KEY

AE 5535 Non-ideal turbofan homework (1)

Consider a non-afterburning turbofan engine with engine parameters as given below.

If τ_λ = 7.3 (ratio of total enthalpy at burner exit to freestream enthalpy), M_0 = 2.0 (flight Mach number), π_c = 12 (compressor total pressure ratio), π_{fan} = 1.64 (fan total pressure ratio), and α = 3.6 (engine bypass ratio), **find the specific thrust and specific fuel consumption of this engine.**

T_0 = 220K (ambient temperature) $\pi_d = 1 - 0.015M_0^2$ (inlet total pressure drop)

e_c = 0.91 (polytropic compressor efficiency) γ_c = 1.4 (ratio of specific heats upstream of burner)

π_b = 0.98 (burner total pressure drop) e_{fan} = 0.90 (polytropic fan efficiency)

$C_{P,c}$ = specific heat at constant pressure upstream of burner = 1000J/kgK (also thru fan stream)

$\pi_N = \pi_{N'}$ = 0.99 (primary and bypass nozzle total pressure drops)

e_T = 0.89 (polytropic turbine efficiency) γ_t = 1.32 (ratio of specific heats downstream of burner)

η_b = 0.99 (burner efficiency) h = 4.5x10^7 J/kg (fuel) (heating value of fuel)

$C_{P,t}$ = specific heat at constant pressure downstream of burner = 1200 J/kgK

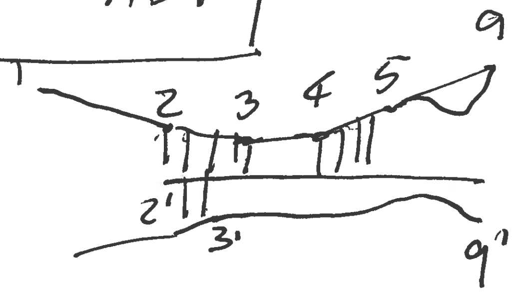
η_m = 0.99 (mechanical efficiency – shaft)

$P_9 = P_{9'} = P_0$ (both primary and bypass nozzles are ideally expanded)

If an efficiency/loss is not given, assume ideal for that particular efficiency/loss

Non-ideal Turbofan (cycle) analysis

Homework 1 AE 5535 KEY



$$\tau_c = \pi_c^{\frac{\gamma_c - 1}{\gamma_c e_c}} = 2.1819$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 = 1.8 = \frac{T_{t0}}{T_0} = \frac{T_{t2}}{T_0}$$

$$\tau_{fan} = \pi_{fan}^{\frac{\gamma_c - 1}{\gamma_c e_c'}} = 1.17005 \qquad (e_c' = e_{fan})$$

since $\tau_\lambda = \dfrac{C_{pt} T_{t4}}{C_{pc} T_0} \Rightarrow T_{t4} = 1338.3\,K$

$$T_{t3} = \tau_c \cdot T_{t2} \quad \& \quad T_{t2} = T_{t0} = T_0\left(1 + \frac{\gamma_c - 1}{2} M_0^2\right) = 396\,K$$

(396 K)

$$T_{t3} = 864.03\,K$$

$$T_{t3}' = T_{t2} \cdot \tau_{fan} = 463.3\,K$$

$$P_{t0} = P_0\left(1 + \frac{\gamma_c - 1}{2} M_0^2\right)^{\frac{\gamma_c}{\gamma_c - 1}} = P_0\,(7.825)$$

$$P_{t2} = \pi_d P_{t0} = P_0\,(7.355) \qquad (\pi_d = 0.94)$$

$$P_{t3} = P_{t2} \cdot \pi_c = 88.26\,P_0$$

$$P_{t4} = P_{t3}\,\pi_b = 86.5\,P_0$$

$$P_{t3}' = P_{t2}\,\pi_{fan} = 12.06\,P_0$$

Turbine Compressor Pwr balance (TCPB)

$$\eta_m(\dot m_c + \dot m_f) C_{pt} (T_{t4} - T_{t5}) = \dot m_c C_{pc} (T_{t3} - T_{t2}) + \dot m_{FAN} C_{pc} (T_{t3}' - T_{t2})$$

