STAT 488: Multivariate Statistical Analysis — Homework 1

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2/3/2022

Problem 1.1

```
x1 <- matrix(c(3, 4, 2, 6, 8, 2, 5))
x2 <- matrix(c(5, 5.5, 4, 7, 10, 5, 7.5))
r1 <- data.frame(mean(x1), mean(x2), sum((x1 - mean(x1))^2)/length(x1), sum((x2 -
    mean(x2))^2)/length(x2), sum((x1 - mean(x1)) * (x2 - mean(x2)))/length(x1))
names(r1) <- c("x1", "x2", "s11", "s22", "s12")
r1

##         x1       x2       s11      s22      s12
## 1 4.285714 6.285714 4.204082 3.561224 3.704082
```

Problem 1.4

```
x1<-matrix(c(108.28,152.36,95.04,65.45,62.97,263.99,265.19,285.06,92.01,165.68))
x2<-matrix(c(17.05,16.59,10.91,14.14,9.52,25.33,18.54,15.73,8.10,11.13))
x3<-matrix(c(1484.10,750.33,766.42,1110.46,1031.29,195.26,193.83,191.11,1175.16,211.15))
plot(x1,x2,main="Problem 1.4(a)")           # Problem 1.4(a)
```
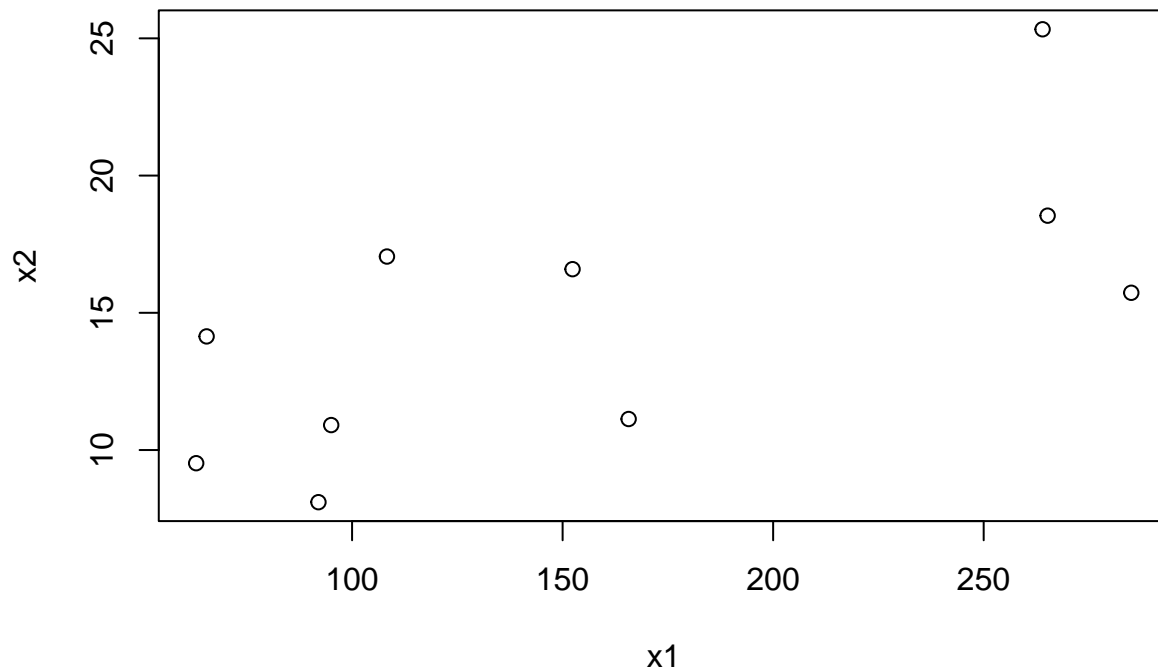
```r
# There appears to be positive correlation between x1 and x2.
s11<-sqrt(sum((x1-mean(x1))^2)/length(x1)) # Problem 1.4(b)
s22<-sqrt(sum((x2-mean(x2))^2)/length(x2))
s12<-s21<-sum((x1-mean(x1))*(x2-mean(x2)))/length(x1)
r4<-data.frame(mean(x1),mean(x2),s11,s22,s12,cor(x1,x2))
names(r4)<-c("x1","x2","s11","s22","s12","cor(x1,x2)")
r4
```

```
##        x1     x2      s11      s22      s12 cor(x1,x2)
## 1 155.603 14.704 82.02931 4.855027 273.2568   0.686136
```

There is moderate correlation ($r = 0.686136$) between x_1 and x_2.

