AE 5535 Non-ideal turbofan homework (1) **Assigned: Feb 1, 2021**

Due: Feb 5, 2021

$\tau_{\lambda} = \frac{C_{P,t}}{C_{P,c}} \frac{T_{t4}}{T_0}$

Consider a non-afterburning turbofan engine with engine parameters as given below.

If τ_λ = 7.3 (ratio of total enthalpy at burner exit to freestream enthalpy), M_0 = 2.0 (flight Mach number), π_C = 12 (compressor total pressure ratio), π_{fan} = 1.64 (fan total pressure ratio), and α = 3.6 (engine bypass ratio), **<u>find the specific thrust and specific fuel consumption of this engine.</u>**

keep P_0

T_0 = 220K (ambient temperature) $\pi_d = 1 - 0.015M_0^2$ (inlet total pressure drop)

e_C = 0.91 (polytropic compressor efficiency) γ_C = 1.4 (ratio of specific heats upstream of burner)

π_b = 0.98 (burner total pressure drop) e_{fan} = 0.90 (polytropic fan efficiency)

$C_{P,c}$ = specific heat at constant pressure upstream of burner = 1000J/kgK (also thru fan stream)

$\pi_N = \pi_{N'}$ = 0.99 (primary and bypass nozzle total pressure drops)

e_T = 0.89 (polytropic turbine efficiency) γ_t = 1.32 (ratio of specific heats downstream of burner)

η_b = 0.99 (burner efficiency) h = 4.5×10^7 J/kg (fuel) (heating value of fuel)

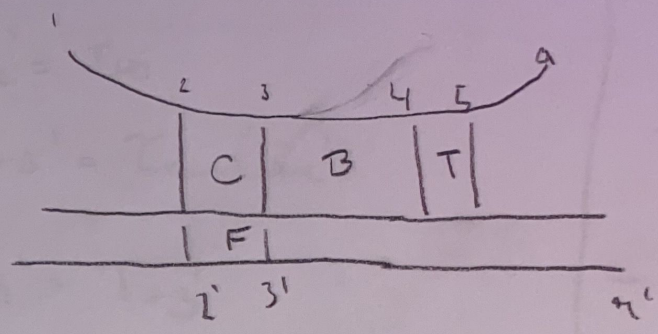
$C_{P,t}$ = specific heat at constant pressure downstream of burner = 1200 J/kgK

η_m = 0.99 (mechanical efficiency – shaft)

$P_9 = P_{9'} = P_0$ (both primary and bypass nozzles are ideally expanded)

If an efficiency/loss is not given, assume ideal for that particular efficiency/loss

HW 1



$$T_{t4} = \frac{c_{p,c}}{c_{p,t}} \tau_\lambda =$$

$$T_{t0} = T_0 \tau_r = T_{t2}$$

$$\tau_c = \pi_c^{\left(\frac{\gamma_c - 1}{\gamma_c e_c}\right)}$$

$$T_{t3} = \tau_c T_{t2}$$

$$P_{t0} = P_0 \overbrace{\left(1 + \frac{\gamma_c - 1}{2} M_0^2\right)^{\gamma/(\gamma-1)}}^{b}$$

$$P_{t2} = P_{t0} \pi_d = P_0 b \pi_d$$

$$P_{t3} = P_{t2} \pi_c = P_0 b \pi_d \pi_c$$

$$P_{t4} = P_{t3} \pi_b = P_0 b \pi_d \pi_c \pi_b$$

Bypass

$\tau_{fan} = \pi_{fan}$ ^ $\left(\frac{\gamma_c - 1}{\gamma_c e_{fan}}\right)$

$T_{t2}' = T_{t0}$

$T_{t3}' = \tau_{fan}(T_{t2}')$

$T_{t9}' = T_{t3}'$

$P_{t2}' = P_{t0}\pi_d$

$P_{t3f} = P_{t2}'\pi_{fan}$

$P_{t9}' = P_{t3}'\pi_N$

turbine - comp - fan pow

$T_{t5} = T_{t4} - \left\{ \begin{array}{l} (c_{pc}(T_{t3} - T_{t2}) + \\ \alpha c_{pc}(T_{t3}' - T_{t2}')) \\ /(\eta_m(1+f)c_{pt}) \end{array} \right.$

