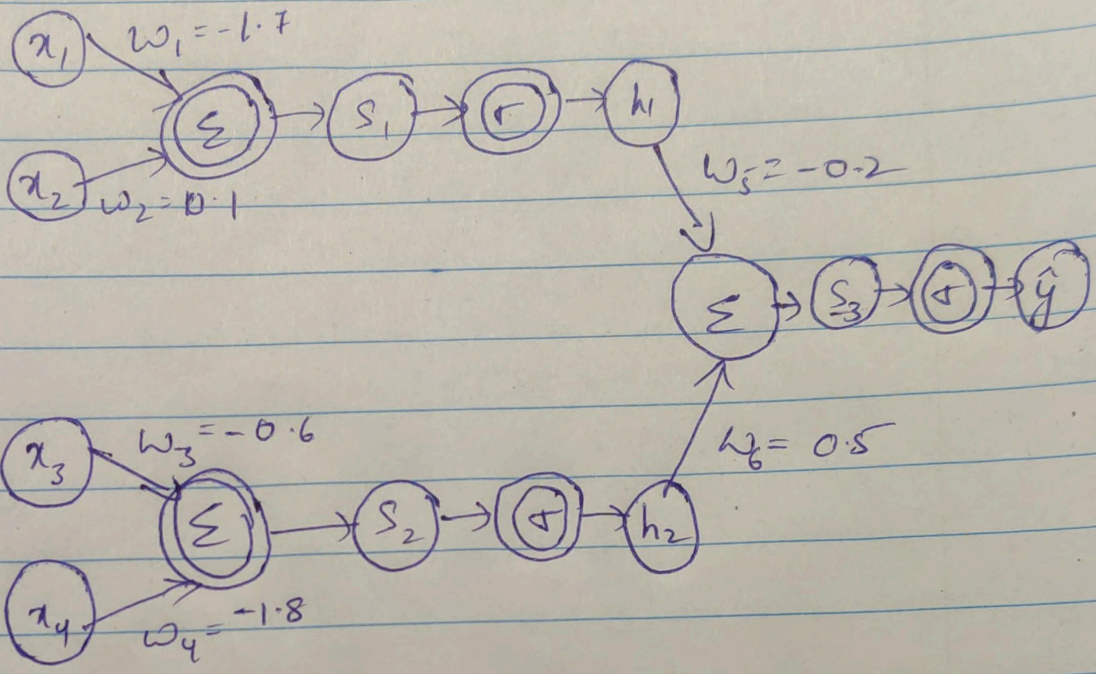
Deep Learning Assignment-2

Pg 1



$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$h_1 = \frac{1}{1+e^{-w_1 x_1 - w_2 x_2}}$$

$$L_2 \text{ loss } L(y, \hat{y}) = \|\hat{y} - y\|^2$$

$x_1 = 0.7$

$x_2 = 1.2$

$x_3 = 1.1$

$x_4 = 2$

$y = 0.5$

Q2

$S_1 = x_1 w_1 + x_2 w_2$

$= (0.7)(-1.7) + (1.2)(0.1)$

$= (-1.19) + (0.12)$

$= -1.07$

$\therefore S_1 = -1.07$

$S_2 = x_3 w_3 + x_4 w_4$

$= (1.1)(-0.6) + (2)(-1.8)$

$= (-0.66) + (-3.6)$

$= -4.26$

$\therefore S_2 = -4.26$

Now, $h_1 = \frac{1}{1 + e^{-x_1 w_1 - x_2 w_2}}$

$h_1 = \frac{1}{1 + e^{-(0.7)(-1.7) - (1.2)(0.1)}}$

$h_1 = \frac{1}{1 + e^{(1.19) - 0.12)}}$

$h_1 = \frac{1}{1 + e^{1.07}}$

$h_1 = \frac{1}{3.915}$

$h_1 = 0.255$

$\therefore h_1 = 0.255$

(Q3)

$$h_2 = \frac{1}{1+e^{-x_3w_3 - x_4w_4}}$$

$$= \frac{1}{1+e^{-(1.1)(0.6)-(2)(-1.8)}}$$

$$= \frac{1}{1+e^{0.66+3.6}}$$

$$= \frac{1}{1+e^{4.26}} = \frac{1}{1+70.5} = \frac{1}{71.5}$$

$$\therefore h_2 = 0.0139$$

$$s_3 = h_1w_5 + h_2w_6$$

$$= (0.255)(-0.2) + (0.0139)(0.5)$$

$$= -0.051 + 0.00695$$

$$s_3 = -0.0445$$

$$\hat{y} = \frac{1}{1+e^{-h_1w_5 - h_2w_6}}$$

$$= \frac{1}{1+e^{-(0.255)(-0.2)-(0.0139)(0.5)}}$$

$$= \frac{1}{1+e^{0.051-0.0695}}$$

$$= \frac{1}{1+e^{0.404405}}$$

$$= \frac{1}{1+0.653}$$

$$\hat{y} = 0.4889$$

(Q.4)

Now, the gradient of L_2 loss function

$$\|\hat{y} - y\|^2 \text{ is } 2\|\hat{y} - y\| - \frac{\partial E}{\partial y}$$

Using Backward propagation

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial s_3} \times \frac{\partial s_3}{\partial h_1} \times \frac{\partial h_1}{\partial s_1} \times \frac{\partial s_1}{\partial w_1}$$

$$\frac{\partial E}{\partial w_1} = 2\|\hat{y} - y\| \times \sigma'(s_3) \times w_5 \times \sigma'(s_1) \times x_1$$

As $\frac{\partial s_3}{\partial h_1} = w_5$ and $\frac{\partial s_1}{\partial w_1} = x_1$ Substituting ↑

$$\sigma'(s_3) = \sigma(s_3)(1 - \sigma(s_3)) \qquad \sigma'(s_1) = \sigma(s_1)(1 - \sigma(s_1))$$

$$\frac{\partial E}{\partial w_1} = 2\|0.4884 - 0.5\| \times [\sigma(s_3)(1 - \sigma(s_3))] \times (-0.2) \times [\sigma(s_1)(1 - \sigma(s_1))] \times (0.7)$$

$$= 2(0.107) \times [(0.2554)(1 - 0.2554)] \times 0.2 \times [(0.4884)(1 - 0.4884)] \times 0.7$$

$$\frac{\partial E}{\partial w_1} = -0.00114$$

$$\frac{\partial E}{\partial w_1} \approx -0.0014$$