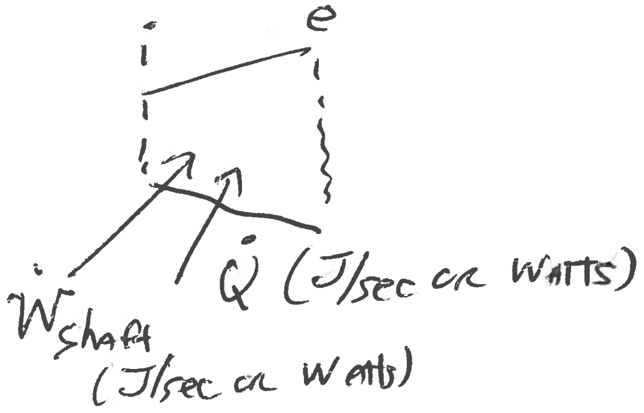
Total conditions Review (quasi-1-D flow)



$P_t, T_t \rightarrow$ total conditions

$\dot m$ = mass flow rate at a station

('reservoir' conditions – pressure & temp. you would get if you slowed flow to $U=0$ isentropically with no work either at a station)

how do P_t, T_t vary in a streamtube? (what 'drives' changes in them?)

$$w_{i\to e} = \frac{\dot W_{shaft\, i\to e}}{\dot m} \qquad q_{i\to e} = \frac{\dot Q_{i\to e}}{\dot m}$$

$$\left(\text{so } w_{i\to e} = \int_i^e \delta w \;\;\&\;\; q_{i\to e} = \int_i^e \delta q \right)$$

$$\frac{T_{te}}{T_{ti}} = 1 + \underbrace{\frac{q_{i\to e}}{c_p T_{ti}}} + \underbrace{\frac{w_{i\to e}}{c_p T_{ti}}}$$

(simply the energy eq.)

Total energy cont.

$$c_p(T_{te} - T_{ti}) = q_{i\to e} + w_{i\to e} \quad \text{(for CPG)}$$

$$\frac{P_{te}}{P_{ti}} = e^{\left[\int_i^e \frac{\delta w}{R T_t} - \frac{s_{irr}/\text{non-ideal heat transfer}}{R}\right]}$$

$s_{\text{non-ideal h.t.}} \rightarrow$ occurs if heat interaction where $M \neq 0$ $(M > 0)$

$s_{irr} \rightarrow$ entropy due to irreversibilities (like friction, shock)

↳ entropy per mass

Aside: P_{te}/P_{ti} expression

Does this make sense for (say) an isentropic compressor?? (just check it)

isentropic compressor, No s_{irr}, No $s_{non\text{-}ideal}$ (since no q!!)

& so $C_p \, dT_t = \delta w$ only

$$\rightarrow \frac{P_{te}}{P_{ti}} = e^{\int_i^e \frac{C_p}{R} \frac{dT_t}{T_t}}$$

but $\frac{C_p}{R} = \frac{\gamma}{\gamma - 1}$ (identity)

so $\frac{P_{te}}{P_{ti}} = \left(\frac{T_{te}}{T_{ti}} \right)^{\frac{\gamma}{\gamma-1}}$ (for isentropic compressor) check

---

Note that P_t, T_t are 'thermodynamic handles'; they don't change (even though cross-section area of a streamtube is changing) unless you have $\dot{Q}$, $\dot{W}$, or internal irreversibilities!

STREAMTUBE FLUID DYNAMICS (Review)

Mass flow rate and Area Relationships in quasi-1-D flow

$$\circledast \quad \dot{m} = \rho u A = \frac{P}{RT} M \sqrt{\gamma R T} A = P_t \sqrt{\frac{\gamma}{R}} \frac{1}{\sqrt{T_t}} M A \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}}$$

$$\left\{ P_t = P\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} ; \quad T_t = T\left(1 + \frac{\gamma-1}{2} M^2\right) \right.$$

So: $\circledast$ gives Mach # (M) at a station in a streamtube, provided $\dot{m}$, P_t, T_t, A known (must solve for M)

Note: 2 solutions exist (sub & supersonic) → 2 roots
... which depends on fluid situation

Apply $\circledast$ at a throat ($* \to$ throat) — could be a reference throat, not a physical throat, where $M = 1$ and $A = A^*$:

$$\dot{m}^* = P_t^* A^* \sqrt{\frac{\gamma}{R T_t^*}} \left(\frac{\gamma+1}{2}\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}}$$

Now define C^* ('characteristic velocity') $= \sqrt{\frac{R T_t^*}{\gamma}} \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$

then $\boxed{\dot{m}^* = \frac{P_t^* A^*}{C^*} ; \text{ Note } C^* = C^*(T_t^*, \gamma, R)}$

If flow is steady in quasi-1-D streamtube
$\dot{m} = \dot{m}^*$

If flow is isentropic (adiabatic & reversible), steady & no external work interaction, then P_t, T_t are const. (between a location of interest & throat $*$ where $A = A^*$, $M = 1$ (again, whether actual or reference throat

no heat interaction → adiabatic; no losses ($\dot{s}_{gen}$ → reversible

Then, since $\frac{\dot{m}}{\dot{m}^*} = 1.0$:

$$\boxed{\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} = G(M)}$$

although this $\left(\frac{A}{A^*}\right)$ is tabulated for $\gamma = 1.4$ in 'isentropic flow tables' versus Mach # M (for both sub & supersonic locuses), it is best to use the equation (find the root) since often in engines $\gamma \neq 1.4$.

If flow is isentropic (no $\dot{Q}$, no losses) & no work interaction ('$\dot{W}_{shaft}$') from surroundings, then

$$P_{t_2} = P_t^* = P_{t_1} \quad \& \quad T_{t_2} = T_t^* = T_{t_1}, \; \& \; A_2^* = A^* = A_1^*$$

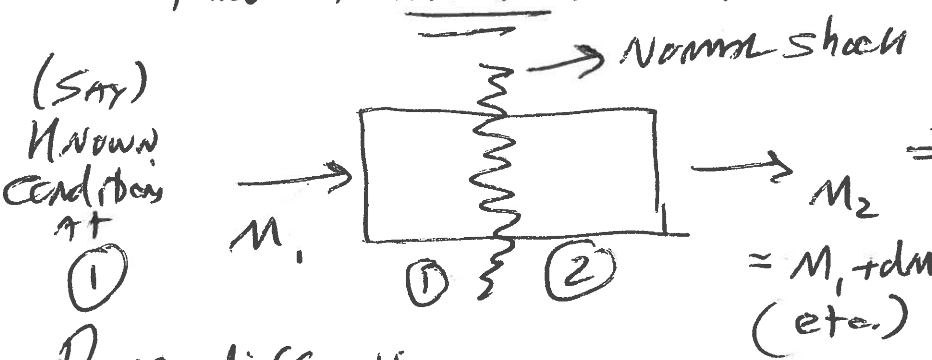
so $\boxed{A_2/A_1 = \frac{G(M_2)}{G(M_1)}}$; hence given $\frac{A_2}{A_1}$ & M_1, you can find M_2 (or any variation)

Also, if no $\dot{Q}$, $\dot{W}$ interaction between 1 & 2 then $C^* =$ constant (since T_t stays constant)

so $\boxed{P_{t_1} A_1^* = P_{t_2} A_2^*}$ Good for non-isentropic flow (but no external energy interactions → $\dot{Q}$, $\dot{W}$)

Normal Shocks:

Apply 1-D control volume (differential C.V.) analysis thru a normal shock



$\Rightarrow$ C.V. yields

$$\left\{ \begin{aligned} &\frac{d\rho}{\rho} + \frac{dv}{v} = 0 \\ &\frac{dP}{\rho} + v\,dv = 0 \\ &C_p T + v\,dv = 0 \\ &P = \rho R T \end{aligned} \right\}$$

These differential equations can be solved (analytically) to yield

$$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma-1} M_1^2 - 1} \; ; \quad \frac{P_2}{P_1} = \left(\frac{2\gamma}{\gamma+1}\right) M^2 - \frac{(\gamma-1)}{(\gamma+1)}$$

No energy interactions from surroundings (no $\dot{Q}, \dot{W}$).

