# THE USE AND DEVELOPMENT OF 'COMPONENT CHARACTERISTICS'
(SOMETIMES CALLED: GAS TURBINE PUMPING CHARACTERISTICS)

TO THIS POINT WE HAVE ESTIMATED ANALYTICALLY COMPONENT CHARACTERISTICS AND FOUND ENGINE PERFORMANCE UNDER ASSUMPTIONS OF CHOKED FLOW AT STATIONS 4 AND 8 AND CONSTANT EFFICIENCIES.
NOW WE BEGIN TO LOOK AT CASES WHERE WE OBTAIN COMPONENT CHARACTERISTICS BY HARDWARE TESTING.

## COMPRESSORS

TWO KINDS IN A TYPICAL JET ENGINE:

(a) AXIAL: COMPOSED OF AXIAL PRESSURE-RAISING STAGES, USUALLY LONGER (MANY STAGES) BUT RELATIVELY SMALL IN CROSS-SECTIONAL AREA.

(b) CENTRIFUGAL: RELIES ON CENTRIFUGING AIR TOWARD SIDES, LARGER CROSS-SECTIONAL AREA BUT SHORTER IN LENGTH.

many aircraft rely on a combination of both types

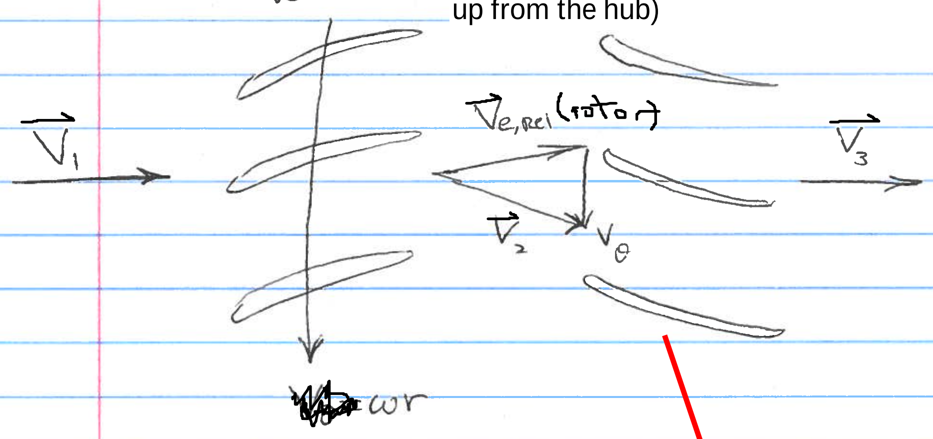
FOR SIMPLICITY, CONSIDER A 'SIMPLE' COMPRESSOR, TRUELY THE FLOW IS 3-D.
AN AXIAL COMPRESSOR IS COMPOSED OF A SERIES OF STAGES, WHERE EACH IS COMPOSED OF TWO BLADE ROWS: THE ROTOR AND THE STATOR. THE ROTOR ROTATES FROM THE HUB. THE PURPOSE OF THE ~~OF THE~~ ROTOR IS TO ADD WORK TO THE FLOW AND RAISE THE TOTAL PRESSURE ($\pi_c$) AND TOTAL TEMP ($\tau_c$). THE PURPOSE OF THE STATOR IS TO TURN THE FLOW BACK INTO THE AXIAL DIRECTION WITH MINIMAL LOSS AND PREPARES THE FLOW FOR THE SUBSEQUENT ROTOR. THE ROTOR INTRODUCES 'SWIRL' OR ANGULAR VELOCITY TO THE FLOW.

The stator turns the flow back to the axial direction so that local flow angle of attack on the next (downstream)stage's rotor blades don't become excessive and cause separation.

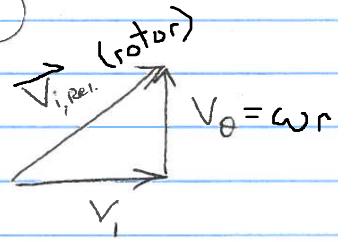
IF WE UNWRAP A SINGLE STAGE, of a compressor AT SOME RADIUS r

THE HUB (called the 'cascade' view - a '2-D' look at the flow-field at a given radial location up from the hub)

$+\theta$ direction




① rotor row ② stator row ③ ready for the next rotor row




$\omega$ = angular velocity [rad/s]

$\vec{V}_{i,Rel}$ = effective incoming velocity vector 'seen' by rotor in the rotor frame.

$\vec{V}_1, \vec{V}_2, \vec{V}_3$ = engine (stator) (inertial) frame velocity vectors.

$\vec{V}_{e,Rel}$ = effective exiting velocity in rotor frame ('as seen' by rotor - in rotor frame)

The blade angles at given r location from hub must be designed correctly to limit the AOA (based on V1, omega, r - to correctly obtain the local blade angles of attack)

usually, $\frac{|\vec{V}_2|}{|\vec{V}_1|} > \frac{|\vec{V}_3|}{|\vec{V}_1|}$ for typical compressors

due to losses (axial velocity 'drift')

and slow down due to adverse pressure gradient fact: If work is added to a 1-D duct (no area variation), Pt, Tt increase (and P, T as well), but the axial velocity drops due to development of adverse pressure gradient (P downstream higher than P upstream), so decreasing the cross-sectional area through compressor stage counters that effect (you should know that decreasing area in a duct in absence of other effects, INCREASES velocity).

A(x) in a compressor decreases to maintain the velocity as the work (energy) is added.
Ideally (repeating stage $\vec{V}_3 = \vec{V}_1$).

# Compressor Performance Map; 'Stand-Alone' characteristics of Compressor

A Compressor is tested alone on a stand in order to determine its individual ('component') characteristics at all possible compressor operating points.

What is best way to approach 'mapping out' a given compressor's performance? $\Pi_c$ is obviously the performance metric!

Dimensional Analysis shows that for a given compressor the achieved $\Pi_c$ ($P_{t3}/P_{t2}$) is a function of 4 dimensionless parameters:

a) Mach # at compressor face, which is directly related to a 'corrected' mass flow rate ('scaled' $\dot{m}$), $\dot{m}_{corr,2}$ (book uses $\dot{m}_{c,2}$)

b) ratio of blade tip speed to speed of sound, which is directly related to a 'corrected' blade speed ('scaled' RPM), $N_{corr,2}$ (book uses $N_{c,2}$)

c) $\gamma$ (ratio of specific heats $C_p/C_v$)

d) Reynolds #, $Re_c$ (based on chord of rotor blade

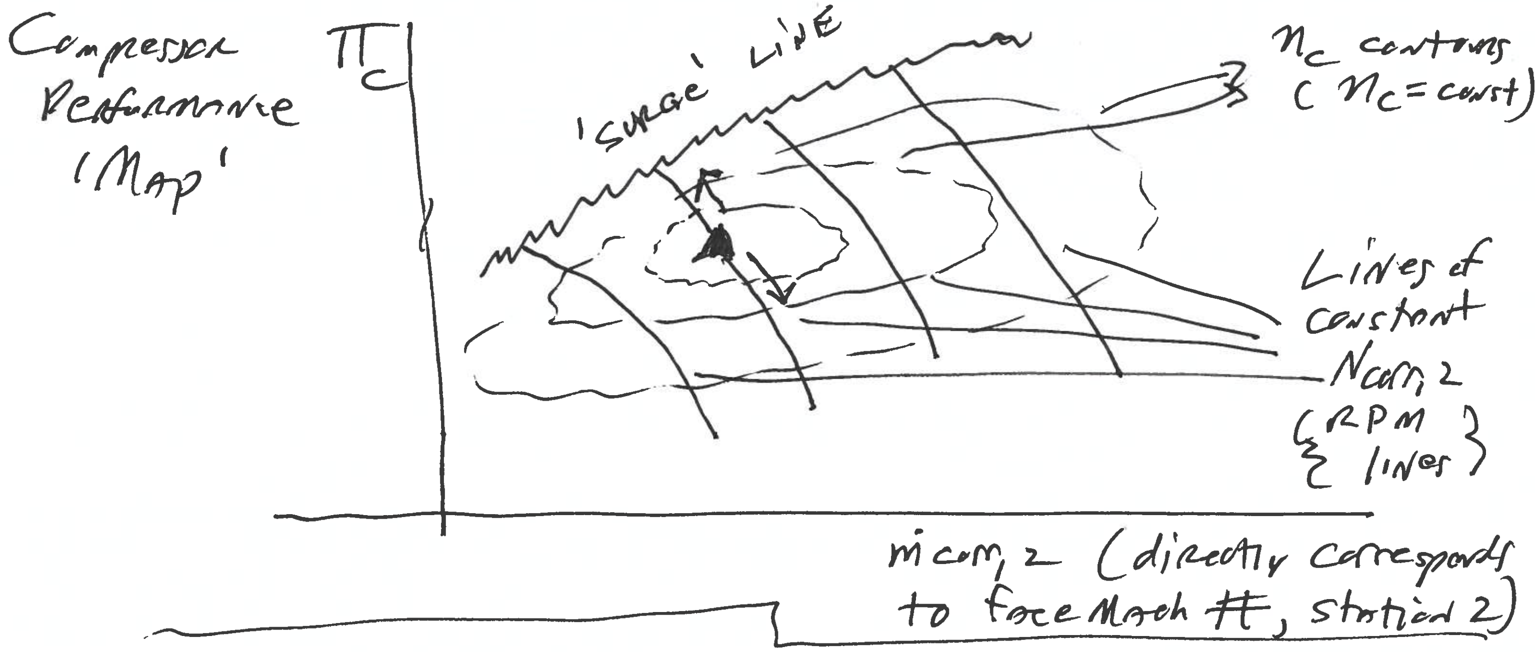
$$Re_c = \frac{\rho_2 U_2 C}{\mu_2} \; ; \; C = \text{chord}, \; \mu = \text{viscosity})$$

Hence, it is useful (customary) to 'map' $\Pi_c$ obtained vs $\dot{m}_{corr,2}$, $N_{corr,2}$ for a given compressor

(Note $\gamma$, Re effects not explicitly considered: more minor, secondary)

To generate its 'pumping' characteristics, a compressor by itself is placed in a closed 'direct-connect' test facility, 'rotated' at some fixed $N_{corr,2}$ (scaled RPM) and then $\dot{m}_{corr,2}$ (think Mach at station 2 → compressor face) is varied, while measuring $\pi_c$ ...

Results of such a test look like this (graphically):




Since at every point on the map, $\pi_c$ is measured for a given (known) amount of supplied external shaft work ($\tau_c$!), $\eta_c$ can be computed at that point & its contours can also be plotted on the map (as shown)

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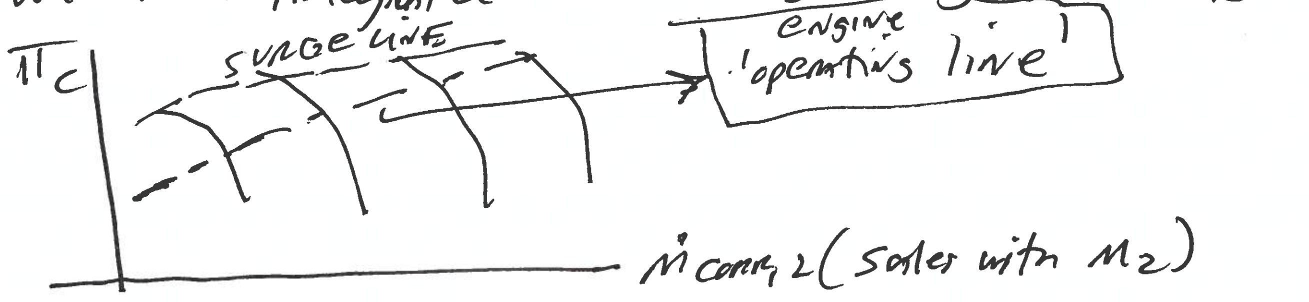
Four points about such a map

* This map represents the <u>entire</u> map of possible compressor '<u>stand-alone</u>' performance, <u>not</u> integrated into a gas turbine/jet engine! (yet)

* An 'operating line' will eventually be developed ON this compressor performance map, once a burner/turbine (& nozzle) are connected to the compressor (i.e. build an engine). This operating line will reflect (give) the actual compressor performance as the complete engine is operated in the 3-space of $\dot{m}_f$, ALT, $M_0$.

