AE_5335 Project: Turbojet Component Matching

Gari M. Pahayo[1]
Missouri University of Science and Technology, Rolla, Missouri, 65409, USA

I. Table of Contents

II. Introduction

This document is the completed project. The project will be given with the results found. A methodology section will also be given for each part. Code and other information will be given in the appendix.

III. Project details

AE 5535 **Assigned: March 5, 2021**
Due: March 29, 2021

<u>Project: Turbojet Engine Design/Off-Design Performance Analysis</u>

Consider a single-spool fixed-area turbojet missile-class engine which has compressor and turbine maps attached. The engine has a simple converging nozzle.
(i.e., station 8 is the exit of the engine). For consistency and simplicity, assume that $\dot{m}_{fuel} << \dot{m}_{air}$ (i.e., f << 1). The engine "design point" values are as follows:

$M_0 = 0.8$ Altitude = sea level (T_0 = 288K, P_0 = 101325 N/m^2)

$\pi_C = 15$ A_1 (inlet face area) = 0.00318 m^2 $\pi_b = 1.0$ $\eta_t = .90$

$M_2 = 0.5$ RPM_{design} = 60,000 RPM T_{t4} = 1500K $\pi_d = .9904$

A. The flow is choked at both turbine inlet and nozzle throats. Assume fuel-air ratio is much less than 1.0 throughout all your work (including in part B and C) and use constant specific heats, etc. (γ=1.4, C_P = 1004.5 J/kg-K, R = 287 J/kg-K.) The

[1] Student, Aerospace Engineering, and AIAA Member Grade (if any) for first author.

heating value of the fuel is $4.45\text{x}10^7$ J/kg (fuel). At the design point, the compressor maintains constant axial velocity (i.e., $u_3 = u_2$) and the turbine exit velocity is 80% of the turbine entrance velocity (i.e., $u_5=0.8u_4$). Also assume that the engine is sized to mandate *full mass capture* at this 'design point'.

Completely define the on-design engine fluid dynamics, areas. and performance. Specifically, fill out the relevant information (data table and other information) relevant to this section as given below.

B. Determine, tabulate (fill in information on the attached data table provided below for this section), and plot on the compressor map the steady-state operating line for this engine. Use a convergence requirement (between right hand side and left-hand side of the nozzle matching criteria) of at least 0.01 for all operating points.

Assume that that the flow remains choked at both *turbine* inlet and nozzle throats for the operating range of interest and that the turbine efficiency (η_t) is constant for the development of the operating line. Perform your analysis if the engine control system prevents compressor ratios higher than 19, shaft RPM greater than 66000 and burner exit temperatures higher than 1800K. After the operating line is developed, comment very briefly on the effect of the approximation of constant turbine efficiency by examining the turbine performance map provided and comparing the approximated turbine characteristics against the actual turbine performance map.

C. Develop the performance envelope of this engine by calculating and plotting raw engine thrust (uninstalled), turbo-machinery RPM, and engine spillage characteristics for flight Mach numbers from zero to 2.4, a full range of possible fuel throttle settings.
(ranging – as possible - from 20% of design throttle to max feasible throttle, in increments of 20% design fuel flow rate), and three altitudes (sea level, 4500m, and 9000m). Assume in this performance analysis that the inlet (diffuser) total pressure drop is given using the following empirical inlet relationship: $\pi_d = 1 - 0.015 * M_o^2$. This will result in nine plots (see attached summary of required plots).

Presentation (besides the requested data/plots): Besides the requested data tables, information, operating line plot, and nine operating envelope plots, also include in appendices work (neatly presented) and code(s) used and methodology description, etc. I am looking for an intelligible summary of your work that is not everything you did, but provides a clear picture of what you did, how you did it.

Note: This project will be graded based on 70% technical content, 30% presentation.

IV. Results

Numbering of the stations follows AIAA conventions. The face of the engine would be station 1 and the nozzle would be station 8.

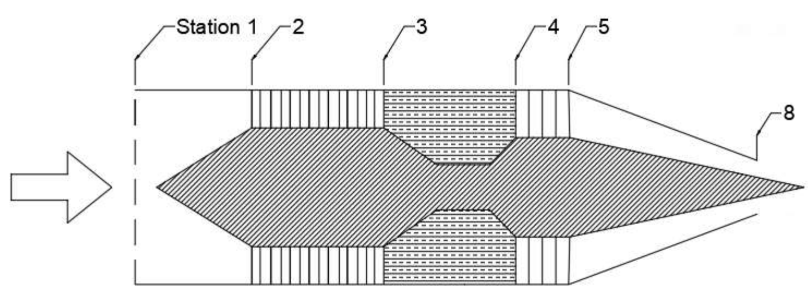


Fig. 1 Numbering Scheme

A. Part A

Table 1 Part A Data

Station	0	1	2	3	4	5	8
u (m/s)	272.139	272.139	176.291	176.291	708.696	566.957	593.784
P (N/m^2)	101325	101325	128957.483	2137470.78	1212176.865	309258.738	296740.897
T (K)	288	288	309.394	755.757	1250	860.6	877.5
Mach (M)	0.8	0.8	0.5	0.3199	1	0.9641	1
ρ (kg/m^3)	1.226	1.226	1.452	9.855	3.379	1.252	1.178
A (m^2)	0.00318	0.00318	0.00414	6.11E-04	4.43E-04	0.001494	0.001492
P_t (N/m^2)	154453.752	154453.8	152970.996	2294564.93	2294564.933	561709.496	561709.496
T_t (K)	324.864	324.864	324.864	771.227	1500	1053	1053
τ_t (T_{t5}/T_{t4})	0.702						
π_t (P_{t5}/P_{t4})	0.2448						
Thrust (uninstalled N)	29946.837						
$mdot_{air}$ (kg/s)	1.0609						
$mdot_{fuel}$ (kg/s)	0.0174						
f (fuel/air ratio)	0.01644						
$mdot_{corr4}$ (kg/s)	0.107						
$mdot_{corr8}$ (kg/s)	0.366						

B. Part B

Table 2 Part B Data

Speed Lines	π_c	τ_c	$mdot_{corr2}$	η_c	$N_c/\sqrt{\theta_2}$	$N_t/\sqrt{\theta_4}$	τ_t	π_t	T_{t4}/T_{t2}	Δ (convergence criteria)
0.76	7.95	1.9917	0.4663	0.815	4.04E+04	2.22E+04	0.7021	0.2449	3.3286	0.000281
0.82	9.195	2.0172	5.32E-01	0.87	4.36E+04	2.36E+04	7.03E-01	2.46E-01	3.43E+00	3.40E-03
0.875	10.695	2.0878	0.599	0.89	4.65E+04	2.44E+04	0.7019	0.2447	3.6494	0.0006876
0.922	12.195	2.1657	0.6621	0.895	4.90E+04	2.49E+04	0.6999	0.2418	3.8843	0.006421
0.964	13.425	2.2646	0.6994	0.87	5.13E+04	2.50E+04	0.7002	0.2423	4.2187	0.005139
1	15	2.373923	0.746	0.85	5.32E+04	2.47E+04	0.7032	0.2464	4.629	0.00257
1.033	16.875	2.5525	0.79	0.8	5.49E+04	2.40E+04	0.7028	0.2459	5.2238	0.001521
1.063	18.9375	2.8816	0.8064	0.7	5.65E+04	2.25E+04	0.702	0.244	6.3137	0.00039296

As seen in table 2, The total temperature ratio across the turbine and total pressure ratio across the turbine do not vary much. This is due to the assumption that the turbine efficiency is constant. This is indicative of the analytical method of solving fixed-area turbojets (FATs). These slight changes/nearly constant values can be seen on the turbine map (figure 3).