Problem 1.5

```r
par(mfrow=c(1,2))                                        # Problem 1.5(a)
plot(x2,x3)
plot(x1,x3)
```
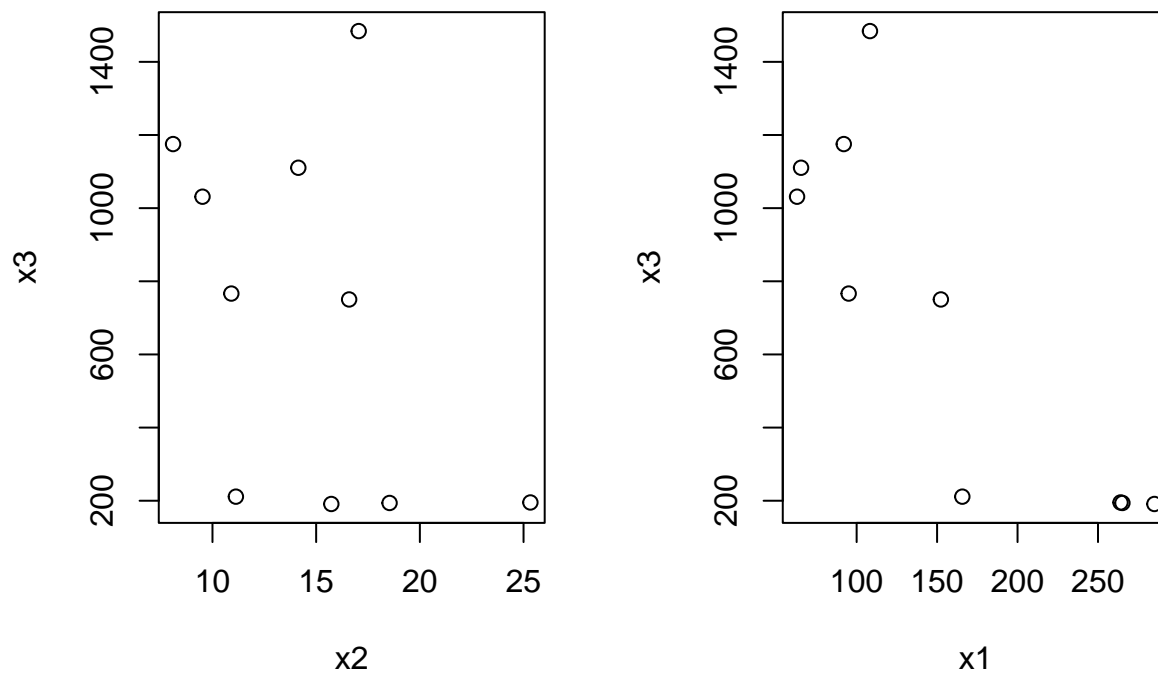


```r
# There appears to be weak negative correlation between x2 and x3
# and a moderately strong negative correlation between x1 and x3.
s13<-s31<-sum((x1-mean(x1))*(x3-mean(x3)))/length(x1) # Problem 1.5(b)
s23<-s32<-sum((x2-mean(x2))*(x3-mean(x3)))/length(x2)
s33<-sqrt(sum((x3-mean(x3))^2)/length(x3))
x<-matrix(c(mean(x1),mean(x2),mean(x3)))
x
```

```
##         [,1]
## [1,] 155.603
## [2,]  14.704
## [3,] 710.911
```

```r
Sn<-matrix(c(s11,s12,s13,s21,s22,s23,s31,s32,s33),nrow=3,ncol=3)