* Note, if $\dot{m}_{corr,2}$ is 'forced' (at same $N_c$) to be greater than that allowed by this steady flow/stable operation map, $\pi_c$ plummets (compressor not operating, $\eta_c$ bad, you are off the useful map

* This info (the performance map) is obviously useful (necessary) to get actual performance of compressor when it is integrated into an engine (gas turbine/jet)

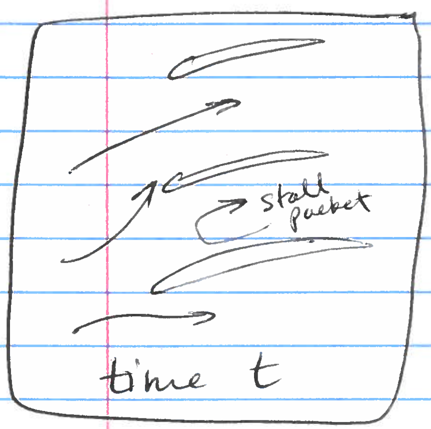


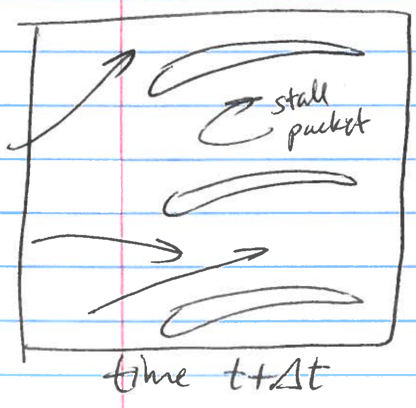

The 'surge line' is the line of maximum total pressure rise across the compressor that is obtainable. At or before this point, 2 primary forms of instability can cause compressor problems!

## SURGE LINE :

at or before compressor reaches surge line, two (primary) forms of instability can cause problems

(a) ROTATING STALL : flow through the compressor is working against an adverse pressure gradient. If the pressure gradient locally becomes adverse enough (severe enough), one (or more) blade passages may stall; having reverse flow and seen as a blockage by the main flow.




The flow above the stall packet increases the angle of attack, whereas the flow below the stall packet reduces the AoA. Due to this, the stall packet 'migrates' upward [in the cascade view] at some frequency → 'rotating stall'. Rotating stall (hard to predict) can cause large, severe, catastrophic engine failure due to large vibratory stresses in the blades and frame.

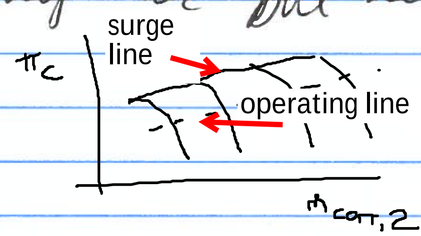
→ One good reason for uniform flow out of the inlet into the compressor being desired. since local non-uniformities can easily cause rotating staill

- Can happen well before/ away from the surge line.

difficult to predict because it is local in character (needs 3-D modeling, experimental data, etc.)

(b) SURGE : Is a large fraction of compressor blades loss in the ability to support pressure rise due to massive flow seperation due too large global pressure gradient → back pressure (downstream) effects.

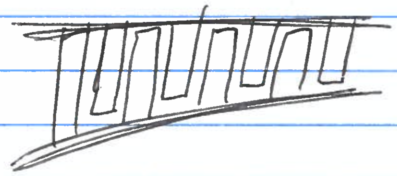
The trick to designing/integrating a compressor into the engine is to reduce the 'surge margin' which the distance between the surge line and the operating line enough to have high $\pi_c$ but not close enough to induce surge.

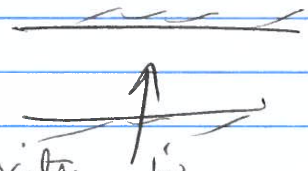



* COMPRESSOR STARTING.

A typical compressor section - - -

(side view)





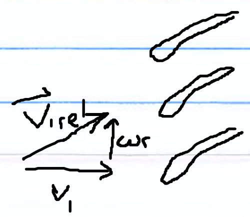
1-D work (energy addition)

→ adverse pressure gradient, velocity ↑ is decreases.

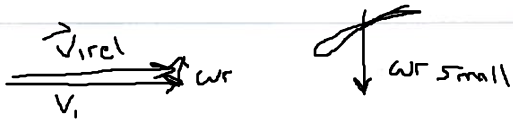
it has an overall contraction in the cross-sectional area in order to maintain the axial velocity to match the design (angles) of the blades. At a 'design' point, flow contraction and pressure rise are matched

(to maintain fairly constant/reasonable velocity)

When the compressor is just starting, the RPM is low, the pressure rise is low, the flow will speed up (due to area restriction), so the rear (aft) blades will 'windmill' mean local AoA on aft blades decreases and the flow can even choke (Mach 1) this will cause the forward (front) blade rows to have too large AoA and they stall.




(downstream blades) on-design operation (blade angles match velocity vectors)




starting operation (downstream blades -angle of attack small or even negative on blade leading edges, in turn can cause early (upstream blade rows) to stall due to upstream interaction

Techniques to deal with the starting problem:

most engines use a combination

(a) Compressor bleed during start (~~reduces~~ removing m, reduces axial velocity).

(b) Change the angle of the stators during starting to redirect the flow; change the AoA.
On the forward blades you want to reduce AoA
On the aft blades " " " increase AoA

(c) Multiple spool compressor with different RPM's for different sections. nested shafts

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## TURBINES

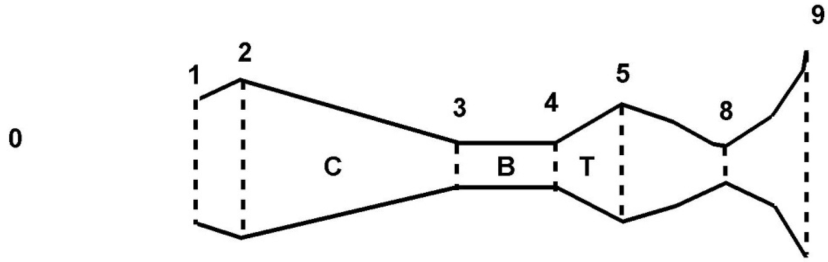
Turbines can be examined like compressors. (turbine performance map). standalone turbine characteristics
→ Similar starting characteristics.

– Turbines are in favorable pressure gradient that's why they are generally shorter than compressors with less stages because for a compressor at an adverse pressure gradient, we need to do things (increase pressure) gently to avoid separation.

turbine cross-sectional area INCREASES due to work extraction (favorable pressure gradient), need to counter tendency of flow to accelerate by decreasing A(x)

***AE 5535  Component Matching and Development of Operating Line***

Consider a single-spooled fixed area turbojet without afterburning . . . with f << 1 ($\dot{m}_f \ll \dot{m}_{air}$)




Each component has its individual performance map.

**A.  Compressor Performance Map**

Dimensional analysis shows that, for a given compressor (hardware), $\pi_C = \pi_C(\gamma, Re, M_2, \frac{\omega_{tip}}{a})$ where Re is the Reynolds number, $\omega_{tip}$ is the compressor blade tip angular speed, and $a$ is the speed of sound. Furthermore, $a \sim \sqrt{T_{t2}}$.

Therefore, $\frac{\omega_{tip}}{a} \sim \frac{blade\ RPM}{\sqrt{T_{t2}}}$

Compressor efficiency = $\eta_C$  $\quad \eta_C = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\tau_c - 1}$

$N_C$ = compressor (raw) RPM

Corrected mass flow rate, $\dot{m}_{corr}$ can be defined at any station.  Corrected mass flow rate at station two is defined as follows:

$T_{STP}$ : 288 K

$$\dot{m}_{corr,2} = \frac{\dot{m}_2 \sqrt{\frac{T_{t2}}{T_{STP}}}}{\frac{P_{t2}}{P_{STP}}}$$

Since $\dot{m} = \frac{P_t}{\sqrt{T_t}} \cdot \sqrt{\frac{\gamma}{R}} AM \left[1 + \frac{\gamma-1}{2} M^2\right]^{\frac{-(\gamma+1)}{2(\gamma-1)}}$ at any station, we can write that corrected mass flow rate is

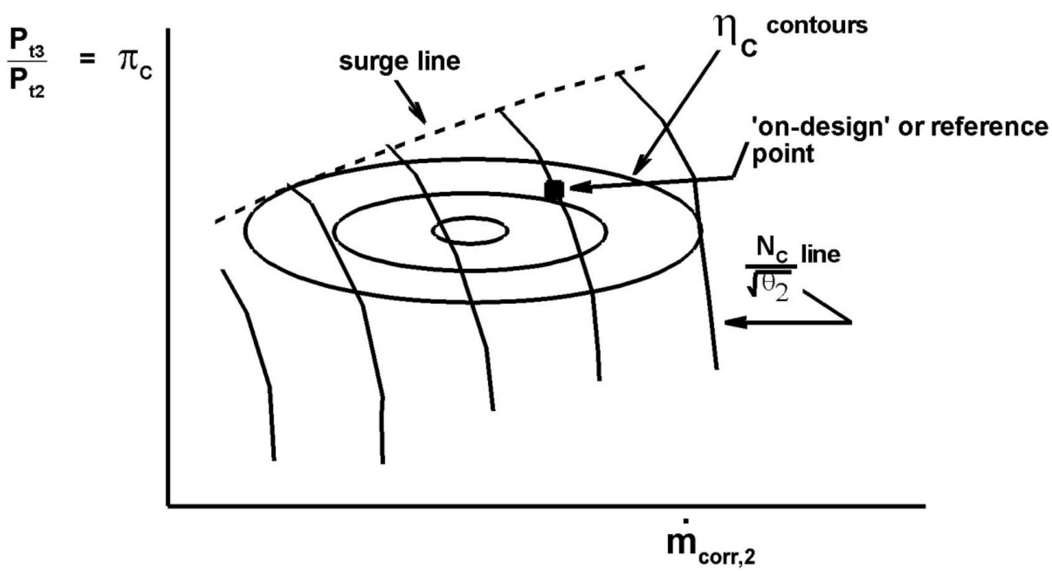
$$\dot{m}_{corr} = \sqrt{\frac{\gamma}{R}} \frac{P_{STP}}{\sqrt{T_{STP}}} AM \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{-(\gamma+1)}{2(\gamma-1)}}$$

Hence $\dot{m}_{corr}$ is a function of only γ, Mach, and area and does NOT change as long as the flow is choked at a given station and area is fixed; hence this is usually true for stations 4 and 8 (turbine entrance and nozzle throat) across much of the operating range (design and off-design) – especially true for turbojets.

We write

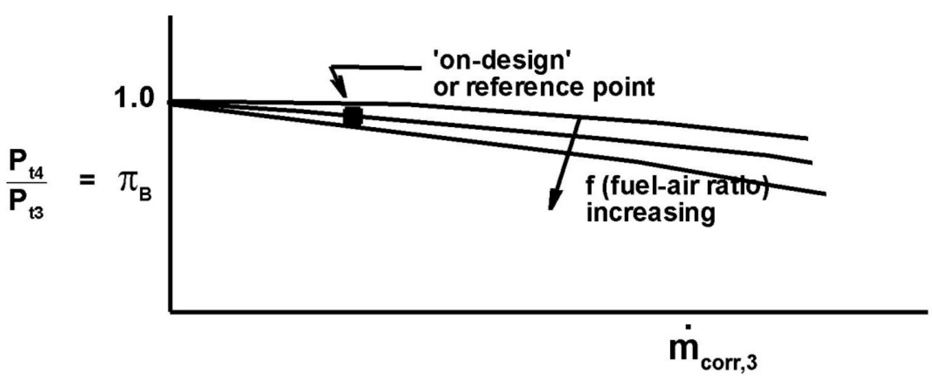
$$\dot{m}_{corr,2} = \frac{\dot{m}_2\sqrt{\theta_2}}{\delta_2}$$

where $\theta = \frac{T_t}{T_{STP}}$ and $\delta = \frac{P_t}{P_{STP}}$, and $T_{STP}$ = 288K and $P_{STP}$ = 101325 N/m$^2$




Note that the 'corrected' blade speed at station 2, $N_{corr,2} = \frac{N_C}{\sqrt{\theta_2}}$ (sometimes notationally this corrected blade speed is represented as $N_C$, - as in Gordon and Oates - so be careful, since in this handout and related work $N_C$ is the raw (actual) compressor blade speed.)

