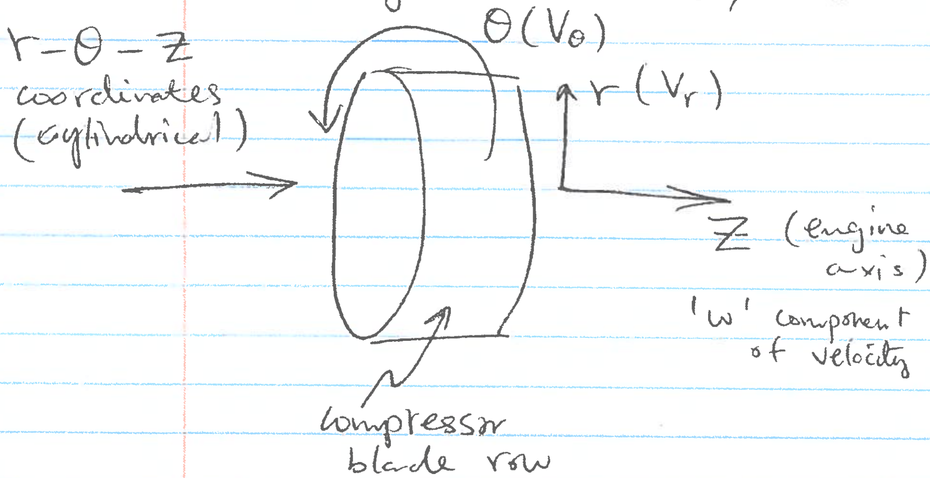
TURBOMACHINERY AERODYNAMICS

* Inlet Guide Vane (IGV): prepares angle of the flow entering the first stage of the compressor gets it ready for first stage rotor blade.

rotor, stator

Fluid moving thru a compressor experiences a torque from the moving rotor blades (in the θ-direction), i.e. a V_θ, component of velocity, is imparted to the flow.

$$V_\theta = r\dot{\theta} = r\omega$$ where ω is angular velocity in radians/sec.



rotor Rotating at $\omega \left(= \frac{d\theta}{dt}\right)$

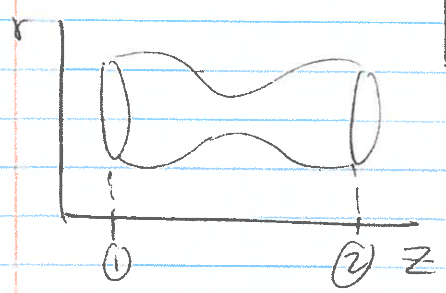
V_θ = "SWIRL" velocity component in θ direction

Torque $= rF_\theta = T$

$\left(F_\theta = \text{local force in } \theta \text{ direction at a } r \text{ location from hub}\right)$

It can be shown using the moment of momentum theorem based on Newton's law that the torque 'T' on the fluid in a stream tube provided by the rotor blades between station 1 & 2 in the z-direction

$$\boxed{T = \dot{m}\left[(rV_\theta)_2 - (rV_\theta)_1\right]}$$



$\dot{m} = \dot{m}$ of AIR in a defined streamtube passing thru a compressor blade row

'rV_θ' is angular momentum at a given station

Furthermore, power considerations demand that the mechanical power between (1) & (2)

$$F_\theta \cdot \frac{c(rd\theta)}{dt} \Leftarrow \boxed{\dot{W} = \text{Power}_{1\to 2} = \omega T = \dot{m}\omega\left[(rV_\theta)_2 - (rV_{\theta_1})\right]}$$

but we also know the energy equation:

$$\dot{m}\,(h_{t_2} - h_{t_1}) = \dot{W}_{\substack{\text{adiabatic}\\ \text{compressor}}} \quad : h_t = C_p T_t$$

$$C_p\,(T_{t_2} - T_{t_1}) = h_{t_2} - h_{t_1} = \frac{\omega T}{\dot{m}}$$

or

$$\boxed{C_p\,(T_{t_2} - T_{t_1}) = h_{t_2} - h_{t_1} = \omega\left[(rV_\theta)_2 - (rV_\theta)_1\right]} \quad *$$

- if blade is not moving (stator), $\omega = 0 \Rightarrow$ no work
- only rotor provides work.

'cascade view'

'unwrap' the compressor at a given r location:

Station 0

$\vec{V}_1 = W_0$

T_{t_0}

P_{t_0}

W_0

z, θ

IGV ($\omega = 0$)

Station 1

$\vec{V}_1$, W_1

$T_{t_1} \cong T_{t_0}$

$P_{t_1} \cong P_{t_0}$

(isentropic)

rotor

ωr

Station 2

$\vec{V}_2$

$T_{t_2} > T_{t_1}$

$P_{t_2} > P_{t_1}$

best you can do is isentropic

$$\frac{P_{t_2}}{P_{t_1}} = \left(\frac{T_{t_2}}{T_{t_1}}\right)^{\frac{\gamma}{\gamma-1}}$$

stator $\omega = 0$

Station 3

$\vec{V}_3$

$T_{t_3} \cong T_{t_2}$

$P_{t_3} \cong P_{t_2}$

next stage rotor row

If the process is isentropic thru rotor

$$** \boxed{\frac{P_{t_2}}{P_{t_1}} = \left(\frac{T_{t_2}}{T_{t_1}}\right)^{\frac{\gamma}{\gamma-1}}}$$

if not isentropic, use loss coefficient i.e. $\frac{P_{t_2}}{P_{t_1}} = \left(\frac{T_{t_2}}{T_{t_1}}\right)^{\frac{\gamma e_c}{\gamma-1}}$ etc...

combine / use * & ** with assumed small Mach numb (≤ 0.5) so ρ does not change much, $\rho \sim \rho_{TOTAL}$