Figure 2 shows the operating points along the speed lines on the compressor map.

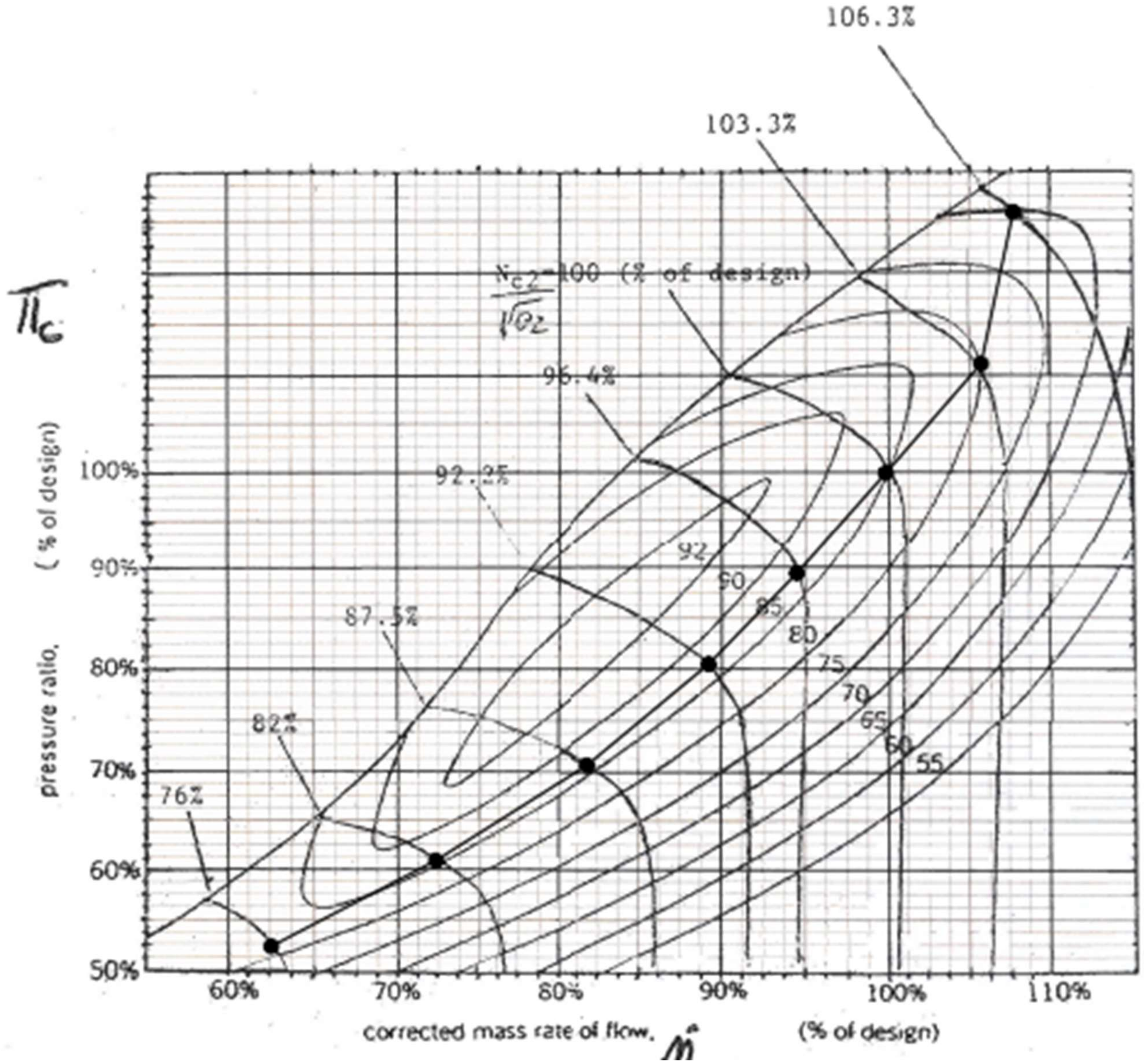


Fig. 2 Operating Line

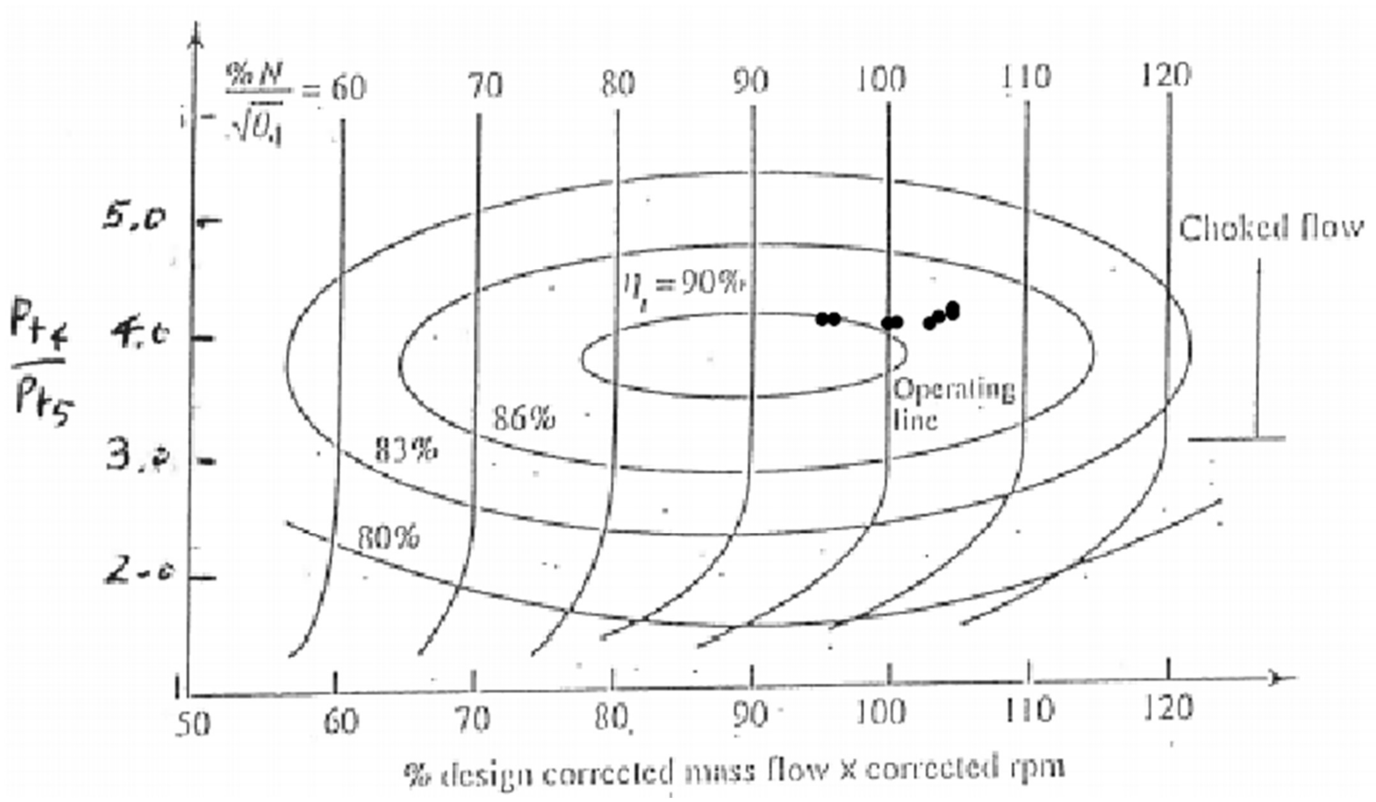


Fig. 3 Turbine Map

As stated above, total pressure ratio across the turbine is more or less constant.