**B. Burner Performance Map**

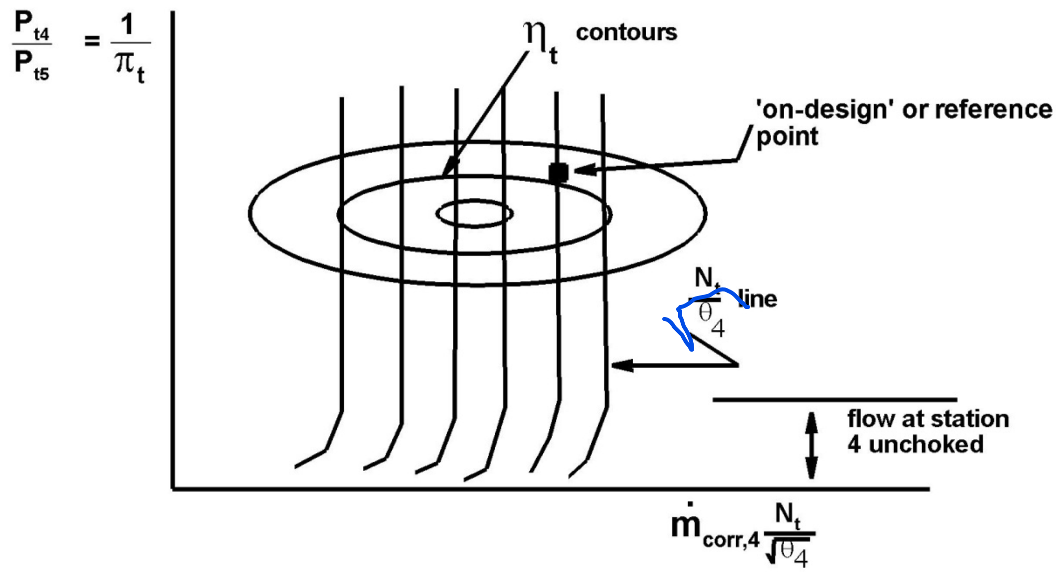


Here

$$\dot{m}_{corr,3} = \frac{\dot{m}_3\sqrt{\theta_3}}{\delta_3}$$

Note that there is another burner map for burner efficiency, $\eta_b$ as well, in general. In the following analysis we will 'ignore' the burner map, although in general it would have to be 'folded into' the analysis.

**C. Turbine Performance Map**




Here $N_t$ is the (raw) turbine RPM and

$$\dot{m}_{corr,4} = \frac{\dot{m}_4\sqrt{\theta_4}}{\delta_4}$$

The reason the $\dot{m}_{corr,4}$ on the x axis in the turbine performance map is multiplied by the corrected turbine blade speed $\left(\frac{N_t}{\sqrt{\theta_4}}\right)$ is because generally the flow is choked at station 4, hence $\dot{m}_{corr,4}$ is constant for all (or much) of the operating range. By multiplying by the corrected turbine blade speed, the lines are simply 'spread out' on the plot, such that one can distinguish variations. This means that the corrected turbine blade speed lines are vertical on the turbine map as shown, as long as the flow is choked at station 4.

*Important note! These component maps are developed for the individual components, i.e. are developed from component-alone testing (maps are produced with the respective component NOT integrated with the other components). Hence ,the 'on-design' or reference point shown on these three component maps (compressor, burner, turbine) is simply demonstrative for future discussion, since this* *on-design point is only located (or fixed) when the three components are actually 'bolted' together along with suitable inlet, downstream nozzle and operating at the appropriate on-design point.*

**<u>Development of gas turbine (jet engine) 'Operating Lines' on the individual component maps – when individual components are 'hooked' together</u>**

The individual component maps give all possible performance information for each individual component as a stand-alone component, in steady flow operation. However, when the compressor, burner, turbine (and nozzle) are 'hooked together', the fluid dynamics, heat release, and work interactions amongst these components couple together such that various steady state specific 'operating points' on these maps result, with specific locations of these various operating points depending on fuel flow rate and compressor inlet conditions (driven by flight Mach number and altitude).

To develop the operating lines on the performance maps, we need to apply the 'matching criteria'.

**<u>Matching Criteria</u>**

Specifically, for steady operation, whether 'on' or 'off' design (and for small fuel air ratio), we note that

$\dot{m}_0 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4 = \dot{m}_5 = \dot{m}_8 \; (= \dot{m}_9)$

$N_C = N_t$ (blade speed matched for single spool shaft, no gearbox)

Turbine-compressor power balance $\tau_t = 1 - \frac{1}{\frac{T_{t4}}{T_{t2}}}(\tau_C - 1)$ (you verify this!)

Note that at the 'on-design' point, all information is considered known and components are matched . . . everything can be obtained, i.e. on design values of

$\pi_C, M_0, T_0, P_0, P_{t2}, T_{t2}, \eta_C, \tau_t, \eta_t, \pi_t, P_{t5}, T_{t5}, \dot{m}_2, \dot{m}_{corr,2}, \dot{m}_{corr,4}, \dot{m}_{corr,3}, \dot{m}_{corr,8}, etc.$

$AND\ raw\ thrust\ and\ fuel\ consumption$
$Uninstalled\ thrust = \dot{m}(u_9 - u_0) + (P_9 - P_0)A_9$
$\dot{m}_f = \frac{c_P}{h}(T_{t4} - T_{t3})$ where h is the heating value of the fuel
… all obtainable on the 'on-design' engine operating point.

One further matching criteria ties in the nozzle with the compressor-burner-turbine (gas turbine core) system:

Define

$$\dot{m}_{corr,8} = \frac{\dot{m}_8\sqrt{\theta_8}}{\delta_8} = \dot{m}_8 \frac{\sqrt{\frac{T_{t8}}{T_{STP}}}}{\frac{P_{t8}}{P_{STP}}}$$

But without an afterburner $T_{t8}$ = $T_{t5}$ and $P_{t8}$=$P_{t5}$ (provided nozzle is isentropic, or use $\pi_N$ if known) and $\dot{m}_8 = \dot{m}_2$ for steady operation, so since

$$\dot{m}_{corr,2} = \frac{\dot{m}_2\sqrt{\theta_2}}{\delta_2}$$

We can write the following (always true)

$$\frac{\dot{m}_{corr,8}}{\dot{m}_{corr,2}} = \sqrt{\frac{T_{t5}}{T_{t2}}}\left[\frac{1}{\frac{P_{t5}}{P_{t2}}}\right]$$

Note also that

$$\frac{\dot{m}_{corr,8}}{\dot{m}_{corr,2}} = \sqrt{\frac{T_{t4}}{T_{t2}}(\tau_t)}\left[\frac{1}{\pi_c\pi_b\pi_t}\right]$$

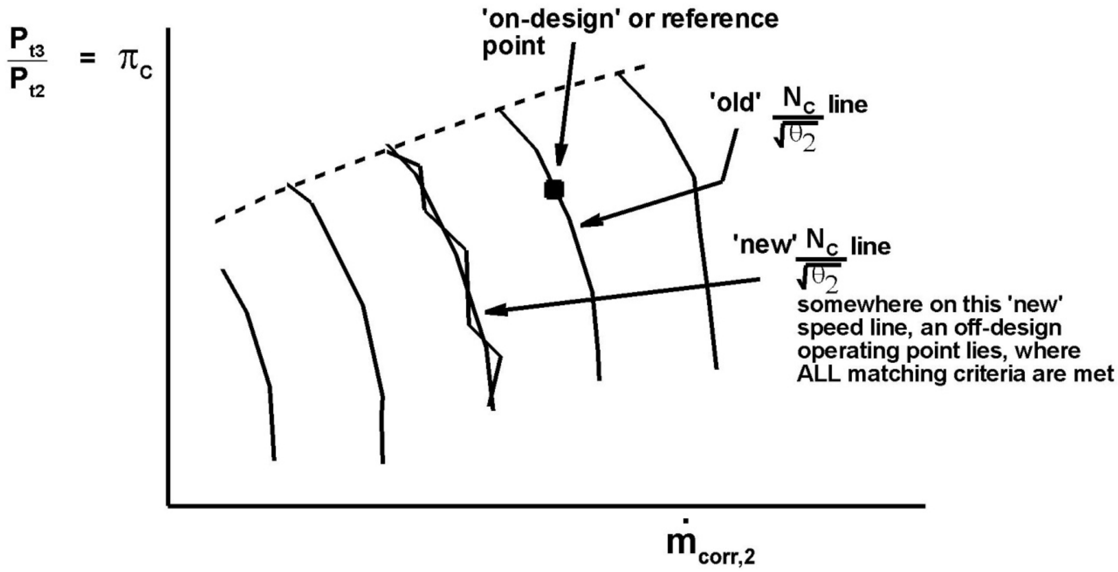
Furthermore, if the flow is choked at station 8 and area at 8 does not change (nozzle throat) for on and off design operation of the turbojet, $\dot{m}_{corr,8}$ does not change; i.e., it is same for both on and off design operation.

**Question: How do/can we determine the steady-state operating lines on the component charts from the matched compressor –burner-turbine-nozzle?**

Or, more specifically, how do we determine ONE off-design operating point? The key is that the matching criteria as given above apply at the off-design point.

**Method for determining operating point**

We must move off the on-design point (i.e., fix one parameter and find all others based on matching criteria). We will therefore pick a new $\frac{N_C}{\sqrt{\theta_2}}$ speed line on the compressor map . . .

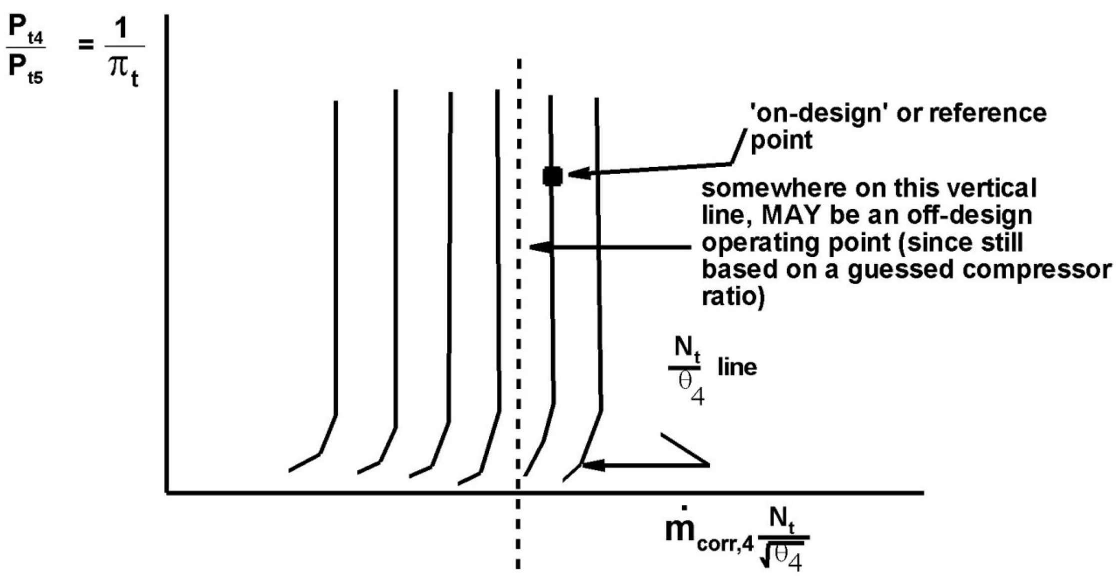



**Iterative procedure:**

a) Guess a $\pi_C$ on this line (hence yielding a trial $\dot{m}_{corr,2}, \eta_C, \tau_C$)

b) Compute a trial $\dot{m}_{corr,4} \frac{N_t}{\sqrt{\theta_4}}$ (which can be formulated as $\dot{m}_{corr,2} \frac{1}{\pi_C \pi_b} \frac{N_C}{\sqrt{\theta_2}}$ if $\dot{m}_2 = \dot{m}_4$ (steady flow) and $N_t = N_C$ (single spool – matched shaft RPM between compressor and turbine)

c) This $\dot{m}_{corr,4} \frac{N_t}{\sqrt{\theta_4}}$ then 'fixes' a vertical line on the turbine map, i.e.




d) Now 'guess' a $\pi_t$ (along with a corresponding $\eta_t$) on this trial vertical line. This 2$^{nd}$ guess of a $\pi_t$ (remember an outer iteration working with a guessed $\pi_C$ from step a) is already in progress) implies that - rigorously - there is an INNER iteration required on the turbine map itself.

