AE 5335 is taught by Dr. Riggins

Midterm

Propulsion 2

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1.0 Results

1.1 question 1

q1	numerical	analytical
P [N]	139527.5698	139568.7153
T [K]	315.5844355	315.593
Pt [N]	140507.529	140548.043
Tt [K]	316.216133	316.224
u [m/s]	35.62415227	35.608
M	0.100041866	0.09995
thrust [N]	58182.05266	58210

1.2 question 2

q2	numerical	analytical
P [N]	74622.2376	74600.60408
T [K]	1210.648204	1210.709
Pt [N]	92112.16809	104190
Tt [K]	1285.719328	1285.824
u [m/s]	388.3527888	388.46228
M	0.556816993	0.55696
thrust [N]	1.352740673	1.775638

1.3 question 3

q3	numerical q = 1/2*ρi*ui^2	numerical q = 1/2*ρ*u^2	analytical
P [N]	66690.08846	56727.02624	56375.659
T [K]	285.0920682	283.1956849	283.1117
Pt [N]	71055.6416	61868.76996	61549.488
Tt [K]	290.3039376	290.3038291	290.304
u [m/s]	102.3261729	119.5000494	120.2047
M	0.302335537	0.354258111	0.3546
thrust [N]	-3177.491386	-4030.554068	-4060.143

Column 2 uses the dynamic pressure at the inlet and column 1 uses the local dynamic pressure. When the dynamic pressure is set to the inlet dynamic pressure, the results differ from the analytical solution.

1.4 question 4

q4	numerical
P [N]	71044.2156
T [K]	1189.44049
Pt [N]	89209.40268
Tt [K]	1269.388887
u [m/s]	400.7696721
M	0.579720321
thrust [N]	-252.8594539
$Qdot_{conv}$ [J/s]	8196672.491

1.5 question 5

q5	numerical eta = 0.9	numerical eta = 1	analytical
P [N]	11446897.46	19420600.42	19522651.36
T [K]	1288.752531	1294.635272	1294.78848
Pt [N]	11731012.58	19588009.41	19688418.31
Tt [K]	1297.811845	1297.814071	1297.92026
u [m/s]	134.9079724	79.91374038	79.319467
M	0.187476952	0.110800769	0.1099703
thrust [N]	46367.59476	85087.27487	86717.77616

1.6 question 6

q6	numerical eta = 1.07	numerical eta =1	analytical
P [N]	12066.95027	17146.9898	17007
T [K]	163.6311913	173.2803479	172.957
Pt [N]	18656.63837	21693.93375	21658.897
Tt [K]	185.324776	185.3254851	185.328
u [m/s]	208.7640095	157.5463435	157.644
M	0.814174726	0.589543997	0.598
thrust [N]	-5342.672315	-3783.766978	-3802.796

1.7 question 7

q7	numerical
P [N]	8181.861346
T [K]	1187.907233
Pt [N]	180270.2974
Tt [K]	2874.202753
u [m/s]	1840.588954
M	2.664161708
thrust [N]	-354.3399952
$Qdot_{conv}$	-17579167.38

2.0 Methodology

First off, the problem is to solve a non-linear set of differential equations. The method that will be used to solve the non-linear set is a Newton-Raphson method for multivariable systems.

$$F\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \{f\} = \begin{bmatrix} f_1(x_1, x_2, x_3, x_4) = 0 \\ f_2(x_1, x_2, x_3, x_4) = 0 \\ f_3(x_1, x_2, x_3, x_4) = 0 \\ f_4(x_1, x_2, x_3, x_4) = 0 \end{bmatrix}$$

In this case or functions are the differential equations for continuity, momentum, energy, and the equation of state.

$$f_1 = \frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

$$f_2 = \frac{dP}{\rho} + udu + \frac{\tau_w c dx}{\rho A} - \eta\delta w = 0$$

$$f_3 = C_p dT + udu - \delta q - \delta w = 0$$

$$f_4 = \frac{dP}{P} - \frac{d\rho}{\rho} + + \frac{dA}{A} = 0$$

The Jacobian is needed for further calculation and is denoted [J].

$$[J] = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} \end{bmatrix}$$

$$[J]^k\{\Delta x\}^k = -\{f\}^k \quad (1)$$

At the first iteration {x} is the values at the inlet. To get the solution vector for equation 1, use an algorithm for solving linear equations. It was chosen that a Gauss elimination algorithm ought to be used. Alternatively, the built-in function in MATLAB may be used instead (linsolve uses LU factorization algorithm).