$\longrightarrow$ you can show algebraically: & small turnings

Non-dimensional ΔP_t thru rotor row $\leftarrow$

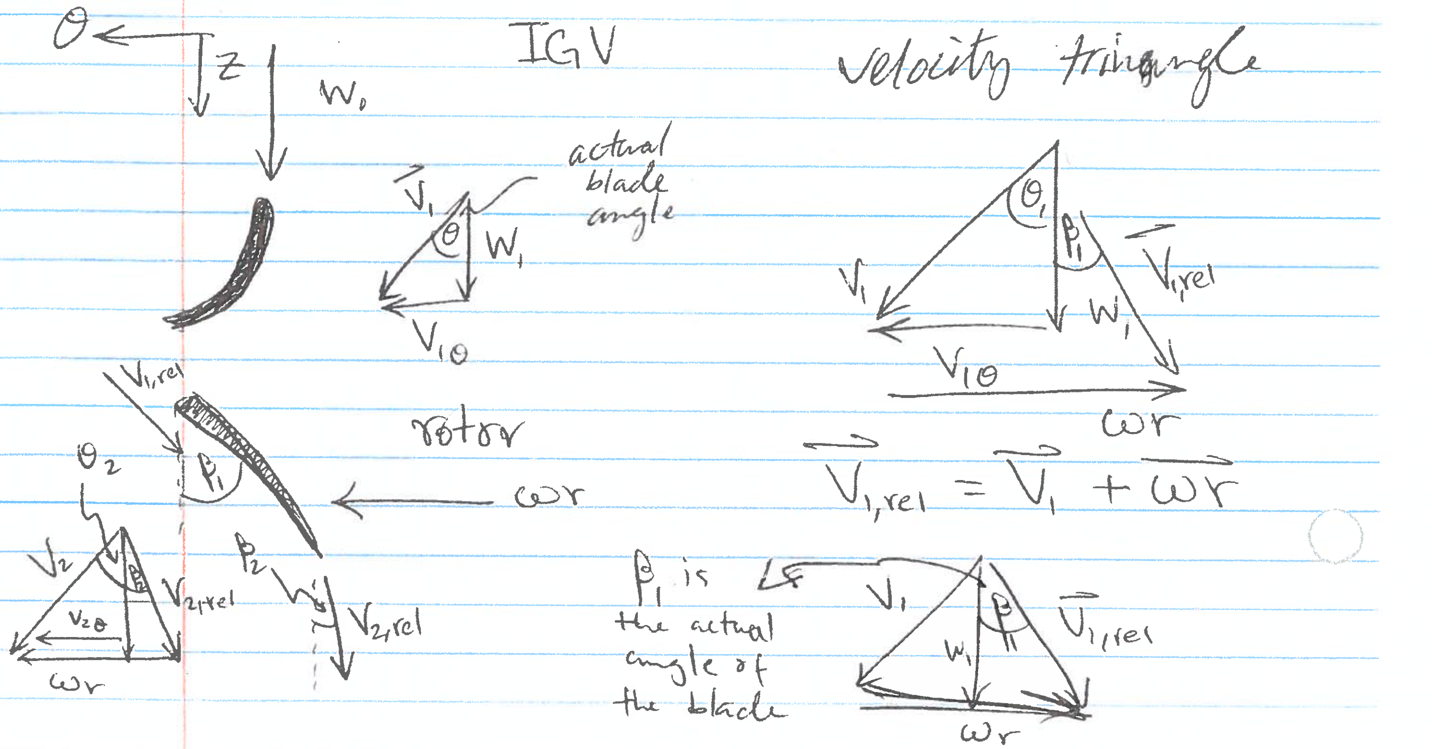
$$\boxed{\frac{P_{t_2} - P_{t_1}}{\rho_0 W_0^2} \cong \frac{\omega}{W_0^2}\left[(rV_\theta)_2 - (rV_\theta)_1\right]}$$

expression for P_t rise in terms of change in angular moment

example: if $\pi_c = 20$ and 13 stages (F-15 F-100 engine) (for $P_{t_0} = 101325$ N/m²)

ΔP_t single stage $\cong 53{,}000$ N/m²

$\longrightarrow$ It would be highly useful to define the local flow turning in terms of actual (physical) local blade angles of IGV, rotor, stator blades.



$$\vec{V}_{1,rel} = \vec{V}_1 + \vec{\omega r}$$

'Repeat'

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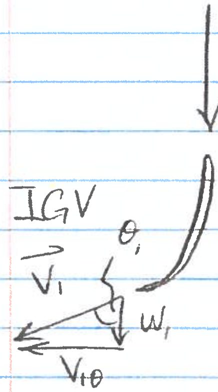
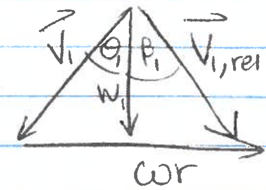
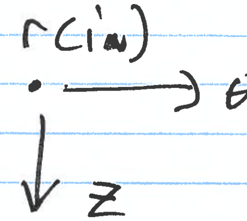
$$\frac{P_{t_2} - P_{t_1}}{\rho_0 W_0^2} = \frac{\omega}{W_0^2}\left[(rV_\theta)_2 - (rV_\theta)_1\right]$$

0

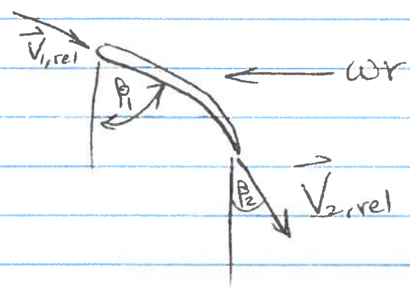






'velocity triangle'

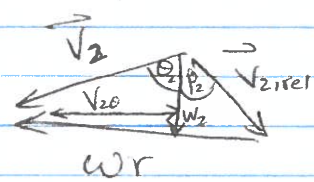
1



in rotor frame → Relative → 'rel' to rotor

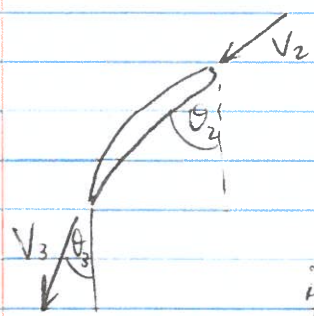
β's are actual physical blade angles (leading edge and trailing edge)

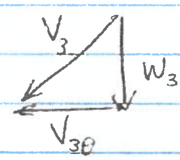
2



→ exit ... velocity triangle

stator





θ_2 & θ_3 are actual (physical) LE & TE angles of stator!

if the stage is repeating: $\vec{V}_3 = \vec{V}_1$

3

$$\theta_3 = \theta_1$$

assume $W_0 = W_1 = W_2 = W_3$ and r doesn't change much thru a stage, ρ does not change (M<.5)

Statement: velocity triangles relate engine frame / rotor relative velocities & angles!