$Tt_1 = Tt_2$ (but since shock is irreversible $\Rightarrow$ losses) (Non-isentropic $Pt_2 < Pt_1$)

Can find T_2, then $U_2 (= M_2 \sqrt{\gamma R T_2})$, etc.

Oblique Shocks:

(2-D)

θ = deflection (turning angle) of flow

Region 2 is uniform region of turned flow (compression)

Analysis shows:

$$\tan\theta = 2\cot\beta \left[\frac{M_1^2 \sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}\right]$$

'θ-β-M' relation (Anderson

(oblique shocks, cont.)

[So, for known θ, M_1, γ, find β that satisfies the $\theta-\beta-M$ relation.

Solve this yourself → do not rely on oblique shock chart!!!

Reminder: oblique shock chart plots $\theta-\beta-M$ relation for $\gamma = 1.4$
'strong shock solution'
'weak shock solution'

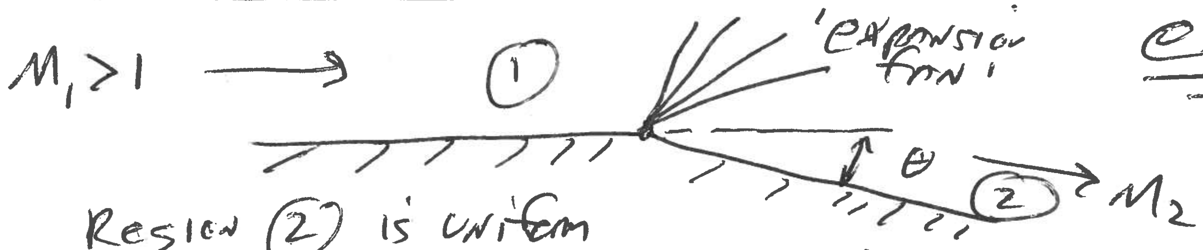
(cannot – even remotely – read chart, especially for high Mach #'s!!!)

Then $M_2 = \dfrac{M_{N2}}{\sin(\beta-\theta)}$ where $M_{N_2}^2 = \dfrac{M_{N_1}^2 + \left(\frac{2}{\gamma-1}\right)}{\left[\frac{2\gamma}{\gamma-1}\right] M_{N_1}^2 -}$

& $M_{N_1} = M_1 \sin\beta$ and $\dfrac{P_2}{P_1} = 1 + \left[\dfrac{2\gamma}{\gamma-1}\right] M_{N_1}^2 -$

& since $T_{t_2} = T_{t_1}$, find T_2, ρ_2, P_{t_2}, etc...

Prandtl-Meyer Expansion fans (inviscid)



Expansion!

$M_2 > M_1$

Region 2 is uniform region of turned flow (expanded)

isentropic (no coalescence of Mach waves → they spread out)

so $\boxed{P_{t_2} = P_{t_1} \quad T_{t_2} = T_{t_1}}$

θ = turn angle

Prandtl-Meyer Expansion Fan (cont.)

Analysis shows:

$$\Theta = \nu(M_2) - \nu(M_1)$$

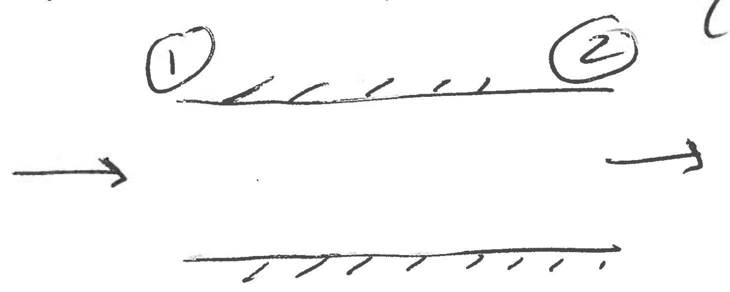
where $\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \arctan\sqrt{\frac{\gamma-1}{\gamma+1}(M_1^2-1)} - \arctan\sqrt{M_1^2-1}$

so: find M_2 (for given M_1, θ) that satisfies this equation! (again, do NOT use tables, especially for high speed!)

Then find T_2, P_2, ρ_2, U_2 etc. (since P_t, T_t con...

Rayleigh flow (1-D, frictionless, no area variation, no work interaction) but with heat addition (non-adiabatic)

1-D → no area change!



energy equation results in $C_p(T_{t_2} - T_{t_1}) = q_{1\to2}$

$$q_{1\to2} = \frac{\dot{Q}_{1\to2}}{\dot{m}} \quad (J/kg\ \text{air}) \text{ ← if air}$$

Analytical solution for this kind of flow shows

$$\frac{T_{t_2}}{T_{t_1}} = \frac{f(M_2^2)}{f(M_1^2)} \left(= 1 + \frac{q_{1\to2}}{C_p T_{t_1}} \right)$$

where $f(M^2) = \left\{ \frac{1+\frac{\gamma-1}{2}M^2}{(1+\gamma M^2)^2} \right\} M^2$

Rayleigh flow (cont.)

So if $q_{1\to 2}$ known (& T_{t_1}, M_1 known)
find $f(M_1^2)$, then find $f(M_2^2)$ & then solve for M_2 from:

$$M = \sqrt{\frac{2f}{1-2\gamma f \pm \left[1-2(\gamma+1)f\right]^{1/2}}}$$

+ subsonic flow
− supersonic flow

& $\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}$ & since M_2 known, now find all other conditions at (2) (or any variance!)

→ There are P_t losses ('Rayleigh losses' due to q added at $M \neq 0$ & these P_t losses get larger the bigger M is!! (quite large for supersonic flow)

'Fanno Flow': 1-D, adiabatic, with friction
[you review!]

$\boxed{\text{Engine and thermal choking (\& friction choking)}}$

See your AE 4535 notes!!

Rayleigh flow shows the 'thermal choking' issue very well; heat addition (say exothermic model for fuel combustion) always drives Mach toward 1.0 (on either subsonic or supersonic locus)
If you try to add more heat anyway (beyond thermal choking), conditions at (1) will necessarily change! (different problem)

AE5535

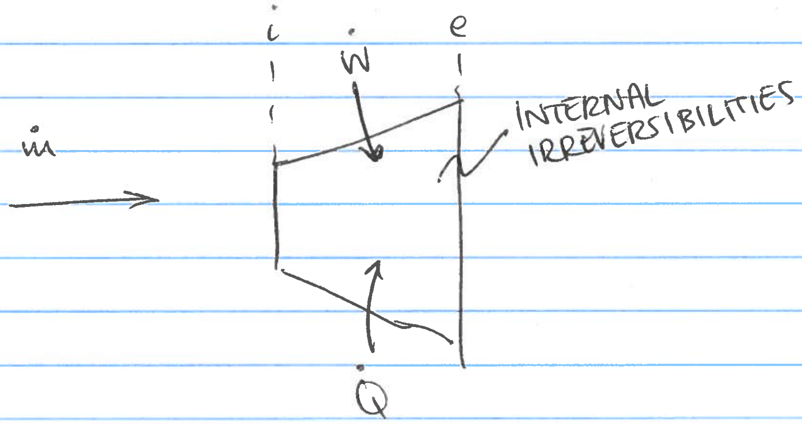
CYCLE ANALYSIS OF JET ENGINES (REVIEW)

BASED ON QUASI-1D : FLOW DIRECTION THRU FLOW PATH IS ALIGNED WITH X-AXIS. ($\vec{V} \rightarrow u$)

CONSIDER ENGINE / ENGINE COMPONENT (GENERAL)

$\dot{W}$ = SHAFT WORK RATE ACROSS BOUNDARY

$\dot{Q}$ = HEAT RATE ACROSS BOUNDARY



$P_t, T_t \Rightarrow$ TOTAL CONDITIONS

* HOW DO P_t, T_t VARY IN DUCT? WHAT DRIVES CHANGE? NOTE STATIC P, T, etc. CHANGE DUE TO AREA CHANGE (EVEN IF $\dot{Q}=0$, $\dot{W}=0$, NO IRREVERSIBILITIES).