C. Part C

Part C is further broken into three sub-parts, where in each part consists of three graphs, meaning that there a total of nine graphs. Each sub-part is altitude constrained and the performance envelopes are examined. The green lines represent the speed lines given in Part B. Only the lowest and highest speed line, and speed lines between the min and max are the speed lines with values between min and max in ascending/descending order. Lines of constant fuel flow rate are marked with red, and follow the same rules as the speed lines, unless explicitly told the so. Additionally, fuel lines are incremented by 20%. (The color of the speed lines and the fuel line were deliberate, as red and green are complimentary colors and have a high contrast).

Sea Level (0m)

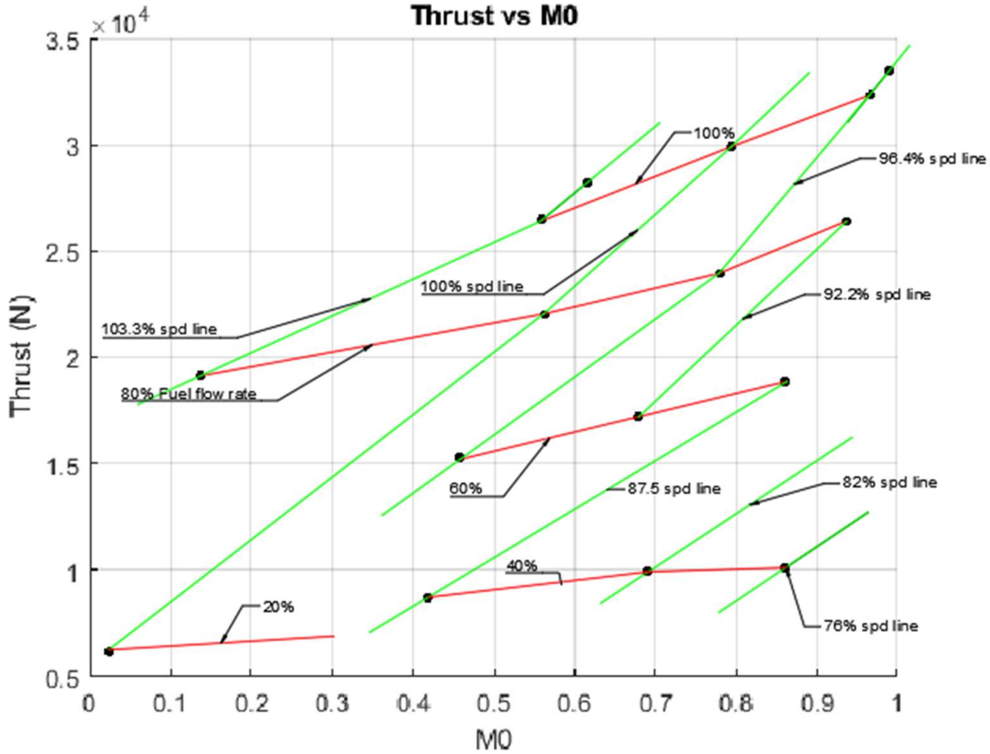


Fig. 4 Thrust vs Flight Mach at Sea Level (0m)