Sn
```

```
##                [,1]         [,2]         [,3]
## [1,]      82.02931   273.256758  -32018.3636
## [2,]     273.25676     4.855027    -948.4447
## [3,]  -32018.36361  -948.444654     461.8970
```
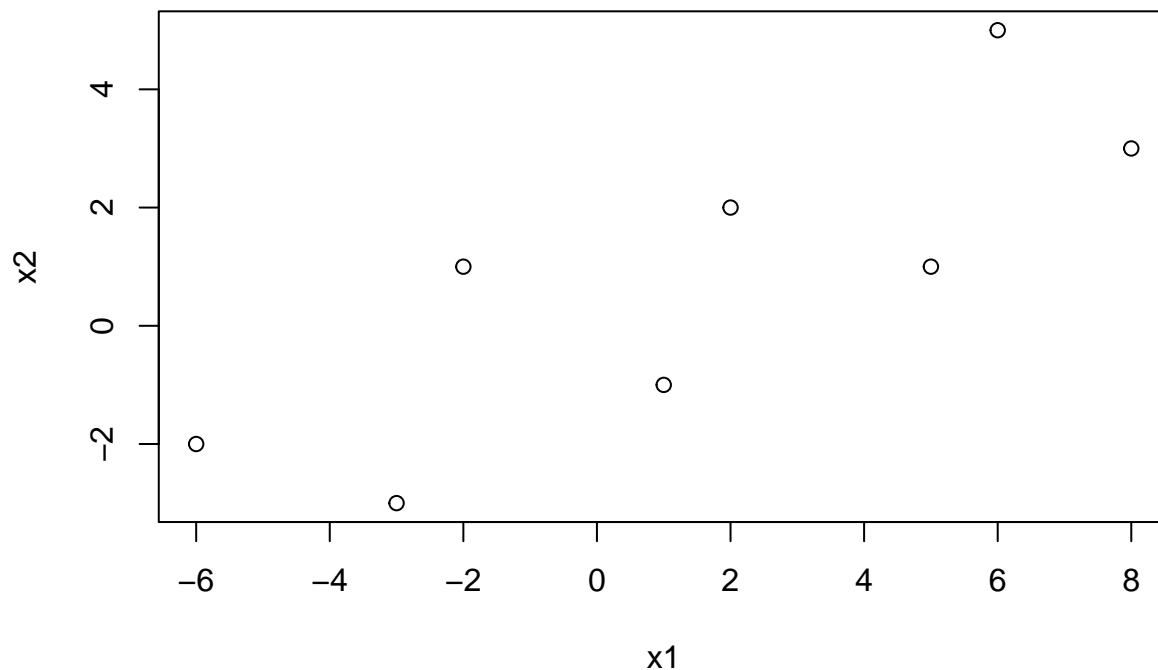
```
R <- matrix(c(cor(x1, x1), cor(x1, x2), cor(x1, x3), cor(x2, x1), cor(x2, x2), cor(x2,
    x3), cor(x3, x1), cor(x3, x2), cor(x3, x3)), nrow = 3, ncol = 3)
R
```

```
##             [,1]        [,2]        [,3]
## [1,]   1.0000000   0.6861360  -0.8450549
## [2,]   0.6861360   1.0000000  -0.4229366
## [3,]  -0.8450549  -0.4229366   1.0000000
```