DEFINE WORK PER MASS (LOWER CASE) w

" HEAT " " " " q

$$w_{i\to e} = \frac{\dot{W}}{\dot{m}} \quad , \quad q_{i\to e} = \frac{\dot{Q}}{\dot{m}}$$

$$\boxed{\frac{P_{t,e}}{P_{t,i}} = \exp\left[\int_i^e \frac{\delta w}{RT_t} - \frac{S_{\text{IRR / NON IDEAL HEAT TRANSFER}}}{R}\right]}$$

where δw = DIFFERENTIAL WORK INTERACTION

$$w_{i\to e} = \int_i^e \delta w$$

NON-IDEAL HEAT INTERACTION OCCURS WHEN HEAT INTERACTION OCCURS AT A FINITE (NON-ZERO) MACH NUMBER ($T < T_t$)

$$ds = \frac{\delta q}{T}$$ $\Rightarrow$ MOST IDEAL TO ADD HEAT AT HIGHEST TEMP WHICH OCCURS AT LOWEST VELOCITIES, M

$$\boxed{\frac{T_{t,e}}{T_{t,i}} = 1 + \frac{q_{i\to e} + w_{i\to e}}{C_p T_{t,i}}}$$

FROM THE ENERGY EQUATION

$$C_p(T_{t,e} - T_{t,i}) = q_{i\to e} + w_{i\to e}$$

* THE DRIVERS :

- P_t CHANGES OCCUR DUE TO WORK INTERACTIONS, INTERNAL IRREVERSIBILITIES, NON-IDEAL HEAT INTERACTION
- T_t CHANGES OCCUR <u>ONLY</u> DUE TO ENERGY INTERACTIONS ACROSS BOUNDARY (NO DIFFERENCE BETWEEN HEAT AND WORK)

<u>DEFINE</u> : $\boxed{\pi = \frac{P_{t,e}}{P_{t,i}}}$; $\boxed{\tau = \frac{T_{t,e}}{T_{t,i}}}$

* COMMONLY USED ENGINE SUBSCRIPTS :

d $\longrightarrow$ DIFFUSER (INLET)

c $\longrightarrow$ COMPRESSOR (MECHANICAL)

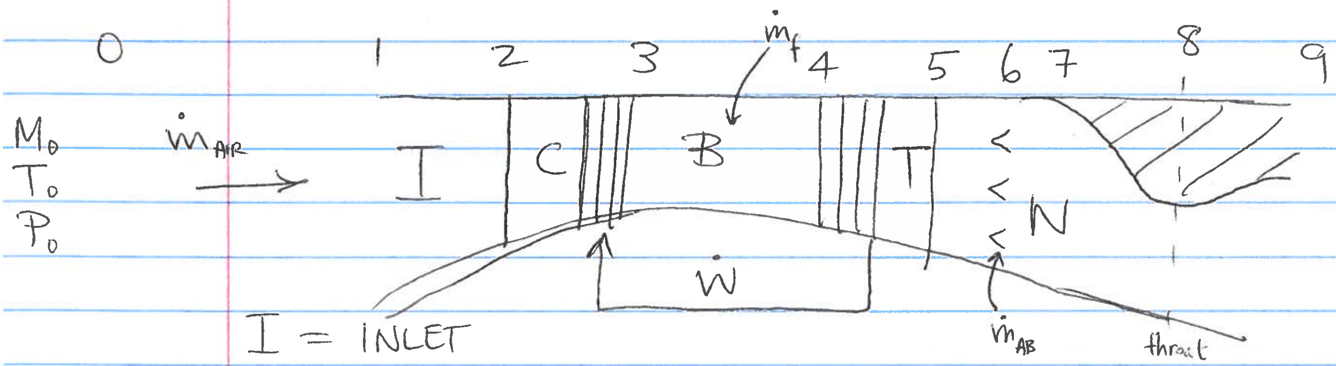
c' OR 'FAN' OR F $\longrightarrow$ FAN (LOW PRESSURE COMPRESSOR)

b $\longrightarrow$ BURNER

t $\longrightarrow$ TURBINE

n $\longrightarrow$ NOZZLE

SCHEMATIC 'IDEAL' TURBOJET & NUMBERING (AIAA STANDARD)



I = INLET

C = COMPRESSOR

B = BURNER

T = TURBINE

N = NOZZLE

$$T_{t,0} = T_0 \left(1 + \frac{\gamma - 1}{2} M_0^2\right)$$

T_{t4}, $\pi_c = \frac{P_{t3}}{P_{t2}}$ ARE GENERALLY PRESCRIBED

h = 'HEATING VALUE' OF FUEL [J/kg] 4.5×10^7

☆ IDEAL TURBOJET (FLOWPATH RULES)

$\pi_d = 1.0$, $\tau_d = 1.0$

$\pi_c = \tau_c^{\frac{\gamma}{\gamma-1}}$ ISENTROPIC

$\pi_b = 1.0$, $\pi_t = \tau_t^{\frac{\gamma}{\gamma-1}}$, $\pi_n = \tau_n = 1.0$

$\dot{m}_f = \dot{m}_{fuel}$,

WE HAVE TO HAVE $P_0 = P_9$ MATCHED

[NO CONIDERATION OF ADDITIVE OR EXTERNAL DRAG.

ALSO, $\dot{m}_f \ll \dot{m}_{AIR}$

f = FUEL-TO-AIR RATIO $= \frac{\dot{m}_f}{\dot{m}_{AIR}} \ll 1$

$f \approx 0.02$, $f_{STOICH} \cong 0.06$

ALSO, $C_p, \gamma, R \Rightarrow$ CONSTANT

$\boxed{\text{ANALYSIS OF AN IDEAL ENGINE}}$

TURBINE – COMPRESSOR POWER BALANCE

$$\dot{m}_{AIR} C_p (T_{t3} - T_{t2}) = (\dot{m}_{AIR} + \dot{m}_f) C_p (T_{t4} - T_{t5})$$

BUT $\dot{m}_f \ll \dot{m}_{AIR}$ SO ...

$$\tau_t = \frac{T_{t5}}{T_{t4}} = 1 - \frac{\tau_r}{\tau_\lambda} (\tau_c - 1)$$

where $\tau_r = \frac{T_{t0}}{T_0} = \frac{T_{t2}}{T_0} = 1 + \frac{\gamma - 1}{2} M_0^2$

AND $\tau_\lambda = \frac{C_{p4} T_{t4}}{C_{p0} T_0} = \frac{T_{t4}}{T_0}$

ALSO $\pi_r = \frac{P_{t0}}{P_0} = \left(1 + \frac{\gamma - 1}{2} M_0^2\right)^{\frac{\gamma}{\gamma - 1}}$

BURNER ENERGY EQUATION:

$$\dot{m}_f h = (\dot{m}_{AIR} + \dot{m}_f) C_p T_{t4} - \dot{m}_{AIR} C_p T_{t3}$$

$$\Rightarrow f = \frac{\dot{m}_f}{\dot{m}_{AIR}} = \frac{C_p T_0}{h} (\tau_\lambda - \tau_r \tau_c) \qquad [\text{FOR } f \ll 1]$$

AE5535

2 ENGINE PERFORMANCE PARAMETERS :

a) SPECIFIC THRUST $= \dfrac{F}{\dot{m}_{AIR}}$

WHERE 'F' IS UNINSTALLED THRUST

$$F = (\dot{m}_{AIR} + \dot{m}_f)\, u_9 - \dot{m}_9 u_0 + (P_9 - P_0) A_9$$

IDEALLY, $P_9 = P_0$ AND $f << 1$

fuel air ratio

$$\therefore F_{IDEAL} = \dot{m}_{AIR}(u_9 - u_0)$$

$$\therefore \frac{F_{IDEAL}}{\dot{m}_{AIR}} = u_9 - u_0$$

b) SPECIFIC FUEL CONSUMPTION $= S = \dfrac{\dot{m}_f}{F}$

$$\longrightarrow S = \frac{f}{(F/\dot{m})} \times 10^6 \quad \left[\frac{mg}{N \cdot sec}\right]$$

$$\left(\tau_\lambda = \frac{T_{t4}}{T_0}, \quad \tau_r = \frac{T_{t0}}{T_0} = 1 + \frac{\gamma - 1}{2} M_0^2, \quad \cdots \right)$$