After solving equation 1, we can get the next value of {x}.

$$\{\Delta x\}^{k+1} = \{\Delta x\}^{k} + \{\Delta x\}^{k} \quad (2)$$

The exit criteria are the L2 norm of the current iteration of {f} scaled with the L2 norm of {f} at the original guess/inlet conditions. To exit this must be less than a tolerance value (ε). Epsilon was chosen as 10^-5. By decreasing epsilon more accurate results are found at the expense of computation time.

$$\frac{\|\{f\}^{k}\|}{\|\{f\}^{0}\|} < \epsilon$$

The amount that the steps are preformed are dependent on the step size used. Since a step size of 10000 was used, the number of times the Newton-Raphson algorithm is 10000. For each step, the {x} vector found at the end of the previous step is the initial guess of the next step.

3.0 Appendix

3.1 IM.m

```
% Iterative Methods class
% used to solve linear and non-linear systems iteratively
classdef IM
    methods (Static)
%==========================================================================%
                      % Gauss-Seidel Method
%==========================================================================%
function [x,w] = gauSei(A,b,n,x,imax,es,lambda)
    for i = 1:n
        dum = A(i,i);
        for j = 1:n
            A(i,j) = A(i,j)/dum;
        end
        b(i) = b(i)/dum;
    end
    for i = 1:n
        sum = b(i);
        for j = 1:n
            if i~= j
                sum = sum - A(i,j)*x(j);
```

```
            end
            x(i) = sum;
        end
    end
    iter = 1;
    sen = 0;
    L2norm_0 = norm(b-A*x);
    while sen == 0
        sen = 1;
        for i = 1:n
            old = x(i);
            sum = b(i);
            for j = 1:n
                if i~= j
                    sum = sum - A(i,j)*x(j);
                end
            end
            x(i) = lambda*sum + (1-lambda)*old;
            L2norm = norm(b-A*x);
            if sen == 1 && x(i) ~= 0
                ea = abs(L2norm/L2norm_0)/1;
                if ea > es
                    sen = 0;
                end
            end

        end
        iter = iter + 1;
        if iter >= imax
            break
        end
    end
    w = [lambda iter];
end
%==========================================================================%
                      % Newton-Raphson Method
%==========================================================================%
function [q] = newRap(f,q,p,kmax)
    % f is the 'A' matrix
    % q is the 'b' vector
    % p is the precision goal
    % kmax is the maximum allowable iterations
    syms x1 x2 x3 x4
    fp(x1,x2,x3,x4) = jacobian(f,[x1 x2 x3 x4]);
    b = transpose(double(f(q(1),q(2),q(3),q(4))));
    b_0 = b;
    k = 0;
     while (norm(b)/norm(b_0)) > 10^p && k<kmax
        A = double(fp(q(1),q(2),q(3),q(4)));
        b = transpose(double(f(q(1),q(2),q(3),q(4))));
        del = gauss(A,-b); % gauss elimination algorithm
        q = q+del;
        k = k + 1;
     end
end
```

```
    end
end
```

3.2 Gauss elimination algorithm

```
function [x] = gauss(a,b)
% gauss elimination

n = length(a);

k = 1 ;
p = k ;
big = abs(a(k,k));

%****************************************************
% pivoting portion
%****************************************************
for ii=k+1:n
    dummy = abs(a(ii,k));
    if dummy > big
       big = dummy;
       p = ii ;
    end
end
if p ~= k
    for jj = k:n
        dummy = a(p,jj);
        a(p,jj) = a(k,jj);
        a(k,jj) = dummy;
    end
    dummy = b(p);
    b(p)=b(k);
    b(k) = dummy;
end

%****************************************************
% elimination step
%****************************************************
for k=1:(n-1)
    for i=k+1:n
        factor = a(i,k)/a(k,k);
       for j=k+1:n
           a(i,j) = a(i,j) - factor*a(k,j);
       end
       b(i) = b(i) - factor*b(k);
    end
end

%****************************************************
% back substitution
%****************************************************
x(n,1) = b(n)/a(n,n);
for i = n-1:-1:1
    sum = b(i);
   for j = i + 1:n
```

```
        sum = sum - a(i,j)*x(j,1);
    end
    x(i,1) = sum/a(i,i);
end
end
```