HOWEVER, if the flow is choked at station 4 on off-design (i.e., $\dot{m}_{corr,4}$ is constant) then note that the $\frac{N_t}{\sqrt{\theta_4}}$ line is vertical; hence $\frac{N_t}{\sqrt{\theta_4}}$ itself is constant (and known from looking at the turbine map) on this trial line (the dashed line established on the turbine map from step c), in the above figure).

(Note that if the flow is NOT choked, $\dot{m}_{corr,4}$ changes, hence one is working in the region on the turbine map corresponding to the curved portions of the corrected speed lines in the above figure. Things are still doable, but with the 2$^{nd}$ – inner - iteration required.)

Therefore, assuming choked flow, then (i.e., removing the necessity of 'picking' a $\frac{N_t}{\sqrt{\theta_4}}$ on this vertical line, i.e. removing the necessity of performing an inner iteration on $\frac{N_t}{\sqrt{\theta_4}}$) you know the value of the (trial) $\frac{N_t}{\sqrt{\theta_4}}$

Then find the (trial) value of $\frac{T_{t4}}{T_{t2}}$ from the following ratio:

$$\sqrt{\frac{T_{t4}}{T_{t2}}} = \frac{\left(\frac{N_C}{\sqrt{\theta_2}}\right)}{\left(\frac{N_t}{\sqrt{\theta_4}}\right)}$$

e) Now apply the turbine-compressor power balance:

$\tau_t = 1 - \frac{1}{\frac{T_{t4}}{T_{t2}}}(\tau_C - 1)$ to find $\tau_t$

(Note that if the flow at station 4 is not choked and you are down in the corresponding region of the turbine map, you would have to then iterate (i.e., perform the inner iteration) between steps d) and e) with changing $\frac{N_t}{\sqrt{\theta_4}}$ until $\frac{T_{t4}}{T_{t2}}$, $\pi_t$, $\eta_t$, $\tau_t$ all match, then proceed to step f).

f) Rigorously (even for the case of choked flow at station 4 – i.e. working at a fixed $\frac{N_t}{\sqrt{\theta_4}}$), you would now have to find the $\pi_t$, $\eta_t$ on the turbine map that actually gives you the $\tau_t$ found in step e), but a nice short-cut is to assume that $\eta_t$ is constant between the on and off design, and then use the $\tau_t$ obtained in step e) to directly find the $\pi_t$ from the definition of turbine efficiency:

$$\eta_t = \frac{1 - \tau_t}{1 - \pi_t^{\frac{\gamma-1}{\gamma}}}$$

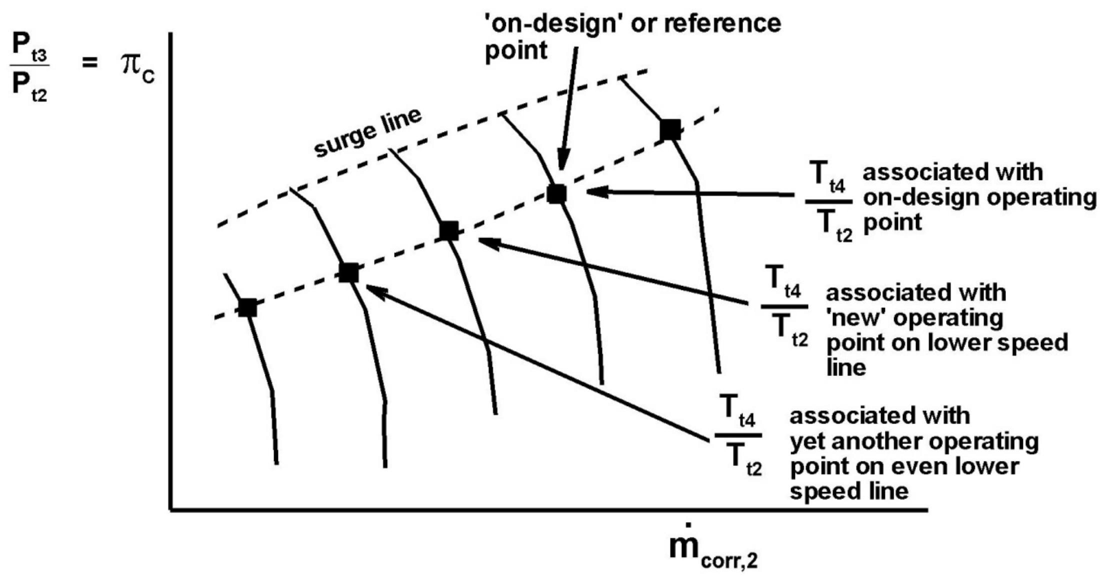
g) Finally, check the last nozzle matching criteria:

$$\frac{\dot{m}_{corr,8}}{\dot{m}_{corr,2}} = \sqrt{\frac{T_{t4}}{T_{t2}}(\tau_t)}\left[\frac{1}{\pi_C \pi_b \pi_t}\right]$$

If this final matching criterion is not satisfied, go back to step a) and pick another compressor pressure ratio $\pi_C$ on the new corrected speed line on the compressor map. Repeat steps a) – g) until all steps are satisfied.

This would then yield a single operating point on the new corrected speed line, $\frac{N_C}{\sqrt{\theta_2}}$ at which all matching criteria are met.

Repeat for the range of $\frac{N_C}{\sqrt{\theta_2}}$ lines available on the compressor map (hence defining a distinct new $\frac{T_{t4}}{T_{t2}}$ value corresponding to each operating point) until the correct OPERATING LINE is established. . .




So you can then tabulate a matrix completely defining this steady-state operating line, where the established operating point located on each respective corrected (compressor) speed lines (each $\frac{N_C}{\sqrt{\theta_2}}$ line on the compressor map) has a specific value of $\frac{T_{t4}}{T_{t2}}$, $\pi_C$, $\eta_C$, $\tau_C$, $\pi_t$, $\eta_t$, $\tau_t$, $\frac{N_t}{\sqrt{\theta_4}}$, $\dot{m}_{corr,2}$.

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**Some important notes:**

A. In actuality, $\frac{T_{t4}}{T_{t2}}$ can be viewed as the 'driving parameter' as the engine is moved off-design in the 'three space' of fuel throttle setting, flight Mach number, and altitude, since

$$\frac{T_{t4}}{T_{t2}} = \frac{T_{t4}}{T_0\left(1 + \frac{\gamma - 1}{2} {M_0}^2\right)}$$

i.e., change the fuel throttle setting (i.e., the mass flow rate of fuel), $T_{t4}$ will obviously change (other things as well, of course!). Or change altitude, $T_0$ obviously changes (other things as well!), or change flight Mach number – all determine the resulting $\frac{T_{t4}}{T_{t2}}$ for a given off design operating point.

B. It is vital to realize that EACH one of the steady state operating points located on the operating line on the compressor map (and the turbine map), actually refer to an entire 'surface' of possible engine operating conditions in the 'three-space' of fuel throttle setting, flight mach number, and altitude for a given engine. In other words, there are many ENGINE fuel throttle settings and flight Mach numbers and altitudes that will give the same $\frac{T_{t4}}{T_{t2}}$, hence the same $\pi_C$, same corrected mass flow rate at station 2, same corrected blade speeds, etc. Therefore, the 'found' operating point for a given engine on the compressor and turbine performance maps applies for ALL possible engine throttle levels, altitudes, and flight Mach numbers that happen to yield the same $\frac{T_{t4}}{T_{t2}}$!

**AE 5535** **Homework 4** **In-Class**

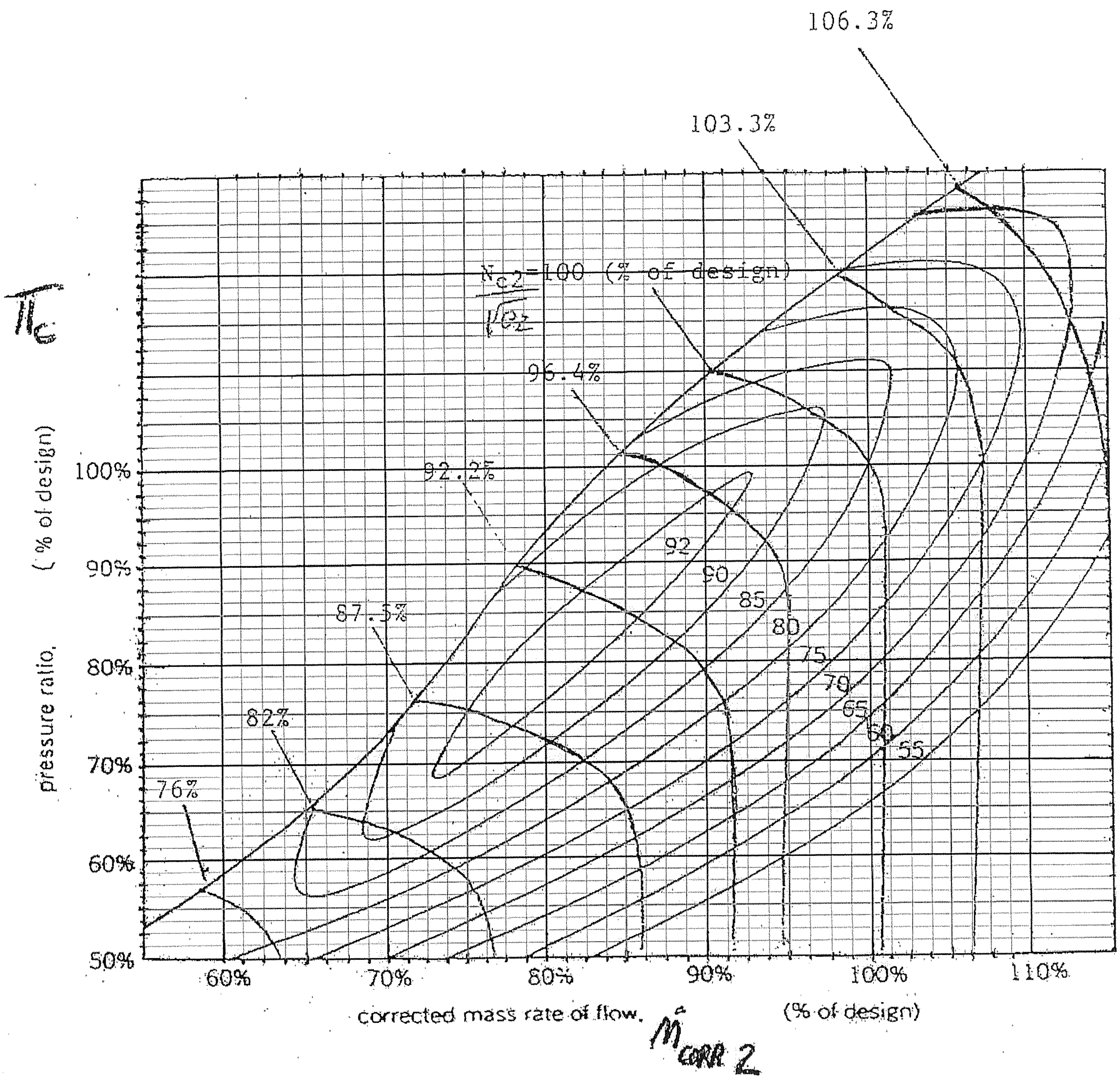
Determine a ***single*** *component matching*/operating point on the 87.5% RPM line as given on the following compressor and turbine maps for a fixed-area single-spool turbojet engine. (No burner map is given so assume ideal burner as noted below.) Make sure that you find all relevant parameters which completely define that operating point, i.e. $T_{t4}/T_{t2}$, $\pi_C$, $\dot{m}_{corr,2}$ etc... The engine design point values are as follows:

$M_0 = 0.8$ Altitude = 9000 meters ($T_0 = 230K$, $P_0 = 30{,}000\ N/m^2$)

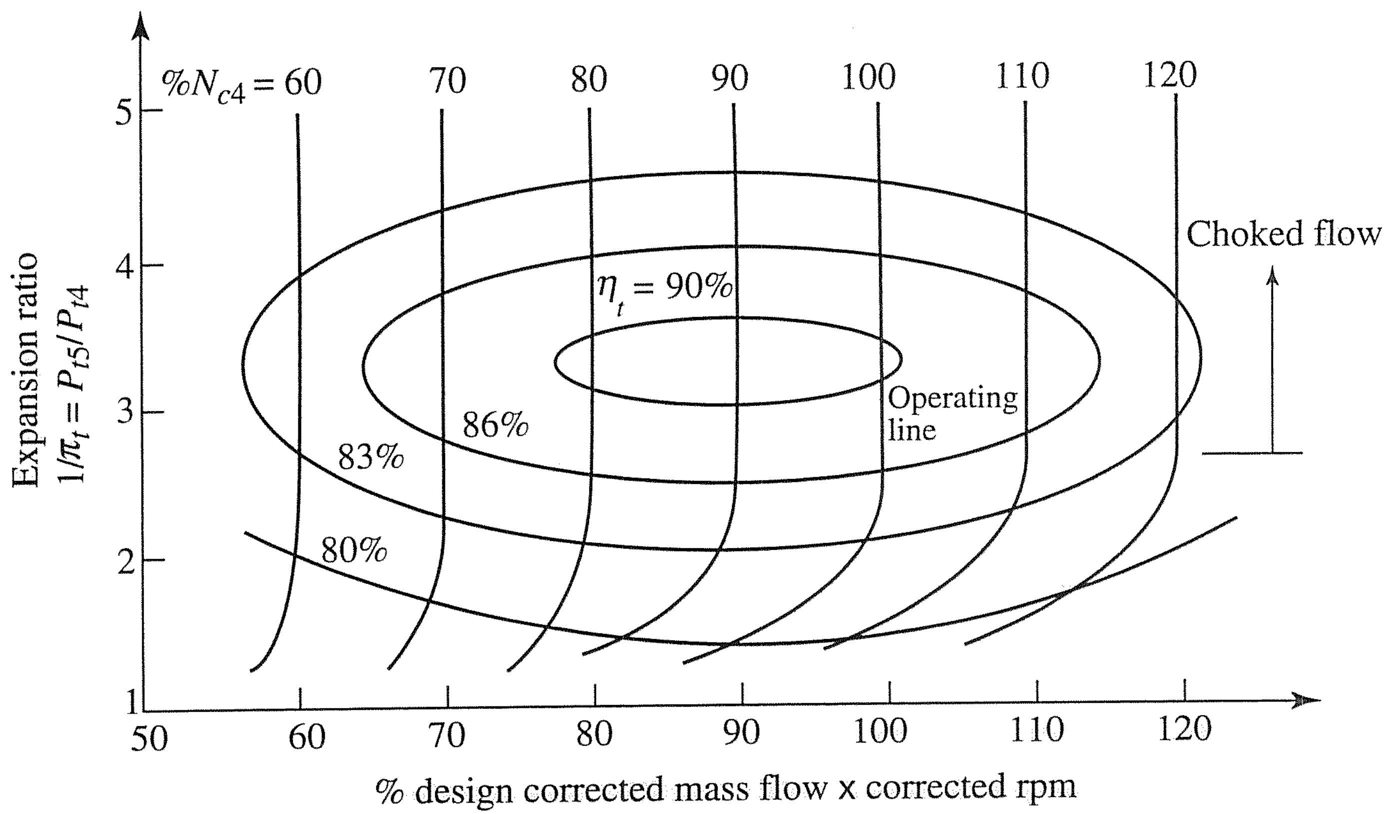
$\pi_C = 15.742$ $\dot{m}_{corr,2} = 35$ kg/s $\pi_b = 1.0$ $\eta_t = .86$

$M_2 = 0.5$ $RPM_{design} = 70{,}000$ RPM $T_{t4} = 1300K$

Neglect the fuel-air ratio and use constant specific heats, etc. Assume that the flow is choked at both turbine inlet and nozzle throats for the operating range of interest and that the turbine efficiency ($\eta_t$) is constant for the development of the operating line. After the operating point on this speed line is found, quantify the effect of the approximation of constant turbine efficiency by examining the turbine performance map provided and comparing the approximated turbine characteristics against the actual turbine performance map.



106.3%
103.3%
$\frac{N_{c2}}{\sqrt{\theta_{c2}}}$=100 (% of design)
96.4%
92.2%
87.5%
82%
76%
92
90
85
80
75
70
65
60
55
$\pi_c$
pressure ratio, (% of design)
100%
90%
80%
70%
60%
50%
60%
70%
80%
90%
100%
110%
corrected mass rate of flow, $\hat{m}_{corr\,2}$ (% of design)



Expansion ratio
$1/\pi_t = P_{t5}/P_{t4}$
$\%N_{c4} = 60$
70
80
90
100
110
120
$\eta_t = 90\%$
86%
83%
80%
Operating line
Choked flow
5
4
3
2
1
50
60
70
80
90
100
110
120
% design corrected mass flow x corrected rpm

[redacted]

HW4

# SOLUTION FOR [redacted] / HANDOUT PROBLEM :

FIRST, completely define the on-design point :

$\dot{m}_{corr,2} = 35\ kg/s$, $\pi_c = 15.742$, $T_{t4} = 1366K$, $\eta_c = 85\%$, $\eta_t = 86\%$

$$T_{t_0} = T_{t_2} = T_0\left(1+\frac{\gamma-1}{2}M_0^2\right) = 259.4K$$

isentropic: $$P_{t_0} = P_{t_2} = \left(1+\frac{\gamma-1}{2}M_0^2\right)^{\frac{\gamma}{\gamma-1}} = 45730.2\ Pa$$

$$\frac{T_{t4}}{T_{t2}} = 5.011$$ important parameter

$$\dot{m}_2 = \frac{P_{t2}}{P_{stp}}\dot{m}_{corr,2}\frac{1}{\sqrt{\frac{T_{t2}}{T_{stp}}}} = 16.643\ kg/s$$

$$\tau_c = 1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}}-1}{\eta_c} = 2.409$$

$$\tau_t = 1 - \frac{T_{t2}}{T_{t4}}(\tau_c - 1) = 0.7187$$

$$\therefore \pi_t = 0.2486$$

$$P_{t3} = P_{t4} = P_{t2}\pi_c = 719885\ Pa$$

$$P_{t8} = P_{t4}\pi_t = 179971\ Pa$$

$$\frac{N_{c2}}{\sqrt{\theta_2}} = \frac{\overset{70\ kRPM}{N_{c2}}}{\sqrt{\frac{T_{t2}}{T_{stp}}}} = 73752\ RPM$$ scaled RPM

$$T_{t8} = T_{t4}\tau_t = 934K$$

$$f = \frac{\tau_r\left(\frac{T_{t4}}{T_{t2}} - \tau_c\right)}{\frac{h\eta_b}{c_pT_0} - \tau_r\frac{T_{t4}}{T_{t2}}} \Rightarrow \dot{m}_f = 0.258\ kg/s$$

for consistency if using assumption f<<1, don't include that term (combustor energy balance solve)

Assume $P_9 = P_0 = 30000\ Pa$ careful, this may not be true if area or Mach number constrained! problem dependent!

$\therefore M_9 = 1.828 \Rightarrow T_9 = 560\ K$

$\Rightarrow u_9 = 867\ m/s, \quad u_0 = 243.2\ m/s$

$A_9 = 0.1028\ m^2, \quad A_0 = 0.15\ m^2$

$(M_8 = 1.0) \Rightarrow \rho_8 = .4255\ kg/m^3, \quad P_8 = 95076\ Pa$

$T_8 = 779\ K, \quad u_8 = 559\ m/s \Rightarrow A_8 = 0.0699\ m^2$

On design uninstalled thrust:

$\text{Thrust} = \dot{m}(u_9 - u_0) = 10.3\ kN$

these will be constant as long as flow stays choked at station 4 and 8!

$\dot{m}_{corr,4} = \frac{\dot{m}\sqrt{\theta_4}}{\delta_4} = 4.977\ kg/s \qquad \theta = \frac{T_{t4}}{T_{STP}} \qquad \delta = \frac{P_{t4}}{P_{STP}}$

$\dot{m}_{corr,8} = \frac{\dot{m}\sqrt{\theta_8}}{\delta_8} = 16.878\ kg/s$

Check the matching criteria:

$\dot{m}_{corr,4} \frac{N_t}{\sqrt{\theta_4}} = \dot{m}_{corr,2} \frac{1}{\pi_c \pi_b} \frac{N_c}{\sqrt{\theta_2}}$ ✓ checks, 'main' matching criteria 1

$\frac{\dot{m}_{corr,8}}{\dot{m}_{corr,2}} = \sqrt{\frac{T_{t4}}{T_{t2}}\tau_t} \frac{1}{\pi_c \pi_b \pi_t}$ ✓ checks, 'main' matching criteria 2

single spool tjet!

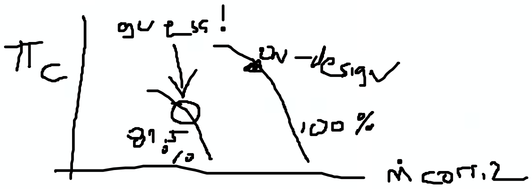
$N_t = N_c \Rightarrow \frac{N_t}{\sqrt{\theta_4}} = \frac{N_c}{\sqrt{\theta_2}\sqrt{\frac{T_{t4}}{T_{t2}}}}$ or $\frac{N_t}{\sqrt{\theta_4}} = 32947\ RPM$

Then $\dot{m}_{corr,4} \frac{N_t}{\sqrt{\theta_4}} = 163973$

this 100% design component matched point on component maps (comp, turbine, here)

→ obtained by analyzing the on-design engine

given single on-design operating point (M0 = 0.8m ALT = 9 km, piec = 15.74, mdotcorr2 = 35 kg/sec, Tt4 = 1300K, RPM = 70000), i.e., for a single point in the '3 space' of engine performance based on mdotfuel, ALT, Mo) but is valid for ANY steady state engine operating condition where Tt4/Tt2 is 5.01 (i.e. the component map matching point found actually corresponds to an entire 'surface' of possible mdotfuel, ALT, Mo values for the engine where Tt4/Tt2 = 5.01. And hence corresponds for all those possible points where Tt4/Tt2 = 5.01 for the given specific mdotcorr2, piec, etac, etaturbine, pieturbine, tauturbine, Nc/sqrt(theta2), Nt/sqrt(theta4), etc. )




Now, tasked is to find a single component matched point on 87.5% speed line.

Guess: $\pi_{c\ guess} = 0.72\ \pi_{c\ design}$

all guessed values now!