$\pi_t = \frac{T_{t4}}{T_{t5}}$ ^ $\left(\frac{\gamma_t}{e_t(\gamma_t - 1)}\right)$

$P_{t5} = P_{t4}\pi_t$

$T_{t9} = T_{t5}$

$P_{t9} = P_{t5}\pi_N$

fuel air ratio [f]

$$f = \frac{c_{p,t} T_{t4} - c_{pc} T_{t3}}{\eta_b h - c_{p,t} T_{t4}}$$

$$\frac{P_{t9}}{P_0} = \pi_b \pi_d \pi_c \pi_b \pi_T \pi_N$$

$$\frac{P_{t9}'}{P_0} = \pi_b \pi_d \pi_{fan} \pi_N$$

u_9 and u_9'

$$M_a = \sqrt{\frac{2}{\gamma_t - 1}\left(\left(\frac{P_{t9}}{P_0}\right)^{\left(\frac{\gamma_t - 1}{\gamma_t}\right)} - 1\right)}$$

$$T_9 = T_{t9}\left(1 + \frac{\gamma_t - 1}{2} M_9^2\right)^{-1} \Rightarrow u_9 = M_9\sqrt{\gamma_t R T_9}$$

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Specific thrust $\quad P_0 = P_a = P_a'$

$$\frac{(\dot{m}_c + \dot{m}_f)\,u_a - \dot{m}_c u_0 + \dot{m}_F (u_a' - u_0')}{\dot{m}_c + \dot{m}_F} \qquad \frac{\frac{1}{\dot{m}_c}}{\frac{1}{\dot{m}_c}}$$

$$\Rightarrow \frac{F}{\dot{m}_{tot}} = 127.5 \, \frac{N\cdot s}{kg}$$

SFC

$$S = f/(F/\dot{m}) = 135.501 \, \frac{mg}{N\cdot s}$$

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Microsoft Windows [Version 10.0.18363.1316]
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C:\Users\Matthew>C:/Users/Matthew/AppData/Local/Programs/Python/Python39/python.exe
d:/Documents/Prop_2/prop2/hw/hw_01/hw1.py
The specific thrust is 127.50978414859875 (N.S)/kg
The specific fuel consumption is 135.5007694489483 mg/(N.S)
Press enter to continue.
```

```
> special variables
  C_pc: 1000
  C_pt: 1200
  M_0: 2
  M_9: 1.8293360443939768
  M_9p: 2.2698283349327872
  PIb: 0.98
  PIc: 12
  PId: 0.94
  PIfan: 1.64
  PIn: 0.99
  PIt: 0.06848269410366387
> P_0: P_0
> P_t0: 7.82444906686726*P_0
> P_t2: 7.35498212285523*P_0
> P_t2p: 7.35498212285523*P_0
> P_t3: 88.2597854742627*P_0
> P_t3p: 12.0621706814826*P_0
> P_t4: 86.4945897647775*P_0
> P_t5: 5.92338253248315*P_0
> P_t9: 5.86414870715832*P_0
> P_t9p: 11.9415489746677*P_0
  R: 287
```

