, M_0 WE NEED TO CALCULATE τ_λ DEPENDENCE ON M_0, ALT, $\dot m_f$

WRITE THE BURNER TOTAL ENTHALPY BALANCE

$$\eta_b \dot m_f h = \dot m C_p (T_{t4} - T_{t3})$$

$$\text{OR} \quad \dot m_f = \frac{\dot m C_p}{\eta_b h} (T_{t4} - T_{t3})$$

$$** \quad \boxed{\dot m_f = \frac{\dot m (\tau_\lambda - \tau_r \tau_c)}{\frac{h \eta_b}{C_p T_0} - \underline{\underline{\tau_\lambda}}}}$$

$\dot m \to$ ALT, M_0, τ_λ

$\tau_r \to M_0$

$\tau_c \to \tau_\lambda, M_0$

$T_0 \to$ ALT

This term keep if $f \ll 1$ Not assumed!

So: FOR A GIVEN $\dot m_f$, M_0, ALTITUDE, YOU CAN FIND τ_λ FROM **

III. PLUG THAT τ_λ INTO * TO FIND $\dot m$ (AIR)

so procedure to find mdotair at any point in 3 space of mdotfuel, ALT, M0 operating condition:

Find   from **
then find associated mdotair from *
This mdotair is the mdot demanded by the engine at that mdotfuel, ALT, M0.

Note:requires knowledge/estimation of turbine and compressor efficiencies rpied, rpieb, A8, A4

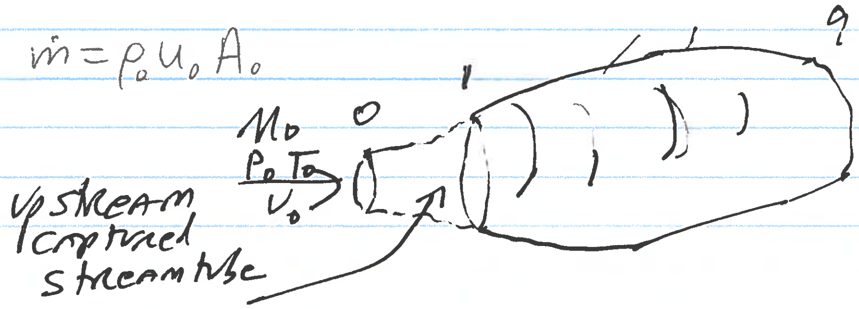
SO, THIS ANALYSIS REQUIRES KNOWLEDGE OF

$\eta_t, \eta_c, \eta_m, \pi_d, \pi_b, A_8, A_4$

NOTE: $f = \dfrac{\dot m_f}{\dot m}$ CAN BE FOUND

AND ALSO A_0 (UPSTREAM CAPTURE AREA) Then found from $\dot m_{air}$, & ρ_0, V

$$\dot m = \rho_0 u_0 A_0$$



[$\dot{m}_{ref}$ is $\dot{m}$(AIR) if NO curvature of upstream streamtube → 'full mass capture']

IN GENERAL THE $A_0 \neq A_{inlet}$, SO WHAT'S DONE IS DEFINE A REFERENCE / NOMINAL MASS CAPTURE AS $\rho_0 u_0 A_{inlet}$ ($\hookrightarrow A_1$)

SPILLAGE = ~~[illegible]~~ $= \rho_0 u_0 \underbrace{A_{inlet}}_{\hookrightarrow A_1} - \rho_0 u_0 A_0 = \dot{m}_{reference} - \dot{m}_{Actual}$

+ spillage for upstream decel ($A_0 < A_1$)

− spillage for upstream accel ($A_0 > A_1$)

NOW USE THE PERFORMANCE EQUATIONS WITH OFF-DESIGN PARAMETERS. (off-design mdotair, piec, etc.)

$$f = \frac{\dot{m}_f}{\dot{m}} = \frac{\tau_\lambda - \tau_r \tau_c}{\frac{h \eta_b}{c_p T_0} - \tau_\lambda}$$