3.3 main.m

```
clc
clear all
close all

format longg

syms x1 x2 x3 x4
% x1 is P; x2 is rho; x3 is T; x4 is u
p = -5.5;
R = 287;
gam =1.4;
kmax = 1000;
rho = 1.225;
T = 288;
T0 = T;
P = 101325;
P0 = P;
rho0 = rho;
cp =1004.5;

l = 1;
M = .2;
M0 = M;
u = M*sqrt(gam*R*T);
u0 = u;
cf = 0.08;
Tw = 3000;
eta = 0;
w = 0;
h = 0;
A = .1*ones(1,100);
% for convective heat transfer set ht to 1 else set it to 0
ht = 1;

mdot = rho*u*A(1);
for i = 1:length(A)-1
    D = sqrt(A(i)/pi*4);
    c = pi*D;
    f = @(x1,x2,x3,x4) ([(x2-rho)/x2+(x4-u)/x4+(A(i+1)-A(i))/A(i) (x1-
P)/x2+x4*(x4-u)+1/2*cf*rho0*u0^2*c*(l/length(A))/rho/A(i)-eta*w/length(A) ...
        cp*(x3-T)+x4*(x4-u)-ht*2*cp*cf*(Tw-T*(1+(gam-
1)/2*M^2))*l/length(A)/D-(h/length(A))-(w/length(A)) (x3-T)/x3+(x2-rho)/x2-
(x1-P)/x1]);
    q = transpose([P,rho,T,u]);
    [q] = IM.newRap(f,q,p,kmax);
    P = q(1)
```

```
    rho = q(2)
    T = q(3)
    u = q(4)
    M = u/sqrt(gam*R*T)
end

Tt2 = T*(1+(gam-1)/2*M^2);
Pt2 = P*(1+(gam-1)/2*M^2)^(gam/(gam-1));
thrust = P/R/T*u*A(end)*(u-u0)+P*A(end)-P0*A(1);
P
T
Pt2
Tt2
u
M
thrust
if ht ==1
    Qdot = mdot*(cp*(Tt2-T0*(1+(gam-1)/2*M0^2))-w)
end
```

3.4 work for q1-q7

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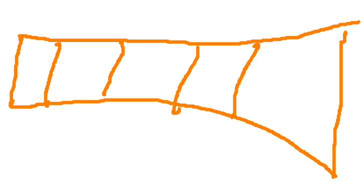
$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

$$\frac{dP}{\rho} + u\,du = \frac{-\tau_w c\, dA}{\rho A} + \eta \delta \dot{w}$$

$$C_p dT + u\,du = \delta q + \delta \dot{w}$$

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}$$



$$\frac{d\rho}{\rho} \Rightarrow \frac{\rho_i - \rho_{i-1}}{\rho_i} + \frac{u_i - u_{i-1}}{u_i} + \frac{A_i - A_{i-1}}{A_i} = 0$$

$$\frac{\rho_i - \rho_{i-1}}{\rho_i} + u_i(u_i - u_{i-1}) = \frac{-\tau_w c (A_i - A_{i-1})}{\rho_i A_i} + \eta\, \delta \dot{w}$$

$$C_p(T_i - T_{i-1}) + u_i(u_i - u_{i-1}) = \delta q + \delta \dot{w}$$

$$\frac{p_i - p_{i-1}}{p_i} = \frac{\rho_i - \rho_{i-1}}{\rho_i} + \frac{T_i - T_{i-1}}{T_i}$$

$$\begin{bmatrix} 1 - \rho_{i-1} & 1 - u_{i-1} & 0 & 0 \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} \rho_i \\ u_i \\ p_i \\ T_i \end{bmatrix} \quad \frac{-(A_i - A_{i-1})}{A_i}$$

① $$[J]^k \{\Delta x\}^k = \{f\}^k$$

② $$\bar{x}^k = \overline{\Delta x}^k + \bar{x}^k$$

Solve ① with gauss-Seidel

$\frac{A_1}{A^*} \div \frac{A_2}{A^*}$ $\frac{A_2}{A_1} \cdot \frac{A_1}{A^*}$ $.8\bar{3} \times 1.09$