Observe these

Relationships from velocity triangles:

$$\omega r - V_{1\theta} = W_0 \tan\beta_1$$

$$\omega r - V_{2\theta} = W_0 \tan\beta_2$$

$$\Rightarrow \quad V_{2\theta} - V_{1\theta} = W_0(\tan\beta_1 - \tan\beta_2)$$

since $\frac{V_{2\theta}}{W_0} = \tan\theta_2$; $\frac{V_{1\theta}}{W_0} = \tan\theta_1$

$$\left.\boxed{\begin{aligned}\tan\beta_1 &= \frac{\omega r}{W_0} - \tan\theta_1 \\ \tan\beta_2 &= \frac{\omega r}{W_0} - \tan\theta_2\end{aligned}}\right\} \text{ relates } \beta\text{'s and } \theta\text{'s!!}$$

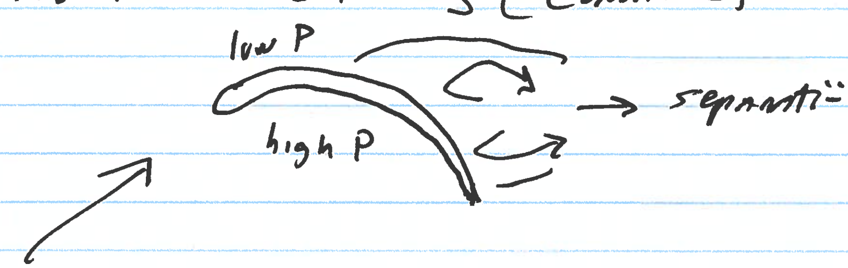
from ✱ then

$$\boxed{\frac{P_{t_2} - P_{t_1}}{\rho_0 W_0^2} = \frac{\omega r}{W_0}(\tan\beta_1 - \tan\beta_2)}$$

→ ΔP_t for a stage in terms of physical LE/TE blade angles of rotor!!

* To increase the ΔP_t across the stage:
  - increase ω, W_0 but limited by blade tip effects
  - Increase amount of turning thru the rotor; $(\tan\beta_1 - \tan\beta_2)$, however, limited by separation due to an adverse pressure gradient. The static pressure, P, goes up, separation may occur, due to too much blade curvature.

Too much blade turning (curvature):



* 2 Engineering coefficients are generally used in the business for assessing seperation risk. based on empirical data.

1) Degree of Reaction, $°R = \frac{\Delta P_{rotor}}{\Delta P_{stage}}$ "defined across a stage"

P → static pressure

Driver separation phenomena: P.G. $= \frac{dP}{dx}$ pressure gradient

$$°R = 1 - \frac{\Delta P_{static,\, stator}}{\Delta P_{stage}}$$ $\because \Delta P_{stage} = \Delta P_{rotor} + \Delta P_{stator}$

$°R \sim 0.5$ is good (equal blade loading, "loading") 'spreads' P.G. between rotor & stator

$°R$ can be shown to be:

$$°R_{stage} = 1 - \frac{(V_{\theta_2} + V_{\theta_1})}{2\omega r} = \frac{w(\tan\beta_2 + \tan\beta_1)}{2\omega r}$$ (show using velocity triangle info)

2) Diffusion Factor, D (aka "blade loading factor")

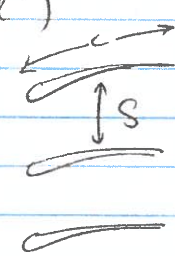
describes both the pressure gradient effect and blade curvature effect on the pressure gradient.

(hence we have D_{stator}, D_{rotor})

$$D = \underbrace{\left(1 - \frac{|V_e|}{|V_i|}\right)}_{\text{pressure gradient effect}} + \underbrace{\frac{|\Delta V_\theta|}{2\sigma |V_i|}}_{\text{blade curvature effect}}$$: i, e denote inlet and exit for a given blade row (stator or rotor)

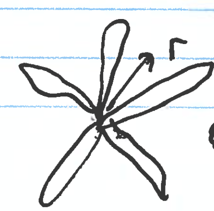
$D \leq 0.6$ is 'good' (small D is 'good')

σ = 'blade solidity' $= \frac{\text{chord}}{\text{Spacing}} = \frac{c}{s}$



at a given r location

- it changes with r



s grows with r ...

* It can be shown: from velocity triangles

$$D_{rotor} = 1 - \frac{\cos\beta_1}{\cos\beta_2} + \frac{1}{2\sigma}\left|(\tan\beta_1 - \tan\beta_2)\right|\cos\beta_1$$

$$D_{stator} = 1 - \frac{\cos\theta_2}{\cos\theta_3} + \frac{|\tan\theta_3 - \tan\theta_2|}{2\sigma\sec\theta_2}$$

This then gives a means to 'assess' whether a compressor blade design is acceptable at a given r

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* How does one begin to design a compressor?

So... (i.e., choose β's, θ's across the r range; hub to tip).

'Free Vortex' compressor stage 'base line'

we know $\frac{\Delta P_t}{\rho_0 W_0^2} \cong \frac{\omega r}{W_0}(\tan\beta_1 - \tan\beta_2)$

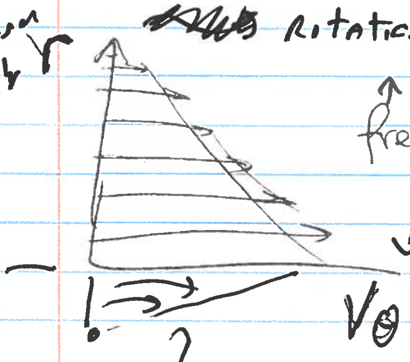
* we want (desirable to have) ΔP_t as constant as possible from hub to tip (across r) at a given axial location ('z'); don't like gradients,

obviously, P_t going up as progress thru compressor axially

↳ true for 'Free Vortex' compressor stage.

