KEY

Homework 2 **AE 5535**

Assigned: 2/15/2021
Due: 2/22/2021

An ideal fixed-area turbojet (FAT-jet) is operated on-design where $\pi_c = 15$, $M_0 = 0.8$, $T_0 = 260K$, $T_{t4} = 2000K$, and $P_0 = 20{,}000\ N/m^2$. Mass flow rate of air processed by this engine at on-design is 100 kg/sec.

What will be the performance of this engine (thrust and mass capture) compared to the on-design conditions if it is flown at a Mach of 0.3 and at an altitude where temperature and pressure are 288K and 101325 N/m^2, respectively. Furthermore, the fuel throttle is set such that fuel flow rate is 21.5% higher than the fuel flow rate at the on-design point. Assume that A_9 is varied to keep $P_9 = P_0$.

What is the ratio of the off to on-design A_9 required to maintain $P_9 = P_0$? Does this seem reasonable? If not, perhaps the analysis needs to be redone with the A_9 'fixed'. (Don't do it, just realize it).

HW 2 'RAT jet' KEY

ON-design engine analysis (R → ON-design)

$$* \quad \dot{m}_{4R} = \frac{\Gamma}{\sqrt{R}} A_4 \left[P_{0R} \pi_{rR} \pi_{dR} \pi_{cR} \pi_{bR} \right] \frac{1}{\sqrt{T t_4}}$$

station 4 choked!

$$\Gamma = \sqrt{\gamma} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} = 0.68473$$

$$\pi_{rR} = \left(1 + \frac{\gamma-1}{2} M_{0R}^2 \right)^{\frac{\gamma}{\gamma-1}} = 1.52434$$

Given
$$\begin{cases} \pi_{cR} = 15 \\ P_{0R} = 20000 \text{ N/m}^2 \\ T t_{4R} = 2000 \text{ K} \\ \dot{m}_R = 100 \text{ kg/sec} \quad (\text{assume } f \ll 1) \end{cases}$$

from $*$ $\quad A_{4R} = A_4 = 0.241954 \text{ m}^2$ (invariant)

General: ON and off-design
Burner enthalpy balance:

$$\dot{m}_f h = \dot{m} c_p (T t_4 - T t_3) \quad (\text{small } f)$$

$$\vdots$$

$$\dot{m}_f = \frac{\dot{m} (\tau_\lambda - \tau_r \tau_c)}{(h / c_p T_0)} \quad **$$

ON-design:

$$\tau_{rR} = 1 + \frac{\gamma-1}{2} M_{0R}^2 = 1.128, \quad \underbrace{T_{0R} = 260 \text{ K}}_{\text{Given}}$$

$$\tau_{cR} = \pi_{cR}^{\frac{\gamma-1}{\gamma}} = 2.1678$$

$$\tau_{\lambda R} = 7.6923 \quad (= T t_{4R} / T_{0R})$$

From $**$ applied at R (ON-design): $\boxed{\dot{m}_{fR} = 3.1 \text{ kg/sec}}$

$$T t_{2R} = T_{0R} \left(1 + \frac{\gamma-1}{2} M_{0R}^2 \right) = 293.3 \text{ K}$$

$$T t_{3R} = \tau_{cR} \cdot T t_{2R} = 635.8 \text{ K}$$

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General TCPB (ON and off-design): $\dot{m} c_p (T_{t3} - T_{t2}) = \dot{m} c_p (T_{t4} - T_{t5})$ (SMALL F)

So, for on-design: from *** $T_{t5R} = 1657.5\,K = T_{t9R}$

$$\boxed{\tau_{tR} = \frac{T_{t5R}}{T_{t4R}} = 0.82875} \text{ \& so } \boxed{\pi_{tR} = 0.518}$$

{ Note that τ_t at any off-design point will equal τ_{tR} since A_8/A_4 fixed (FAT jet!) }

So, since $\dfrac{\tau_{tR}^{\frac{1}{2}}}{\pi_{tR}} = \dfrac{A_8}{A_4} \Rightarrow \boxed{\begin{array}{l} A_8 = 0.42522\,m^2 \\ \text{\& } A_8 = A_{8R} \text{ (invariant)} \end{array}}$

$$P_{t9R} = P_{t5R} = P_{0R} \pi_{rR} \pi_{cR} \pi_{tR} = 236882\ N/m^2$$

$\dfrac{P_{t9R}}{P_{9R}} = \left(1 + \frac{\gamma-1}{2} M_{9R}^2\right)^{\frac{\gamma}{\gamma-1}}$ but $P_{9R} = P_{0R}$ (matched pressure nozzle) (ideally expanded)

So $M_{9R} = 2.265$

$T_{9R} = 817.9\,K$ $\left(T_{9R} = T_{t9R} \Big/ \left(1 + \frac{\gamma-1}{2} M_{9R}^2\right)\right)$

$U_{9R} = M_{9R} \sqrt{\gamma R T_{9R}} = 1298.7\ m/s$

$P_{9R} = P_{0R} = 20{,}000\ N/m^2$ so $\rho_{9R} = \dfrac{P_{9R}}{R T_{9R}} = .0852\ kg/m^3$

Hence $\boxed{F_R \text{ (Thrust)} = \dot{m}_R (U_{9R} - U_{0R}) = 104{,}000\ N}$

($\dot{m}_{fR} = 3.1$ kg/sec from before)

$$\boxed{\begin{array}{l} A_{9R} = \dfrac{\dot{m}_R}{\rho_{9R} U_{9R}} = 0.90374\ m^2 \\ \text{\& } A_{0R} = \dfrac{\dot{m}_R}{\rho_{0R} U_{0R}} = 1.443\ m^2 \end{array}}$$

