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In-Class Assignment 2

9.19. $E(Y) = \alpha / (\alpha + \beta) = \theta / (\theta + 1)$
$\text{Var}(Y) = \alpha\beta / (\alpha + \beta)^2(\alpha + \beta + 1) =$
$\text{Var}(Y) = \theta / ((\theta + 1)^2(\theta + 2))$
$\text{Var}(\bar{y}) = [\theta / ((\theta + 1)^2(\theta + 2))](1 / \text{n})$
$\lim_{n\to\infty} \theta / (\text{n}(\theta + 1)^2(\theta + 2)) = 0$

After some examination of $f(y)$, we can see that $Y \sim \text{Beta}(\theta,1)$.

We can see that $\lim_{n\to\infty} \text{Var}(\bar{y}) = 0$. Per Theorem 9.1, $\bar{y}$ is a consistent estimator for $\theta / (\theta + 1)$.

9.20. $E(Y / n) = np / n = p$
$\text{Var}(Y / n) = np(1 - p) / n^2 = p(1 - p) / n$
$\lim_{n\to\infty} p(1 - p) / n = 0$

$E(Y) = np$, $\text{Var}(Y) = np(1 - p)$
We can see that $\lim_{n\to\infty} \text{Var}(Y / n) = 0$. Per Theorem 9.1, Y / n is a consistent estimator for p.

9.30. $E(Y) = y(3y^2) = 3y^3 \rightarrow$
$\int E(Y)\, dy = \int_0^1 3y^3 = |_0^1\, 3y^4/4 =$ **3/4**

9.31. By the Law of Large Numbers, we can see that $\bar{y}$ converges *in probability* to $\alpha\beta$.