AE 5335 is taught by Dr. Riggins

Final Project

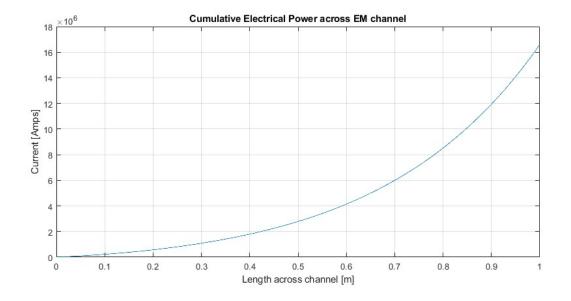
Propulsion 2

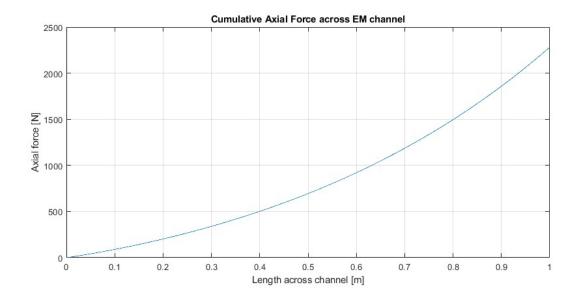
Matt Pahayo

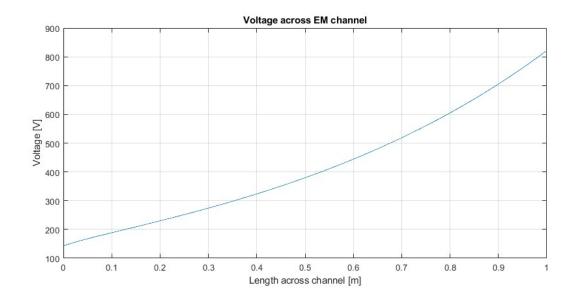
1.0 Results

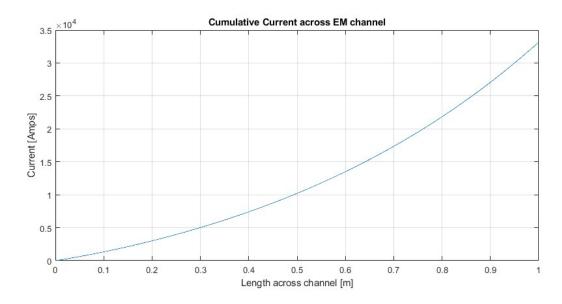
| Fluid Dynamics | | | | | |
|--------------------|------------------|-----------------|----------------|--|--|
| х | Inlet of channel | exit of channel | exit of nozzle | | |
| u [m/s] | 1631.293965 | 9388.876206 | 10596.17541 | | |
| Mach | 1.2 | 3.938143876 | 11.43758172 | | |
| T [K] | 320 | 984.2197599 | 148.6202101 | | |
| T _t [K] | 412.16 | 4037.06812 | 4037.06812 | | |
| P [Pa] | 100000 | 53447.75804 | 71.5121369 | | |
| Pt [Pa] | 242496 | 7470307.593 | 7470307.593 | | |

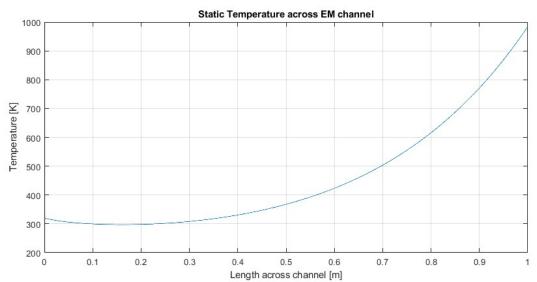
| Requirements | | | | | |
|----------------------|-------------|--|----------------------------|-------------|--|
| B [T] | 1.4 | | Thrust [N] | 3275.002102 | |
| σ ₀ [mho] | 400 | | Isp [sec] | 1080.140205 | |
| η | 0.8 | | mdot [kg/s] | 0.308957 | |
| payload mass [kg] | 500 | | electrical power rate [kW] | 16593.87356 | |
| engine mass [kg] | 4976.64 | | heat rate [kW] | -424.752 | |
| propellant mass [kg] | 8595.944039 | | | | |
| initial mass [kg] | 14072.58404 | | | | |