(rV_θ) = const. with r → maintains constant P_t with r

~~[illegible]~~ Rotational Core' (~~[illegible]~~)



free vortex ('irrotational')

(model in low-speed flow)

Rotational 'core'

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Free Vortex Compressor "A place to start"

– blades are designed to ensure $(rV)_\theta = \text{const.}$

ensures $P_t(r) = \text{const.}$ at a z-position

From continuity: $u_r = 0$ (radial velocity component) $= 0$

$\therefore$ no radial movement of the streamtubes.

Example:

$\rho = 1.225\ kg/m^3$, $RPM = 10{,}000$

so $\therefore \omega = 1047.2\ rad/s$

$W_0 = 100\ m/s$

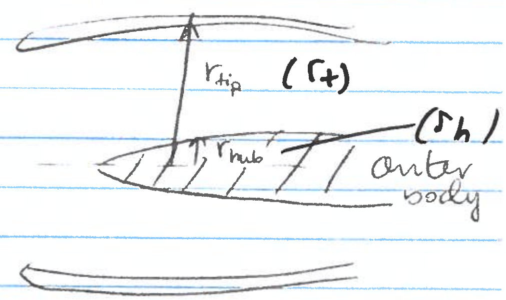
require $V_{\theta_1} = 10\ m/s$ at the hub $= V_{\theta_{1,h}}$

$1 \longrightarrow 2$ rotor

$2 \longrightarrow 3$ stator

let $r_{hub} = 0.4m$ (hub radius)

$r_{tip} = 0.55m$ (tip radius)

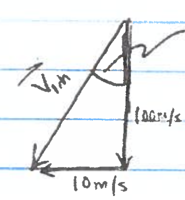


require $\Delta P_t = 50{,}000\ Pa$ (single stage)

hence $(\omega r)_h = 418.7\ m/s$, $(\omega r)_t = 576.0\ m/s$

A. at hub:

At exit of the upstream stator

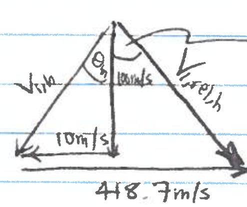


$\theta_{1,hub} = 5.71° = \arctan\left(\frac{10}{100}\right)$

$\therefore V_{1,h} = 100.5\ m/s$

B. at hub:

entering the rotor



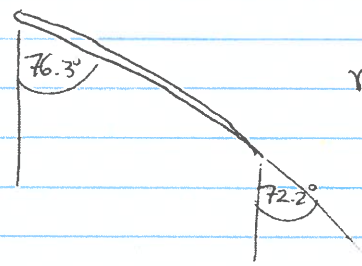
$\beta_{1,h} = 76.3° = \arctan\left(\frac{408.7}{100}\right)$

$\therefore V_{1,rel,h} = 420.8\ m/s$

we know $\dfrac{P_{t2}-P_{t1}}{\rho W_0^2} = \dfrac{\omega r}{W_0}(\tan\beta_1 - \tan\beta_2)$

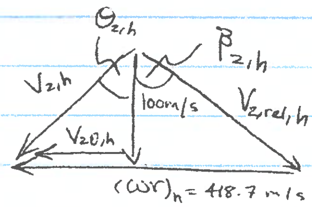
50 $\therefore \beta_2 = 72.2°$ (for $\Delta P_t = 50\,kPa$)

rotor sketch at hub



relatively flat

C. at hub:
exiting the rotor



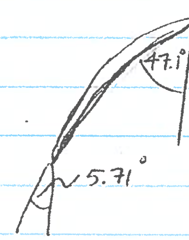
$\therefore V_{2,rel,h} = 327\ m/s$

$V_{\theta,2h} = 107.6\ m/s$

$\theta_{2,h} = 47.1°$

$V_{2,h} = 147\ m/s$

stator sketch at hub (for repeating stage)



$\therefore \theta_{3h} = \theta_{1h}$ repeating

$^{\circ}R_h = \dfrac{\Delta P_{rotor}}{\Delta P_{stage}} = 0.86$ (rotor a bit 'loaded' in comparison to the stator)

– the flow over the rotor is more likely to seperate.

from before

$$^{\circ}R_h = \left\{\frac{W_0(\tan\beta_2 + \tan\theta_1)}{2\omega r}\right\}_{hub} = 0.86$$

D. Repeat at tip where $(\omega r)_t = 5.76$ m/s

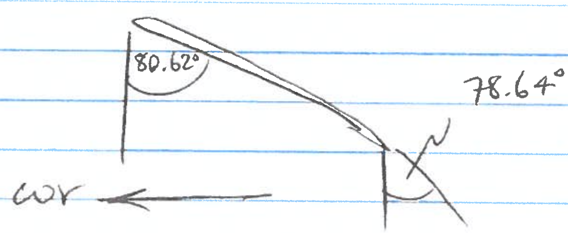
(since $rV_\theta = \text{const.}$, so $r_h V_{\theta_1,h} = r_t V_{\theta_1,t}$)

$\therefore\ V_{\theta_1,t} = 7.3$ m/s

at (tip) :
(worked thru analysis)

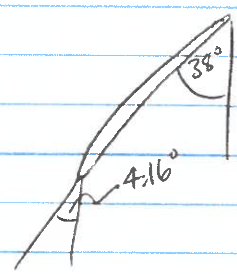
$\theta_{1,t} = 4.16°$
$\beta_{1,t} = 80.62°$
$\beta_{2,t} = 78.64°$
$\theta_{2,t} = 38.0°$

rotor sketch at tip



Virtually flat-!

stator sketch at tip



And

$^o\!R_{tip} = 0.925 \quad \left(= \frac{\Delta P_{rotor}}{\Delta P_{stage}}\right)$

Rotor is 'loaded' with more static pressure rise than stator

So rotor and stator geometry are 'rotating' clockwise from tip to hub. (with ωr direction as shown)

Example: pretty representitive numbers

$\sigma = \frac{c}{s}$

Given $\frac{\Delta P_t}{\rho_0 W_0^2} = 0.9$, $\frac{r_t}{r_h} = 2.646$

${}^oR_m = 0.5$, $\sigma_m = 1.0$ (typical value for gas-turb.)