Problem 1.9

```
x1<-c(-6,-3,-2,1,2,5,6,8)                                           # Problem 1.9(a)
x2<-c(-2,-3,1,-1,2,1,5,3)
plot(x1,x2,main="Problem 1.9(a)")
```

Problem 1.9(a)


```
s11<-sum((x1-mean(x1))^2)/length(x1)
s22<-sum((x2-mean(x2))^2)/length(x2)
s12<-sum((x1-mean(x1))*(x2-mean(x2)))/length(x1)
r9a<-data.frame(s11,s22,s12)
r9a
```

```
##         s11     s22     s12
## 1 20.48438  6.1875 9.09375
```

```
x1t<-x1*.899+x2*.438
x2t<-x2*.899-x1*.438
```

3

```
r9b<-data.frame(t(data.frame(x1t,x2t)))                              # Problem 1.9(b)
names(r9b)<-NULL
r9b
```

```
##
## x1t -6.27 -4.011 -1.360  0.461 2.674  4.933 7.584  8.506
## x2t  0.83 -1.383  1.775 -1.337 0.922 -1.291 1.867 -0.807
```

```
s11t<-sum((x1t-mean(x1t))^2)/length(x1t)                            # Problem 1.9(c)
s22t<-sum((x2t-mean(x2t))^2)/length(x2t)
r9c<-data.frame(s11t,s22t)
r9c
```

```
##       s11t      s22t
## 1 24.90407 1.769002
```

```
sqrt((4*.899+(-2)*.438)^2/s11t+((-2)*.899-4*.438)^2/s22t)           # Problem 1.9(d)
```

```
## [1] 2.724179
```

```
a11 <- 0.899^2/(0.899^2 * s11 + 2 * 0.438 * 0.899 * s12 + 0.438^2 * s22) +
    0.438^2/(0.899^2 * s22 - 2 * 0.438 * 0.899 * s12 + 0.438^2 * s11)  # Problem 1.9(e)
a22 <- 0.438^2/(0.899^2 * s11 + 2 * 0.438 * 0.899 * s12 + 0.438^2 * s22) +
    0.899^2/(0.899^2 * s22 - 2 * 0.438 * 0.899 * s12 + 0.438^2 * s11)
a12 <- 0.438 * 0.899/(0.899^2 * s11 + 2 * 0.438 * 0.899 * s12 + 0.438^2 * s22) -
    0.438 * 0.899/(0.899^2 * s22 - 2 * 0.438 * 0.899 * s12 + 0.438^2 * s11)
sqrt(a11 * (4)^2 + 2 * a12 * (4) * (-2) + a22 * (-2)^2)
```

```
## [1] 2.724179
```

We can see this is the same distance as the one using the equation in Problem 1.9(d).
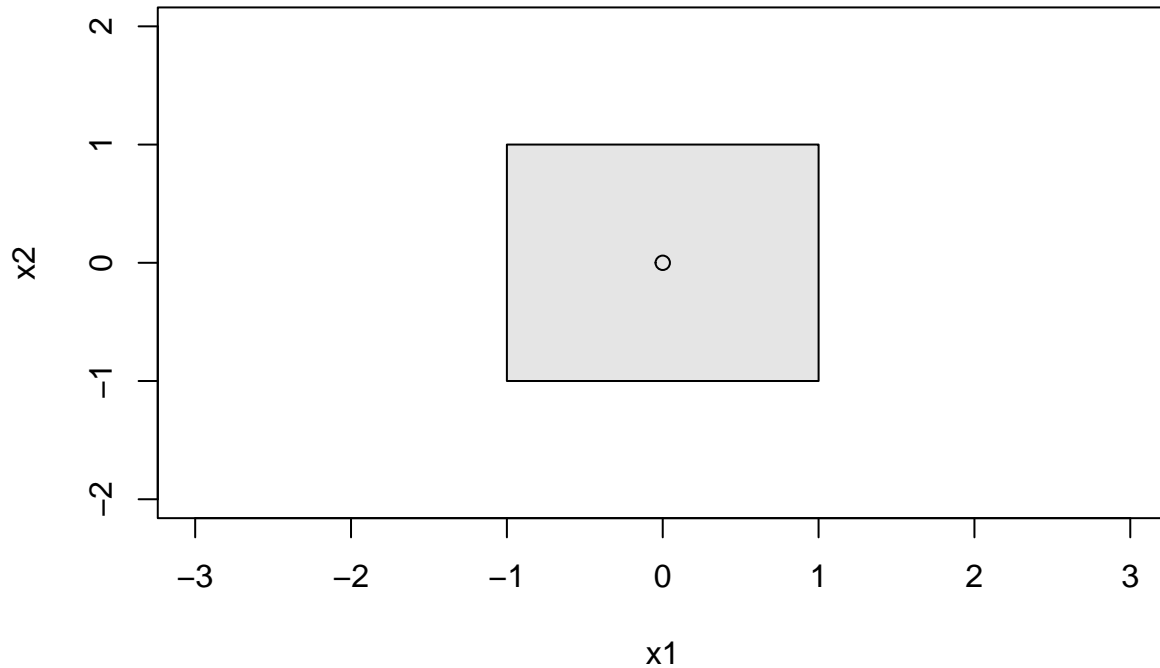
Problem 1.12

```
max(abs(-3),abs(4))                                                  # Problem 1.12(a)
```

```
## [1] 4
```

```
plot(NA,xlim=c(-3,3),ylim=c(-2,2),xlab="x1",ylab="x2",main="Problem 1.12(b)") # Problem 1.12(b)
rect(-1,-1,1,1,border="black",col="gray90")
points(0,0)
```