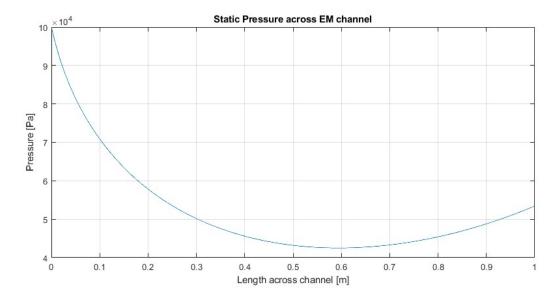


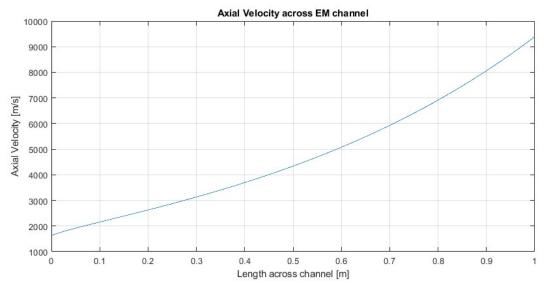


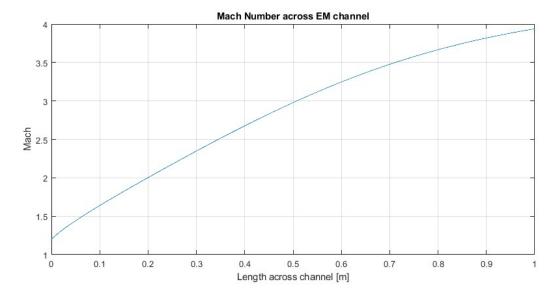


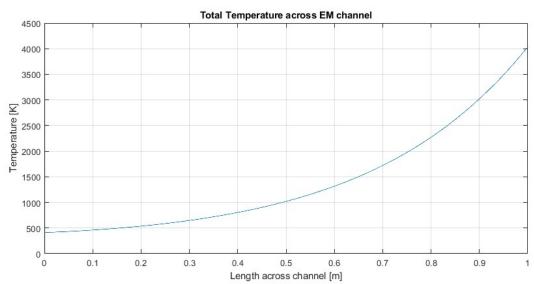


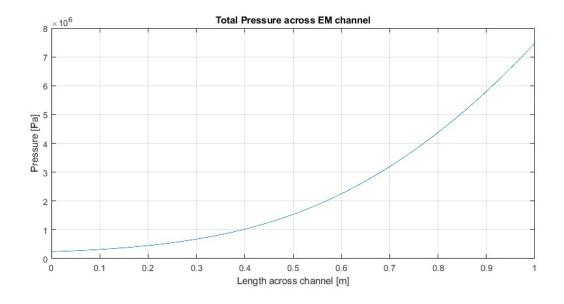












2.0 Methodology

First off, the problem is to solve a non-linear set of differential equations. The method that will be used to solve the non-linear set is a Newton-Raphson method for multivariable systems.

$$F\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \{f\} = \begin{bmatrix} f_1(x_1, x_2, x_3, x_4) = 0 \\ f_2(x_1, x_2, x_3, x_4) = 0 \\ f_3(x_1, x_2, x_3, x_4) = 0 \\ f_4(x_1, x_2, x_3, x_4) = 0 \end{bmatrix}$$

In this case or functions are the differential equations for continuity, momentum, energy, and the equation of state.

$$f_1 = \frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

$$f_2 = \frac{dP}{\rho} + udu + \frac{\tau_w cdx}{\rho A} - \eta \delta w = 0$$

$$f_3 = C_p dT + udu - \delta q - \delta w = 0$$

$$f_4 = \frac{dP}{P} - \frac{d\rho}{\rho} + \frac{dA}{A} = 0$$

The Jacobian is needed for further calculation and is denoted [J].

$$[J] = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} \end{bmatrix}$$
$$[J]^k \{ \Delta x \}^k = -\{f\}^k \quad (1)$$

At the first iteration $\{x\}$ is the values at the inlet. To get the solution vector for equation 1, use an algorithm for solving linear equations. It was chosen that a Gauss elimination algorithm ought to be used. Alternatively, the built-in function in MATLAB may be used instead (linsolve uses LU factorization algorithm).