'm' denotes mass-averaged radius

The number of blades determines σ really

remember: $D \leq 0.6$ is good

Consider two values of blade speed $\frac{\omega r_h}{W_0} = 0.5, 0.7$

$\frac{\omega r_h}{W_0}$	$D_{rotor\ hub}$	$D_{rotor\ tip}$	$D_{stator\ hub}$	$D_{stator\ tip}$	${}^oR_{tip}$	${}^oR_{hub}$
0.5	−0.179	0.56	problem 0.74	0.55	5/7	−1.0
0.7	−0.35	0.38	0.57	0.43	—	—

weren't calculated

$D_{stator\ hub}$ is too high for $\frac{\omega r_h}{W_0} = 0.5$, so put more 'loading' on the rotor by adjusting the blade angles near the hub, etc.

* You can lower D by increasing $\frac{\omega r_h}{W_0}$ to some extent (blade tip effects), decrease $\frac{r_t}{r_h}$ (reduces in through ~~reduces~~ reduce ΔP_t (more stage for desired π_c).

So, free vortex machine is just a 'place to start'.

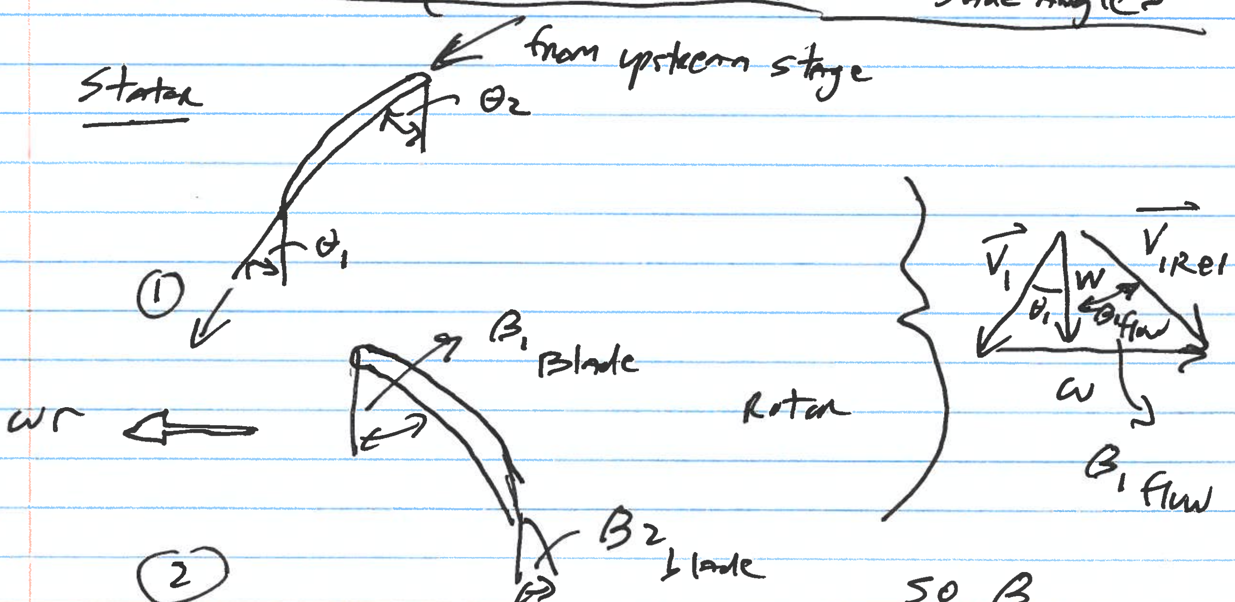
^{o}R, D change too dramatically with r.

The best thing to do is to modify the design to allow r variation in streamtube path especially at hub and radius! ($U_r \neq 0$) (helps ^{o}R & D)

You can always change the stator angles during startup to avoid unwanted phenomena (unstarting windmilling, etc.).

Compressor Starting (Axial velocity W thru compressor small)

"Normal" flow: (Blade angles, W, ωr 'matched') such that that $\beta_{1_{flow}} \sim \beta_{1_{blade\ angle}}$

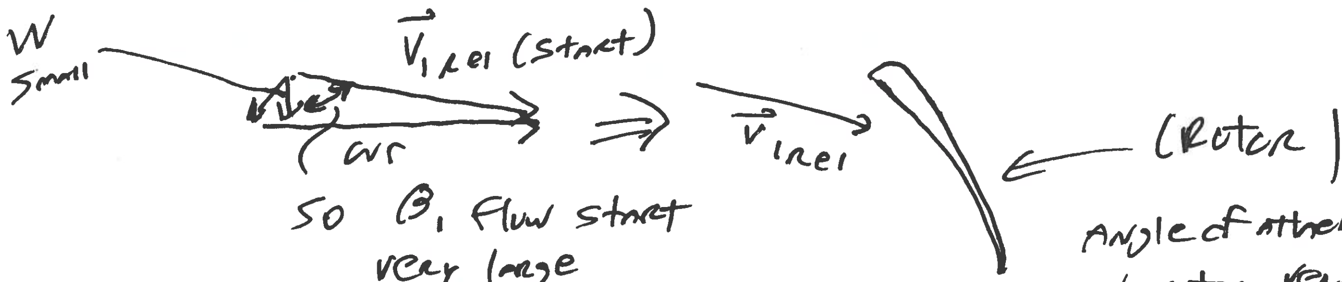


so $\beta_{1_{flow}}$

$\sim \beta_{1\ blade}$

Angle of attack on Rotor blade small, little risk of separation

But when compressor is starting, W is small on initial stages. So → if stator blade angles are not changed from 'normal run':