UNINSTALLED $F = (\dot{m} + \dot{m}_f) u_9 - \dot{m} u_0 + (P_9 - P_0) A_9$

$$\frac{F}{\dot{m}} = (1+f) u_9 - u_0 + \left(1 - \frac{P_0}{P_9}\right)(1+f) R \frac{T_9}{u_9}$$

AE5535

$$\boxed{\frac{F}{\dot m} = (1+f)u_9 - u_0 + \left(1-\frac{P_0}{P_9}\right)(1+f)\frac{RT_9}{u_9}}$$

SO, WHAT IS u_9, T_9?

$$\frac{P_{t9}}{P_9} = \left(1+\frac{\gamma-1}{2}M_9^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow M_9 = \left\{\left[\left(\frac{P_{t9}}{P_9}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\frac{2}{\gamma-1}\right\}^{1/2}$$

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\frac{P_{t0}}{P_0}\frac{P_{t2}}{P_{t0}}\frac{P_{t3}}{P_{t2}}\frac{P_{t4}}{P_{t3}}\frac{P_{t5}}{P_{t4}}\frac{P_{t9}}{P_{t5}}$$

$$\therefore \frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r\pi_d\pi_c\pi_b\pi_t\pi_n$$

$$u_9 = M_9\sqrt{\gamma R T_9} \quad ; \quad T_9 = T_{t9}\left(1+\frac{\gamma-1}{2}M_9^2\right)^{-1}$$

AND $T_{t9} = \tau_\lambda \tau_t T_0$

IF $\frac{A_8}{A_4}$ IS FIXED, WE HAVE A FIXED TURBOJET, FIXED AREA TURBOJE
(FAT-JET) MEANS:

$$\frac{\tau_t^{1/2}}{\pi_t} = \frac{A_8}{A_4} = \text{CONST.} \Rightarrow \tau_t = \text{CONST.} \quad [\eta_t = \text{CONS}$$

π_t = CONST

(across operating range of engine, all mdotfuel, ALT, M0)

FAT-JET ASSUMPTIONS

a) $M_4 = M_8 = 1.0$

b) NO AFTERBURNER

c) η_t = CONST.

d) $\frac{A_8}{A_4}$ = CONST.

PROCEDURE TO ANALYZE FAT-JET

NOTE 'R' SUBSCRIPT DENOTES ON-DESIGN POINT

1. PERFORM 'ON-DESIGN' ANALYSIS. (SELECT $\pi_{c_R} \longrightarrow$ SOLVE FOR τ_{t_R}) from TCPB, do full single-point on-design analysis, chaining Ptot, Ttot thru engine, etc., finding thrust, etc.

2. MAKE FAT-JET ASSUMPTIONS, THEN $\tau_t = \tau_{t_R}$ FOR ANY POINT in operational 3-space

3. VARY ($\dot{m}_f$, ALT, M_0) FROM ON-DESIGN AND FIND OFF-DESIGN τ_λ, $\dot{m}$, π_c, τ_c etc. AS NECESSARY WITH FIXED τ_t

4. COMPUTE OFF-DESIGN PERFORMANCE (INCLUDING $\dot{m}$) USING THIS τ_λ, π_c, π_t, etc. AND AVAILABLE OFF-DESIGN LOSS COEFFICIENTS LIKE π_d, π_b, η_c, π_n, etc.

---

* SOLUTION TO IN-CLASS EXAMPLE (HAND-OUT) [ARIZONA TO GREENLAND]

R subscript - on design point

$P_{0_R} = P_0$; $T_{t4_R} = T_{t4}$; $M_{0_R} = M_0$ BUT $T_{0_R} \neq T_0$

IDEAL, γ, C_p CONST.