Problem 1.12(b)

Problem 1.12(b)

x2

x1

Observing the equation for distance, $d(O,P) = \max(|x_1|,|x_2|)$, we can clearly see "the locus of points" is simply a square bounded by $x = $ -1, 1 and $y = $ -1, 1.
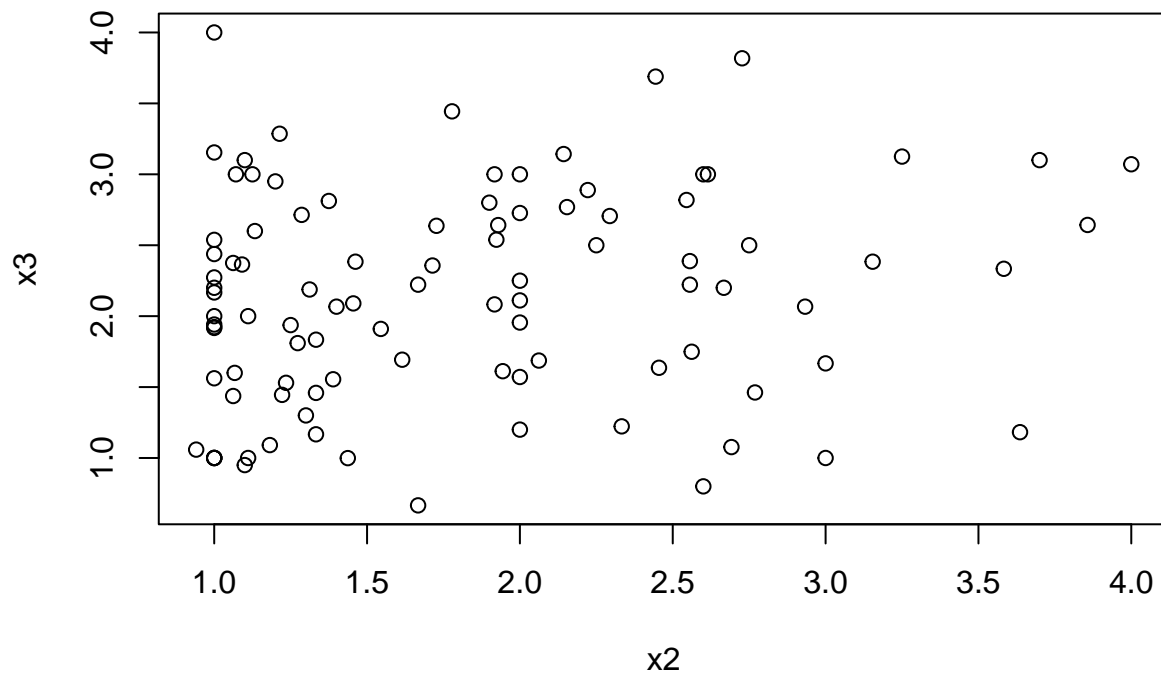
Problem 1.15

```
x1 <- c(0.889, 2.813, 1.454, 0.294, 2.727, 3.937, 2.786, 5.231, 1.15, 6.5, 0.8,
    4.6, 3.5, 3.444, 4.071, 3.692, 5.167, 0.5, 2.385, 2.1, 5, 4.571, 2.733, 4.235,
    0, 0.75, 3.077, 1.6, 6.273, 2.625, 1.25, 2.437, 4.454, 0.133, 0.222, 2.467,
    4, 5.385, 0.773, 3.786, 1.923, 1, 5.8, 6.062, 3.706, 2.444, 6.111, 2.533, 2.167,
    2.375, 1.875, 1.75, 7.333, 5.25, 5.182, 1.875, 5.4, 1.154, 6.375, 9.454, 1,
    1.444, 1.8, 2.818, 10.461, 4.143, 1.227, 5.667, 4.111, 4.444, 3.714, 7.4, 3.182,
    5.2, 2.333, 3.333, 5.25, 7.714, 3.846, 2.444, 5.333, 1.556, 3.182, 6.222, 7.231,
    3.857, 3.778, 6, 2.333, 7.571, 3.667, 3.6, 3.364, 4.1, 0.125, 6.231, 3, 0.889)
x2 <- c(1.389, 1.437, 1.091, 0.941, 2.545, 1.25, 1.714, 2.692, 1.1, 2.562, 1, 2,
    1.286, 2.556, 1, 1, 3, 1, 1.923, 1.3, 3.25, 1.214, 1.133, 2.294, 1, 1.125,
    1.462, 1.2, 3.636, 1, 1, 2.062, 1.727, 1, 1.222, 2.667, 1, 3.154, 1, 2, 1.615,
    1.333, 2.6, 1, 1.235, 2.333, 2.222, 1.067, 1, 1.062, 1.312, 1.333, 1.333, 1.375,
    2, 2, 2, 1, 2.25, 2.727, 1, 1.111, 1.1, 2, 2.154, 1.929, 1.182, 3, 2.556, 1.667,
    3.857, 3.7, 2.455, 2.6, 1.667, 1.917, 2.75, 4, 2.615, 1.111, 1.917, 1.778,
    1.545, 2.444, 1, 1.071, 1.944, 1.4, 3.583, 2.143, 2, 2.933, 1.273, 1.9, 1.062,
    2.769, 1.455, 1)
x3 <- c(1.555, 0.999, 2.364, 1.059, 2.819, 1.937, 2.357, 1.077, 0.95, 1.749, 2.2,
    3, 2.714, 2.388, 1, 2.538, 1, 1, 2.539, 1.3, 3.125, 3.286, 2.6, 2.706, 1.941,
    3, 2.384, 2.95, 1.182, 2.438, 2, 1.687, 2.637, 1, 1.445, 2.2, 4, 2.384, 2.273,
    1.571, 1.693, 1.834, 3, 1.562, 1.53, 1.223, 2.889, 1.6, 2.167, 2.375, 2.188,