After solving equation 1, we can get the next value of $\{x\}$.

$$\{\Delta x\}^{k+1} = \{\Delta x\}^k + \{\Delta x\}^k$$
 (2)

The exit criteria are the L2 norm of the current iteration of $\{f\}$ scaled with the L2 norm of $\{f\}$ at the original guess/inlet conditions. To exit this must be less than a tolerance value (ϵ). Epsilon was chosen as 10^-5. By decreasing epsilon more accurate results are found at the expense of computation time.

$$\frac{\|\{f\}^k\|}{\|\{f\}^0\|} < \epsilon$$

The amount that the steps are preformed are dependent on the step size used. Since a step size of 10000 was used, the number of times the Newton-Raphson algorithm is 10000. For each step, the $\{x\}$ vector found at the end of the previous step is the initial guess of the next step.

To find the optimum values of B, sigma0, and eta to minimize the initial mass. A brute force method was used. There are 75 different combinations; at each combination of B, sigma0, and eta was put in to the MHD solver with 100 axial steps. For each combination initial mass was tabulated also total temperature was tabulated. The tabulated results will be given in the appendix. A minimum value of the initial mass was chosen with the constraint that the total temperature must not exceed 6000 K.

$$\frac{df}{f} + \frac{du}{u} + \frac{dA}{A} = 0$$

$$\frac{df}{f} + udu = -\frac{2w}{f} \cdot \frac{dx}{A} + G(I_{m})(\frac{1}{n} - 1)$$

$$\frac{df}{f} = \frac{df}{f} \cdot \frac{dT}{T}$$

$$\frac{df}{f} = \frac{df}{f} \cdot \frac{dT}{f}$$

$$\frac{df}{f} = \frac{df}{f} \cdot \frac{df}{f} \cdot \frac{df}{f}$$

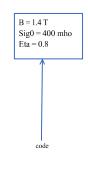
$$\frac{df}{f} = \frac{df}{f} \cdot \frac{df}{f} \cdot \frac{df}{f}$$

$$\frac{df}{f} = \frac{df}{f} \cdot \frac{df}{f} \cdot \frac{df}{f}$$

$$\frac{df}{f} = \frac{df}{f} \cdot \frac{df}{f$$

NUM exical.





$$G(M_1^2, \frac{A_{UL}}{A_{UL}} = G(N_2^2)$$

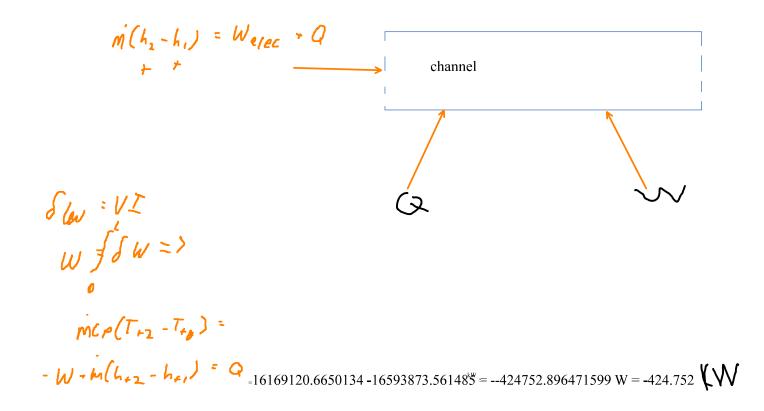
$$G(M_1^2) = M_1^2 \left(\frac{2}{r_{+1}} \left(1 + \frac{r_{-1}}{2} M_1^2\right)\right) \frac{1}{2D(4)} = 13.4117833559$$

$$= G(M_1^2) = 100.113781359 = M_2 = 1.61783720784$$

$$T_2 = \frac{T_L}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} = \frac{402.00131099}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} = \frac{104.200701091}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)}$$

$$P_2 = \frac{P_L}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{71.511180000408}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} = \frac{1000.174444000418}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.17444400041741460040741}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.1744440004174160040741}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.1744440004174160040741}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.1744440004174160040741}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.174444000417416004074}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.174444000417416004074}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.174444000417416004074}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.1744440004174}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.1744440004174}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.1744440004174}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.1744440004174}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.174440004004}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.174440004004}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.174440004004}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.174440004004}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.174440004004}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.17440004004}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1000.17440004004}{\left(1 + \frac{r_{-1}}{2} M_1^2\right)} \times 1/7 - \frac{1}{2} = \frac{1}{$$