1) $\frac{A_2}{A_1} = 5.32 \Rightarrow M_2 = .99$

302.4 K 5)

2) M2 = 0.55696
3) M2 = 035467

$\frac{T_{te}}{T_{ti}} = 4.29$

$T_{te} = 1297\,K$

$T_{ti} = 334.656\,K$

$T_{te} = 185.328$

$\frac{T_{te}}{T_{Ti}} = .5587$

$P_{ti} = 171371\,pa$

$\frac{P_{te}}{P_{ti}} = .126385$

7) $\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$

$\frac{dP}{\rho} + u\,du = -\frac{\tau_w\, c\, dx}{\rho A} + \eta\, \delta w$

$c_p\, dT + u\,du = \delta q + \delta w$

$\delta q_{conv} = C_p C_f (T_w - T_t) \frac{dx}{D}$

$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}$

Q1

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$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

$$\frac{dP}{\rho} + u\,du = 0$$

$$c_p\,dT + u\,du = 0$$

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

$\delta w = 0$

$\delta q = 0$, revers.

$U_i = M_i\sqrt{\gamma R T_i} = 238.1219$ m/s

$\dot{m} = \rho U A = 29.1699$ kg/s

$$\frac{G(M_e^2)}{G(M_i^2)} = \frac{A_e}{A_i} \Rightarrow M_e$$

$G(M_i^2) = 1.09437$

$M_e = .099945$

$T_{ti} = \left(1 + \frac{\gamma-1}{2} M_i^2\right) T_{ti} = 316.224$ K

$P_{ti} = \left(1 + \frac{\gamma-1}{2} M_i^2\right)^{\gamma/(\gamma-1)} P_i = 140548.043$ pa

$$P_e = \frac{P_{te}}{\left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\gamma/(\gamma-1)}} = 139568.7153 \text{ pa}$$

$$T_e = \frac{T_{te}}{1 + \frac{\gamma-1}{2} M_e^2} = 315.593$$

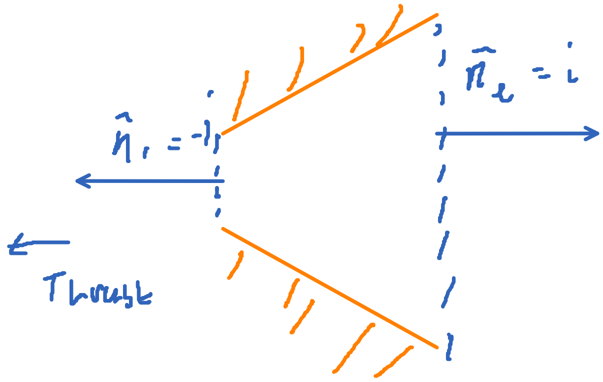
$U_e = M_e\sqrt{\gamma R T_e} = 35.608$ m/s

$\text{Thrust} = -\left[\dot{m}(u_e - u_i) + P_e A_e - P_i A_i\right]$

$= -[29.1699\,(35.608 - 238.1219) + 139568.7153 \times .532$

$- 101325 \times .1]$

$= -58210$ N



$$\sum F_x = \int_{cs} u\rho\,\bar{u}\cdot\hat{n}\,dA = \int_{S_{cv}} p\,\hat{n}\,dA + \int_{S_{cv}} \tau_w\,dA$$

$$\text{Thrust} = -\int_{\substack{cs\\ wet}} p\,\hat{n}\,dA - \int_{\substack{cs\\ wet}} \tau_w\,dA$$

$$-\text{Thrust} = \int_{cs} u\rho\,\bar{u}\cdot\hat{n}\,dA + \int_{\substack{cs\\ \text{inlout}}} p\,\hat{n}\,dA$$