a) FIRST, ON-DESIGN

$$\tau_{\lambda_R} = \frac{1500}{300} = 5 \quad , \quad \pi_{c_R} = 20$$

$$\tau_{c_R} = (\pi_{c_R})^{\frac{\gamma-1}{\gamma}} = 2.3535$$

$$u_{0_R} = M_{0_R}\sqrt{\gamma R T_{0_R}} = 694.4 \text{ m/s}$$

$$\tau_{r_R} = 1 + \frac{\gamma-1}{2} M_{0_R}^2 = 1.8$$

$$\pi_{r_R} = (\tau_{r_R})^{\frac{\gamma}{\gamma-1}} = 7.824$$

on-design TCPB

$$\tau_{t_R} = 1 - \frac{\tau_{r_R}}{\tau_{\lambda_R}}(\tau_{c_R} - 1) = 0.513$$

$$\pi_{t_R} = 0.0965$$

AE 5535 *In-Class Example* ***Fixed Area Turbojet (FATjet) Engine Off-Design***

A low altitude ideal fixed area turbojet or FATjet (π_C = 20) is flown 'on-design' at M_0 = 2.0, T_0 = 300K in **Arizona** where is produces 222,500 N of thrust (probably actually a number of engines on a given vehicle). The maximum temperature (T_{t4}) in the engine is 1500K.

The vehicle on which the engine is installed is moved to **Greenland** where T_0 = 230K. Calculate the thrust obtained from this same engine when it is flown in Greenland at the same flight Mach number of 2.0 and with T_{t4} unchanged at 1500K. Assume that P_0 (ambient pressure) is the same between the two locations and that the nozzle is ideally expanded in both nozzles. (P_0 = 101,325 N/m^2). Also calculate the air mass flow rates, the fuel flow rates, and the capture areas (A_0) at both locations.

FROM PERFORMANCE EQUATIONS i.e. by chaining Ptot, Ttot thru engine, etc. obtain . . .

$U_{9R} = 913.14 \ m/s$

$M_{9R} = 2.421$

$T_{9R} = 354K \ , \ \rho_{9R} = 0.9843 \ kg/m^3$

on-design thrust

$\frac{T_R}{\dot{m}_R} = U_{9R} - U_{0R} = 218.74 \ \frac{N\text{-}s}{kg}$

$T_R = 222500 N$ GIVEN

$\therefore \ \dot{m}_R = 1017.2 \ kg/s$

$f_R = \frac{\tau_{\lambda R} - \tau_{rR}\tau_{cR}}{\frac{h}{C_p T_{0R}} - \tau_{\lambda R}} = 0.0053$

$\dot{m}_f = 5.385 \ kg/s$

$A_{0R} = \frac{\dot{m}_R}{U_{0R}\rho_{0R}} = 1.2613 \ m^2$ [capture area]

$A_{9R} = 1.1317 \ m^2$

$h = 4.5 \times 10^7 \ \frac{J}{kg_{fuel}}$

b) OFF-DESIGN – GREENLAND

$\tau_r = \tau_{rR}$ SINCE $M_0 = M_{0R}$ $\therefore \ \pi_r = \pi_{rR}$

note that since Taulambda can be directly found from givens, you don't need (for this case!) to find it from a mdotfuel (could be necessary however depending on what problem is . ...)

$\tau_\lambda = \frac{1500}{230} = 6.522$, $[\tau_t = \tau_{tR}$ & $\therefore \ \pi_t = \pi_{tR}]$ FAT-jet!

(efficiencies - ideal so pied = pied(R) = 1 (etc)

$\pi_d = \pi_{dR}$

$\therefore$ FIND τ_c AT THIS τ_λ, τ_t

off-design off-design

$\tau_c = 1 + \frac{\tau_\lambda}{\tau_r}(1 - \tau_t) = 2.765$

THEN $\pi_c = \tau_c^{\frac{\gamma}{\gamma-1}} = 35.2$ (spooling a lot harder than in Arizona!)