m(h2-h1) = Welec + Q



16593873.561485 W = 16593.873561485 kW

0.309057107275532*14437.5(4037.06812031099-412.16) =

16169120.6650134 W =16169.1206650134 kW

cooling

3.0 Appendix

3.1 IM.m

```
% Iterative Methods class
% used to solve linear and non-linear systems iteratively
classdef IM
   methods (Static)
% Gauss-Seidel Method
%==========
function [x,w] = gauSei(A,b,n,x,imax,es,lambda)
   for i = 1:n
      dum = A(i,i);
      for j = 1:n
         A(i,j) = A(i,j)/dum;
      end
      b(i) = b(i)/dum;
   end
   for i = 1:n
      sum = b(i);
      for j = 1:n
          if i~= j
             sum = sum - A(i,j) *x(j);
          x(i) = sum;
      end
   end
   iter = 1;
   sen = 0;
   L2norm 0 = norm(b-A*x);
   while sen == 0
      sen = 1;
      for i = 1:n
          old = x(i);
          sum = b(i);
          for j = 1:n
             if i~= j
                 sum = sum - A(i,j) *x(j);
             end
          end
          x(i) = lambda*sum + (1-lambda)*old;
          L2norm = norm(b-A*x);
          if sen == 1 && x(i) \sim= 0
             ea = abs(L2norm/L2norm 0)/1;
             if ea > es
                 sen = 0;
             end
          end
      end
      iter = iter + 1;
      if iter >= imax
```

```
break
      end
   end
   w = [lambda iter];
% Newton-Raphson Method
function [q] = newRap(f,q,p,kmax)
   % f is the 'A' matrix
   % q is the 'b' vector
   % p is the precision goal
   % kmax is the maximum allowable iterations
   syms x1 x2 x3 x4
   fp(x1, x2, x3, x4) = jacobian(f, [x1 x2 x3 x4]);
   b = transpose(double(f(q(1),q(2),q(3),q(4))));
   b 0 = b;
   k = 0;
    while (norm(b)/norm(b 0)) > 10^p && k<kmax
      A = double(fp(q(1), q(2), q(3), q(4)));
      b = transpose(double(f(q(1),q(2),q(3),q(4))));
      del = gauss(A,-b); % gauss elimination algorithm
      q = q + del;
      k = k + 1;
    end
end
   end
end
```

3.1 Gauss elimination algorithm

```
function [x] = gauss(a,b)
% gauss elimination
n = length(a);
k = 1 ;
p = k;
big = abs(a(k,k));
§*****************
% pivoting portion
for ii=k+1:n
   dummy = abs(a(ii,k));
   if dummy > big
     big = dummy;
     p = ii ;
   end
end
if p \sim = k
   for jj = k:n
      dummy = a(p,jj);
      a(p,jj) = a(k,jj);
```

```
a(k,jj) = dummy;
  end
  dummy = b(p);
  b(p) = b(k);
  b(k) = dummy;
end
8**********
% elimination step
for k=1:(n-1)
  for i=k+1:n
     factor = a(i,k)/a(k,k);
     for j=k+1:n
       a(i,j) = a(i,j) - factor*a(k,j);
    b(i) = b(i) - factor*b(k);
  end
end
§******************
% back substitution
x(n,1) = b(n)/a(n,n);
for i = n-1:-1:1
  sum = b(i);
  for j = i + 1:n
    sum = sum - a(i,j) *x(j,1);
  x(i,1) = sum/a(i,i);
end
end
```