$\eta_c = 88.5\%$ , $\pi_c = 11.33$

$\dot{m}_{corr,2} = 28.175$ kg/s (80.5% of on design value)

64533 RPM

$\dfrac{N_c}{\sqrt{\theta_2}} = 64533$ RPM , $\tau_c = 1 + \dfrac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} = 2.131$

to match, this must be true: $\underbrace{\dot{m}_{corr,4}}_{\text{same as on-design (choked)}} \dfrac{N_t}{\sqrt{\theta_4}} = \dot{m}_{corr,2} \dfrac{1}{\pi_c \pi_b} \dfrac{N_c}{\sqrt{\theta_2}}$

(main matching criteria 1)

$\left.\dfrac{N_t}{\sqrt{\theta_4}}\right)_{guess} = 32233$ RPM

$\sqrt{\left.\dfrac{T_{t4}}{T_{t2}}\right)_{guess}} = \dfrac{\boxed{N_c/\sqrt{\theta_2}}}{N_t/\sqrt{\theta_4}} = 2.002 \Rightarrow \left.\dfrac{T_{t4}}{T_{t2}}\right)_{guess} = 4.0084$

$\tau_t = 1 - \dfrac{1}{\left(\frac{T_{t4}}{T_{t2}}\right)}(\tau_c - 1) = 0.718 \Rightarrow \pi_t = 0.2486 \ : \eta_t = 0.86$

Now, check to see how the nozzle matching criteria..

(main matching criteria 2)

same as on design

$\dfrac{\dot{m}_{corr,8}}{\dot{m}_{corr,2}} = \sqrt{\dfrac{T_{t4}}{T_{t2}} \tau_t} \dfrac{1}{\pi_c \pi_t}$ ✓ (Riggins is one heck of a guesser!)

$.59902 \cong .60198$ closely matched!

We found the operating point on 87.5% Speed line

$\longrightarrow$ found that point by assuring matching criteria are met

'guessed' $\pi_c = .72\,\pi_{c\,design} = 11.33$

$\eta_c = 88.5\%$ , $\dot{m}_{corr,2} = 28.175$ kg/s

$\frac{N_c}{\sqrt{\theta_2}} = 64533.3$ RPM , $\frac{T_{t4}}{T_{t2}} = 4.0084$

$\frac{N_t}{\sqrt{\theta_4}} = 32233$ RPM

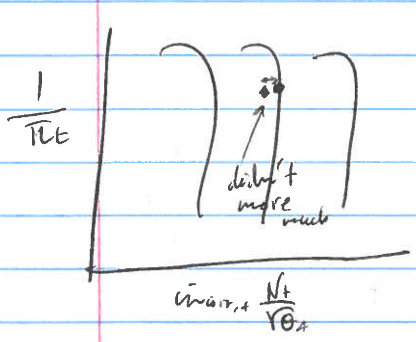
final matching criteria : $\frac{\dot{m}_{corr,8}}{\dot{m}_{corr,2}} = \sqrt{\frac{T_{t4}}{T_{t2}}\tau_t}\ \frac{1}{\pi_c \pi_t}$

was met ...

$.599 \approx .602$

* if $\pi_{c\,guess}$ is not good, $\longrightarrow$ guess another $\pi_c$

So $\dot{m}_{corr,4}\left.\frac{N_t}{\sqrt{\theta_4}}\right)_{87.5\%RPM\ line} = 0.98\ \dot{m}_{corr,4}\left.\frac{N_t}{\sqrt{\theta_4}}\right)_{100\%RPM}$ doesn't move much on turbine map!



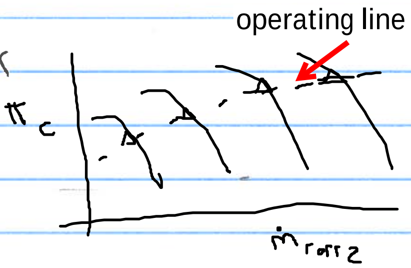

So, we have located the operating point on 87.5% RPM line;
it always has $\frac{T_{t4}}{T_{t2}} = 4.0084$, $\pi_c = 11.33$, ... etc

* Repeat process for other lines.

- The end product of such a process is a table 'completely' defining the engine steady state operating line on the individual component maps.

| | $\pi_c$ | $\dot{m}_{corr,2}$ | $\eta_c$ | $\tau_c$ | $\frac{T_{t4}}{T_{t2}}$ | $\tau_t$ | $\eta_t$ | $\frac{N_t}{\sqrt{\theta_4}}$ | ... |
|---|---|---|---|---|---|---|---|---|---|
| $\left.\frac{N_c}{\sqrt{\theta_2}}\right)_1$ | x | x | ✓ | ✓ | ✓ | x | ✓ | | |
| $\left.\frac{N_c}{\sqrt{\theta_2}}\right)_2$ | ✓ | | | | | | | | |
| $\left.\frac{N_c}{\sqrt{\theta_2}}\right)_3$ | ✓ | ✓ | ✓ | | | | | | |
| ⋮ | | | | | | | | | |

TABLE / MATRIX



note also the usefulness of presenting the compressor map (for instance) in terms of mdotcorr2, Nc/sqrt (theta2) - it allows us to have a single data base (map) for the compressor that is valid across the entire operating range of the engine!

* **Note**: each of these operating points (rows) on a given speedline correspond to a 'surface' of possible a $\dot{m}_f$, ALT, $M_0$ combinations for a given engine.

**In-Class Example (Homework 5)** **AE 5535**

**Engine Performance Analysis from Component Performance Map with Operating Line Defined – From Previous Example (In-class Homework 4) and Notes**

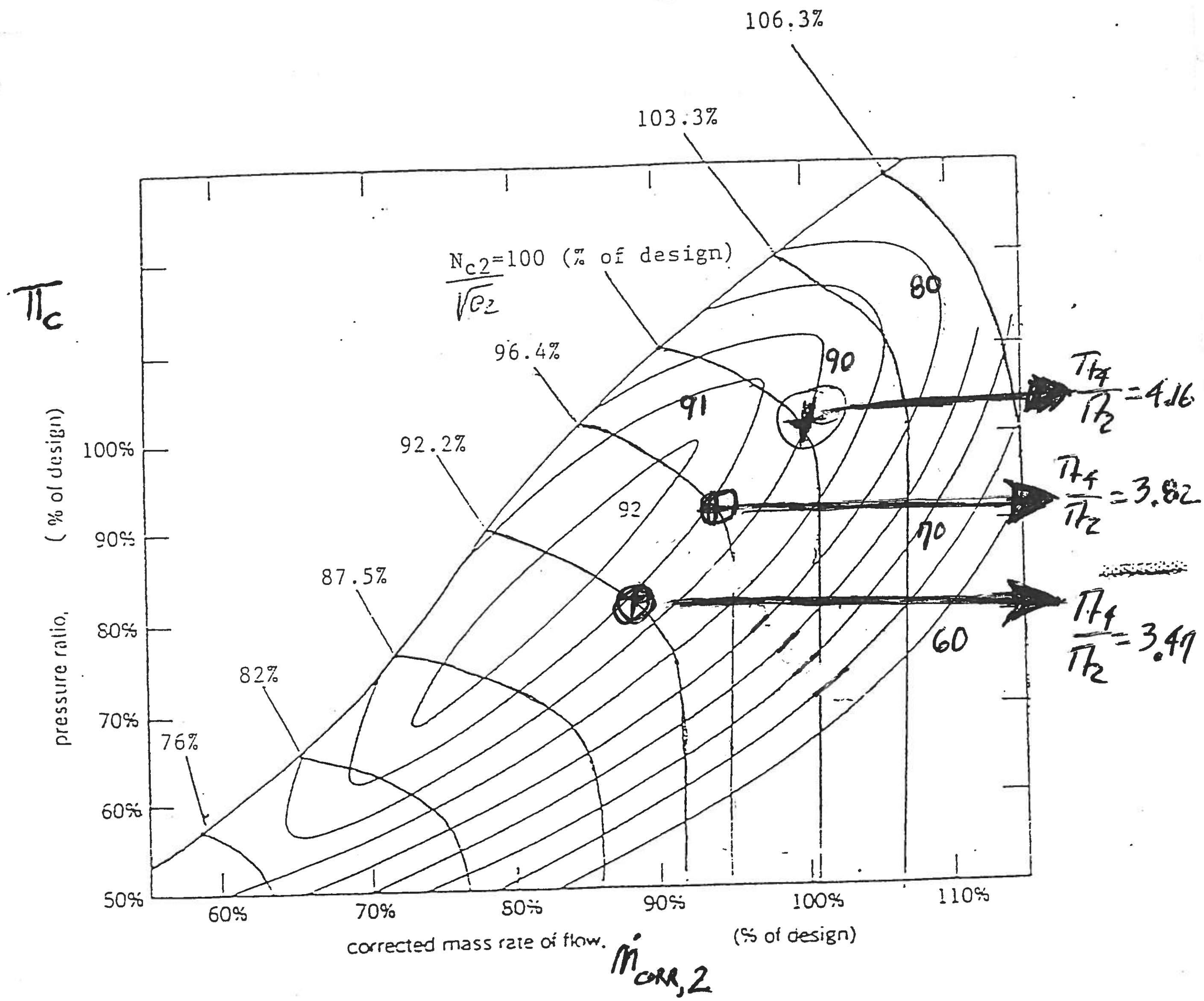
Consider a simple single-spool fixed-area turbojet engine with converging nozzle (choked flow at station 8). Let the engine be sized for full mass capture (no spillage) at $M_0 = 0.8$. The following are given as on-design conditions for this engine:

$$M_0 = 0.8 \quad T_0 = 220\ K \quad P_0 = 20{,}000\ N/m^2$$

$$\eta_t = 0.9\ (constant) \quad \eta_b = 1.0 \quad \pi_b = 1.0$$

$$\eta_C = 0.9 \quad \frac{T_{t4}}{T_{t2}} = 4.16 \quad \pi_C = 5.0 \quad \dot{m}_{corr2} = 50\ lb_m/sec = 22.68\ kg/sec$$

Given the attached compressor performance map with defined operating line (note that off-design operating points located on operating line are found by methodology described in previous handout and worked in previous in-class example), fully describe the engine flow-field, geometry, and performance at on-design. Then find the engine flow-field and performance for a case where flight Mach number and altitude remain the same but fuel throttle is adjusted such that $\frac{T_{t4}}{T_{t2}} = 3.47$.



★ Solution to Homework 5 / In-Class example:

Assume converging nozzle with $\eta_t = 0.9$

engine sized for $M_0 = 0.8$ (full mass capture)

On design:

$T_0 = \cancel{2\,2}0\,K$, $P_0 = 20\,kPa$, $\frac{T_{t4}}{T_{t2}} = 4.16$, $\eta_c = 0.9$, $\pi_c = 5.0$

$\dot{m}_{corr,2} = 22.68\ kg/s$

$T_{t_0} = T_{t_2} = 248.2\,K$ ; $P_{t_2} = 30487\,Pa$ (isentropic) $P_{t_2} = P_{t_0}$

since $\dot{m}_{corr,2} = \frac{\dot{m}_2\sqrt{\theta_2}}{\delta_2}$ solve for $\dot{m}_2 = \dot{m}$ ondesign $= 7.355\,kg/s$ (no spillage)

size for no spillage!
at 'on-design'

since Tt4/Tt2 = 4.16

$\therefore T_{t4} = 1032\,K$

$$\tau_c = 1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} = 1.6487$$

$$\tau_t = 1 - \frac{T_{t2}}{T_{t4}}(\tau_c - 1) = 0.844$$

$$\tau_r = 1 + \frac{\gamma-1}{2}M_0^2 = 1.128$$

$$f = \frac{\tau_r\left(\frac{T_{t4}}{T_{t2}} - \tau_c\right)}{\frac{h\eta_b}{c_p T_0} - \tau_r\frac{T_{t4}}{T_{t2}}} = 0.01425$$

$\therefore \dot{m}_{fuel} = 0.105\ kg/s$

$$\pi_t = \left(1 - \frac{1-\tau_t}{\eta_t}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\eta_t = \frac{1-\tau_t}{1-\pi_t^{\frac{\gamma-1}{\gamma}}} \Rightarrow \pi_t = 0.5191 \quad 0.514$$

$P_{t3} = P_{t2}\pi_c = 152435\ N/m^2$

$P_{t4} = P_{t3} = 152435\ Pa$ (assume $\pi_b = 1.0$ since not given)