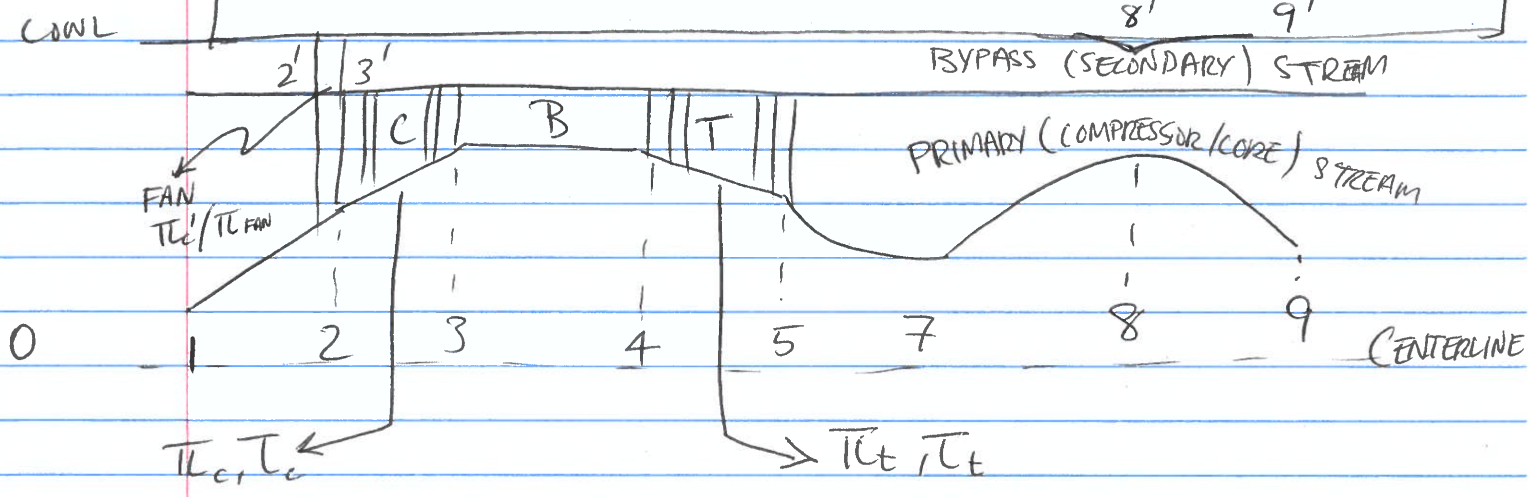
FOR INSTANCE, FOR IDEAL RAMJETS : $\pi_c = \tau_c = 1.0$

$\pi_t = \tau_t = 1.0$

RAMJET $\dashrightarrow \dfrac{F}{\dot{m}} = a_0 M_0 \left[\sqrt{\dfrac{\tau_\lambda}{\tau_r}} - 1\right]$; $a_0 = \sqrt{\gamma R T_0}$

* ALLOWS PARAMETRIC STUDIES OF ENGINES (i.e. FIX M T_{t4}, T_0, h AND VARY π_c TO STUDY IMPACT OF COMPRESSOR SIZE ON $\frac{F}{\dot{m}}$, S)

IDEAL TURBOFAN WITH SEPERATE EXHAUST STREAMS



- OUTER OR BYPASS STREAM DESIGNATED BY 'FAN', 'F', OR C'
- COMPRESSOR OR CORE STREAM DESIGNATED BY C (SOMETIMES 'P' FOR PRIMARY)

FOR THE IDEAL TURBOFAN:

$$\pi_c' = \tau_c'^{\frac{\gamma}{\gamma-1}} \quad \text{OR} \quad \pi_{FAN} = \tau_{FAN}^{\frac{\gamma}{\gamma-1}}$$

$$\alpha \text{ (OR } \beta) = \text{BYPASS RATIO} = \frac{\dot{m}_{FAN}}{\dot{m}_{COMPRESSOR}}$$

TOTAL CONDITIONS AT 2' SAME AS 2

TURBINE - COMPRESSOR - FAN POWER BALANCE:

$$\underbrace{(\dot{m}_c+\dot{m}_f)\, C_{p_4} T_{t_4} - (\dot{m}_c+\dot{m}_f)\, C_{p_5} T_{t_5}}_{\text{TURBINE POWER MAGNITUDE}}$$

$$= \underbrace{\dot{m}_c C_{p_0}(T_{t_3} - T_{t_2})}_{\text{COMPRESSOR POWER}} + \dot{m}_{FAN} C_{p_0} (T_{t_3'} - T_{t_2'})$$

C_p = CONST FOR IDEAL

$$\boxed{\begin{aligned} F_{\substack{TURBO\\FAN\,(IDEAL)}} &= \dot{m}_c(u_9 - u_0) + (P_9 - P_0)A_9 \\ &+ \dot{m}_F(u_9' - u_0) + (P_9' - P_0)A_9' \end{aligned}}$$

IDEAL Turbofan (see next page)

SOLUTION TO HANDOUT EXAMPLE: (DIRECT ANALYSIS)

a) $\dot{m}_f h = \dot{m}_c C_p (T_{t4} - T_{t3})$

b) $\dot{m}_c C_p (T_{t3} - T_{t2}) + \dot{m}_F C_p (T_{t3}' - T_{t2}') = \dot{m}_c C_p (T_{t4} - T_{t5})$

where $T_{t2} = T_{t2}'$

$$\therefore \boxed{T_{t5} = T_{t4} - T_{t3} - 3 \cdot T_{t3}' + 4 T_{t2}}$$

c) $P_{t2}' = P_{t0} = P_0 \left(1 + \frac{\gamma-1}{2} M_0^2\right)^{\frac{\gamma}{\gamma-1}} = 191{,}801\ Pa$

$T_{t0} = T_{t2}' = T_0 \left(1 + \frac{\gamma-1}{2} M_0^2\right) = 345.6\ K$

$P_{t3}' = P_{t2}' \pi_{FAN} = 575\ 403\ Pa = P_{t9}'$

$T_{t3}' = T_{t2}' (\pi_{FAN})^{\frac{\gamma-1}{\gamma}} = 473\ K = T_{t9}'$

d) $P_{t2} = P_{t0} = P_0 \left(1 + \frac{\gamma-1}{2} M_0^2\right)^{\frac{\gamma}{\gamma-1}} = 191{,}801\ Pa$