3.2 main.m

```
clc
clear all
close all
height = .05;
width = .05;
steps = 100;
cf = 0.002;
eta = .8;
Tw = 220;
w = 0;
h = 0;
A = ones(1, steps) *height*width;
% for convective heat tranmdot0sfer set ht to 1 else set it to 0
ht = 1;
1 = 1;
sig0 = 400;
B = 1.4;
R = 4125;
```

```
gam = 1.4;
M = 1.2
Pi = 100000
Ti = 320
[P,T,Pt2,Tt2,rho,u,M,thrust,F,mdot0,mdot,V,Ic,Powerc,forcec] =
MHD(Pi,Ti,M,cf,Tw,eta,w,h,ht,A,l,sig0,B,steps,R,gam,height,width);
Gm1 = 1/M(end)*(2/(gam+1)*(1+(gam-1)/2*M(end)^2))^((gam+1)/(2*(gam-1)))
Gm2 = Gm1 * 100
syms X
func = symfun(1/X*(2/(gam+1)*(1+(gam-1)/2*X^2))^((gam+1)/(2*(gam-1)))-Gm2,X)
Me = rootFind.newRap(func,7)
Te = Tt2 (end) / (1+ (gam-1) / 2*Me^2)
ue = Me*sqrt(gam*R*Te)
ml = me(B, sig0, eta)
mp = exp(10000/ue)*(ml+500)-(ml+500)
m0 = m1+500+ mp
Tt2(end)
function m e = me(B, sig0, eta)
    N B = (B-.6)/.2;
    N \text{ sig0} = (\text{sig0-100})/100;
    N = ta = (1-eta)/.1;
    m = 1000*1.2^{(N B-1)}*1.2^{(N sig0-1)}*2^{(N eta-1)};
end
3.4
% initialMass.m
clc
clear all
close all
y = 1
for i = 1:5
    sig0 = i*100+100;
    for j = 1:5
        B = j*.2+.6;
        for k = 1:3
            eta = k*.1+.6;
            height = .05;
            width = .05;
            steps = 100;
            cf = 0.002;
응
              eta = .9;
```

