GP

Homework 2 **AE 5535**

Assigned: 2/15/2021
Due: 2/22/2021

An ideal fixed-area turbojet (FAT-jet) is operated on-design where $\pi_C = 15$, $M_0 = 0.8$, $T_0 = 260K$, $T_{t4} = 2000K$, and $P_0 = 20{,}000\ N/m^2$. Mass flow rate of air processed by this engine at on-design is 100 kg/sec.

What will be the performance of this engine (thrust and mass capture) compared to the on-design conditions if it is flown at a Mach of 0.3 and at an altitude where temperature and pressure are 288K and 101325 N/m^2, respectively. Furthermore, the fuel throttle is set such that fuel flow rate is 21.5% higher than the fuel flow rate at the on-design point. Assume that A_9 is varied to keep $P_9 = P_0$.

What is the ratio of the off to on-design A_9 required to maintain $P_9 = P_0$? Does this seem reasonable? If not, perhaps the analysis needs to be redone with the A_9 'fixed'. (Don't do it, just realize it).

$$\tau_{\lambda R} = \frac{T_{t4}}{T_0} = 7.692$$

$$\tau_{cR} = \pi_{cR}^{\frac{\gamma-1}{\gamma}} = 2.168$$

$$\tau_{rR} = 1 + \frac{\gamma-1}{2} M_{0R}^2 = 1.128$$

$$\tau_{tR} = 1 - \frac{\tau_{rR}}{\tau_{\lambda R}}(\tau_{cR}-1) = .829$$

$$T_{t2R} = T_{0R}\,\tau_{rR} = 293.28\,K$$

$$T_{t3R} = \tau_{cR}\,T_{t2R} = 635.782\,K$$

$$T_{t5R} = T_{t4} - T_{t3R} + T_{t2} = 1657.498\,K = T_{t9R}$$

$$\pi_{rR} = (\tau_{rR})^{\frac{\gamma}{\gamma-1}} = 1.524$$

$$\pi_{tR} = \tau_{tR}^{\frac{\gamma}{\gamma-1}} = .518$$

$$\frac{P_{t9R}}{P_{9R}} = \pi_r \pi_d \pi_c \pi_b \pi_t = \pi_r \pi_c \pi_t = 11.848$$

$$M_{9R} = \left\{\left[\left(\frac{P_{t9R}}{P_{9R}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\frac{2}{\gamma-1}\right\}^{1/2} = 2.266$$

$$T_{9R} = \frac{T_{t9R}}{1+\frac{\gamma-1}{2}M_{9R}^2} = 817.89\,K$$

$$u_{9R} = M_{9R}\sqrt{\gamma R T_{9R}} = 1298.757\ m/s$$

$\left(\frac{F}{\dot{m}}\right)_R = U_{aR} - U_{oR} = 1040.186\ \frac{N}{kg \cdot s}$

$$f_R = \frac{\tau_{\lambda R} - \tau_{rR}\tau_{cR}}{\frac{h}{c_p T_{oR}} - \tau_{\lambda R}} = .0318 \Rightarrow \dot{m}_{fR} = 3.18\ kg/s$$

$$S_R = \left(\frac{f}{F/\dot{m}}\right)_R = 30.64\ \frac{mg}{N \cdot s}$$

$\rho_{aR} = P_{aR}/R/T_{aR} = .0852\ \frac{kg}{m^3}$

$$A_{aR} = \frac{\dot{m}_e}{\rho_{aR} U_{aR}} = .904\ m^2$$

Off - design

$\tau_R = 1.08$

$\pi_R = 1.044$

$M_4 = M_8 = 1$

$\frac{A_8}{A_4} = \text{Const}$

$\tau_t = \tau_{tR} \Rightarrow \pi_t = \pi_{tR}$

$\dot{m}_f = 121.5\% \; \dot{m}_{fR} = 3.872 \text{ kg/s}$

$\tau_\lambda = \frac{T_{t4}}{T_0} = 6.944$

$\tau_c = 1 + \frac{\tau_\lambda}{\tau_r}(1-\tau_t) = 2.168$

$\pi_c = \tau_c^{\frac{\gamma}{\gamma-1}} = 15$

$\frac{P_{t9}}{P_9} = \pi_c \pi_r \pi_t = 8.27$

$\hookrightarrow M_9 = 2.026$

$T_{t2} = T_0 \tau_r = 293.184 \text{ K}$

$T_{t3} = T_{t2} \tau_c = 635.686$

$T_{t9} = T_{t4} - T_{t3} + T_{t2} = 1657.498 \text{ K}$

$$T_9 = \frac{T_{t9}}{1+\frac{\gamma-1}{2}M_9^2} = 906.107\,K$$

$$u_9 = M_9\sqrt{\gamma R T_9} = 1228.634 \quad m/s$$

$$\frac{F}{\dot m} = u_9 - u_0 = 1126.813 \quad \frac{N\cdot s}{kg}$$

$$\dot m = \frac{\Gamma}{\sqrt{R}} A_4 \left[P_0 \pi_r \pi_c\right] \frac{1}{\sqrt{T_{t4}}}$$