So β_1 flow start very large

Angle of attack on rotor very large... blade stalls

(Rotor LE blade angle badly mismatched to the $\vec{V}_{1\,rel}$)

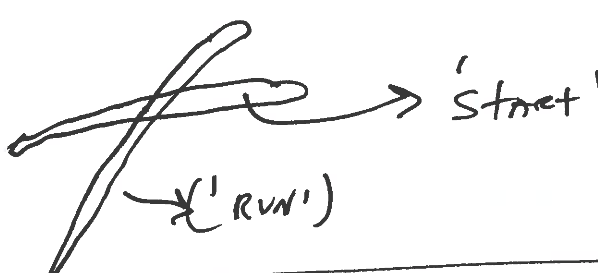
$\beta_{1\,\text{flow start}} \gg \beta_{1\,\text{blade}}$

So, during starting, rotate upstream stator (clockwise in sketch) to make

$$\beta_{1\,\text{flow starting}} \sim \beta_{1\,\text{blade}}$$

(upstream) Stator orientation:

(upstream Stator)



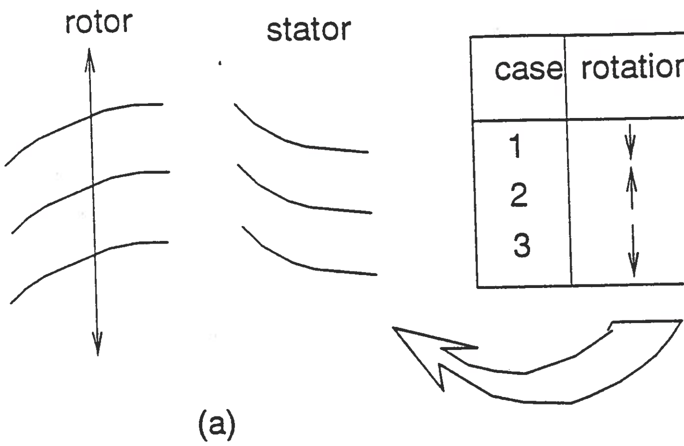
For rear stages, since overall area constriction thru a compressor is based on 'design' ('run') conditions (to maintain axial velocities thru compressor), when compressor is starting, power into flow does not match the area contraction so axial flow tends to speed up

... speed up too much on rear stages; rear stages can 'wind-mill' (develop negative angles of attack - trying to act like a turbine)

So for starting (& 'off-design operation'), ECS must schedule both stator orientation and axial bleed to ensure effective operation. Also 'spooling' used (compressor - turbine are 'segmented' and run sections at different RPM)...

Turbine aerodynamics analyzed in similar fashion. The primary function of the stator in turbine is to provide a 'high' velocity impinging on downstream rotor (to provide adequate force & hence torque on rotor)...

1. Shown schematically in part a) of the figure is the blading of a single-stage axial turbomachine. What kind of machine is represented by cases 1 and 2? What would happen in case 3? Would it be desirable to build a compressor stage as in part b) of the figure?

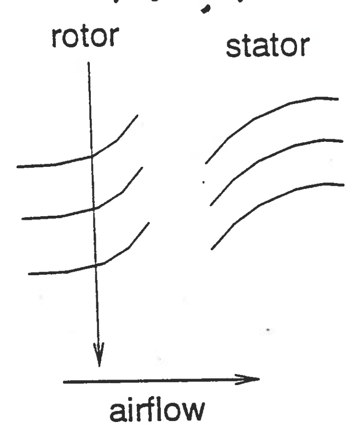


case	rotation	inlet flow direction
1	↓	→
2	↑	←
3	↓	←

(a)

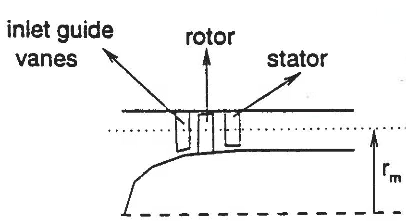
see next page

NO !!

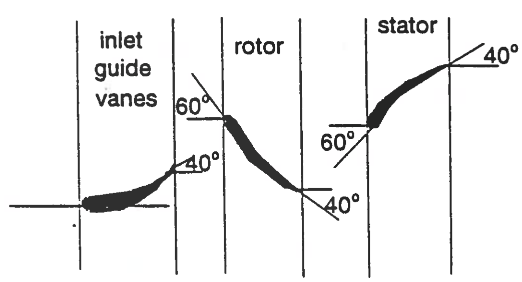


(b)

2. Estimate the power required to drive a single stage compressor shown schematically as in parts a) and b) of the sketch below. At the mean radius r(mean) = .3048m. The blade configuration at this mean radius is as shown in part b). For simplicity it is assumed that the air and the blade angles are identical. The overall efficiency of the stage is .8. The hub-tip radius ratio is 0.8 and is high enough so that conditions at the mass-averaged radius are a good average of conditions from hub to tip. Axial velocity component at design flow rate is uniformly 122 m/s and the inlet air is at 1 atm and 288K. What should the shaft speed be under these conditions? What is the ΔP_t through the stage?



(a)



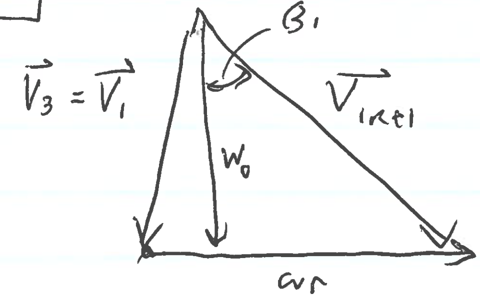
(b)

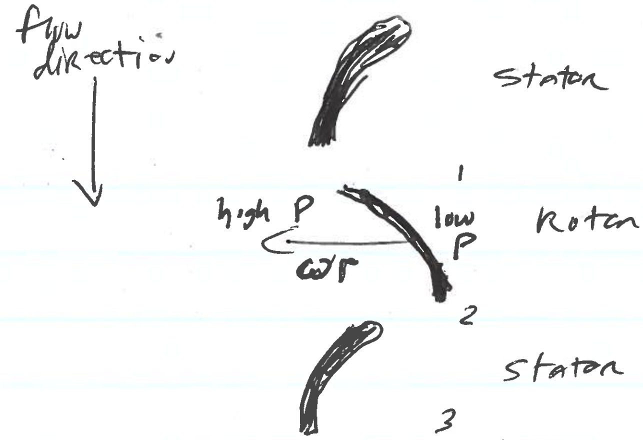
following pages ...