Tw = 220w = 0;

```
h = 0;
            A = ones(1, steps) *height*width;
            % for convective heat transfer set ht to 1 else set it to 0
            ht = 1;
            1 = 1;
응
              sig0 = 600;
응
              B = .8;
            R = 4125;
            gam = 1.4;
            M = 1.2;
            Pi = 100000;
            Ti = 320;
             [P,T,Pt2,Tt2,rho,u,M,thrust,F,mdot0,mdot,V,Ic,Powerc,forcec] =
MHD(Pi, Ti, M, cf, Tw, eta, w, h, ht, A, l, sig0, B, steps, R, gam, height, width);
            Gm1 = 1/M(end)*(2/(gam+1)*(1+(gam-
1)/2*M(end)^2))^((gam+1)/(2*(gam-1)))
            Gm2 = Gm1 * 100
            syms X
             func = symfun(1/X*(2/(gam+1)*(1+(gam-1)/2*X^2))^((gam+1)/(2*(gam-1)/2*X^2))
1)))-Gm2,X)
            Me = rootFind.newRap(func,7)
            Te = Tt2 (end) / (1+ (gam-1) / 2*Me^2)
            ue = Me*sqrt(gam*R*Te)
            ml = me(B, sig0, eta)
            mp = exp(10000/ue)*(ml+500) - (ml+500)
            m0(y,1) = m1+500+ mp
            maxTt(y,1)=Tt2(end)
            s(y,1)=sig0
            b(y, 1) = B
           e(y,1) = eta
            y = y+1
        end
    end
end
function m e = me(B, sig0, eta)
    N B = (B-.6)/.2;
    N = (sig0-100)/100;
    N = (1-eta)/.1;
    m = 1000*1.2^{(N B-1)}*1.2^{(N sig0-1)}*2^{(N eta-1)};
3.5 MHD.m
function [Pv,Tv,Pt2,Tt2,rho,uv,Mv,thrust,F,mdot0,mdot,V,Ic,Powerc,forcec] =
MHD(P0,T0,M,cf,Tw,eta,w,h,ht,A,l,sig0,B,steps,R,gam,height,width)
% Author: Matt P
응 {
```

```
MHD.m must include IM.m and gauss.m
PO is the static pressure at the inlet
TO is the static temperature at the inlet
M is the Mach # at the inlet
cf is the coefficient of skin friction
Tw is the wall temperature
eta is the thermodynamic efficiency
w is the work interaction per mass
h is the heat interaction per mass
ht is for convective heat transfer - 1 if present, 0 if not
A is the area along the length of the channel/nozzle and is a ROW VECTOR
l is the length of the device
응 }
format longg
syms x1 x2 x3 x4
% x1 is P; x2 is rho; x3 is T; x4 is u
p = -7.0;
kmax = 1000;
T = T0;
P = P0;
cp = R*gam/(gam-1);
rho = P/R/T;
rho0 = rho;
MO = M;
u = M*sqrt(gam*R*T);
u0 = u;
mdot = rho*u*A(1);
totalPower = 0;
Pv = zeros(steps-1,1);
rhov = zeros(steps-1,1);
Tv = zeros(steps-1,1);
uv = zeros(steps-1,1);
Mv = zeros(steps-1,1);
V = zeros(steps-1,1);
I = zeros(steps-1,1);
Power = zeros(steps-1,1);
F = zeros(steps-1,1);
Tt2 = zeros(steps-1,1);
Pt2 = zeros(steps-1,1);
Ic = zeros(steps-1,1);
Powerc = Ic;
forcec = Ic;
Pv(1,1) = P;
rhov(1,1)=rho;
Tv(1,1) = T;
uv(1,1)=u;
Mv(1,1) = M;
Pow = 0;
F = 0;
Icumulative = 0;
for i = 2: length(A) - 1
    G = A(i) *sig0*B^2*u^2*1/length(A)/mdot;
    c = 2*width+2*height;
```

```
f = 0(x_1, x_2, x_3, x_4) (((x_2-rho)/x_2+(x_4-u)/x_4+(A(i+1)-A(i))/A(i)) (x_1-rho)/x_2+(x_4-u)/x_4+(A(i+1)-A(i))/A(i))
P)/x^2+x^4(x^4-u)+1/2*cf*rho0*u0^2*c*(1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/leta-1)-1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)-G*(1/length(A))/rho/A(i)
eta*w/length(A) ...
                     cp*(x3-T)+x4*(x4-u)-ht*1/2*cp*cf*(Tw-T*(1+(gam-
1)/2*M^2) \times (A(i)*(1/length(A))-G*(1/eta-1)*1/eta-(h/length(A))-(w/length(A))
(x3-T)/x3+(x2-rho)/x2-(x1-P)/x1]);
          q = transpose([P, rho, T, u]);
           [q] = IM.newRap(f,q,p,kmax);
          P = q(1);
          rho = q(2);
          T = q(3);
          u = q(4);
          M = u/sqrt(qam*R*T);
          Pv(i, 1) = P;
          rhov(i,1) = rho;
          Tv(i,1) = T;
          uv(i,1)=u;
          Mv(i, 1) = M;
          F = Pv(i)/R/T*uv(i,1)*A(i)*(uv(i,1)-uv(i-1,1))+Pv(i)*A(i)-Pv(i-1)*A(i-1)
+ F;
          forcec(i-1,1) = F;
          V(i-1,1) = B*u/eta*height;
          I(i-1,1) = siq0*(B*u/eta-B*u)*width*(1/length(A));
          Icumulative = Icumulative + I(i-1,1);
          Ic(i-1,1) = Icumulative;
          Power(i,1) = V(i-1,1)*I(i-1,1);
          totalPower = totalPower + Power(i,1)
          Powerc(i-1,1) = totalPower;
          Pow = (G*(1/eta-1)*1/eta*mdot + Pow)
          Tt2(i,1) = T*(1+(gam-1)/2*M^2);
          Pt2(i,1) = P*(1+(qam-1)/2*M^2)^(qam/(qam-1));
          md = rho*u*A(i)
end
thrust = P/R/T*u*A(end)*(u-u0)+P*A(end)-P0*A(1);
mdot0 = mdot;
mdot = rho*u*A(end);
if ht ==1
          Qdot = mdot*(cp*(Tt2-T0*(1+(gam-1)/2*M0^2))-w);
end
3.6 rootFind.m
classdef rootFind
          %rootFind is a class of functions that find the root of a function /
          %data set
          methods (Static)
                     function x = Bisect(f,a,b,tol)
                               %Bisect uses the bisection algoritm using the interval
                               iter = 0;
```

while (b-a)/2 >= tol

```
c = (a+b)/2;
            if f(c) > 0
               b = c;
            end
            if f(c) < 0
               a = c;
            iter = iter + 1;
         end
         x = (a+b)/2
      end
%-----%
      function x = newRap(f, x0)
         %newRap is a function that utilizes the Newton-Raphson
         % \operatorname{algorithm} to find the roots of the function
         %x0 is the initial guess
         fp = diff(f);
         x=x0;
         nmax=25;
         eps=1;
         n=0;
         while eps>=1e-5&&n<=nmax</pre>
            y=x-double(f(x))/double(fp(x));
            eps=abs(y-x);
            x=y;
            n=n+1;
         end
     end
 end
end
```