Q2

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$$\frac{d\rho}{\rho} + \frac{du}{u} + \cancel{\frac{dA}{A}} = 0$$

$$\frac{dp}{\rho} + u\,du = \cancel{-\frac{\tau_w C\, dx}{\rho A}} + \cancel{\eta\, \delta w}$$

$$c_p\, dT + u\,du = \delta q + \cancel{\delta w}$$

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

$$f(M_1^2) = \left\{\frac{1+\frac{\gamma-1}{2}M_1^2}{(1+\gamma M_1^2)^2}\right\} M_1^2 = .036175$$

$$T_{t_i} = t_i\left(1+\frac{\gamma-1}{2}M_1^2\right) = 290.304 \qquad P_{t_i} = 104190.5846\ pa$$

$$\frac{T_{te}}{T_{ti}} = 1 + \frac{q_{1\to2}}{c_p T_{ti}} = 1 + \frac{10e6}{1004.5 \cdot 290.304} = 4.4292$$

$$f(M_e^2) = .160147 \quad \Rightarrow \quad M_e = .5569$$

$$T_{te} = 1285.824\ K \qquad u_e = M_e\sqrt{\gamma R T_e} = 388.46228$$

$$T_e = \frac{T_{te}}{1+M_e^2\frac{\gamma-1}{2}} = 1210.709\ K$$

$$\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2} = .7362507 \Rightarrow P_2 = 74600.60408\ Pa$$

$$P_{t2} = P_2\left(1+\frac{\gamma-1}{2}\right)^{\gamma/(\gamma-1)} = 92095.48146\ pa$$

$$\dot m = \rho u A = 83.3426\ kg/s \qquad U_i = 68.0348\ m/s$$

$$\text{thrust} = -\left[\dot{m}(u_e - u_i) + (P_e - P_i)A_e\right]$$

$$= -\left[8.33426\,(388.465 - 68.0348) + (74600.60408 - 101325) \times 0.1)\right]$$

$= -1.775638$ N thrust

Q3

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$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

$$\frac{dp}{\rho} + u\,du = -\frac{\tau_w\, c\, dx}{\rho A} + \cancel{\eta\,\delta w}^{\,0}$$

$$c_p\,dT + u\,du = \cancel{\delta q} + \cancel{\delta w}$$

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

$$f_v(M_1^2) = \frac{\gamma+1}{2} \ln\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{M_1^2}\right] - \frac{1}{M_1^2} = -21.128$$

$$\gamma C_f \frac{C}{A} L = f_{v_2} - f_{v_1}$$

$$\gamma C_f \frac{4}{D} L + f_{v_1} = 5.4327 \qquad f(M_2^2) \Rightarrow M_2 = .3564$$

$$T_{t1} = T_{t2} = (1+.2^3) \cdot 288 = 290.304\ K$$

$$T_2 = \frac{T_{t2}}{\left(1+\frac{\gamma-1}{2}M_2^2\right)} = \frac{290.304}{1+.2 \cdot .3564^2} = 283.1117K$$

$$P_2 = P_1 \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{1/2} = 56375.659\ pa$$

$$Pt_2 = P_2\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1} = 61549.488\ Pa$$

$$u_2 = M_2\sqrt{\gamma R T_2} = 120.2047\ m/s$$

$$u_0 = 68.0348\ m/s \qquad \dot{m} = 8.3343\ kg/s$$

$$\text{thrust} = -\left[\dot{m}(u_2 - u_1) + (p_2 - p_1)A_2\right]$$

$$\cdot [8.3343(120.2047 - 68.0348) + (56375.654 - 101325) \cdot 0.1]$$

$$= 4060.134 \text{ N}$$

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Q4

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$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

$\Rightarrow$

$$\frac{dp}{\rho} + u\,du = -\frac{\tau_w\, c\, dx}{\rho A} + \cancel{\eta\, \delta w}$$

$$c_p\, dT + u\,du = \delta q + \cancel{\delta w}$$

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

$$\frac{d\rho}{\rho} + \frac{du}{u} + \cancel{\frac{dA}{A}} = 0$$

$$\frac{dp}{\rho} + u\,du = -\frac{\tau_w\, C\, dx}{\rho A}$$

$$c_p\, dT + u\,du = \delta q = 2 c_p c_f (T_w - T_t)\frac{dx}{D}$$

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

No work interaction or Area variation

Q5

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$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

$$\frac{dp}{\rho} + u\,du = \eta\,\delta w \;;\; \eta = 1$$

$$c_p\,dT + u\,du = \delta w$$

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

$$\frac{T_{T2}}{T_{t1}} = 1 + \frac{W_{1\to 2}}{c_p T_{t1}} \;;\; T_{t1} = T_1\left(1 + \frac{\gamma-1}{2}M_1^2\right) = (1 + .2 \cdot .5^2) \cdot 288 = 302.4\,K$$