$\uparrow \omega r$

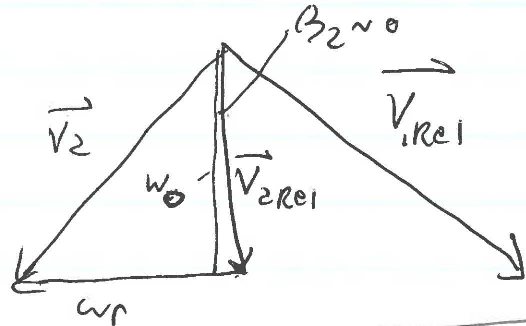
#1

Case 1:



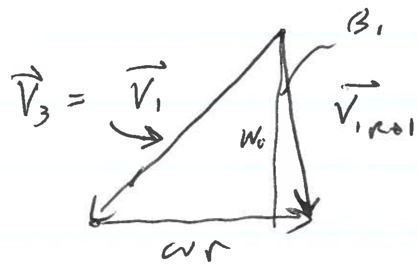


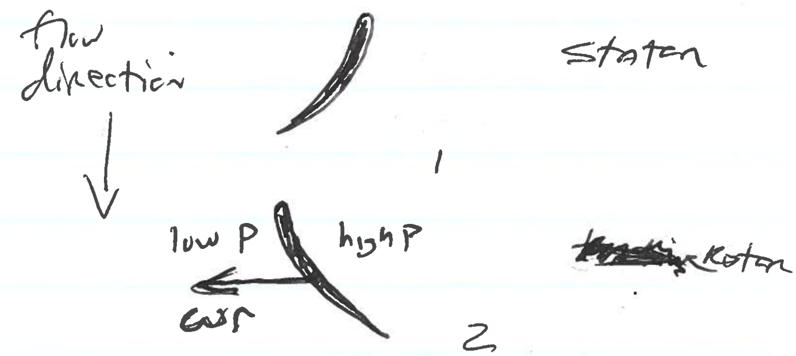
Compression: Rotor works on the flow, stator turns flow back toward axial direction

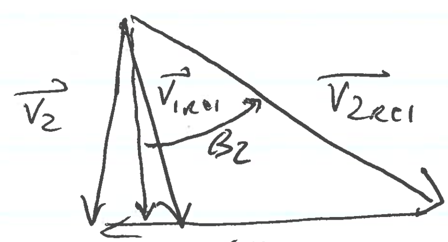


Note: Rotor "pushing" against higher P side of Rotor blade - takes work!

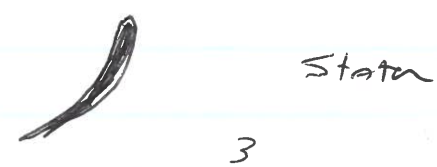
Case 2:





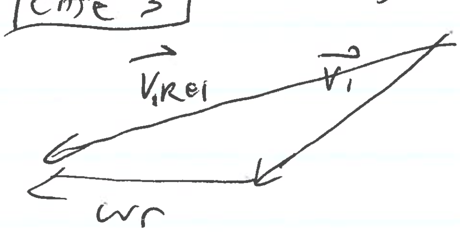


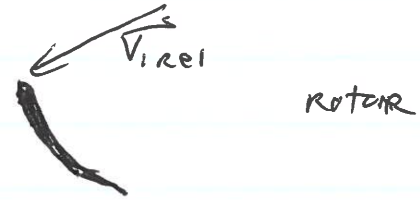
Turbine:



Note: Rotor being pushed by high P side - produces work!

Case 3 (ωr changes sign)





Massive mismatch between blade angle & flow direction

Massive Stall

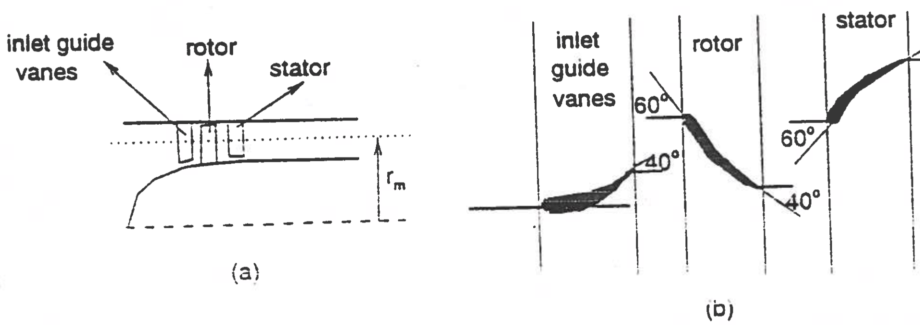
Solution: (#2)

$P_0 = 101325\ N/m^2$
$T_0 = 288\ K$
$(\rho_0 = 1.225\ kg/m^3)$
$W_0 = 122\ m/s$
$\eta_c = 0.8$

$\left.\right\} M_0 = \dfrac{W_0}{\sqrt{\gamma R T_0}} = 0.359$
$T_{t0} = 295.41\ K$
$P_{t0} = 110745\ N/m^2$

$\dfrac{r_t}{r_h} = \dfrac{1}{.8} = 1.25$

r_m = mass averaged radius $= 0.3048\ m$



$\xrightarrow{\quad} z$
engine Axis (main flow direction)

$\uparrow \omega r$ (Rotor rotation direction)

$$\boxed{\text{Power required} = \omega\left[(rV_\theta)_2 - (rV_\theta)_1\right]\dot{m}}$$

(Only assumptions are adiabatic blades ~~[illegible]~~)