$$\Gamma = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = .6847$$

$$A_4 = \frac{\dot m R\, R\, \sqrt{T_{t4}}}{\Gamma\, [P_0 \pi_r \pi_c]}$$

$$= \frac{100\sqrt{287}\sqrt{2000}}{.6847\,(200000)(5.24)(15)} = .2424\ m^2$$

$$\dot M = \frac{.6847}{\sqrt{287}}(.2424)(101325)(1.064)(15)\frac{1}{\sqrt{1600}}$$

$$= 353.705\ kg/s$$

$$A_9 = \frac{\dot m}{\rho u_9} = \frac{353.705}{.39(1228.634)} = .738\ m^2$$

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	on	off
T_0 (K)	260	288
P_0 (Pa)	20000	101325
M_0	.8	.3
π_c	15	15
τ_t	.829	.829
P_a (Pa)	20000	101325
$\dot{m}_{air}$ (kg/s)	100	353.705
$\dot{m}_f$	3.18	3.872
τ_λ	7.692	6.944
A_a (m^2)	.904	.738

$$\boxed{\frac{A_a}{A_{aR}} = \frac{.738}{.904} = .817}$$

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# Matthew Pahayo
# 2/22/2021
# propulsion 2
# hw 2

import functions as f
import math

#*******************************************************
# on_design analysis starts here!
#*******************************************************

# initialize given values
R = 287
P_0R = 20000
mdot_R = 100
h = 4.5*10**7
gamma = 1.4
PIc_R = 15
T_T4 = 2000
T_0R = 260
M_0R = .8
cp = 1004.5
P_9R = P_0R

# Total Temperature ratios
Tau_lR = T_T4/T_0R
Tau_cR = PIc_R**((gamma-1)/gamma)
Tau_rR = f.Tau(M_0R,gamma)
Tau_tR = 1 - Tau_rR/Tau_lR*(Tau_cR - 1)

# Total Temps [K]
T_T2R = T_0R*Tau_rR
T_T3R = T_T2R*PIc_R**((gamma-1)/gamma)

# burner Tau
Tau_bR = T_T4/T_T3R

# total temp at station 9 [K]
T_T9R = T_T4 - T_T3R + T_T2R

# Velocity of freestream [m/s]
U_0R = f.U(M_0R,gamma,R,T_0R)

# pressure ratios
PIr_R = Tau_rR**(gamma/(gamma-1))
PIt_R = Tau_tR**(gamma/(gamma-1))

# total pressusre by pressure at station 9
P_T9RbyP_9R = PIr_R*PIc_R*PIt_R
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# Mach at station 9
M_9R = math.sqrt(((P_T9RbyP_9R)**((gamma-1)/gamma)-1)*2/(gamma-1))

T_9R = T_T9R/(1+(gamma-1)/2*M_9R**2)
U_9R = f.U(M_9R,gamma,R,T_9R)

# specific thrust
ST_R = U_9R - U_0R

# fuel flow rate
f_R = f.f(Tau_lR,Tau_rR,Tau_cR,h,cp,T_0R)
mdot_fuelR = f_R*mdot_R

SFC_R = f_R/ST_R*10**6
# density [kg/m^3]
rho_0R = P_0R/R/T_0R
rho_9R = P_9R/R/T_9R

# area at station [m^2]
A_0R = mdot_R/rho_0R/U_0R
A_9R = mdot_R/rho_9R/U_9R

#*********************************************************
# off_design analysis starts here!
#*********************************************************
M = .3
T_0 = 288
P_0 = 101325
mdot_fuel = 1.215*mdot_fuelR
P_9 = P_0
Tau_t = Tau_tR
PIt = PIt_R
Tau_r = f.Tau(M,gamma)
PIr = Tau_r**(gamma/(gamma-1))

U_0 = f.U(M,gamma,R,T_0)

Tau_l = T_T4/T_0

Tau_c = 1 + Tau_l/Tau_r*(1-Tau_t)
PIc = Tau_c**(gamma/(gamma-1))

P_T9RbyP_9 = PIr*PIc*PIt

# Total Temps [K]
T_T2 = T_0*Tau_r
T_T3 = T_T2*PIc**((gamma-1)/gamma)

# burner Tau
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Tau_b = T_T4/T_T3

# total temp at station 9 [K]
T_T9 = T_T4 - T_T3 + T_T2

# Mach at station 9
M_9 = math.sqrt(((P_T9RbyP_9)**((gamma-1)/gamma)-1)*2/(gamma-1))

T_9 = T_T9/(1+(gamma-1)/2*M_9**2)
U_9 = f.U(M_9,gamma,R,T_9)

# specific thrust
ST = U_9 - U_0
f = f.f(Tau_l,Tau_r,Tau_c,h,cp,T_0)
SFC = f/ST*10**6


G = math.sqrt(gamma)*(2/(gamma+1))**((gamma+1)/2/(gamma-1))

rho_9 = P_9/R/T_9
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```
import math as m


def U(M,gamma,R,T):
   U = M*m.sqrt(gamma*R*T)
   return U

def Tau(M,gamma):
    Tau = 1+(gamma-1)/2*M**2
    return Tau

def f(Tau_l,Tau_r,Tau_c,h,cp,T_0):
    f = (Tau_l - Tau_r*Tau_c)/(h/cp/T_0 - Tau_l)
    return f
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