$P_{t5} = P_{t4}\pi_t = \cancel{79\ldots}\ Pa$ 78300 pa

$P_{t9} = P_{t8} = P_{t5} = \cancel{79\ldots}\ Pa$ ($\pi_n = 1.0$) 78300 Pa

$T_{t3} = T_{t2}\tau_c = 409\,K$, $T_{t4} = 1032\,K$ ($M_4 = 1.0$)

$\therefore T_4 = \frac{T_{t4}}{\left(1 + \frac{\gamma-1}{2}\right)} = 860\,K$, $u_4 = \sqrt{\gamma R T_4} = 588.9\ m/s$

$T_{t5} = T_{t4}\tau_t = 873.62K = T_{t9} = T_{t8}$

$M_8 = 1.0 \Rightarrow T_8 = 728K$

$\therefore P_8 = \cancel{47802}\ Pa$, $\rho_8 = \cancel{0.1994}\ kg/m^3$

41363 pa .198

$u_8 = \sqrt{\gamma R T_8} = 542\ m/s$

$$A_8 = \frac{\dot{m}(1+f)}{\rho_8 u_8} = 0.069\ m^2$$

Let $M_2 = 0.5$ ('good' on-design compressor face Mach. No.)

$$T_2 = \frac{T_{t2}}{(1+\frac{\gamma-1}{2}M_2^2)} = 236.4K$$

$\therefore P_2 = 25701\ Pa$, $\rho_2 = 0.3776\ kg/m^3$

$u_2 = 154.3\ m/s$

$$\Rightarrow A_2 = \frac{\dot{m}}{\rho_2 u_2} = 0.126\ m^2$$

whereas $\underbrace{A_0 = A_1}_{\text{full mass capture}} = \frac{\dot{m}}{\rho_0 u_0} = 0.0976\ m^2$

(U0 = M0*sqrt (T0*gam*r) = 237.85 m/s)

0.0364 m2

$M_4 = 1.0 \Rightarrow A_4 = 0.\cancel{0384}\ m^2$

uninstalled thrust $= \dot{m}_8 u_8 - \dot{m}_0 u_0 + (P_8 - P_0)A_8 = 3.793\ kN$

Note: Rules of thumb:

Note that $A_3$ & $A_5$ can also be found with additional assumptions (design info):

a) Assume that the axial velocity is maintained thru the compressor: $u_3 = u_2$

b) Similarly, $\boxed{u_5 = 0.8 u_4}$

AE5535

Using the assumption that $u_3 = u_2 = 154.\ m/s$

$$\left(C_p T_3 + \cancel{\frac{u_3^2}{2}}\right) - \left(C_p T_2 + \cancel{\frac{u_2^2}{2}}\right) = W_{2\to3} = C_p(T_{t3} - T_{t2})$$

$$\therefore T_3 - T_2 = T_{t3} - T_{t2} \Rightarrow T_3 = 397.2\,K$$

$$M_3 = \frac{u_3}{\sqrt{\gamma R T_3}} = 0.386$$

$$\dot{m} = P_t A M \sqrt{\frac{\gamma}{R T_t}} \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}}$$

$$\Rightarrow A_3 = 0.0396\,m^2 \qquad T_5 = \frac{0.18}{C_p} u_4^2 + T_4 + T_{t5} - T_{t4}$$

Similarly, $T_5 = 764\,K$, $M_5 = 0.85$

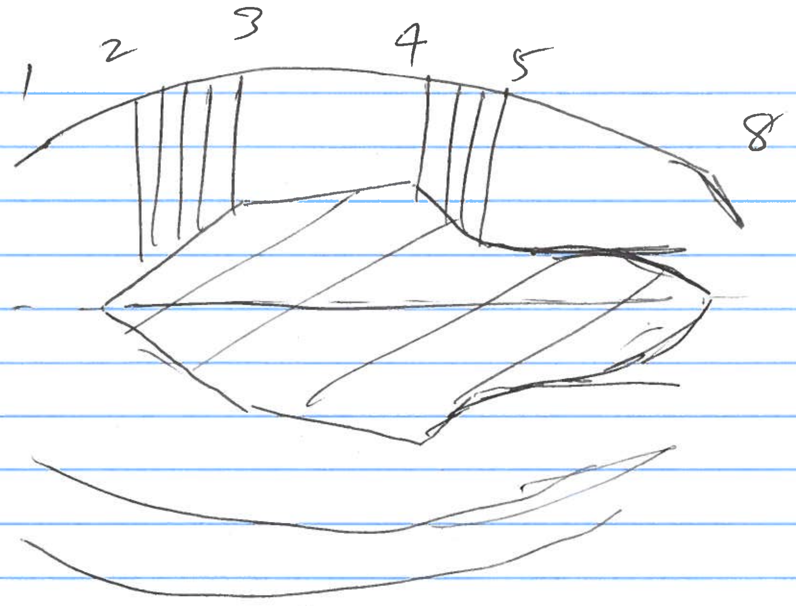
$\dot{m}$ considerations, $A_5 = 0.0694\,m^2$

So:

$A_1 = 0.0976\,m^2$ (inlet face) $= A_0$
$A_2 = 0.128\,m^2$ (compressor enterance)
$A_3 = 0.0396\,m^2$ (compressor exit)
$A_4 = 0.0364\,m^2$ (turbine entrance)
$A_5 = 0.07\,m^2$ (turbine exit)
$A_8 = 0.069\,m^2$ (Nozzle throat & engine nozzle exit)

$\therefore$ The engine is '<u>sized</u>'
Keep in mind ... $P_0 = 20\,kPa$ and $P_8 = 41.8\,kPa$

So, station 8 is choked $P_8 > P_0$ is good
if $P_0 > P_8$ the exit will unchoke!




Engine Schematic

Now, move engine to the given off-design point in the 3-space. Let $M_0 = .8$, $T_0$, $P_0$ stay the same. So $T_{t2}$ & $P_{t2}$ won't change.
But move to $\frac{T_{t4}}{T_{t2}} = 3.47$ (you have throttled the engine back down). this is 92.2% speed line on compressor map

so $T_{t4} = 861.1\,K$ $\left(\frac{T_{t4}}{T_{t2}} = 3.47\right.$ corresponds to 92.2% speed line on comp. map$\left.\right)$.

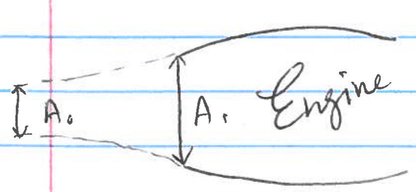
$\pi_c)_{off} = 3.9$ $(.78\,\pi_c$ on design$)$

$\eta_c)_{off} = 0.905$, $\dot{m}_{corr,2})_{off} = .88\ \dot{m}_{corr,2})_{on} = 20\ kg/s$

mdotcorr2 = mdot2*sqrt(theta2)/delta2

$\dot{m} = 6.472\ kg/s \Rightarrow$ spillage $= \underset{\substack{\uparrow \\ \text{complete} \\ \text{mass} \\ \text{capture}}}{\overset{7.355}{\dot{m}_{on}}} - \overset{6.472}{\dot{m}_{off}} = 0.885\ kg/s$

(mdot air design is the 'reference' mdot used for spillage def. since M0, ALT same, would change if M0, ALT changed!)




$\Rightarrow A_0 = 0.0859\ m^2$

whereas $A_1 = 0.0976\ m^2$

off-design

$\tau_c = 1.525$

$\tau_t = 1 - \frac{T_{t2}}{T_{t4}}(\tau_c - 1) = 0.849$

$f = 0.011$ so $\dot{m}_f = 0.07$ kg/s (make sense cause you throttled back)

compared to $\dot{m}_{f_{on}} = 0.105$ kg/s

$P_{t3} = 118895$ Pa

$P_{t8} = 61719$ Pa

$T_{t3} = 378.2$ K

$T_{t8} = 731$ K

still assuming choked flow at station 8

$M_8 = 1.0 \Rightarrow T_8 = 609$ K , $P_8 = 32{,}605$ Pa

$\rho_8 = 0.1858$ kg/m$^3$ , $u_8 = 495.6$ m/s

$\text{Thrust} = \dot{m}_8 u_8 - \dot{m}_0 u_0 + (P_8 - P_0) A_8$

Thrust = 2.57 kN dropped!

if $\left.\text{RPM}\right)_{\text{on design}} = 70{,}000$

$\therefore \left.\text{RPM}\right)_{\text{off design}} = (.922)(70{,}000) = 64{,}540$ RPM [$T_{t2}$ = const. between on & off

'Homework 5 / in class example' : Recap

→ Found engine areas at stations 1, 2 ... 8 using 'on-design' info & requiring good characteristics at that on-design point (i.e, complete mass capture, $M_2 = 0.5$, etc.)

→ Then found 'off-design' performance at a single operating point in the 3-space of $\dot{m}_f$, ALT, $M_0$ based on the ('previously determined') operating line on the compressor where $\frac{T_{t4}}{T_{t2}} = 3.47$ (where $M_0$, ALT not changed from 'on-design' but throttled back since $T_{t4}/T_{t2}$ was 4.16.)

→ Found 'off-design' thrust

$F_{\text{'off-design'}} = 2570\ N$ ($F_{\text{'on-design'}} = 3793\ N$)

$\dot{m}_f|_{\text{'off-design'}} = 0.07$ kg/sec ($\dot{m}_f|_{\text{'on-design'}} = 0.105$ kg/sec)

etc.

→ Keep in mind, this is engine performance at the given $M_0$, ALT, $\dot{m}_f$! (steady-state)

→ Of course, if engine integrated on vehicle, & vehicle is initially cruising at $M_0 = 0.8$ where $\dot{m}_f = 0.105$ kg/sec

Then Thrust = 3793 N = External Drag.

→ But throttle back to $\dot{m}_f = 0.07$ kg/sec, the thrust drops immediately to 2570 N as calculated (but 'at first' external drag 'stays the same'... but then $M_0$ will begin to drop since for the vehicle

$$dV\Big]_{vehicle} = \frac{(F_{engine} - D_{external})\,dt}{MASS_{vehicle}}$$

but then, of course, thrust will begin to respond (change) due to the lowering $M_0$ (as vehicle decelerates).

## Engine Control System Limits (ECS)

Controls limit maximum $T_{t4}$, $\pi_c$, $N_c$, $N_t$, $T_{t3}$, $P_{t3}$, etc. obtainable … This then drives the operating line & possible engine performance …

## Performance Accuracy

Things that affect component performance maps (& hence operating line accuracy)

1) Test accuracy
2) Reynolds # effects (size); fluid dynamic separation
   can change results dramatically;
   sub-scale (small) engine designed from given component maps could have completely laminar flow on first blade rows (with increased risk of separation) (might not be reflected by 'larger-scale' actual compressor used in developing ~~oper~~ component maps!
   (could be mostly turbulent flow)
3) humidity ($\gamma$) effects

Not generally as significant as $M_2$ & $N_c$! for preliminary design, 'neglect'

## Transient (acceleration) effects!



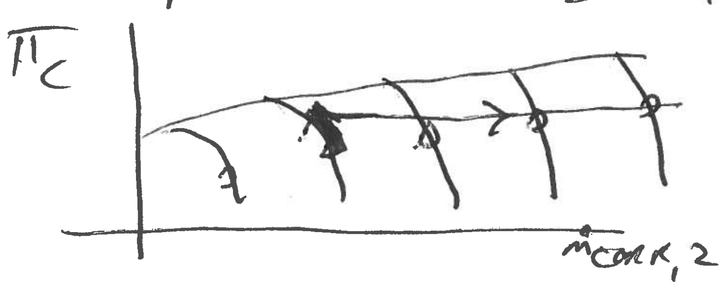
We develop a 'steady state' operating line based on steady state matching criteria, i.e. $\dot{m}_2 = \dot{m}_4$, etc., steady state TCPB assumed, etc.