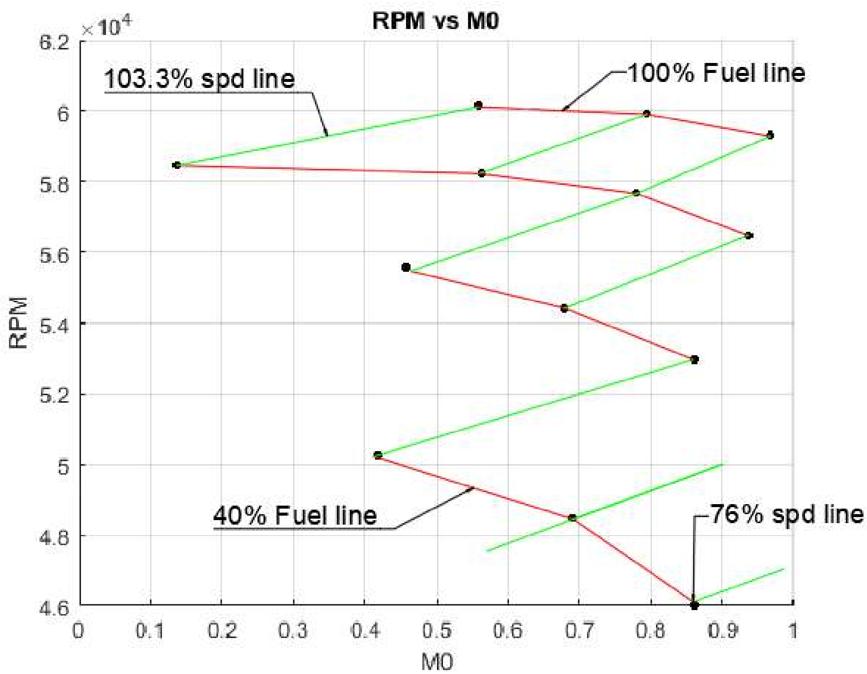


Fig. 5 RPM vs Flight Mach at 0m

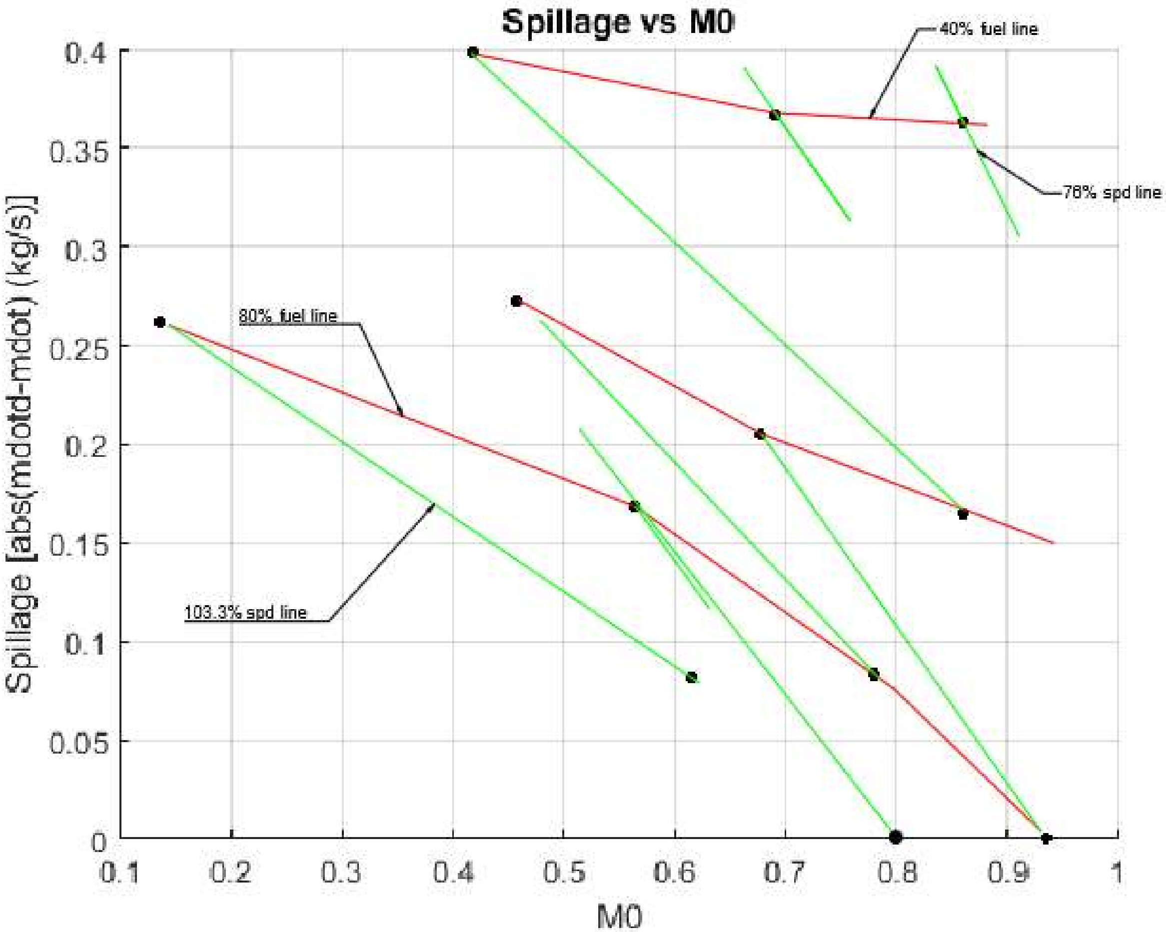


Fig. 6 Spillage vs Flight Mach At Sea Level

4500 m

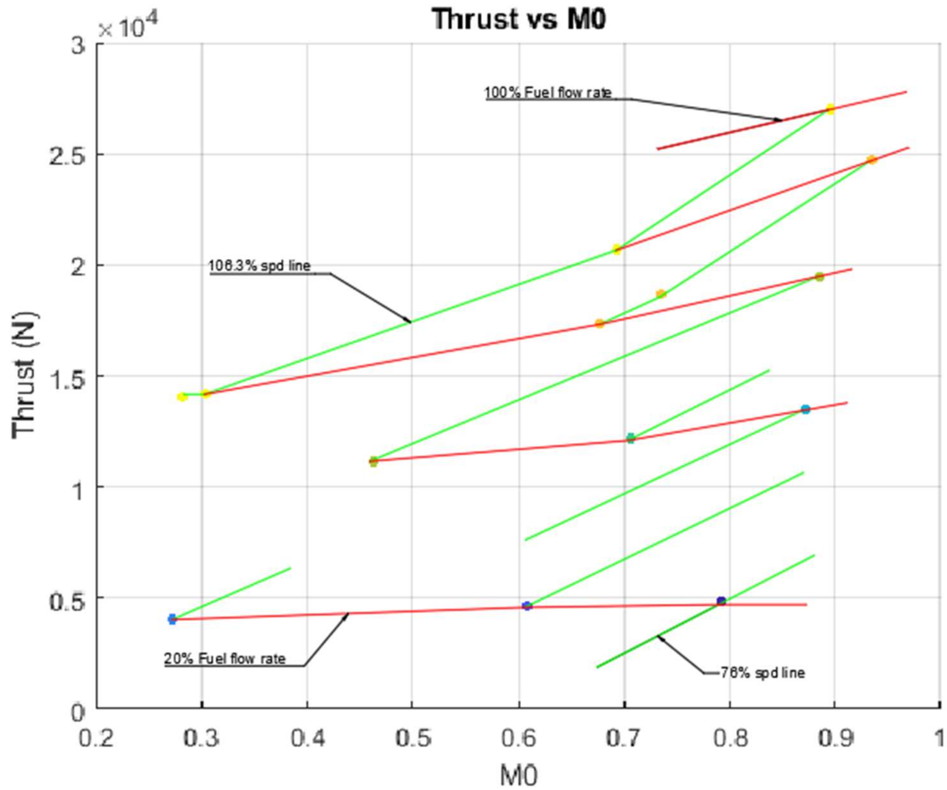