Now perform off-design analysis where
$\dot{m}_f = 1.215\, \dot{m}_{fR} = 3.7665$ kg/sec

We know (berner enthalpy balance, see eq. ##)

$$\dot{m}_f = \frac{\dot{m}\left(\tau_\lambda - \tau_r \tau_c\right)}{h/c_p T_0}$$

but

$$\dot{m} = \frac{\Gamma}{\sqrt{R}} A_4 \left[P_0 \pi_r \pi_c\right] \frac{1}{\sqrt{\tau_\lambda T_0}} \quad (*)$$

$$\tau_c = 1 + \frac{\tau_\lambda}{\tau_r}(1 - \tau_t) \quad \underset{TCPB}{(\#\#\#)}$$

$$\pi_c = \tau_c^{\frac{\gamma}{\gamma-1}}$$

so ...

$$\dot{m}_f = \left\{\frac{\Gamma}{\sqrt{R}} A_4 \left[P_0 \pi_r \left(1 + \frac{\tau_\lambda}{\tau_r}(1-\tau_t)\right)^{\frac{\gamma}{\gamma-1}}\right] \frac{1}{\sqrt{\tau_\lambda T_0}}\right\} \cdot \left\{\frac{\tau_\lambda - \tau_r\left(1 + \frac{\tau_\lambda}{\tau_r}(1-\tau_t)\right)}{h/c_p T_0}\right\}$$

~~From on-design analysis:~~

from on-design

$A_4 = .24195\ m^2$

$P_0 = 101325\ N/m^2$

$\tau_r = 1 + \frac{\gamma-1}{2} M_0^2 = 1.018 \quad (M_0 = 0.3)$

$\pi_r = 1.06443$

$\tau_t = 0.82875$

$T_0 = 288K$

$\dot{m}_f = 3.7665$ kg/sec

Solve $\dot{m}_f$ equation for τ_λ ...
(only unknown)

· continuing off-design analysis:

(4)

$\tau_\lambda = 4.5139$

then $\tau_c = 1.75933$

$\pi_c = 7.2231$

$\dot{m} = 211.3$ kg/sec

Now 'chain' thru engine at this off-design point:

$T_{t2} = T_0\left(1+\frac{\gamma-1}{2}M_0^2\right) = 293.184\ K$

$T_{t3} = T_{t2}\cdot\tau_c = 515.81\ K$

$\frac{\dot{m}_f h}{\dot{m} c_p} = (T_{t4} - T_{t3}) \Rightarrow T_{t4} = 1300.17\ K$

$T_{t5} = \tau_t T_{t4} = 1077.4\ K = T_{t9}$

$P_{t2} = P_{t0} = P_0\left(1+\frac{\gamma-1}{2}M_0^2\right)^{\frac{\gamma}{\gamma-1}} = 107853.4\ N/m^2$

$P_{t3} = P_{t2}\pi_c = 779036\ N/m^2 = P_{t4}$

$P_{t5} = P_{t9} = P_{t4}\cdot\pi_t = 403541\ N/m^2$

$\frac{P_{t9}}{P_9} = \frac{P_{t9}}{P_0}$ (achieved by flaring A_9!) (ideally expanded nozzle at off-design) $= \left(1+\frac{\gamma-1}{2}M_9^2\right)^{\frac{\gamma}{\gamma-1}}$

$\Rightarrow M_9 = 1.5559$

$T_9 = T_{t9} \big/ \left(1+\frac{\gamma-1}{2}M_9^2\right) = 725.9\ K$

$U_9 = M_9\sqrt{\gamma R T_9} = 840.3$ m/s $\qquad \rho_9 = \frac{P_9}{R T_9}$

$U_0 = M_0\sqrt{\gamma R T_0} = 102.05$ m/s

$F(\text{off-design}) = \dot{m}(U_9 - U_0) =$ ~~[illegible]~~ $155989.5\ N$

$A_9 = \frac{\dot{m}}{\rho_9 U_9} = 0.51703\ m^2$

so $\boxed{\frac{A_9}{A_{9R}} = 0.57213}$

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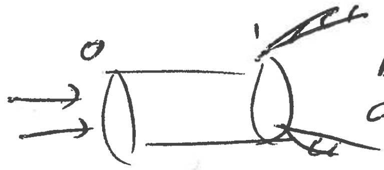
Comments:

I. If we design for full mass capture ($D_{add}=0$, no curvature of upstream streamtube) at on-design:

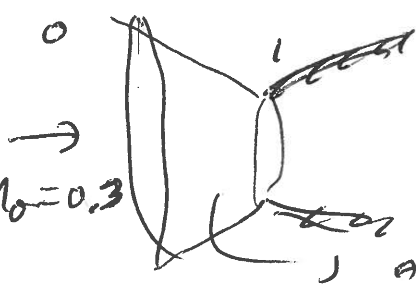
$$A_{0R} = A_{1R} = \frac{\dot m_R}{\rho_{0R} U_{0R}} = 1.443\ m^2$$



& then since $A_1 = A_{1R}$ (inlet face area would be invariant)

& off-design $A_0 = \dfrac{\dot m}{\rho_0 U_0} = 1.69025\ m^2$,

captured streamtube is accelerating from 0 to 1



II. However, often engine is sized to 'spill' (over-sized) at 'high-speed cruise' (here treated as the 'on-design' point ... This in order to ~~maintain~~ minimize upstream acceleration of captured streamtube at low Mach (take-off)

(reduces adverse pressure gradient in inlet process, reduces risk of separation)