$$\frac{T_{t2}}{T_{T1}} = 1 + \frac{10e6}{1004.5(302.4)} = 4.2921 \implies T_{T2} = 1297.92026\,K$$

$$P_{t1} = P_1\left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\gamma/\gamma-1} = 120192.9955\,pa$$

$$P_{t2} = P_{t2}\left(\frac{T_{t2}}{T_{t1}}\right)^{\gamma/\gamma-1} = 19688418.31\ pa$$

$$M_e \Rightarrow P_{te}\sqrt{\gamma/R\,\frac{1}{T_{te}}}\,M_e A_e \left(1 + \frac{\gamma-1}{2}M_e^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} = \dot{m} = 20.8357\,kg/s$$

$M_e =$ .1099703

$$P_2 = \frac{P_{t2}}{\left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1}} \simeq 19522651.36\ pa$$

$$T_2 = \frac{T_{t2}}{\left(1 + \frac{\gamma-1}{2}M_2^2\right)} = 1294.78848\ K$$

$$u_2 = M_2\sqrt{\gamma R T_2} = 79.319467\ m/s$$

$$u_1 = 170.087\ m/s$$

$$thrust = \dot{m}(u_e - u_1) + P_e A_e - P_i A_i$$

$=$ -(8.334(79.319467 -170.87)+ 19640903.66 (.005)-101325*.1)=-86717.77616 N

Q6

Friday, April 9, 2021 9:56 PM

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

$$\frac{dp}{\rho} + u\,du = -\frac{\tau_w \cancel{c}\, dx}{\cancel{\rho} A} + \eta\, \delta w$$

$$c_p\, dT + u\,du = \cancel{\delta q} + \delta w$$

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

$$T_{ti} = T_i\left(1 + \frac{\gamma-1}{2} M_i^2\right) = 288 \cdot (1 + .2 \times .9^2) = 334.654\ k$$

$$\frac{T_{te}}{T_{ti}} = 1 + \frac{w_{i\to e}}{c_p T_{ti}}$$

$$= 1 + \frac{-1.5 \times 10^5}{1004.5 \cdot T_{ti}} = .554 \Rightarrow T_{te} = 185.328\ k$$

$$P_{ti} = p_i\left(1 + \frac{\gamma-1}{2} M_i^2\right)^{\gamma/(\gamma-1)} = 101325(1 + .2 \times .9^2)^{1.4/.4} \approx 171371\ pa$$

$$\frac{P_{te}}{P_{ti}} = \left(\frac{T_{te}}{T_{ti}}\right)^{1.4/.4} = .12639 \Rightarrow P_{te} = 21658.897\ pa$$

$$\dot{m}_e = P_{te}\sqrt{\gamma/R}\,\frac{1}{\sqrt{T_{te}}}\, M_e A_e \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} \Rightarrow M_e = .598$$

$$T_e = T_{te} / \left(1 + \frac{\gamma-1}{2}\right) \Rightarrow T_e = 172.957\ k \quad P_e = 17007\ N/m^2$$

$$u_e = M\sqrt{\gamma R T_e} = 157.644\ m/s$$

$$-\text{Thrust} = \dot{m}(u_e - u_i) + P_e A_e - P_i A_i$$

$$-[37.5042(157.644 - 306.1367) + 17007 \times .7 - 101325 \cdot .1 = +3223.148\ N$$

drag

$$\dot{m}\, c_p (T_{te} - T_{ti}) =$$

$$\left(\frac{\dot{w}}{\dot{m}}\right)\dot{m} = \dot{w}$$

$\Rightarrow \dot{m} C_p (T_{ti} - T_{t1})$

Q7

Tuesday, April 13, 2021 4:06 PM

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

$$\frac{dp}{\rho} + u\,du = -\frac{\tau_w\, c\, dx}{\rho A} + \eta\, \delta w$$

$$c_p\, dT + u\,du = 2 c_p c_f (T_w - T_t)\frac{dx}{D} + \delta w$$

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

Work interaction with area variation with irreversibility with convective heat transfer, i.e., all the terms

$$C_p\, dT_t = q + w$$

$$q = C_p\, dT_t - w$$

$$\dot{Q} = (C_p\, dT_t - w)\,\dot{m}$$