Fig. 7 Thrust vs Flight Mach at 4.5 km

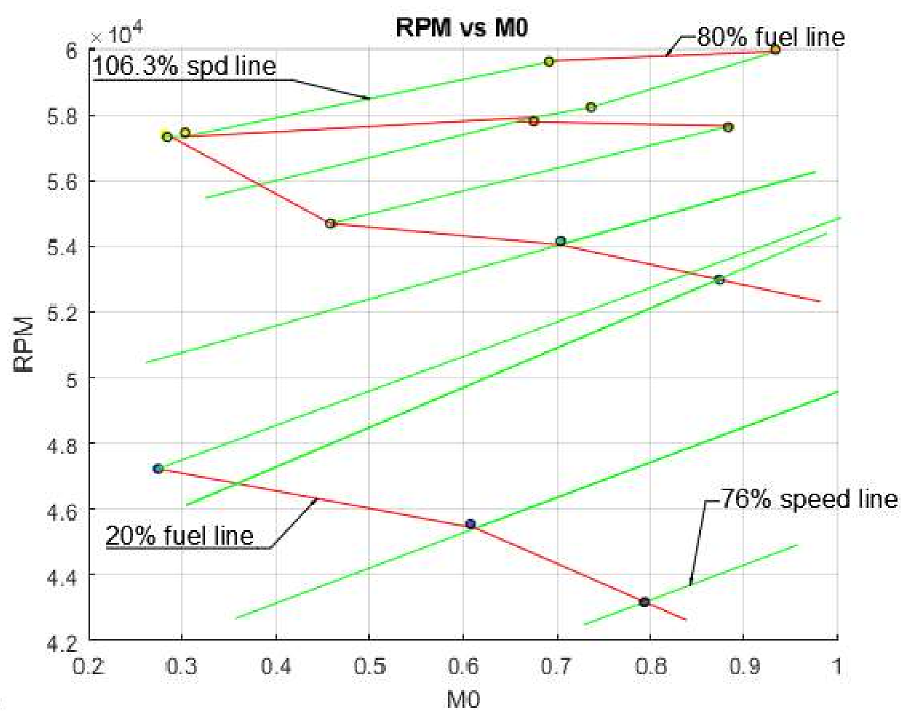


Fig. 8 RPM vs Flight Mach at 4.5 km

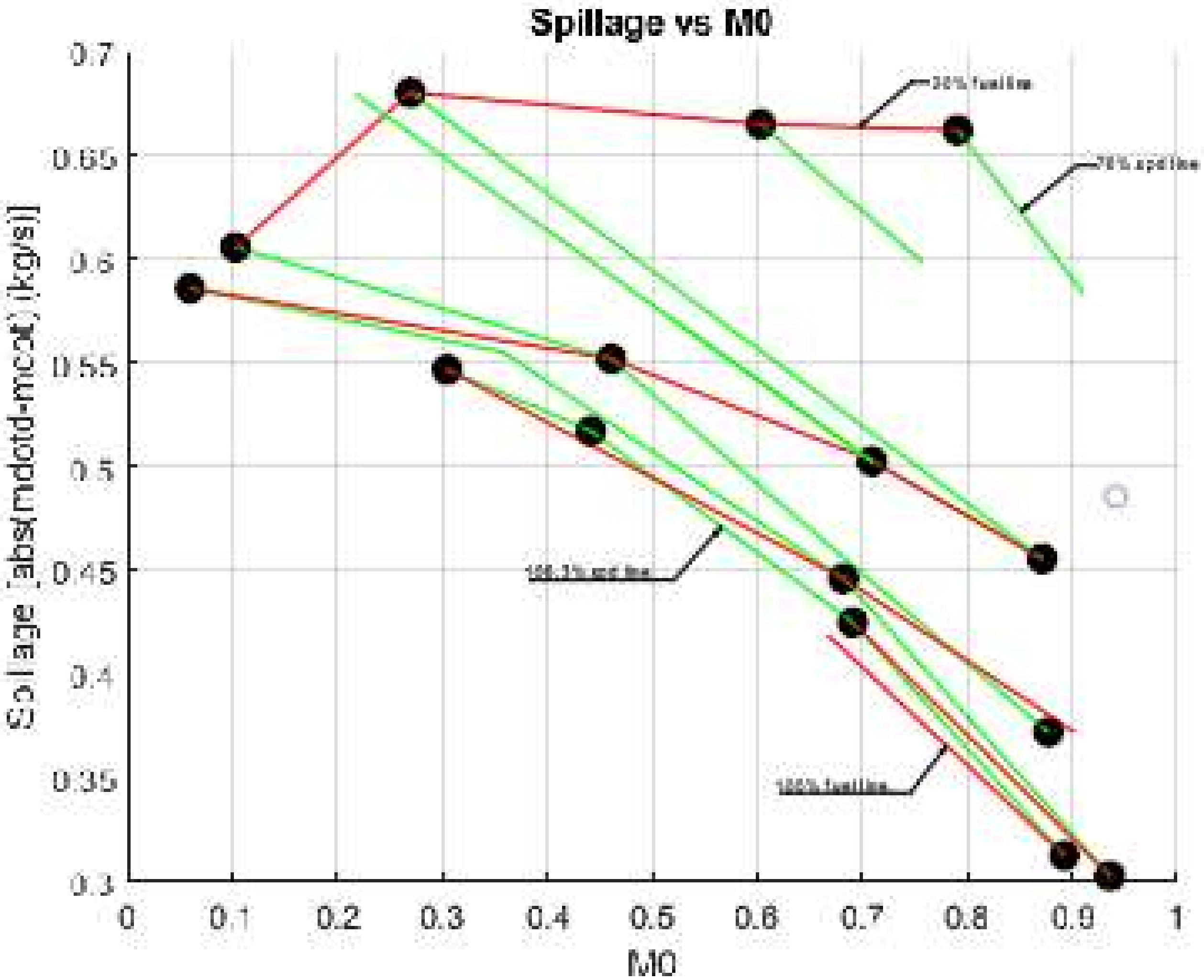
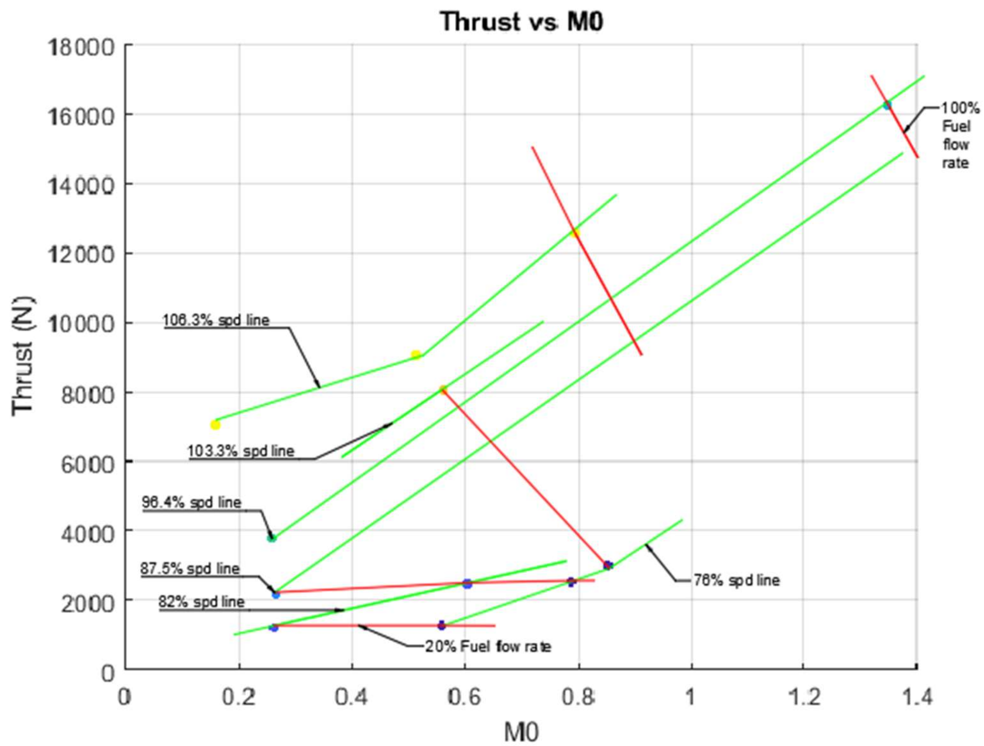


