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question @28

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Actions

Problem 5.14

14. Hierarchical Poisson model: consider the dataset in the previous problem, but suppose only the total amount of traffic at each location is observed.

  (a) Set up a model in which the total number of vehicles observed at each location j follows a Poisson distribution with parameter θ_j, the 'true' rate of traffic per hour at that location. Assign a gamma population distribution for the parameters θ_j and a noninformative hyperprior distribution. Write down the joint posterior distribution.

  (b) Compute the marginal posterior density of the hyperparameters and plot its contours. Simulate random draws from the posterior distribution of the hyperparameters and make a scatterplot of the simulation draws.

  (c) Is the posterior density integrable? Answer analytically by examining the joint posterior density at the limits or empirically by examining the plots of the marginal posterior density above.

  (d) If the posterior density is not integrable, alter it and repeat the previous two steps.

  (e) Draw samples from the joint posterior distribution of the parameters and hyperparameters, by analogy to the method used in the hierarchical binomial model.

I'm having trouble finding the joint posterior and noninformative hyperprior (and thus can't proceed with the rest of the problem). I've been trying to replicate problem 5.13(a) but I'm not sure if I have the notation right. I currently have the following:

1. $p(\theta, \alpha, \beta|y) \propto p(\alpha, \beta)p(\theta|\alpha, \beta)p(y|\theta, \alpha, \beta) \propto$
2. $p(\alpha, \beta)(\prod_{j=1}^{J} \frac{\beta^\alpha}{\Gamma(\alpha)} \theta_j^{\alpha-1} e^{-\beta\theta_j})(\prod_{j=1}^{J} \frac{\theta_j^{y_j} e^{-\theta_j}}{y_j!}) \propto$
3. $p(\alpha, \beta)(\prod_{j=1}^{J} \theta_j^{\alpha-1} e^{-\beta\theta_j})(\prod_{j=1}^{J} \theta_j^{y_j} e^{-\theta_j}) \propto$
4. $p(\alpha, \beta) \prod_{j=1}^{J} \theta_j^{y_j+\alpha-1} e^{-(\beta+1)\theta_j} \propto$
5. $p(\alpha, \beta) \prod_{j=1}^{J} Gamma(y_j + \alpha, \beta + 1)$

However, I'm not sure if I'm way off? I'm using the rate parameter for the gamma distribution. How do I get the noninformative hyperprior?

Also, are we supposed to use θ_j or λ? This was throwing me off with the products and summations.

hw3

Edit good question 0

Updated 2 years ago by Charles Hwang

S **the students' answer,** *where students collectively construct a single answer*

Actions

I was going to ask about this during class tonight. I have somewhat similar work to you so far (I'm not using the rate parameter, though), but there are some differences in mine. I changed the capital J to 10 since there are 10 groups (this probably doesn't make a difference), and I didn't get rid of the $\frac{1}{\beta^\alpha \Gamma(\alpha)}$. I thought we had to keep it since α and β are (hyper)parameters now. But, if that term is kept, I'm not sure what the distribution is now since there's an α and an $\alpha + y_j$ now. After this, I'm also stuck.