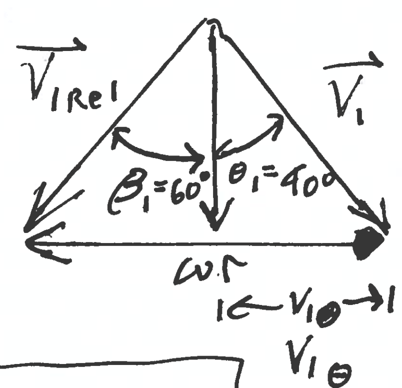
0-1 IGV
1-2 Rotor
2-3 stator
$\downarrow$
1

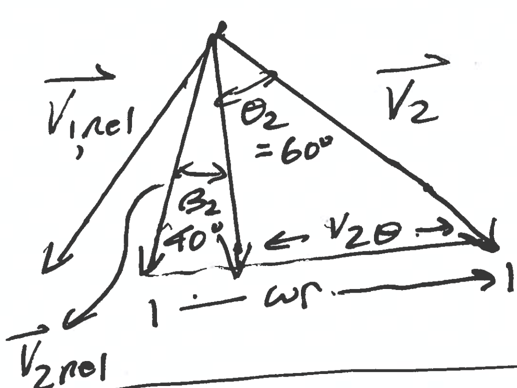
Velocity triangles:

engine Axis
$z \downarrow$

(W velocity constant) = 122 m/s

$W_0 = W_1 = W_2$





$\theta_1 = 40°$
$\beta_1 = 60°$

$\left.\begin{array}{l}\theta_2 = 60° \\ \beta_2 = 40°\end{array}\right\}$ $V_{1\theta} = 102.4\ m/s$
$V_{2\theta} = 211.3\ m/s$

$$\left\{ \begin{array}{ll} \dfrac{\omega r - V_{\theta 1}}{W_0} = \tan \beta_1 & \dfrac{\omega r - V_{\theta 2}}{W_0} = \tan \beta_2 \\[2ex] \dfrac{V_{\theta 1}}{W_0} = \tan \theta_1 & \dfrac{V_{\theta 2}}{W_0} = \tan \theta_2 \end{array} \right\}$$

so $\dfrac{\omega r}{W_0} = 2.57 \Rightarrow \boxed{\begin{array}{l}\omega = 1029 \text{ rad/sec} \\ (= 9826 \text{ RPM})\end{array}}$

Need $\dot{m}$ thru stage (for $*$):

$r_m = 0.3048$ if W uniform with r ...

mass average radius $\longrightarrow$ $\dfrac{r_t^2 + r_h^2}{2} = r_m$ & since $\dfrac{r_t}{r_h} = 1.25$ (given)

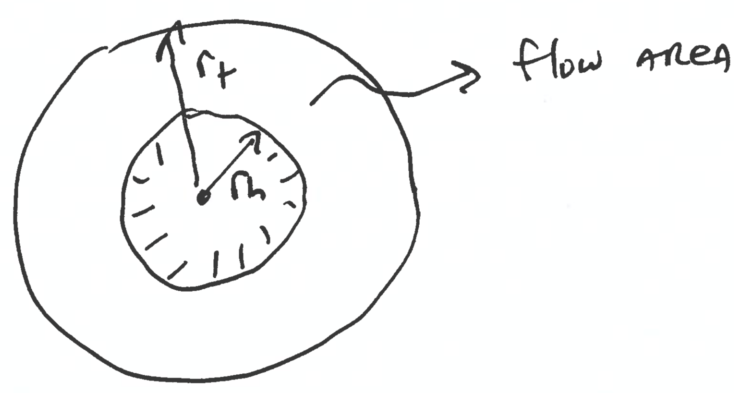
so $\left\{ \begin{array}{l} r_h = 0.2693 \text{ m} \\ r_t = 0.3366 \text{ m} \end{array} \right\}$

so A (cross-sectional area of flow path) $= A_{tip} - A_{hub}$

$$= \pi r_t^2 - \pi r_h^2$$

$A = 0.128 \text{ m}^2$

$\dot{m} = \rho_0 W_0 A = 19.16$ kg/sec



from * Power $= \dot{m}(h_{t_2} - h_{t_1}) = \dot{m}\omega\left[(rV_\theta)_2 - (rV_\theta)_1\right]$
$(= \dot{m} c_p (T_{t_2} - T_{t_1}))$

$\boxed{\text{Power} = 65~~0,000~~ 416 \text{ Watts}}$ ((Power necessary for this stage))

(also $T_{t_2} = 329.2$ K)

How do we find ΔP_t (across stage) Not isentropic → $\eta < 1$ given!!

let $\tau = \frac{T_{t_2}}{T_{t_0}}$ (recall $T_{t_2} = 329.2$ K)

stage based → $\pi = \frac{P_{t_2}}{P_{t_0}}$ (& $P_{t_0} = 110745$ N/m²)

& by definition of η_{stage}

$$\tau = 1 + \frac{\pi^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} \Rightarrow \pi = 1.3582$$

(across stage)

so $P_{t_2} = 150412$ N/m²

& $\boxed{\Delta P_{t\,\left\{\substack{\text{actual} \to \text{for}\\ \text{given } \eta}\right\}} = 39669 \text{ N/m}^2}$

Now, let us compare that to an isentropic stage & simply use blade angles info to get ΔP_t ...

$$** \quad \frac{\Delta P_t (\text{isentropic})}{\rho_0 W_0^2} = \frac{\omega r}{W_0}(\tan\beta_1 - \tan\beta_2)$$

→ {recall that we derived ** based on isentropic flow thru IGV, rotor, stator; AND small M_0 AND small flow turning ...}

BIG ASSUMPTION

so ... $\Delta P_t(\text{isentropic}) = 41873\ N/m^2$

(Compared to ΔP_t actual $= 39669\ N/m^2$)

... But the assumption that M_0 is small leads to a pretty large approximation (in the development of **) & hence in the ΔP_t(isentropic) ** equation ...

take ** as approximate only

** is really strictly valid for incompressible ($\rho \sim \rho_{TOTAL}$)($M \to 0$) & small turning ...
Note $\Delta\beta$ thru rotor is ~ 20° (Not really small)