Fig. 9 Spillage vs Flight Mach at 4.5 km

9000 m



Fig, 10 Thrust vs Flight Mach at 9 km

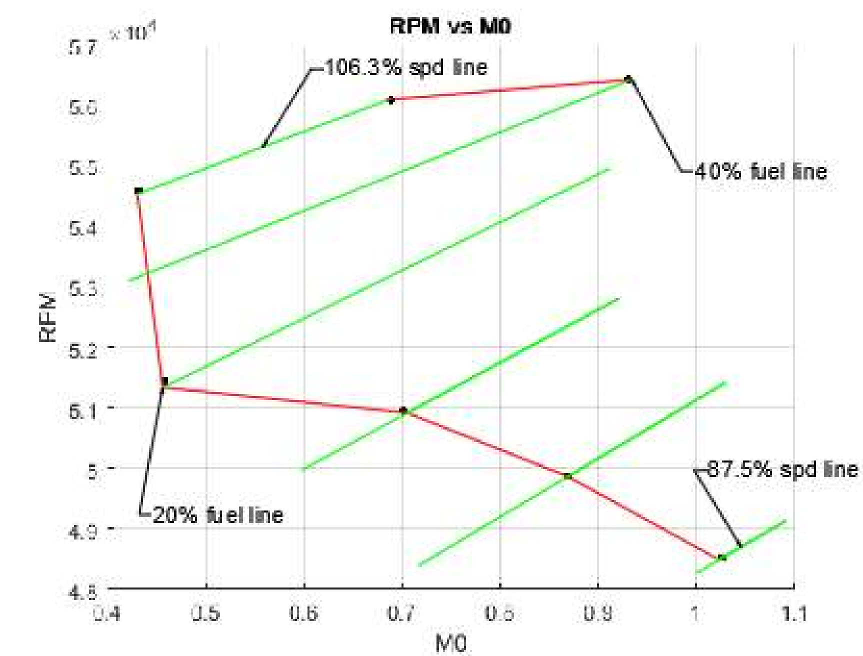


Fig. 11 RPM vs Flight Mach at 9 km

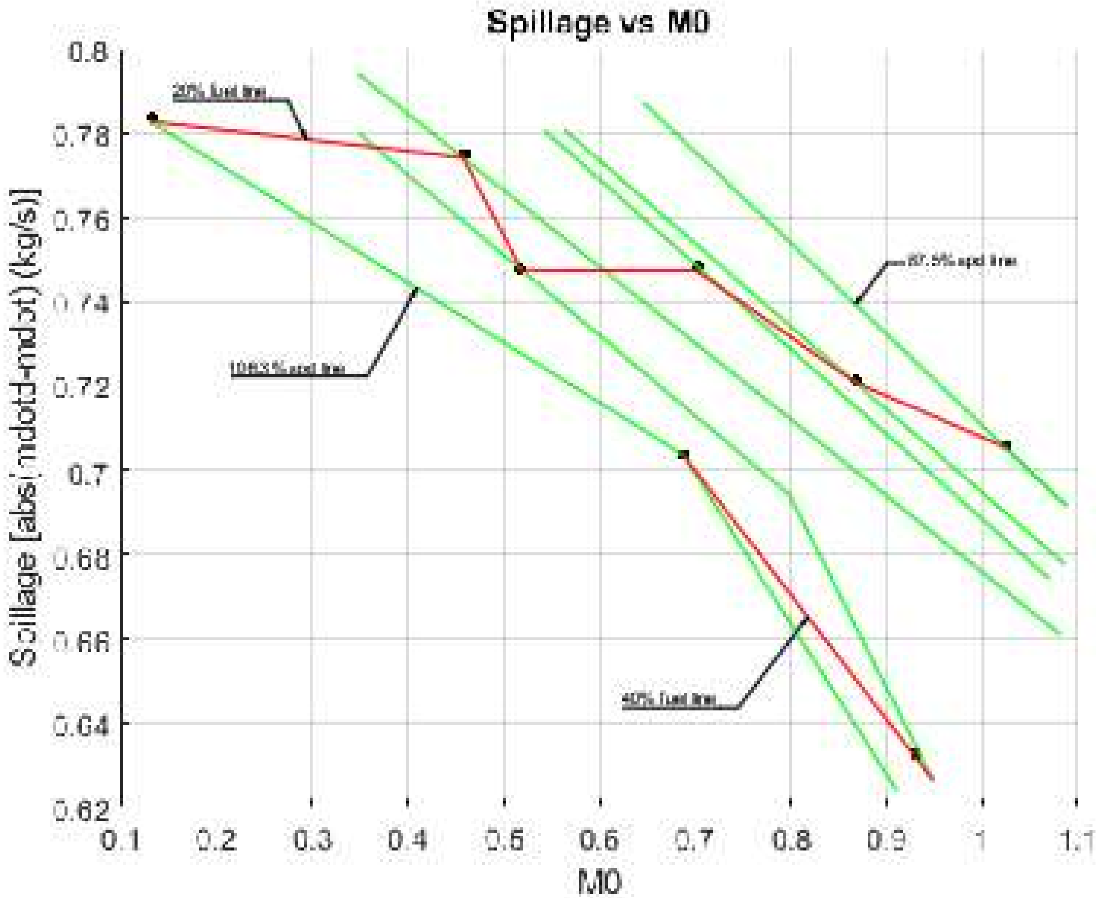


Fig. 12 Spillage vs Flight Mach at 9 km

V. Methodology

Part A

To get the values to part one, first chain through the engine, getting the total pressures and total temperatures at each station. The values that should now be acquired are the total pressures, total temperatures, and the corresponding ratios.

$$\tau_c = 1 + \frac{\pi_c{}^{(gam-1)/gam}{}_{-1}}{\eta_c} \quad (1)$$

$$\tau_t = 1 - \frac{T_{t2}}{T_{t4}}(\tau_c - 1) \quad (2)$$

$$\pi_t = (1 - (1 - \tau_t)/\eta_t)^{gam/(gam-1)} \quad (3)$$

The static temperatures and pressures should now be able to find under the assumption that station 4 and 8 are chocked (Mach is 1) and with the additional velocity requirement through the compressor and the turbine; namely, velocity at the entrance of the compressor is the same as the velocity exiting the compressor and the velocity at the exit of the turbine is 80% of the velocity at the entrance of the turbine. Now velocities should also be tabulated. Finally get densities at each station to size the engine. Now performance should be able to be solved for. Use the thrust equation and fuel-air ratio definition under the assumption fuel-air ratio is much less than one. Also, see appendix A for scratch work.

Part B

To get the data from part 2, an iterative method must be used. First the design corrected RPM at the compressor face must be calculated. At the desired speed line, a guess for the compressor ratio is made by examining the compressor map. The total temperature across the compressor is found by using the equation 1. The corrected RPM at station 4 is calculated with equation 4. The total temperature at station 2 divided by the total temperature at station 4 is equal to the square of the quantity of the corrected RPM to the corrected RPM at station 4. Use turbine compressor balance to get the total temperature ratio across the turbine (use equation 2 and equation 3). Since the engine is chocked at station 4 and 8, corrected mass flow rate does not change. The corrected mass flow rate at station 8 over the corrected mass flow rate a station 2 must equal the right-hand side of equation 5.

$$\frac{N_t}{\sqrt{\theta_4}} = \frac{\dot{m}_{corr2}}{\dot{m}_{corr4}} \frac{1}{\pi_c} \frac{N_c}{\sqrt{\theta_2}} \quad (4)$$

$$\frac{\dot{m}_{corr8}}{\dot{m}_{corr2}} = \sqrt{\frac{T_{t4}}{T_{t2}}\tau_t} \frac{1}{\pi_c \pi_t} \quad (5)$$

A singular guess the 76% speed line is given in appendix B and code is given in appendix B (Match.m).

Part C

The first thing to do is to find the flight Mach number. Once the Mach number is found everything can be solved for. Values from part are used. An outline for the procedure and code (partC.m) are given in appendix C.

Appendix A

A. Scratch work

Wednesday, March 17, 2021   7:52 AM

$\eta_{c_p} = .85$

$M_0 = .8$

$\pi_c = 15$

$M_2 = .5$

Altitude = sea level (T_0 = 288K, P_0 = 101325 N/m²)

A_1 (inlet face area) = 0.00318 m² $\pi_b = 1.0$ $\eta_t = .90$

RPM_{design} = 60,000 RPM T_{t4} = 1500K $\pi_d = .9904$

$\tau_r = \frac{T_{t2}}{T_0}$

$A_1 = A_\infty \rightarrow$ design

$T_{t2} = T_0 \left(1 + \frac{\gamma-1}{2} M_0^2\right)$

$P_{t2} \Rightarrow \left(1 + \frac{\gamma-1}{2} M_0^2\right)^{\frac{\gamma}{\gamma-1}} P_0 = P_{t1}$ $P_{t2} = P_{t1} \pi_d$

$\dot{m} = \rho u A$

$\tau_c = 1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c}$

$P_{t3} = P_{t4}$

$\tau_t = 1 - \frac{T_{t2}}{T_{t4}} (\tau_c - 1)$

π

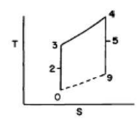


Fig. 5.20 Temperature-entropy diagram.

$M_8 = 1.0 \Rightarrow T_8 =$

Mach number=	1	Mach angle=	90	P-M angle=	0
p/p0=	0.52828178	rho/rho0=	0.63393814	T/T0=	0.83333333
p/p*=	1	rho/rho*=	1.00000000	T/T*=	1

$\frac{N_{c2}}{\sqrt{\theta_2}}$

$M_4 =$ $M_8 = 1 \Rightarrow T_8 \Rightarrow P_8$

$T_{t8} = \cancel{T_{t4}} T_{t4} \tau_t$

Velocity $\rightarrow$ temps. $\rightarrow$ machs $\rightarrow$

$\rho_0 u_0 A_0$

$\frac{P_0}{R T_0} M_0 \sqrt{\gamma R T_0} A_0 = \frac{101325}{287 \cdot 288} .8\sqrt{1.4(287)288} (.00318) = \dot{m} = 1.0609 \text{ kg/s}$

$\tau_r = 1 + \frac{\gamma-1}{2} M_0^2 = 1 + .2(.8)^2 = 1.128$

$T_{t3} = \tau_c T_{t2} = 2.374(324.864) = 771.227 K$

$T_{t0} = \tau_r T_0 = 324.864 \text{ K} = T_{t2}$

$P_{t0} = P_0 [\tau_r]^{\frac{\gamma}{\gamma-1}} = 154458.752 \text{ pa}$

$P_{t2} = \pi_d P_{t0} = 152970.996 \text{ p.}$

$\dot{m}_{corr_2} = \frac{\dot{m}\sqrt{T_{t2}/T_{tsp}}}{P_{t2}/P_{tsp}} = .746 \text{ kg/s}$