FROM PERFORMANCE ANALYSIS,

$T_{t0R} = T_{t2R} = 540K$

$T_{t0} = T_{t2} = 414K$

$T_{t3R} = 1271K$

$T_{t3} = 1145K$

RECALL, $P_9 = P_0 = P_{0R}$

$u_9 = 969.4 \text{ m/s}$

$M_9 = 2.786$

$T_9 = 301.3K$

$\rho_9 = 1.1564 \text{ kg/m}^3$

$u_0 = 608 \text{ m/s}$

off-design thrust

$$\frac{T}{\dot{m}} = u_9 - u_0 = 361.4 \frac{N\cdot s}{kg}$$

NOTE, $\dot{m} = \frac{\Gamma}{\sqrt{R}} A_4 \left[P_0 \pi_r \pi_d \pi_c \pi_b \right] \frac{1}{\sqrt{T_{t4}}}$

FOR THIS OFF-DESIGN, $\pi_r = \pi_{rR}, \pi_d = \pi_{dR} \; --- = 1.0$

$T_{t4} = T_{t4R}, \; A_4 = A_{4R}$

$$\therefore \frac{\dot{m}}{\dot{m}_R} = \frac{\pi_c}{\pi_{cR}} = 1.759$$

$\therefore \dot{m} = 1778.7 \text{ kg/s}$ (Greenland)

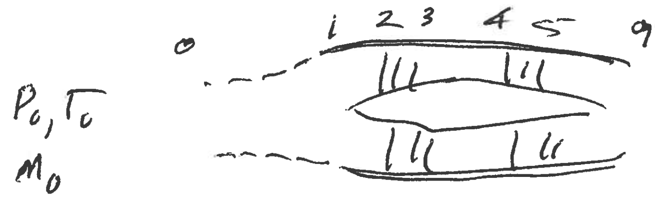
$\therefore T = 646{,}453 \text{ N}$ (Greenland thrust)

$\dot{m}_f = 14.23 \text{ kg/s}$ greenland mdotfuel

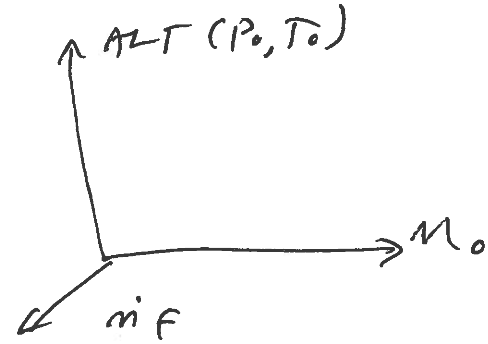
$A_0 = 1.93 \text{ m}^2$ in greenland

$A_9 = 1.585 \text{ m}^2 \Rightarrow \frac{A_9}{A_{9R}} = 1.4$ for $P_9 = P_0 = P_{9R}$

'Re-cap' Ideal FanJet example (Arizona to Greenland)



$A_4 = A_{4R}$
$A_8 = A_{8R}$



	Arizona	Greenland
T_0 (K)	300	230
P_0 (N/m²)	100000	100000
M_0	2	2
Thrust (N)	225000	646 453
π_c	20	35.17
τ_t	0.5127	0.5127
P_9 (= P_0) (N/m²)	100000	1000000
$\dot{m}_{air}$ (kg/s)	1017.2	1778.8
$\dot{m}_F$ (kg/s)	5.385	14.23
τ_λ	5.0	6.522
A_0 (m²)	1.2613	1.93
A_1 (m²)	1.2613 *	1.2613 *
A_9 ** (m²)	1.1317	1.5853

* A_1 chosen for full mass capture in Arizona (& fixed!)

** A_9 will change due to $P_9 = P_0$ requirement (between on-off design points

* COMMENTS

BIGGEST EFFECT:

(1) NOTE THE LARGE CHANGE IN π_c (FROM 20 TO 35.2)

→ ASSUMES $\pi_c = 35.2$ IS POSSIBLE / ACHIEVABLE WITH THE GIVEN COMPRESSOR; WE ARE NOT FACTORING IN THE ACTUAL COMPRESSOR LIMITATIONS OR CAPABILITIES. ALSO η_c

Engine control system (ECS) would limit! (RPM too)

(2) FLEXIBLE A_9 (POSSIBLE?) to make P9 = P0 (even though we call this a fixed area tjet - recall that means A4 and A8 are fixed!, A9 here is 'flexed'