$f = \dfrac{\dot{m}_f}{\dot{m}_c}$ so TCPB results in:

$$* \quad \boxed{\eta_M (1+f) \frac{c_{pt}}{c_{pc}} (T_{t4} - T_{t5}) = T_{t3} - T_{t2} + \alpha(T_{t3}' - T_{t2})}$$

but what is f? $\nearrow$ (TCPB)

Burner enthalpy balance (energy eq. thru burner):

$$(\dot{m}_c + \dot{m}_f) c_{pt} T_{t4} - \dot{m}_c c_{pc} T_{t3} = \eta_b \dot{m}_f h$$

$$(1+f) c_{pt} T_{t4} - c_{pc} T_{t3} = (\eta_b h) f$$

or $$f = \frac{c_{pc} T_{t3} - c_{pt} T_{t4}}{c_{pt} T_{t4} - \eta_b h} = 0.017277$$

Now, from $*$, calculate T_{t5} ... $T_{t5} = 750.4K$

$$\tau_t = \frac{T_{t5}}{T_{t4}} = 0.5607$$

$$\pi_t = \tau_t^{\frac{\gamma_t}{(\gamma_t - 1)e_t}} = \frac{P_{t5}}{P_{t4}} = 0.0685$$

$$P_{t5} = P_{t4} \cdot \pi_t = 5.923 P_0$$

$$P_{t9} = \pi_N \cdot P_{t5} = 5.863 P_0$$

$$P_{t9}' = \pi_N' P_{t3}' = 11.94 P_0$$

$$T_{t9} = T_{t5} = 750.43 K$$

$$T_{t9}' = 463.3 K$$

For this problem (generally does not have to be!)

$$P_9 = P_0 \quad , \quad P_{9'} = P_0$$

$$\frac{P_{t9}}{P_9} = \left(1 + \frac{\gamma_t - 1}{2} M_9^2\right)^{\frac{\gamma_t}{\gamma_t - 1}} = \frac{P_{t9}}{P_0} = 5.8635$$

$$\Longrightarrow M_9 = 1.8293$$

$$\frac{P_{t9'}}{P_{9'}} = \left(1 + \frac{\gamma_c - 1}{2} M_{9'}^2\right)^{\frac{\gamma_c}{\gamma_c - 1}} \Rightarrow M_{9'} = 2.2706$$

$$\text{so...} \quad T_9 = \frac{T_{t9}}{\left(1 + \frac{\gamma_t - 1}{2} M_9^2\right)} = 788.8 \text{ K}$$

$$\& \quad T_{9'} = \frac{T_{t9'}}{\left(1 + \frac{\gamma_c - 1}{2} M_{9'}^2\right)} = 228.1 \text{ K}$$

$$U_9 = M_9 \sqrt{\gamma_t R T_9} = 787.13 \text{ m/s}$$

$$U_{9'} = M_{9'} \sqrt{\gamma_c R T_{9'}} = 687.43 \text{ m/s}$$

(Thrust) $F = (\dot{m}_c + \dot{m}_f) U_9 + \dot{m}_{FAN} U_{9'} - \dot{m}_c U_0 - \dot{m}_{FAN} U_0$

(No pressure terms!)
(ideally expanded)

$$\text{so...} \quad \frac{F}{\dot{m}_c + \dot{m}_{FAN}} = \frac{F}{\dot{m}_{TOT}} \quad \text{(specific Thrust)}$$

$$\alpha = \frac{\dot{m}_{FAN}}{\dot{m}_c} \qquad = \frac{1}{(1+\alpha)} \left\{ (1+f) U_9 + \alpha U_{9'} - (1+\alpha) U_0 \right\}$$

$$V_0 = M_0 \sqrt{\gamma_c R T_0} = 594.63 \text{ m/s}$$

$$\boxed{F/\dot{m}_C + \dot{m}_{FAN} = 117.43 \ \frac{N-s}{kg}}$$

$$S = \text{specific fuel consumption} = \frac{\dot{m}_f}{F}$$

note f = fuel-air ratio = $\dot{m}_f/\dot{m}_c$!

so $$S = \frac{f}{(1+\alpha)\left(F/\dot{m}_C + \dot{m}_{FAN}\right)} \times 10^6 \ \left(\frac{mg}{N\text{-sec}}\right)$$

$$\boxed{S = 31.98 \ mg/N\text{-sec}}$$