$\tau_c = 1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c}$; $\eta_c = .85$ $\Rightarrow \tau_c = 1 + \frac{15^{\frac{.4}{1.4}} - 1}{.85} = 2.374$

T 1500

$$\frac{T_{t4}}{T_{t2}} = \frac{1500}{324.864} = 4.617$$

$$\tau_t = 1 - \frac{T_{t2}}{T_{t4}}(\tau_c - 1) = 1 - \frac{1}{4.617}(2.374 - 1) = .702$$

$$\pi_t = \left(1 - \frac{1-\tau_t}{\eta_t}\right)^{\gamma/(\gamma-1)} = \left[1 - \frac{1-.702}{.9}\right]^{1.4/.4} = .2448$$

$$T_{t8} = \tau_t T_{t4} = 1053\,K$$

$$P_{t8} = \pi_t P_{t4} = 561709.496\ pa$$

$$P_{t4} = P_{t3} = \pi_c P_{t2} = 2294564.433\ pa$$

$$M_8 = 1 \Rightarrow \frac{T_{t8}}{T_8} = 1 + \frac{\gamma-1}{2}M_8^2 \Rightarrow \frac{T_{t8}}{1.2} = T_8 = 877.5$$

$$\frac{P_{t8}}{P_8} = \left(1 + \frac{\gamma-1}{2}M_8^2\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow P_8 = 296740.897$$

$$\rho_8 = 1.178$$

$$u_8 = M_8\sqrt{\gamma R T_8} = 593.784\ m/s$$

$$u_0 = M_0\sqrt{\gamma R T_0} = 272.139\ m/s$$

$$T = \dot{m}(1+f)(u_8 - u_0) + (P_8 - P_0)A_8$$

$$f = \frac{\tau_r\left(T_{t4}/T_{t2} - \tau_c\right)}{\frac{h}{c_p T_0}} = .01644 \Rightarrow \dot{m}_f = .0174$$

$$\dot{m} = \rho_8 u_8 A_8 \Rightarrow \frac{\dot{m} R T_8}{P_8 u_8} = .00515\,m^2$$

$$T = \dot{m}(u_8 - u_0) + (P_8 - P_0)A_8 =$$

$$A_2 = \frac{\dot{m}}{\rho_2 u_2} = \frac{\dot{m} R T_2}{P_2 M_2\sqrt{\gamma R T_2}}$$

$$\frac{P_{t2}}{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\wedge}\left(\frac{\gamma}{\gamma-1}\right)} = P_2 = 128957.483\ pa$$

$$\frac{T_{t2}}{1+\frac{\gamma-1}{2}M_2^2} = T_2 = 309.394$$

$$\rho_2 = \frac{P_2}{RT_2} = 1.452 \ kg/m^3$$

$$u_2 = M_2\sqrt{\gamma R T_2} = 176.291 \ m/s$$

$$u_3 = u_2$$

$$A_2 = \frac{\dot{m}}{\rho_2 u_2} = 0.00414$$

$$A_4 = \frac{\dot{m}}{\rho_4 u_4}$$

$$\frac{P_{t4}}{\left(1+(\gamma-1)/2\right)^{\frac{\gamma}{\gamma-1}}} = P_4 = \frac{2294564.933}{(1.2)^{1.4/.4}} = 1212176.865 \ pa$$

$$\frac{T_{t4}}{1.2} = T_4 = 1250K$$

$$\rho_4 = \frac{P_4}{RT_4} = 3.379$$

$$u_4 = \sqrt{\gamma R T_4} = 708.696 \ m/s$$

$$A_4 = \frac{1.0609}{3.379 \cdot 708.696} = 4.43E{-4} \ m^2$$

$$\frac{\dot{m}\sqrt{T_{t4}/T_{tSP}}}{P_{t4}/P_{tSP}} = .107$$

$$\frac{N_t}{\sqrt{\theta_4}} = 26290.683 \qquad \frac{N_c}{\sqrt{\theta_2}} = 56493.268$$

$$\dot{m}_{corr_2}\frac{1}{\pi_c}\frac{N_c}{\sqrt{\theta_2}} = 2809$$

$$\dot{m}_{corr_4}\frac{N_t}{\sqrt{\theta_4}} = 2810$$

$$A_3 = \frac{\dot{m}}{\rho_3 u_3} \qquad u_2 = u$$

$$T_3 = T_{t3} - T_{t2} + T_{t2} = 753.757$$

$$M_3 = \frac{u_3}{\sqrt{\gamma R T_3}} = .3199$$

$$P_3 = \frac{P_{t3}}{\left(1+M_3^2 \cdot .2\right)^{\frac{\gamma}{\gamma-1}}} = \frac{2294564.933}{\left[1+.2(.3199)^2\right]^{1.4/.4}}$$

$$= 2137470.78 \ pa$$

$$\rho = \frac{P_3}{RT_3} = 9.855 \ kg/m^3$$

$$\dot{m}_{corr_8} = \frac{\dot{m}\sqrt{T_{t8}/T_{STD}}}{P_{t8}/P_{tSP}} = .366$$

$$\frac{\dot{m}_{corr8}}{\dot{m}_{corr2}} = .491$$

$$\sqrt{T_{t4}/T_{t2} \cdot \tau_t}\ \frac{1}{\pi_c \pi_t} = .491$$

$$A = \frac{1.0609}{\quad} = 6.107E{-4} \ m^2$$

RT_3

$$A_3 = \frac{1.0609}{9.855 \cdot 176.291} = 6.107E-4\,m^2$$

$$A_5 = \frac{\dot{m}}{\rho_5 u_5}$$

$$u_5 = .8\,u_4 = 566.957$$

$$T_5 = \frac{.18 u_4^2}{c_p} T_4 + T_{t5} - T_{t4} = 860.6\,K$$

$$M_5 = \frac{u}{\sqrt{\gamma R T_5}} = .9641$$

$$P_5 = \frac{P_{t5}}{(1+ \gamma M^2)^{\frac{\gamma}{\gamma-1}}} = 309258\ 738\,pa$$

$$\rho_5 = \frac{P_5}{RT} = 1.252\,Kg/m^3$$

$$A_5 \approx .001494\,m^2 \Rightarrow A_8 = .001492\,m^2$$

Appendix B

A. Guess procedure

PART B

$$\left.\frac{N_c}{\sqrt{\theta_2}}\right)_{DES} = 60000/(324.864/288) = 53191.489 \text{ RPM}$$

$$.76 \left.\frac{N_c}{\sqrt{\theta_2}}\right)_{DES} = 40425.532 \text{ RPM}$$

$$\dot{m}_{corr2} = .6125\, \dot{m}_{corr2_D} : .746 \cdot .6125 = .45$$

$$\pi_c = .55\pi_{c_{des}} = .15 \times 55 = 8.25 \Rightarrow \eta_c = .825$$

$$\tau_c = 1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} = 2.00296$$

Wednesday, March 24, 2021 1:29 PM

$$\frac{N_t}{\sqrt{\theta_4}} = \frac{\dot{m}_{corr2}}{\dot{m}_{corr4}} \frac{1}{\pi_c} \frac{N_c}{\sqrt{\theta_2}} = 20924.878$$

$$\sqrt{\frac{T_{t4}}{T_{t2}}} = 1.932 \Rightarrow T_{t4}/T_{t2} = 3.732$$

$$\dot{m}_{corr4} \frac{N_t}{\sqrt{\theta_4}} = 2238.962$$

$$\tau_t = 1 - \frac{T_{t2}}{T_{t4}}(\tau_c - 1) = .731$$

$$\eta_t = \frac{1 - \tau_t}{1 - \pi_t^{\frac{\gamma-1}{\gamma}}}, \quad \eta_t = .9$$