$T_{t2} = T_{t0} = T_0 \left(1 + \frac{\gamma-1}{2} M_0^2\right) = 345.6\ K$

$P_{t3} = P_{t2} \pi_c = 5\ 754\ 030\ Pa = P_{t4}$

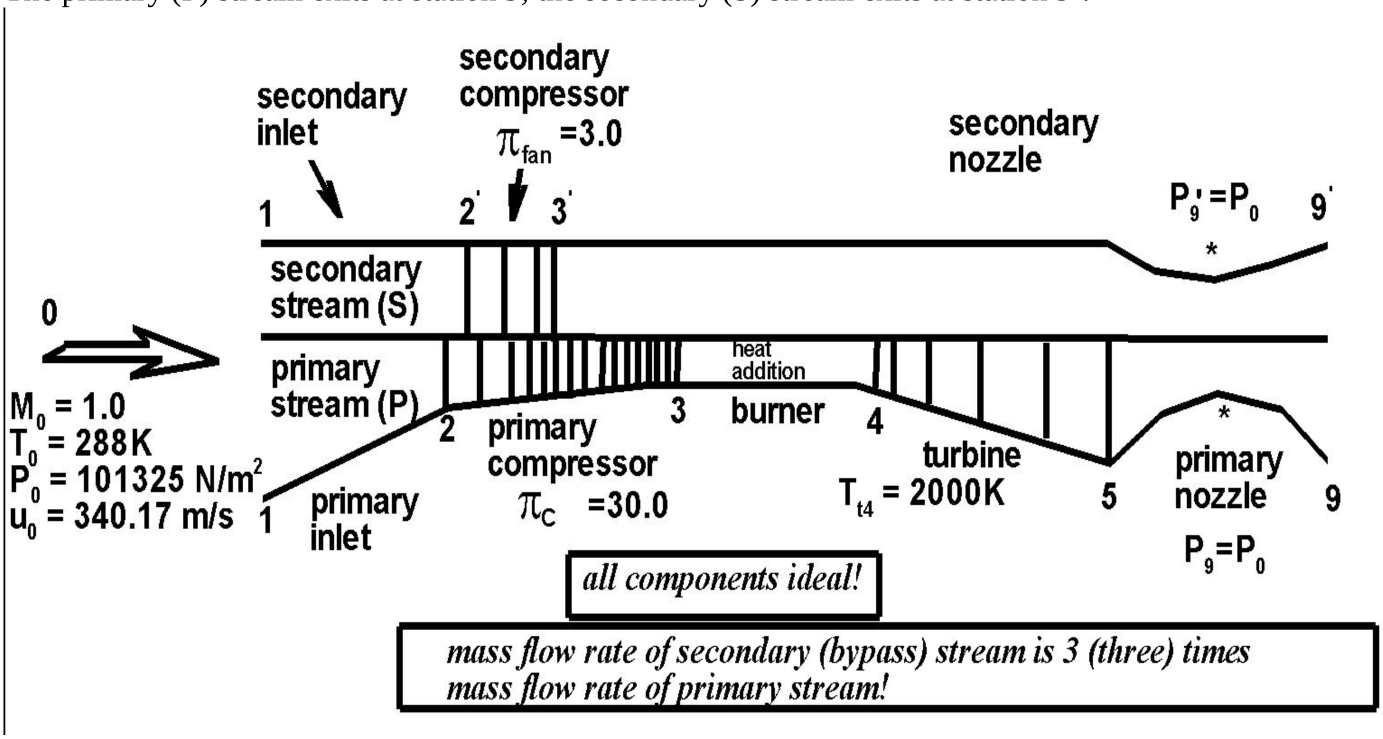
$T_{t3} = T_{t2} (\pi_c)^{\frac{\gamma-1}{\gamma}} = 913.3\ K$

$T_{t5} = 1050\ K = T_{t9}$

$P_{t5} = P_{t4} \left(\frac{T_{t5}}{T_{t4}}\right)^{\frac{\gamma}{\gamma-1}} = 683\ 275\ Pa = P_{t9}$

AE 5535 **Review: Direct analysis of ideal engine (turbofan)**

Consider an engine composed of two streams; a primary (inner or core) stream and a secondary (outer or bypass) stream (see the sketch below). In the primary stream (often identified using a 'P' or sometimes a 'c' – for compressor) there is a compressor (stations 2 to 3), burner (3 to 4), and a turbine (4 to 5). In the secondary stream (designated 'fan' or sometimes 'S' or even c^1) there is a fan which is a low-pressure ratio compressor (located from stations 2' to 3' (where the ' notation - the prime notation - on stations simply designate the secondary (S) stream). The primary (P) stream exits at station 9; the secondary (S) stream exits at station 9'.



The turbine supplies power to BOTH compressor and fan. All components are ideal (including ideally expanded nozzles). NOTE THAT THE AIR MASS FLOW RATE PROCESSED BY THE SECONDARY 'S' STREAM IS 3 TIMES (3X) THE AIR MASS FLOW RATE PROCESSED BY THE PRIMARY 'P' STREAM.

The engine is operating at flight Mach, $M_0 = 1.0$, $T_0 = 288K$, $P_0 = 101,325\ N/m^2$.

$$\frac{P_{t3'}}{P_{t2'}} = \pi_{fan} = 3.0 \qquad \frac{P_{t3}}{P_{t2}} = \pi_C = 30.0 \qquad T_{t4} = 2000K$$

$$h = 4.42x10^7\ J/kg(fuel) \quad C_P = 1004.5\ J/kg-K \quad R = 287\ J/kg-K$$

$$\gamma = 1.4 \qquad P_9 = P_{9'} = P_0 \qquad \dot{m}_{fan} = 3\dot{m}_{C(\text{Primary})}$$

a) write the expression for the burner enthalpy balance (using *h* and mass flow rates of air and fuel)

b) write the expression for the power balance between the compressors and the turbine

c) calculate $P_{t2'}, T_{t2'}, P_{t3'}, T_{t3'}, P_{t9'}, T_{t9'}$ (i.e. total pressures, total temperatures in secondary stream, S)

d) calculate $P_{t2}, T_{t2}, P_{t3}, T_{t3}, P_{t4}, P_{t5}, T_{t5}, P_{t9}, T_{t9}$ (i.e. total pressure, total temperatures in primary stream, P)

e) find M_9 and $M_{9'}$

f) find T_9 and $T_{9'}$

g) find u_9 and $u_{9'}$

h) find the specific thrust of this engine

i) find the specific fuel consumption of this engine

* IT'S TYPICAL FOR A JET ENGINE TO HAVE TOTAL CONDITIONS AT EXHAUST THAT ARE HIGHER THAN FREE STREAM TOTAL CONDITIONS COMING IN.

e)
$$\frac{P_{t9}}{P_9} = \frac{P_{t9}}{P_0} = \left(1+\frac{\gamma-1}{2}M_9^2\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow M_9 = 1.823$$

$$\frac{P_{t9}'}{P_9'} = \frac{P_{t9}'}{P_0} = \left(1+\frac{\gamma-1}{2}M_9'^2\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow M_9' = 1.79$$

f)
$$T_9 = \frac{T_{t9}}{\left(1+\frac{\gamma-1}{2}M_9^2\right)} = 631\,K$$

$$T_9' = \frac{T_{t9}'}{\left(1+\frac{\gamma-1}{2}M_9'^2\right)} = 288\,K = T_0$$

AS EXPECTED FOR THIS CASE (IDEAL)

g)
$$u_9 = M_9\sqrt{\gamma R T_9} = 918 \text{ m/s}$$

$$u_9' = M_9'\sqrt{\gamma R T_9'} = 610 \text{ m/s}$$

h)
$$\frac{F}{\dot{m}_{AIR,TOT}} = \frac{\dot{m}_c(u_9-u_0)+\dot{m}_F(u_9'-u_0)}{\dot{m}_c+\dot{m}_F}$$

$$\dot{m}_{AIR,TOT} = (\alpha+1)\dot{m}_c$$

$$F/\dot{m}_{tot} = 346.6 \ \frac{N\cdot s}{kg}$$

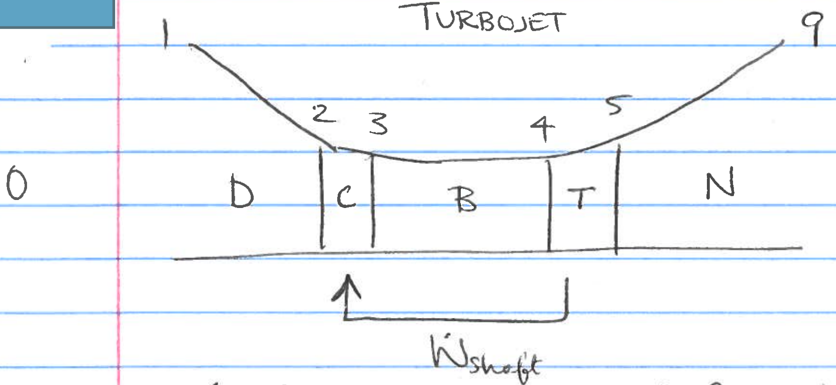
i)
$$S = \frac{\dot{m}_f}{F} = \frac{\dot{m}_f/\dot{m}_{TOT}}{F/\dot{m}_{TOT}} = \frac{\dot{m}_f/\dot{m}_c}{(1+\alpha)\,F/\dot{m}_{tot}} = \frac{f}{(1+\alpha)\,F/\dot{m}_{tot}} \times 10^6$$

$$f = \frac{c_p(T_{t4}-T_{t3})}{h}$$

$$\therefore S = 17.82 \ \frac{mg}{N\cdot s}$$

NON-IDEAL ENGINE EFFECTS

TURBOJET



$\leftarrow \gamma_c, C_{p_c} \rightarrow$ $\quad$ $\leftarrow \gamma_t, C_{p_t} \rightarrow$