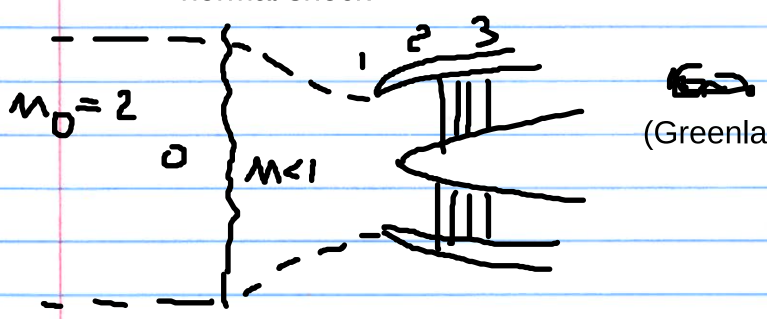
(3) WE ASSUMED $\pi_d = 1.0$ (BUT THERE SHOULD BE SHOCKS!)

NO WAY I HAVE A CAPTURE AREA BIGGER THAN THE ENGINE GOBBLING UP THAT $\dot{m}$ AT $M_0 = 2.0$ WITHOUT HAVING A SHOCK THAT WILL REDUCE YOUR TOTAL PRESSURE; $\pi_d < 1$

especially in Greenland.

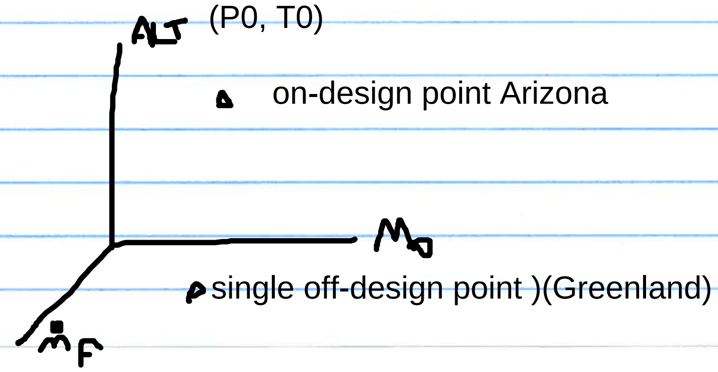
Note A0 (greenland) >> A1 (engine face area) - this implies subsonic flow MUST be entering the inlet face at station 1 in Greenland,, hence shock waves in captured streamtube upstream of engine in Greenland will happen! So there will be big total pressure ratio drop aross those shocks (but analysis assuming none).

normal shock (large π_d drop in reality!)



(Greenland situation)

4. we have found a SINGLE off-design point in M0, ALT, mdotfuel operational 3-space
In general, we want the ENTIRE operating range information!



AE 5535

NOTE, THAT WE COULD COMPUTE PERFORMANCE WHEN THE OFF-DESIGN EXIT AREA $A_9 = A_{9_R}$ (GETS RID OF THE ASSUMPTION $P_0 = P_9$).
WE FOUND THAT A_9 (GREENLAND) REQUIRED TO MAKE $P_9 = P_0$ WAS
$A_9 = 1.585\,m^2$ WHEREAS $A_{9_R} = 1.132\,m^2$
IT'S OBVIOUS THAT WE WANT TO RECOMPUTE THE OFF-DESIGN THRUST WITH $A_9 = 1.132\,m^2$

OFF-DESIGN $P_{t5} = P_{t9} = 2656369\ Pa$, $T_{t5} = T_{t9} = 769\ K$

NOW, $\dot{m} = 1779\ kg/s$

$$\dot{m} = \frac{P_{t9} A_9}{\sqrt{T_{t9}}} \sqrt{\frac{\gamma}{R}}\, M_9 \left(1 + \frac{\gamma-1}{2} M_9^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}}$$ SOLVE FOR M_9

$M_9 = 2.43$ (AS OPPOSED TO M_9 WITH $P_9 = P_0$, $M_9 = 2.79$)