    1.167, 1.459, 2.812, 2.727, 2.25, 1.2, 1.923, 2.5, 3.818, 1.917, 2, 3.1, 1.955,
    2.769, 2.642, 1.091, 1.667, 2.222, 2.222, 2.643, 3.1, 1.636, 0.8, 0.666, 2.083,
    2.5, 3.071, 3, 1, 3, 3.444, 1.91, 3.689, 3.154, 3, 1.612, 2.067, 2.334, 3.143,
    2.111, 2.067, 1.81, 2.8, 1.437, 1.462, 2.09, 1)
```

```
x4 <- c(2.222, 2.312, 2.455, 2, 2.727, 2.937, 2.071, 1.846, 2, 2.562, 2.267, 2.5,
    1.286, 2.389, 2.357, 2.154, 2.667, 2, 2.154, 1.8, 2.375, 2.571, 1.933, 2.176,
    2, 1.875, 2, 2, 2.545, 1.937, 2, 1.875, 2.636, 2, 2, 1.933, 2.167, 2.846, 1.909,
    1.786, 2, 2, 2.8, 2.375, 2.118, 2.444, 2.889, 2, 2, 2, 2.125, 1.75, 1.958,
    2.125, 2.818, 2.813, 1.8, 1.846, 2.125, 2.455, 1.833, 2.111, 2.2, 2.045, 2,
    2.429, 2.227, 2.667, 2.778, 2, 2.286, 2.5, 2.273, 1.8, 1.667, 1.917, 2, 2.929,
    2.692, 2.111, 2.25, 2.667, 2.273, 2.444, 2.308, 2.071, 1.611, 2.267, 2.333,
    2.571, 2.778, 2.2, 2, 2, 1.875, 2.385, 2.273, 2)
x5 <- c(1.945, 2.312, 2.909, 1, 4.091, 3.749, 2, 2.539, 1, 2.499, 2.466, 3.4, 1.252,
    3, 1.572, 2.615, 3.666, 1, 2.461, 2.6, 3.375, 3.572, 1.667, 1.883, 2, 2, 1.846,
    2.75, 3.364, 2.062, 3, 1.375, 3.546, 1, 1, 1.8, 2.5, 2.539, 2.091, 1.285, 1.846,
    1.917, 4.2, 1.75, 2.294, 1.776, 3.555, 1.333, 2.5, 2.125, 2.062, 1, 1.542,
    2.563, 4, 2.437, 1.4, 2.462, 3, 3.272, 2.167, 2, 2.6, 2.546, 2.923, 3.142,
    3.182, 5, 3.778, 2.444, 3.285, 4.2, 3, 2, 2.166, 3, 4, 4.428, 3.693, 1.667,
    1.917, 3.333, 3, 3.445, 4.384, 2.286, 1.945, 2.866, 2.667, 3.929, 4, 2.867,
    2.273, 2.6, 1.563, 4, 3.272, 1)
x6 <- c(1, 2, 3, 1, 0, 1, 2, 1, 1, 1, 2, 1, 3, 1, 1, 1, 0, 0, 1, 1, 0, 1, 1, 1,
    0, 3, 2, 1, 0, 2, 1, 1, 1, 0, 1, 3, 0, 1, 0, 3, 1, 1, 1, 0, 1, 3, 2, 1, 1,
    3, 2, 1, 3, 3, 2, 2, 2, 1, 1, 3, 1, 1, 1, 2, 0, 3, 1, 1, 1, 0, 0, 1, 1, 0,
    0, 1, 0, 3, 2, 2, 1, 1, 1, 1, 2, 1, 1, 2, 2, 1, 3, 0, 0, 2, 0, 2, 2, 2)
plot(x2, x3, main = "Problem 1.15(a)")
```