R $\quad$ R $\quad$ R $\rightarrow$ DOESN'T CHANGE APPRECIABLY (FUEL LEA

COULD HAVE $\gamma_{AB}, C_{p_{AB}}$ IF WE HAVE AB

P_9 = NOZZLE EXIT PRESSURE $\neq P_0$

$\Rightarrow$ INCLUDE P TERMS IN THRUST

DON'T ASSUME $\dot{m}_f \ll \dot{m}_a$ OR $f \ll 1$ (KEEP IT, THAT IS)

* INLET $\pi_d < 1.0$, $\tau_d = 1.0$ [UNLESS VERY HIGH SPEEDS]
* COMPRESSOR/FAN $\pi_c = \tau_c^{\frac{e_c \gamma_c}{\gamma_c - 1}}$: e_c = POLYTROPIC COMPRESSOR EFFICIENCY

OR $\tau_c = 1 + \dfrac{\pi_c^{\frac{\gamma_c - 1}{\gamma_c}} - 1}{\eta_c}$: η_c = GENERAL COMPRESSOR EFFICIENC

e_c', η_c' FOR FAN

* BURNER $\pi_b < 1.0$ DUE TO IRREVERSIBILITIES AND NON-IDEAL HEA

η_b = BURNER EFFICIENCY $\quad \eta_b = \dfrac{(\dot{m}_a + \dot{m}_f) C_{p_t} T_{t4} - \dot{m}_a C_{p_c} T_{t3}}{\dot{m}_f h}$

* TURBINE $\pi_t = \tau_t^{\frac{\gamma_t}{(\gamma_t - 1) e_t}}$: e_t = POLYTROPIC TURBINE EFFICIENCY

$\tau_t = 1 - \eta_t \left[1 - \pi_t^{\frac{\gamma_t - 1}{\gamma_t}} \right]$: η_t = GENERAL TURBINE EFFICIENCY

* NOZZLE $\pi_N < 1.0$, $\tau_N = 1.0$

MECHANICAL EFFICIENCY η_M:
(DUE TO LOSSES) IN SHAFT WORK AND/OR AUXILARY POWER SUPPLY TO SUBSYSTEMS

$$\eta_M C_{P_t} (\dot{m}_a + \dot{m}_f)(T_{t4} - T_{t5}) = C_{P_c} \dot{m}_a (T_{t3} - T_{t2})$$

CHAIN P_t, T_t THROUGH THE ENGINE TO GET $\frac{F}{\dot{m}}$ & S

IF P_9, P_9' GIVEN ; FIND M_9 (ALSO KNOW P_{t9}, T_{t9})

SUPPOSE A_9, A_9', $\dot{m}_9$ GIVEN INSTEAD $\Rightarrow$ FIND M_9

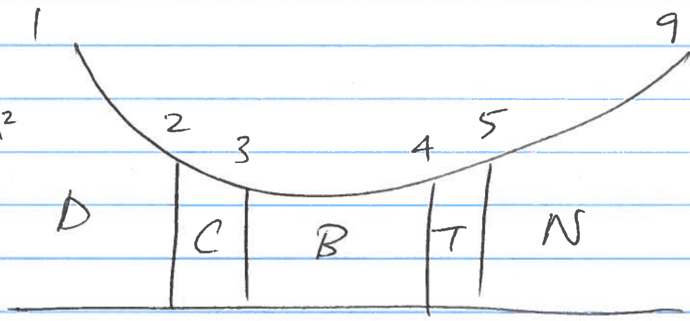
$$\dot{m} = \frac{A P_t}{\sqrt{T_t}} \sqrt{\frac{\gamma}{R}}\, M \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} \longrightarrow \text{FIND } M$$

IF $A_9 = A_8 = A_{throat}$ (ONLY CONVERGENT NOZZLE)
THEN $M_9 = M_8 = 1.0$

EXAMPLE OF NON-IDEAL TURBOJET

$M_0 = 1.0$
$T_0 = 240K$
$P_0 = 10\,000\ N/m^2$

RIGGING' ALTITUDE (DOESN'T CORRESPOND TO TABLES)



$\frac{P_9}{P_0} = 1.1$

GIVEN

$T_{t4} = 1800K$
$\pi_c = 20$
$\pi_d = .98$
$e_c = 0.9$
$\pi_b = .98$
$e_t = 0.9$
$\eta_M = 0.95$

$\pi_N = .99$
$h = 4.42 \times 10^7\ J/kg$
$C_{p_t} = 1098.2\ J/kg$
$C_{p_c} = 996.5\ J/kg$
$\gamma_t = 1.35$
$\gamma_c = 1.4$

ANALYSIS:

$$T_{t_0} = T_0\left(1 + \frac{\gamma - 1}{2} M_0^2\right) = 288K$$

$$P_{t_0} = 18\,930\ Pa$$

$$\frac{T_{t3}}{T_{t2}} = \tau_c = \pi_c^{\frac{\gamma_c - 1}{\gamma_c e_c}} = 2.588$$

$$T_{t_2} = T_{t_0} = 288K \Rightarrow T_{t_3} = T_{t_2}\tau_c = 745.5K$$

$$P_{t_2} = P_{t_0}\pi_d = 18\,551\ Pa$$

$$P_{t_3} = P_{t_2}\pi_c = 371\,014\ Pa$$

BURNER ENERGY BALANCE: (ENTHALPY)

$$(\dot{m}_a + \dot{m}_f) C_{p_t} T_{t_4} - \dot{m}_a C_{p_c} T_{t_3} = \dot{m}_f h \eta_b$$ DIVIDE BY $\dot{m}_a$

$$f = \frac{\dot{m}_f}{\dot{m}_a} = \frac{C_{p_t} T_{t_4} - C_{p_c} T_{t_3}}{\eta_b h - C_{p_t} T_{t_4}} = 0.0298$$

$$P_{t_4} = P_{t_3}\pi_b = 363\,594\ Pa$$

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TURBINE / COMPRESSOR POWER BALANCE:

$$\dot{m}_a C_{p_c} (T_{t_3} - T_{t_2}) = (\dot{m}_a + \dot{m}_f) C_{p_t} (T_{t_4} - T_{t_5}) \eta_M$$

$$\Rightarrow T_{t_5} = T_{t_4} - \frac{C_{p_c}(T_{t_3} - T_{t_2})}{\eta_M (1+f) C_{p_t}} = 1375.7\ K$$

$$\left(\frac{P_{t_5}}{P_{t_4}}\right) = \left(\frac{T_{t_5}}{T_{t_4}}\right)^{\frac{\gamma_t}{(\gamma_t - 1) e_t}} \Rightarrow \pi_t = 0.316$$

$$P_{t_5} = \pi_t P_{t_4} = 114\,893\ Pa$$

$$P_{t_9} = P_{t_5} \pi_N = 113\,744\ Pa$$

$$T_{t_9} = T_{t_5} \tau_N = T_{t_5} = 1375.7 K$$

$$P_9 = P_0 (1.1) = 11\,000\ Pa$$

$$\frac{P_{t_9}}{P_9} = \left(1 + \frac{\gamma_t - 1}{2} M_9^2\right)^{\frac{\gamma_t}{\gamma_t - 1}} \Rightarrow M_9 = 2.181$$

$$T_9 = T_{t_9}\left(1 + \frac{\gamma_t - 1}{2} M_9^2\right)^{-1} = 750.76 K$$

$$u_9 = M_9 \sqrt{\gamma_t R T_9} = 1171.6\ m/s$$

$$\frac{F_x}{\dot{m}_a} = (1+f) u_9 - u_0 + \frac{(P_9 - P_0) A_9}{\dot{m}_a}$$

where $\frac{(P_9 - P_0) A_9}{\dot{m}_a} = \frac{(P_9 - P_0)}{P_9}$ ---

$$\left(1 - \frac{P_0}{P_9}\right) \frac{\rho_9 R T_9 A_9 u_9}{\dot{m}_a u_9} \quad --- \quad \left(1 - \frac{P_0}{P_9}\right)(1+f) R \frac{T_9}{u_9}$$

$$\therefore \frac{F}{\dot{m}_a} = 914.4\ \frac{N \cdot s}{kg}$$

$$S = \frac{f \times 10^6}{F/\dot{m}_a} = 32.6\ \frac{mg}{N \cdot s}$$

FOR <u>TURBOFANS</u>:

WE CAN FIND AN ABSOLUTE MINIMUM SPECIFIC FUEL CONSUMPTION.