During 'transient' operation, not necessarily good assumption (although since flow 'residence times' on order of 0.01 seconds through a typical engine & performance transients are generally over several seconds, steady-state operation is usually a pretty good assumption ... But if departure from operating line large enough in transients, trouble!

but some engines can accelerate from idle to takeoff pwr in ~6 seconds

### example 'fuel-throttle' slam!

Sudden $\dot{m}_f$ increase causes $P_{t4}/P_{t2}$ to ↑ suddenly ($P_{t2}$ same initially); engine does NOT stay on steady-state operating line (accelerating spool takes some of power so TCPB not steady).




Hence ECS will limit rate of $\dot{m}_f$ increase during acceleration of spool to keep engine from surge

Unchoked Conditions at station '8' (exit) for converging Nozzle turbojets (very rare),
Turboprops (most of the time),
Turbofans (some of the time, but usually choked)
{ can 'unchoke' at low-throttle, low speed conditions where $P_8$ is low }

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If $M_8 \neq 1.0$, $\dot{m}_{corr,8}$ not fixed between
→ on & off design; Analysis is slightly complicated but you can assume $P_8 = P_0$ (always true if engine exit flow is subsonic); then given

$\frac{P_{t8}}{P_8} \rightarrow M_8$ determined, hence $\dot{m}_{corr,8}$ can be found at given operating point

→ Properly, would factor in Nozzle / turbine losses (like $\pi_N$) due to subsonic back pressure effects!

# Project Hints / Requirements, actually

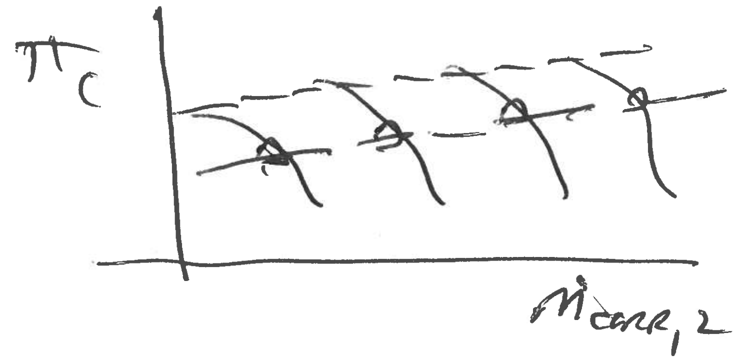
A. Perform on-design analysis (show all fluids / engine performance, Areas, etc.)

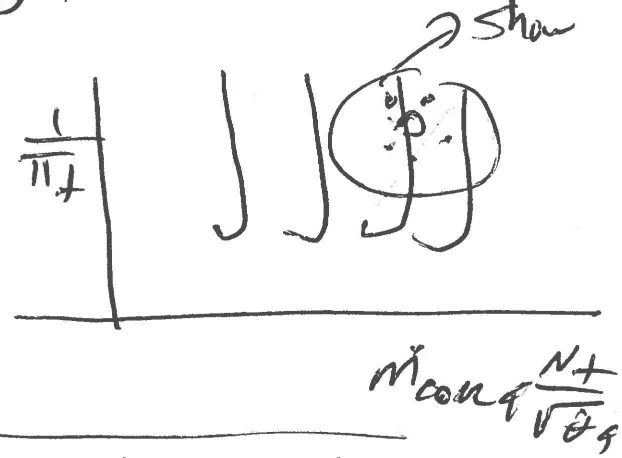
B. Develop complete engine operating line based on finding the 'matched' $\pi_c$, $\dot m_{corr,2}$, $\tau_c$, $\eta_c$, $\tau_t$, $\pi_t$, $\frac{P_{t4}}{P_{t2}}$, $N_t/\sqrt{\theta_4}$ for each speed line on the compressor map (so 'n' operating points will be found where n is total # of speed lines

Required Table

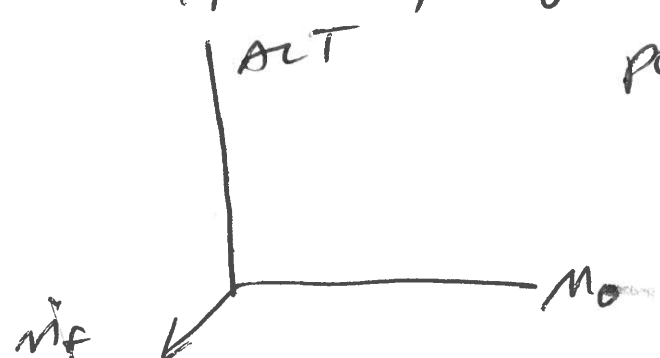
| | $N_{c2}/\sqrt{\theta_2})_1$ | $N_{c2}/\sqrt{\theta_2})_2$ | $\cdots$ | $N_{c2}/\sqrt{\theta_2})_n$ |
|---|---|---|---|---|
| $\pi_c$ | | | | |
| $\dot m_{corr,2}$ | | | | |
| $\dot m_{corr,4}$ | | | | |
| $\dot m_{corr,8}$ | | | | |
| $\eta_c$ | | | | |
| $\tau_c$ | | | | |
| $\pi_t$ | | | | |
| $\tau_t$ | | | | |
| $P_{t4}/P_{t2}$ | | | | |
| $N_t/\sqrt{\theta_4}$ | | | | |
| $\Delta$ Convergence (match) | | | | |

Required plots (Part B):



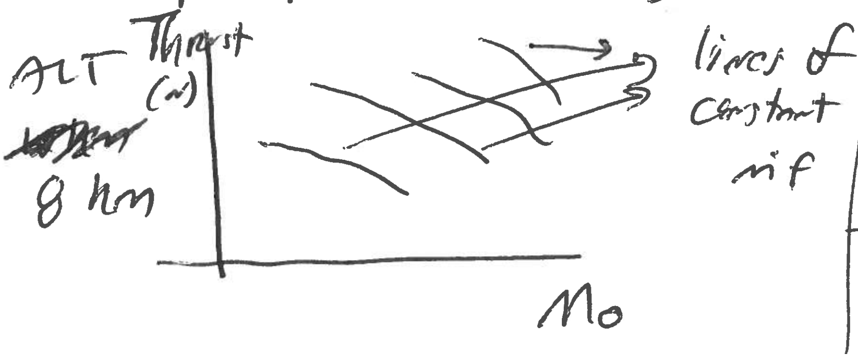

C. Then use the operating line (table) info to develop engine performance 'envelope' in '3-space' of $\dot{m}_f$, ALT, $M_0$

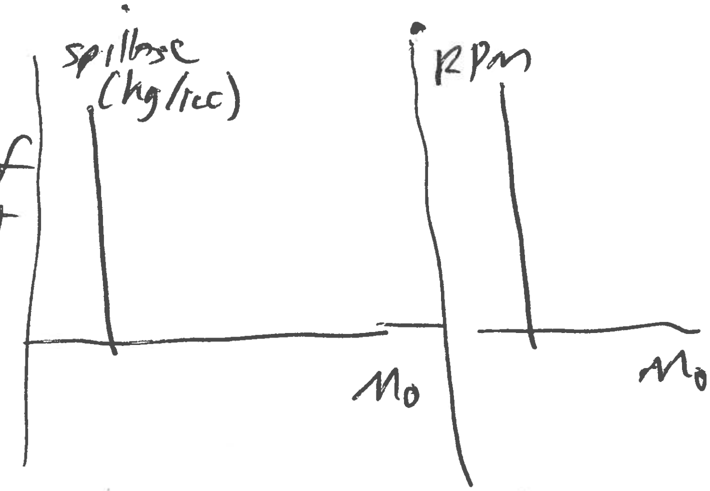



performance is:

a) uninstalled thrust
b) actual ('raw') RPM
c) spillage

Req'd plots (Part C)




+ same at ALT 4 km, & ALT sea level (9 plots here total)

for $\dot{m}_f$ increments, see project requirements

Issue:

Per requirements you need to provide Thrust, RPM, spillage at specific values of $\dot{m}_f$, $M_0$ at specified altitudes ... but this is 'hard to do' for specified values of $\dot{m}_f$, $M_0$, ALT ....

Note $$\frac{T_{t4}}{T_{t2}} = \frac{T_{t4}}{T_0\left(1+\frac{\gamma-1}{2}M_0^2\right)} = \frac{\tau_\lambda}{1+\frac{\gamma-1}{2}M_0^2}$$

& since $\tau_\lambda$ can be found for a given $M_0$, ALT, $\dot{m}_f$ (we did this!) you can find a $T_{t4}/T_{t2}$ at a $M_0$, $\dot{m}_f$, ALT

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But the $\frac{T_{t4}}{T_{t2}}$ thus found for a given $\dot{m}_f$, ALT, $M_0$ will most likely ~~fal~~ fall between the 'known' $T_{t4}/T_{t2}$ values on the operating lines


This would require interpolation on operating line (or in tabulated data) ... very complex, errors, etc.

Instead, do this: To eliminate interpolation between known operating points on speed lines (matrix data from part B) ...

You know/have 8 'exact' operating points and <u>All</u> <u>info</u> ($\pi_c$, $\eta_c$, etc) at <u>each</u> of those points.

Look at burner enthalpy balance

$$\dot{m}_f h = \dot{m}\, c_p (T_{t4} - T_{t3})$$

: you check this (verify!)

$$* \quad \dot{m}_f h = \frac{P_{t2}}{P_{STD}} \sqrt{T_{STD}}\, \dot{m}_{corr,2}\, c_p \sqrt{T_{t2}} \left( \frac{T_{t4}}{T_{t2}} - \tau_c \right)$$

& remember $\tau_c$ & $\dot{m}_{corr,2}$ have discrete values corresponding to the (8) known values of $T_{t4}/T_{t2}$

& also

$$P_{t2} = P_0\, \pi_d \left(1 + \frac{\gamma-1}{2} M_0^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$T_{t2} = T_0 \left(1 + \frac{\gamma-1}{2} M_0^2\right)$$

$P_0, T_0 \rightarrow$ ALT!

This (*) can be explicitly (analytically) solved for <u>$M_0$</u> at a given $\dot{m}_f$, given ALT, and at each known (tabulated) operating point.

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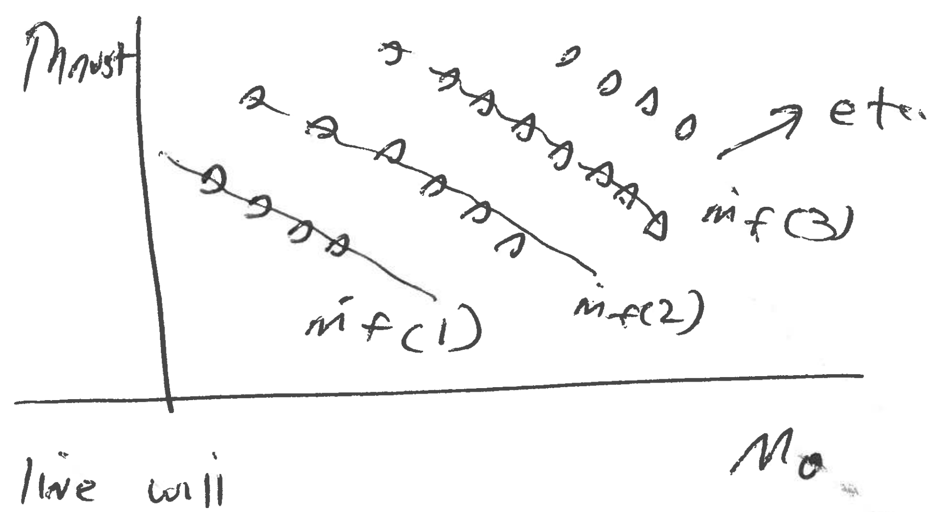
For {$\dot{m}_f$, ALT} specified, use operating point 1
to find $M_0$ result
... then use operating point 2
to find $M_0$ result
⋮ etc ...
to operating point 8

Also find corresponding Thrust, spillage, RPM
at each of these points

Then plot




Each $\dot{m}_f$ line will
have 8 or < 8 points ...

if $M_0$ found from * less than zero
point not physical, discard

Also apply ECS limits (will limit 'envelope' performance); may eliminate (reduce possible points on a given $\dot{m}_f$ line (so 0 to 8 points possible)