3.7 Tabulated data for finding m0

| initial mass [kg] | Max total temp [K] σ | В | η | |
|-------------------|----------------------|-----|-----|-----|
| 54649.57493 | 601.826366 | 200 | 0.8 | 0.7 |
| 41520.31975 | 478.691559 | 200 | 0.8 | 0.8 |
| 30275.62944 | 422.0662108 | 200 | 0.8 | 0.9 |
| 43066.21768 | 845.901369 | 200 | 1 | 0.7 |
| 38709.56885 | 557.5064317 | 200 | 1 | 0.8 |
| 31864.83859 | 441.7417878 | 200 | 1 | 0.9 |
| 32515.892 | 1356.24459 | 200 | 1.2 | 0.7 |
| 33962.39008 | 695.6193473 | 200 | 1.2 | 0.8 |
| 32847.46885 | 471.3179735 | 200 | 1.2 | 0.9 |
| 25052.55585 | 2468.79628 | 200 | 1.4 | 0.7 |
| 28526.47711 | 938.9907038 | 200 | 1.4 | 0.8 |
| 33047.15636 | 514.9597241 | 200 | 1.4 | 0.9 |
| 20609.43319 | 5035.017163 | 200 | 1.6 | 0.7 |
| 23601.65864 | 1375.871824 | 200 | 1.6 | 0.8 |
| 32403.82875 | 578.828545 | 200 | 1.6 | 0.9 |
| 44991.66076 | 812.4738334 | 300 | 0.8 | 0.7 |
| 39697.84885 | 547.3867717 | 300 | 0.8 | 0.7 |
| 32142.30117 | 439.3608828 | 300 | 0.8 | 0.8 |
| 30766.3062 | 1450.893311 | 300 | 0.8 | 0.9 |
| 32664.27005 | 718.7549842 | 300 | 1 | 0.7 |
| 32369.05699 | 475.8524933 | 300 | 1 | 0.8 |
| 21860.41126 | 3126.494861 | 300 | 1.2 | 0.9 |
| 25258.64394 | 1062.794396 | 300 | 1.2 | 0.7 |
| 31323.57362 | 534.5263868 | 300 | 1.2 | 0.8 |
| | | | | |
| 17328.47113 | 7932.262715 | 300 | 1.4 | 0.7 |
| 19376.1976 | 1775.558293 | 300 | 1.4 | 0.8 |
| 29103.01841 | 627.969451 | 300 | 1.4 | 0.9 |
| 15538.88203 | 23199.8501 | 300 | 1.6 | 0.7 |
| 15511.2556 | 3321.423047 | 300 | 1.6 | 0.8 |
| 26123.69347 | 777.0936353 | 300 | 1.6 | 0.9 |
| 37952.32532 | 1136.450104 | 400 | 0.8 | 0.7 |
| 37686.11371 | 639.2931423 | 400 | 0.8 | 0.8 |
| 34127.70066 | 459.8274624 | 400 | 0.8 | 0.9 |
| 24348.96666 | 2587.776102 | 400 | 1 | 0.7 |
| 27831.30225 | 962.2277593 | 400 | 1 | 0.8 |
| 32698.3109 | 518.7427566 | 400 | 1 | 0.9 |
| 17763.9227 | 7382.833081 | 400 | 1.2 | 0.7 |
| 19962.05984 | 1704.627929 | 400 | 1.2 | 0.8 |
| 29604.33285 | 619.7186666 | 400 | 1.2 | 0.9 |
| 15255.87387 | 25509.77356 | 400 | 1.4 | 0.7 |
| 15064.99396 | 3514.735968 | 400 | 1.4 | 0.8 |
| 25536.53446 | 792.9188825 | 400 | 1.4 | 0.9 |
| 14996.27341 | 104709.1646 | 400 | 1.6 | 0.7 |
| 12509.70009 | 8263.284652 | 400 | 1.6 | 0.8 |
| 21420.67832 | 1094.004166 | 400 | 1.6 | 0.9 |
| 33330.82213 | 1625.445377 | 500 | 0.8 | 0.7 |
| 35834.73764 | 759.9418648 | 500 | 0.8 | 0.8 |
| 36262.67803 | 483.6910647 | 500 | 0.8 | 0.9 |
| 21261 60846 | 4686 729116 | 500 | 1 | 0.7 |