IN $(\underbrace{\pi_c, \pi_c'}_{\text{FIND BEST COMBINATION}}, \alpha)$ SPACE

(OPTIMIZATION)

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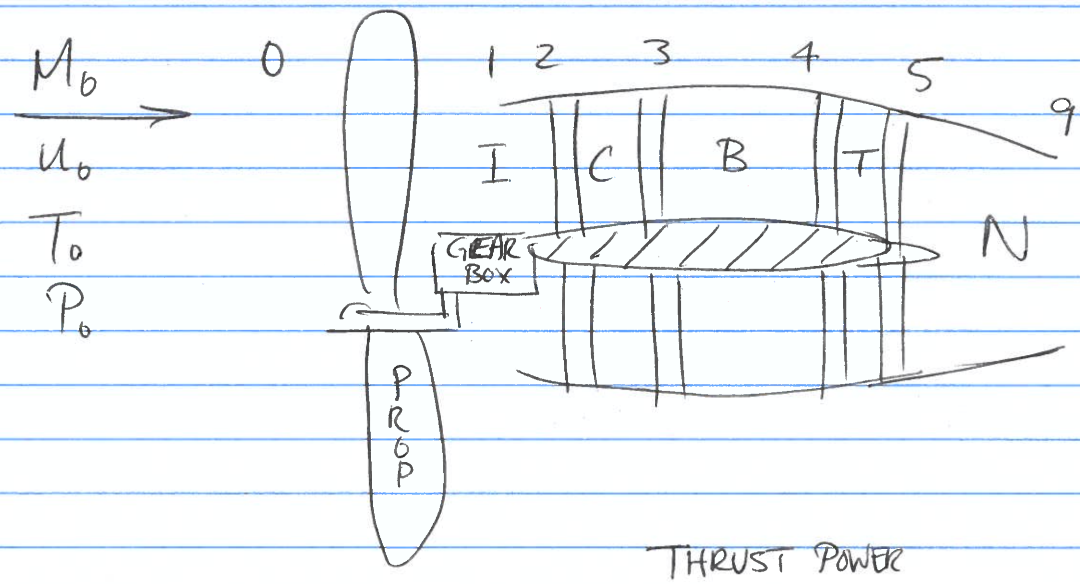
TURBOPROP ANALYSIS



THRUST POWER

$*$ PROPULSIVE EFFICIENCY $= \eta_p = \dfrac{\text{THRUST POWER}}{\text{RATE CHANGE IN KE ACROSS ENGINE}}$

$$\eta_p = \frac{F_x u_0}{\frac{1}{2}\dot{m} u_e^2 - \frac{1}{2}\dot{m} u_0^2}$$

(RECALL : $F_x = \dot{m}(u_9 - u_0)$ SUBSONIC)

WITH ASSUMPTIONS $\eta_p = \dfrac{2u_0}{u_9 + u_0}$ IMPLIES : $u_9 \to u_0$, $\eta_p \to 1.0$

WITH A FIXED F_x REQUIREMENT $\to$ WE WANT TO PROCESS LARGE $\dot{m}$ IF $u_9 \approx u_0$

$*$ PROPELLER EFFICIENCY $= \eta_{prop} = \dfrac{F_x u_0 \text{ (THRUST POWER DELIVE}}{\text{SHAFT POWER TO PROP (MECHANICAL FROM GEAR BOX)}}$

$$\eta_{prop} = \frac{F_x u_0}{\dot{W}_{prop}}$$

NOTES :

* SOME TIMES (MOST OF THE TIME), TURBINE HAS 'SPOOLING' SUCH THAT A SEPERATE SEGMENT OF THE TURBINE RUNS THE PROP (COMMON FOR GAS TURBINES IN GENERAL FOR REDUNDANCY, STATIC STARTING)
* DUE TO THE LARGE DIAMETER OF PROPS, IT MUST GENERALLY ROTATE AT LOWR RPMs THAN CORE SHAFT. TO MINIMZE BLADE STRESSES ESPECIALLY WHEN IT REACHES TRANSONIC $\longrightarrow$ WE NEED GEAR BOX.
* TURBOPROPS LIMITED TO $M_0 \lesssim 0.6$ OR 0.7 DUE TO BLADE TIP SPEEDS.
* CAN BE VERY FUEL EFFICIENT.
* TURBOSHAFT.

CYCLE ANALYSIS NON-IDEAL TURBOPROP (SINGLE-SPOOL

THRUST $\longrightarrow$ HAS TWO CONTRIBUTIONS, ONE FROM CORE STREAM, ONE FROM PROP STREAM.

a) CORE STREAM WILL HAVE SAME EXPRESSION FOR $\frac{F_{core}}{\dot m_{core}}$ INSTEAD OF $\frac{F_{core}}{\dot m_{tot}}$ (LIKE TURBOJET)

$$\frac{F_{core}}{\dot m_c} = a_0\left[(1+f)M_0\frac{u_9}{u_0} - M_0\right]$$

ASSUME $P_9 = P_0$ (NOT BAD ASSUMPTION)

$$\frac{\dot m_f}{\dot m_c} = f = \frac{\tau_\lambda - \tau_r \tau_c}{\frac{\eta_b h}{C_{pc} T_0} - \tau_\lambda}$$

$$: \tau_\lambda = \frac{T_{t4} C_{pt}}{T_0 C_{pc}}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2$$

$$\pi_r = \tau_r^{\frac{\gamma_c}{\gamma_c - 1}} \quad , \quad \tau_c = \pi_c^{\frac{\gamma_c - 1}{\gamma_c e_c}}$$

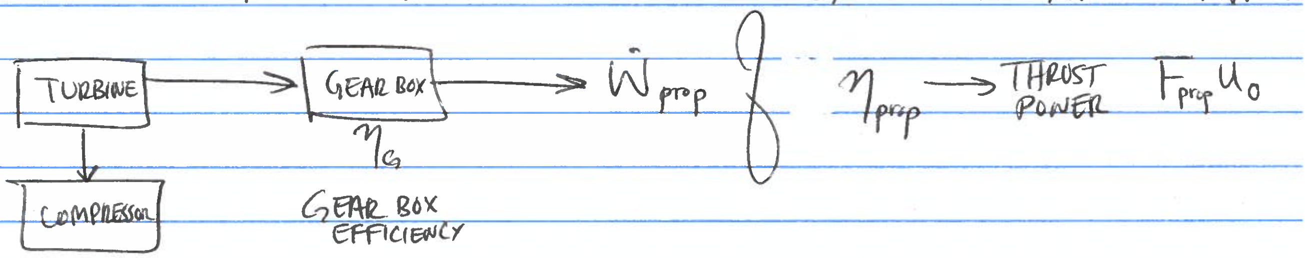
FROM THE TURBOJET ANALYSIS (CHAIN P_t, T_t THROUGH) CAN SHOW:

$$\boxed{M_0 \frac{u_9}{u_0} = \left\{ \frac{2\tau_\lambda}{\gamma_c - 1} \left[\tau_t - \frac{\tau_t^{\frac{-(1-e_t)}{e_t}}}{(\pi_r \pi_d \pi_c \pi_b \pi_n)^{\frac{\gamma_t - 1}{\gamma_t}}} \right] \right\}^{1/2}}$$