SO $T_9 = 353\,K$, $u_9 = 914.2\ m/s$, $\rho_9 = 1.72\ kg/m^3$

$$F = \dot{m}(u_9 - u_0) + (P_9 - P_0) A_9$$

$F = 628{,}533\ N$ (AS OPPOSED TO A MORE THRUST)

SO, EFFECT IS $-18\ kN$ OF THRUST

VARIABLE AREA TURBOJET

RECALL THAT (FOR NO AB, $M_4 = M_8 = 1.0$)

$$\frac{\tau_t^{1/2}}{\pi_t} = \frac{A_8}{A_4}$$

τ_t & π_t related thru η_t

SUPPOSE WE ALLOW $\frac{A_8}{A_4}$ TO VARY to obtain a desired engine characteristic i.e. KEEP THE π_c, τ_c CONSTANT, FOR EXAMPLE, ACROSS THE '3-SPACE'. WE CALL IT A VAT-JET.

* A SIMPLE VAT-JET EXAMPLE (π_c = CONST.)

CONSIDER, A TURBOJET AT FIXED ALTITUDE AND M_0, CONTROL $\frac{A_8}{A_4}$ (SO τ_t, π_t, WILL now VARY) IN ORDER TO MAKE π_c, τ_c = CONST. AS WE VARY τ_λ ($\dot{m}_f$) (OR just T_{t4}) (T_0 const)

$$\tau_c = 1 + \eta_m \left(\frac{\tau_\lambda}{\tau_r}\right)(1 - \tau_t) \qquad \text{OFF-DESIGN POWER BALANCE}$$

$$\tau_{cR} = 1 + \eta_m \left(\frac{\tau_{\lambda R}}{\tau_{rR}}\right)(1 - \tau_{tR}) \qquad \text{ON-DESIGN " "}$$

want to enforce

SET THEM EQUAL AND SOLVE FOR τ_t

$\tau_c = \tau_{cR} \Rightarrow \tau_t = ??$ ($\underline{\tau_r = \tau_{rR}}$, $\underline{\eta_m = \text{CONST.}}$ FOR THIS $\underline{\text{EXAMPLE}}$)

$$\tau_t = \cancel{\tau_{tR}\left(\frac{\tau_{\lambda R}}{\tau_\lambda}\right)} \Rightarrow \text{:GET } \frac{A_8}{A_4} \text{ REQUIRED} = \frac{\tau_t^{1/2}}{\pi_t}$$

$$= 1 - (1 - \tau_{tR})\left(\frac{\tau_{\lambda R}}{\tau_\lambda}\right)$$

EXAMPLE : VAT-JET

A DESIGNER WANTS TO HAVE A VAT-JET THAT MAINTAINS FULL MASS CAPTURE (i.e. NO SPILLAGE/NO CURVATURE; ZERO ADD. DRAG i.e. $A_0 = A_{inlet}$ ALWAYS ACROSS '3-SPACE'!

SELECT A_8 TO BE VARIED, A_4 FIXED

'INLET SWALLOWS ITS PROJECTED AREA'

FIND OUT HOW $\frac{A_8}{A_{8R}}$ WILL NEED TO VARY WITH ANY VARIATIONS IN τ_λ, M_0, T_0 ..

A) INLET HAS NO SPILLAGE --- $A_0 = A_1$ ACROSS '3-SPACE'

$$\frac{\dot m_R}{\dot m} = \frac{\rho_{0R} u_{0R} \cancel{A_{0R}}}{\rho_0 u_0 \cancel{A_0}} = \frac{\rho_{0R} M_{0R}\sqrt{\gamma R T_{0R}}}{\rho_0 M_0 \sqrt{\gamma R T_0}} = \frac{\rho_{0R} M_{0R}}{\rho_0 M_0}\sqrt{\frac{T_{0R}}{T_0}}$$

B) KNOW, $A_4 = A_{4R}$

$$\Rightarrow \frac{A_8}{A_{8R}} = \frac{\tau_t^{1/2}}{\pi_t}\frac{\pi_{tR}}{\tau_{tR}^{1/2}}\ ; \text{ TO GET THE REQUIRED } \frac{A_8}{A_{8R}}$$