$$\pi_t = \left(1 - \frac{1 - \tau_t}{\eta_t}\right)^{\gamma/(\gamma-1)} = .289$$

$$\frac{\dot{m}_{corr8}}{\dot{m}_{corr2}} = .800875$$

$$\sqrt{\frac{T_{t4}}{T_{t2}}\tau_t}\ \frac{1}{\pi_c \pi_t} = .692$$

B. code (Match.m)

```
% Match.m
function [a,b,Pic,tauC,mdotcorr2,etaC,spdLine,M,tauT,Pit,G,del] =
Match(ps,pp,pm,etaC)
%Match function for project 2
%   will match components
gam =1.4;
mdotcorr4 = .107;
mdotcorr8 = .366;
etaT = .9;
spdLined = 56493.268;
spdLine = ps*spdLined;
Picd = 15;
Pic = pp*Picd;
mdotcorr2d = .746;
mdotcorr2 = pm*mdotcorr2d;
tauC = 1 + ((Pic^((gam-1)/gam)-1)/etaC);
M = mdotcorr2/mdotcorr4/Pic*spdLine;
G = (spdLine/M)^2;
tauT = 1-1/G*(tauC-1);
Pit = (1-(1-tauT)/etaT)^(gam/(gam-1));
a = mdotcorr8/mdotcorr2;
b = sqrt(G*tauT)/Pic/Pit;
del = abs(a-b);
end
```

Appendix C

A. outline

Part c

Thursday, March 25, 2021 48 PM

$$\dot{m}_{fh} = \frac{P_0 \pi_d \left(1+\frac{\gamma-1}{2}M_0^2\right)^{\frac{\gamma}{\gamma-1}}}{P_{tsp}} \cdot \sqrt{T_{tsp}}$$

$$\dot{m}_{corr_2} \quad C_p\sqrt{T_0\left(1+\frac{\gamma-1}{2}M_0^2\right)}\left[\frac{T_{t4}}{T_{t2}} - \tau_c\right]$$

$\Rightarrow M_0$

$P_{t3} = \pi_c P_{t2}$

$T_{t3} = T_{t2}\tau_c$

$T_{t4} = \frac{T_{t4}}{T_{t2}} T_{t2}$

$T_{t5} = \tau_t T_{t4}$

$T_{t8} = T_{t5}$

$T_8 = \frac{T_{t8}}{\left(1+\frac{\gamma-1}{2}M_8^2\right)}$

$U_8 = M_8\sqrt{\gamma R T_8}$

$U_0 = M_0\sqrt{\gamma R T_0}$

$P_{t4} = P_{t3}$

$P_{t5} = \pi_t P_{t4}$

$P_{t8} \Rightarrow P_8$

A_8 is known

$\dot{m} = \dot{m}_{corr_2}\frac{\delta_2}{\sqrt{\theta_2}}$

$T = f(m_f, alt) = m(U_8 - U_0) + (P_8 - P_0)A_8$

$T = f\left(\frac{N_c}{\sqrt{\theta_2}}, M_0, alt\right)$

$$\frac{N_c}{\sqrt{\theta_2}} = \frac{N_c}{\sqrt{T_0\left(1+\frac{\gamma-1}{2}M_0^2\right)/T_{STP}}} \approx \% \frac{N_c}{\sqrt{\theta_2}}_d$$

B. Part C code

```
classdef partC
    % partC class

    properties (Constant)
        data = xlsread('part b copy.xlsx');
        mdotfueld = .0174;
        A8 = .1515;
        h = 4.45*10^7;
        gam = 1.4;
        Tstp = 288;
        Pstp = 101325;
        cp = 1004.5;
        R = 287;
        RPMd = 56493.268;
        mdotd = 1.0609;
    end

    methods (Static)
        function [M0,thrust,RPM,spill,Tt4] = thrustM(P0,T0,perc,i)
            syms x
            f(x) = P0*(1-.015*x^2)*(1+(partC.gam-
1)/2*x^2)^(partC.gam/(partC.gam-1))/partC.Pstp*...
                sqrt(partC.Tstp)*partC.data(i,4)*partC.cp*...
                sqrt(T0*(1+(partC.gam-1)/2*x^2))*(partC.data(i,10)-
partC.data(i,3))-...
                partC.mdotfueld*perc*partC.h;
            M0 = newRap(f)
            if M0<2.4 && M0>0
            Pt2 = P0*(1-.015*M0^2)*(1+(partC.gam-
1)/2*M0^2)^(partC.gam/(partC.gam-1));
            Pt4 = partC.data(i,2)*Pt2;
            Pt8 = Pt4*partC.data(i,9);
            P8 = Pt8/(1+(partC.gam-1)/2)^(partC.gam/(partC.gam-1));
            Tt2 = T0*(1+(partC.gam-1)/2*M0^2);
            Tt4 = partC.data(i,10)*Tt2
            Tt8 = Tt4*partC.data(i,8);
            T8 = Tt8/(1+(partC.gam-1)/partC.gam);
            mdot = partC.data(i,4)*Pt2/partC.Pstp/sqrt(Tt2/partC.Tstp);
            u8 = sqrt(partC.gam*partC.R*T8);
            u0 = M0*sqrt(partC.gam*partC.R*T0);
            thrust = mdot*(u8-u0)+(P8-P0)*partC.A8;
            RPM = partC.data(i,1)*partC.RPMd*sqrt(Tt2/partC.Tstp);
            spill = abs(mdot-partC.mdotd);

            else
                thrust = 0;
                RPM = 0;
                spill = 0;
                Tt4=0;
            end
        end
```

Appendix D

A. Newton's Method

```
function x = newRap(f)
syms x

fp(x) = diff(f(x));

x=.1;
nmax=25;
eps=1;
n=0;

while eps>=1e-5&&n<=nmax
    y=x-double(f(x))/double(fp(x));
    eps=abs(y-x);
    x=y;
    n=n+1;
end
```

B. main

```
% matthew Pahayo
% main.m
clc
clear all
close all

format longg

% [a,b,Pic,tauC,mdotcorr2,etaC,spdLine,M,tauT,Pit,G,del] = ...
%     Match(1,1,1,.85)

%
syms x

p=partC;
% % for k = 1:5
% %     for i = 1:8
% %         [M0,thrust,RPM,spill] = p.thrustM(30742.5,229.365,1/5*k,i)
% %         sz = 15;
% %         c = 20*i;
% %         if M0<2.4 && M0>0
% %             hold on
% %             scatter(M0,thrust,sz,c,'filled')
% %             hold off
% %         end
% %     end
```

```
% % end
% % grid on
% % xlabel('M0')
% % ylabel('Thrust (N)')
% % title('Thrust vs M0')
%
% for k = 1:5
%     for i = 1:8
%         [M0,thrust,RPM,spill,Tt4] = p.thrustM(57728.3,258.9,1/5*k,i)
%         sz = 15;
%         c = i;
%         if M0<2.4 && M0>0 &&Tt4<1800 && RPM<66000
%             hold on
%             scatter(M0,RPM,sz,c,'filled')
%             hold off
%         end
%     end
% end
% grid on
% xlabel('M0')
% ylabel('RPM')
% title('RPM vs M0')
%
for k = 1:5
    for i = 1:8
        [M0,thrust,RPM,spill,Tt4] = p.thrustM(101325,288,1/5*k,i)
        sz = 15;
        c = k;
        if M0<2.4 && M0>0 &&Tt4<1800 && RPM<66000
            hold on
            scatter(M0,spill,sz,c,'filled')
            hold off
        end
    end
end
grid on
xlabel('M0')
ylabel('Spillage [abs(mdotd-mdot) (kg/s)]')
title('Spillage vs M0')
```