```
ST: 127.50978414859875

T_0: 220

T_9: 488.7654852206957

T_9p: 253.9767529960808

T_t0: 395.99999999999994

T_t2: 395.99999999999994

T_t2p: 395.99999999999994

T_t3: 864.0275735670157

T_t3p: 463.3397736083661

T_t4: 1338.3333333333333

T_t5: 750.467758965382

T_t9: 750.467758965382

T_t9p: 463.3397736083661

Tau_c: 2.181887812037919

Tau_fan: 1.17004993335446

Tau_l: 7.3

Tau_n: 1

alpha: 3.6

c: 710450.758557134

ec: 0.91

efan: 0.9

et: 0.89

eta_b: 0.99

eta_m: 0.99
```

```
Tau_c: 2.181887812037919
Tau_fan: 1.17004993335446
Tau_l: 7.3
Tau_n: 1
alpha: 3.6
c: 710450.758557134
ec: 0.91
efan: 0.9
et: 0.89
eta_b: 0.99
eta_m: 0.99
f: 0.017277673864404444
h: 45000000.0
> math: <module 'math' (built-in)>
n: 1
> sim: <module 'sympy' from 'C:\\Users\\Matthew\\AppData\\Local\\Programs\\Python
u_0: 594.6292962846684
u_0p: 594.6292962846684
u_9: 787.1756847483166
u_9p: 704.07335701242
y: 1.4
yc: 1.4
yt: 1.32
```

```python
# Matthew Pahayo
# 2/5/2021
# propulsion 2
# hw 1

import math
import sympy as sim
P_0 = sim.Symbol('P_0')

# Engine parameters and efficiencies
y = 1.4
yc = 1.4
yt = 1.32
ec = .91
et = .89
efan = .9
alpha = 3.6
M_0 = 2
T_0 = 220
PIfan = 1.64
PIc = 12
PId = 1-.015*M_0**2
PIb = .98
PIn = .99
C_pt = 1200
C_pc = 1000
h = 4.5*10**7
eta_b = .99
eta_m = .99
Tau_l = 7.3
Tau_n = 1
R = 287

# total temps and pressures between 1-4 (core)
T_t4 = Tau_l*C_pc/C_pt*T_0
T_t0 = T_0*(1+(y-1)/2*M_0**2)
T_t2 = T_t0
Tau_c = PIc**((yc-1)/(yc*ec))
T_t3 = Tau_c*T_t0
P_t0 = P_0*(1+(y-1)/2*M_0**2)**((y)/(y-1))
P_t2 = P_t0*PId
P_t3 = P_t2*PIc
P_t4 = P_t3*PIb

# Bypass duct
Tau_fan = PIfan**((yc-1)/yc/efan)
T_t2p = T_t0
T_t3p = Tau_fan*T_t2p
T_t9p = T_t3p
P_t2p = P_t0*PId
P_t3p = P_t2p*PIfan
P_t9p = P_t3p*PIn

# fuel air ratio [f]
f = (C_pt*T_t4-C_pc*T_t3)/(eta_b*h-C_pt*T_t4)

# turb-comp-fan balance
c = C_pc*(T_t3-T_t2)+alpha*C_pc*(T_t3p-T_t2p)
T_t5 = T_t4 - c/eta_m/(1+f)/C_pt
```

```
PIt = (T_t5/T_t4)**(yt/(yt-1)/et)
P_t5 = PIt*P_t4
P_t9 = P_t5*PIn
T_t9 = T_t5*Tau_n

# core
M_9 = math.sqrt(2/(yt-1)*((P_t9/P_0)**((yt-1)/yt)-1))
T_9 = T_t9*(1+(yt-1)/2*M_9**2)**-1
u_9 = M_9*math.sqrt(yt*R*T_9)
u_0 = M_0*math.sqrt(yc*R*T_0)

#  bypass
M_9p = math.sqrt(2/(yt-1)*((P_t9p/P_0)**((yt-1)/yt)-1))
T_9p = T_t9p*(1+(yt-1)/2*M_9p**2)**-1
u_9p = M_9p*math.sqrt(yt*R*T_9p)
u_0p = M_0*math.sqrt(yc*R*T_0)

# Specific thrust
ST = (u_9-u_0+alpha*(u_9p-u_0p))/(1+alpha)

# Specific fuel consumption
S = f/ST*10**6

print("The specific thrust is", ST, "(N.S)/kg")
print("The specific fuel consumption is", S,"mg/(N.S)" )
n = 1
while n == 1:
    n = input("Press enter to continue.")
```