WE NEED τ_t OR τ_c OR π_c

C) NOTE

$$\frac{\dot m_{4R}}{\dot m_4} = \frac{P_{t4R}}{\sqrt{T_{t4R}}}\cdot\frac{\sqrt{T_{t4}}}{P_{t4}} = \sqrt{\frac{\tau_\lambda T_0}{\tau_{\lambda R} T_{0R}}}\cdot\frac{P_{0R}\,\pi_{rR}\,\pi_{dR}\,\pi_{cR}\,\pi_{bR}}{P_0\,\pi_r\,\pi_d\,\pi_c\,\pi_b}$$

WE KNOW $\left.\frac{\dot m_R}{\dot m}\right)_{inlet} = \frac{\dot m_{4R}}{\dot m_4}$; $f \ll 1$

$$\frac{\rho_{0R}}{\rho_0}\frac{M_{0R}}{M_0}\sqrt{\frac{T_{0R}}{T_0}} = \sqrt{\frac{\tau_\lambda T_0}{\tau_{\lambda R} T_{0R}}}\ \frac{P_{0R}\,\pi_{rR}\,\pi_{dR}\,\pi_{cR}\,\pi_{bR}}{P_0\,\pi_r\,\pi_d\,\pi_c\,\pi_b}$$

from a) (from b)

SOLVE FOR π_c

$$\boxed{\pi_c = \sqrt{\frac{\tau_\lambda}{\tau_{\lambda R}}}\ \frac{M_0}{M_{0R}}\left[\frac{\left(1+\frac{\gamma-1}{2}M_{0R}^2\right)}{\left(1+\frac{\gamma-1}{2}M_0^2\right)}\right]^{\frac{\gamma}{\gamma-1}}\frac{\pi_{dR}\,\pi_{bR}}{\pi_d\,\pi_b}\,\pi_{cR}}$$

this is comp. press. ratio that will ensure straight streamlines on captured streamtube (full mass capture) whereever the engine is flying in off-design 3-space . . .

THEN, SOLVE FOR τ_t FROM $\tau_c = 1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c}$

$$\tau_t = 1 - \frac{(\tau_c - 1)}{\eta_m}\left(\frac{\tau_r}{\tau_\lambda}\right), \quad \pi_t = 1 - \frac{1}{\eta_t}(1-\tau_t)^{\frac{\gamma}{\gamma-1}}$$

$$\therefore \frac{A_8}{A_{8R}} = \frac{\tau_t^{1/2}}{\pi_t}\cdot\frac{\pi_{tR}}{\tau_{tR}^{1/2}}$$

and can analyze performance of VATjet at off-design condition by 'regular' chaining Ptot, Ttot through engine, etc.

AE 5535

(maintain full mass capture in off-design flight)

EXAMPLE

$$\pi_c = \sqrt{\frac{\tau_\lambda}{\tau_{\lambda R}} \frac{M_0}{M_{0R}} \frac{\left(1+\frac{\gamma-1}{2}M_{0R}^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1+\frac{\gamma-1}{2}M_0^2\right)^{\frac{\gamma}{\gamma-1}}}} \left[\frac{\pi_{dR}}{\pi_d}\frac{\pi_{bR}}{\pi_b}\right] \pi_{cR}$$

$$\frac{A_8}{A_{8R}} = \frac{\tau_t^{1/2}}{\pi_t} \frac{\pi_{tR}}{\tau_{tR}^{1/2}}$$

FEW THINGS ON VAT-JETS

1) $P_{9R} = P_{0R}$ $(A_9 = A_{9R})$

2) ALL IDEAL ANALYSIS

3) $h = 4.42 \times 10^7$ J/kg

$$F = \dot m (u_9 - u_0) + (P_9 - P_0) A_9$$

INPUTS
$\pi_{cR} = 20$, $M_{0R} = 0.8$, $P_{0R} = 100$ kPa, $T_{0R} = 280$K
$T_{t4R} = 1800$K, $A_i = 1.8\ m^2$