DUE TO 'RAYLEIGH LOSS

b) PROPELLER STREAM : FIND $\dot{W}_{prop}$

TURBINE SPLITS POWER BETWEEN GEARBOX AND COMPRESSOR



POWER TO GEARBOX:

$$\underbrace{\dot{m}_c (1+f) \eta_m c_{p_t} (T_{t4} - T_{t5})}_{\text{TURBINE PWR MAGNITUDE}} - \underbrace{\dot{m}_c c_{p_c} (T_{t3} - T_{t2})}_{\text{COMP PWR}}$$

$$\dot{W}_{prop} = \dot{m}_c c_{p_c} T_0 \eta_G \left[\eta_m (1+f) \tau_\lambda (1 - \tau_t) - \tau_r (\tau_c - 1) \right]$$

Then $\eta_{prop} = \dfrac{F u_0}{\dot{W}_{prop}} \Rightarrow F u_0 = \eta_{prop} \dot{W}_{prop}$

$$\frac{F_{prop}}{\dot{m}_c} = \frac{\eta_{prop} \eta_G c_{p_c} T_0 \left[\eta_m (1+f) \tau_\lambda (1 - \tau_t) - \tau_r (\tau_c - 1) \right]}{u_0}$$

at $u_0 = 0$, $\eta_{prop} = 0$ So $\dfrac{F_{prop}}{\dot{m}_c}$ DOESN'T GO TO ∞ AT u_0

$$\boxed{\left.\frac{F}{\dot{m}_{core}}\right)_{tot} = \frac{F_{core}}{\dot{m}_c} + \frac{F_{prop}}{\dot{m}_c}} \Rightarrow \boxed{S = \frac{f}{(F/\dot{m}_c)_{tot}}}$$

τ_t IS EITHER PROVIDED OR IS FOUND BY REQUIRING S TO BE MINIMAL.

SET $\dfrac{\partial S}{\partial \tau_t} = 0$ TO GET τ_t^*

WHICH IS τ_t AT MINIMUM S

DO THIS AND OBTAIN (FOR $\gamma = 1.4$):

$$\tau_t^* = \left\{ \frac{1}{K} \tau_t^{*\,\frac{e_t - 1}{e_t}} \right\} + \left\{ \frac{M_0^2 \left[1 + \frac{(1-e_t)}{e_t} \frac{1}{K} \tau_t^{*\,-\frac{1}{e_t}} \right]^2}{5\,\tau_\lambda (\eta_{prop} \eta_G \eta_m)^2} \right\}$$

WHERE $K = (\pi_r \pi_d \pi_c \pi_b \pi_n)^{\frac{\gamma_t - 1}{\gamma_t}}$

YOU NEED TO FIND τ_t^* THAT SATISFIES THE EQN.

THE BYPASS RATIO α DOES NOT DIRECTLY APPEAR BECAUSE ONCE τ_t IS DETERMINED, THEN THE WORK SPLIT BETWEEN THE COMPRESSOR AND PROPELLER IS FIXED AND THE SIZE OF THE PROPELLER (DIAMETER, NUMBER OF BLADES, etc) DETERMINE α.

☆ OPTIMIZED TURBOFANS CAN HAVE ~15% HIGHER S THAN AN OPTIMIZED TURBOPROP FOR LOW SUBSONIC FLIGHT.

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EVALUATING $\frac{P_9}{P_0}$ OR $\frac{P_9'}{P_0}$ IN CYCLE ANALYSIS

IN USUAL SUBSONIC TRANSPORT APPLICATIONS WITH TURBOJET OR SEPERATE STREAM TURBOFAN OR TURBOPRO[P] WITH NO AFTERBURNER, YOU CAN OFTEN GET AWAY WITH A SIMPLE CONVERGING NOZZLE SUCH THAT M_9 OR M_9' ARE UNITY $(M_9 = 1.00)$

IF THAT'S TRUE, BY DEFINITION $\frac{P_{t9}}{P_9} = \left(\frac{\gamma_t + 1}{2}\right)^{\frac{\gamma_t}{\gamma_t - 1}}$

AND $\frac{P_{t9}'}{P_9'} = \left(\frac{\gamma_c + 1}{2}\right)^{\frac{\gamma_c}{\gamma_c - 1}}$

AND THEN YOU KNOW $\frac{P_{t9}}{P_0} = \frac{P_{t0}}{P_0}\frac{P_{t2}}{P_{t0}}\frac{P_{t3}}{P_{t2}}\frac{P_{t4}}{P_{t3}}\frac{P_{t5}}{P_{t4}}\frac{P_{t9}}{P_{t5}}$

$$\therefore \frac{P_{t9}}{P_0} = \pi_r \pi_d \pi_c \pi_b \pi_t \pi_n$$

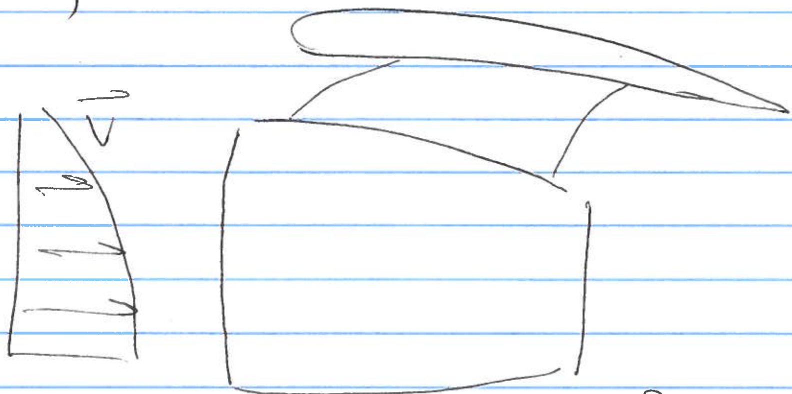
$$\frac{P_{t9}'}{P_0} = \pi_r \pi_d \pi_c' \pi_n'$$

$$\text{So } \frac{P_9}{P_0} = \frac{\pi_r \pi_d \pi_c \pi_b \pi_t \pi_n}{\left(\frac{\gamma_t + 1}{2}\right)^{\frac{\gamma_t}{\gamma_t - 1}}}$$

$$\frac{P_9'}{P_0} = \frac{\pi_r \pi_d \pi_c' \pi_n'}{\left(\frac{\gamma_c + 1}{2}\right)^{\frac{\gamma_c}{\gamma_c - 1}}}$$

ONE-DIMESIONALIZATION OF 3-D FLOW FIELDS

(FLOW AVERAGING)



IF WE WANT TO TAKE A 3D FLOW (SAY FROM CFD) AND PUT IT THROUGH CYCLE ANALYSIS, WE WANT TO ONE-DIMENSIONALIZE IT. HOW?
HOW TO OBTAIN THE BEST 1D PARAMETERS?
i.e. DESIRE 1-D 'EQUIVALENT' FLOW FIELD.

- TO USE FLUX-BASED FLOW RATE CONSERVATION PRINCIPLES / METHODS.

SOLVE FOR ρ_0, u_0, P_0, T_0 WHERE '0' SUBSCRIPT DESIGNATES 1-D PARAMETER

SO AT STATION OF INTEREST, MASS FLOW RATE:

CONTINUITY
$$\underbrace{\int_A \rho u \, dA}_{\text{KNOWN (3-D)}} = \rho_0 u_0 A$$

MOMENTUM
$$\int_A (\rho u^2 + P) \, dA = (\rho_0 u_0^2 + P_0) A$$

ENERGY (TOTAL ENTHALPY CONSERVED)
$$\int_A \rho u \left(C_p T + \frac{u^2}{2}\right) dA = \rho_0 u_0 \left(C_p T_0 + \frac{u_0^2}{2}\right) A$$

TO CLOSE THE SYSTEM
$$P_0 = \rho_0 R T_0$$

$\therefore$ 4 EQUATIONS, 4 UNKNOWNS, FIND: ρ_0, u_0, P_0, T_0

AND YOU ARE READY TO GO!

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[redacted]

ORDER OF PREFERENCE: OF ONE DIMENSIONALIZING FLOW

1. PREVIOUS METHOD (OR BETTER YET, DISTORTION METHODOLOGY)
2. MASS-AVERAGED --- $u_0 = \int \rho u^2 dA / \int \rho u dA$

   OR AREA-AVERAGED
3. OTHER

[redacted]