ON-DESIGN ANALYSIS

$u_{0R} = 268.3$ m/s, $P_{t0R} = 152\,434$ Pa

$T_{t0R} = 315.84$K, $T_{t4R} = 1800$K

$\therefore \tau_{cR} = 2.3535$, $\tau_{rR} = 1.128$, $\tau_{\lambda R} = 6.43$

$P_{t3R} = P_{t0R}\pi_{cR} = 3.05 \times 10^6$ Pa

$T_{t3R} = T_{t0R}\tau_{cR} = 743.3$K

$P_{t4R} = P_{t3R} = 3.05 \times 10^6$ Pa

$\tau_{tR} = 1 - (\tau_{cR} - 1)\left(\frac{\tau_{rR}}{\tau_{\lambda R}}\right) = 0.7625$

$\pi_{tR} = 0.38718$

$P_{t9R} = P_{t4R}\pi_{tR} = 1180731$ Pa

$T_{t9R} = T_{t4R}\tau_{tR} = 1372.5$K

$$\frac{P_{t9R}}{P_{9R}} = \left(1 + \frac{\gamma-1}{2} M_{9R}^2\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow M_{9R} = 2.263$$

$\therefore T_{9R} = 678K$

$u_{9R} = M_{9R}\sqrt{\gamma R T_{9R}} = 1181\ m/s$

$\rho_{9R} = 0.514\ kg/m^3 \implies A_{9R} = 1m^2$

$F_{thrust_R} = 553.917\ kN \quad , \dot{m}_{fR} = 14.58\ kg/s$

OFF-DESIGN ANALYSIS

GIVEN $\rightarrow [\, M_0 = 0.3 \,,\ T_{t4} = 1600K \,,\ T_0 = 230K \,,\ P_0 = 20\ kPa \,]$

$\rho_0 = 0.303\ kg/m^3 \,,\ T_{t_2} = 234.14\ K \,,\ P_{t_2} = 212886\ Pa$

$\dot{m} = \rho_0 u_0 A_0 = 49.74\ kg/s$

π_c (from the big equation) $= 11.172$

$\therefore \tau_c = 1.993$

$P_{t_3} = P_{t_2}\pi_c = 237836\ Pa$

$T_{t_3} = T_{t_2}\tau_c = 466.6\ K$

$\tau_r = 1.018 \,,\ P_{t4} = 237836\ Pa$

$T_{t4} = 1600\ K \longrightarrow \tau_t = 1 - (\tau_c - 1)\left(\frac{\tau_r}{\tau_\lambda}\right) = 0.8547$

$\pi_t = 0.5772$

$P_{t5} = P_{t4}\pi_t = 137278\ Pa$

$T_{t5} = T_{t9} = 1367.5K$

$A_9 = A_{9R} = 1m^2$

$$\dot{m} = \frac{A_9 P_{t9}}{\sqrt{T_{t9}}}\sqrt{\frac{\gamma}{R}}\left(1 + \frac{\gamma-1}{2}M_9^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} \cdot M_9$$

$\implies M_9 = 2.64$

$\therefore T_9 = 571.3K \,,\ P_9 = 6467.2\ Pa \,,\ \rho_9 = 0.03925\ kg/m^3$

$u_9 = 1265\ m/s$

$F_{thrust} = 44.807\ kN \quad , \dot{m}_f = 1.28\ kg/s$

$\frac{A_8}{A_{8R}} = 0.71$

NOTE:

- CONSTANT $\pi_c \longrightarrow$ MAINTAIN η_{th}
- FULL MASS CAPTURE $\longrightarrow$ ZERO ADDITAVE DRAG.
- VARYING THE A_8 FROM A_{8R} WHILE KEEPING $A_9 = A_{9R}$

* SUPERIOR PERFORMANCE OF VAT OVER FAT JET.

- DRAWBACK OF VATJET IS ADDED COMPLEXITY (WEIGHT).