Problem 1.15(a)


```
x <- matrix(c(mean(x1), mean(x2), mean(x3), mean(x4), mean(x5), mean(x6)))  # Problem 1.15b
x
```

```
##          [,1]
## [1,] 3.542347
## [2,] 1.809357
## [3,] 2.137602
## [4,] 2.209000
```

```
## [5,] 2.574827
## [6,] 1.275510
```

```r
Sn <- matrix(c(sum((x1 - mean(x1))^2)/length(x1), sum((x1 - mean(x1)) * (x2 - mean(x2)))/length(x1),
    sum((x1 - mean(x1)) * (x3 - mean(x3)))/length(x1), sum((x1 - mean(x1)) * (x4 -
        mean(x4)))/length(x1), sum((x1 - mean(x1)) * (x5 - mean(x5)))/length(x1),
    sum((x1 - mean(x1)) * (x6 - mean(x6)))/length(x1), sum((x1 - mean(x1)) * (x2 -
        mean(x2)))/length(x1), sum((x2 - mean(x2))^2)/length(x2), sum((x2 - mean(x2)) *
        (x3 - mean(x3)))/length(x2), sum((x2 - mean(x2)) * (x4 - mean(x4)))/length(x2),
    sum((x2 - mean(x2)) * (x5 - mean(x5)))/length(x2), sum((x2 - mean(x2)) * (x6 -
        mean(x6)))/length(x2), sum((x1 - mean(x1)) * (x3 - mean(x3)))/length(x1),
    sum((x2 - mean(x2)) * (x3 - mean(x3)))/length(x2), sum((x3 - mean(x3))^2)/length(x3),
    sum((x3 - mean(x3)) * (x4 - mean(x4)))/length(x3), sum((x3 - mean(x3)) * (x5 -
        mean(x5)))/length(x3), sum((x3 - mean(x3)) * (x6 - mean(x6)))/length(x3),
    sum((x1 - mean(x1)) * (x4 - mean(x4)))/length(x1), sum((x2 - mean(x2)) * (x4 -
        mean(x4)))/length(x2), sum((x3 - mean(x3)) * (x4 - mean(x4)))/length(x3),
    sum((x4 - mean(x4))^2)/length(x4), sum((x4 - mean(x4)) * (x5 - mean(x5)))/length(x4),
    sum((x4 - mean(x4)) * (x6 - mean(x6)))/length(x4), sum((x1 - mean(x1)) * (x5 -
        mean(x5)))/length(x1), sum((x2 - mean(x2)) * (x5 - mean(x5)))/length(x2),
    sum((x3 - mean(x3)) * (x5 - mean(x5)))/length(x3), sum((x4 - mean(x4)) * (x5 -
        mean(x5)))/length(x4), sum((x5 - mean(x5))^2)/length(x5), sum((x5 - mean(x5)) *
        (x6 - mean(x6)))/length(x5), sum((x1 - mean(x1)) * (x6 - mean(x6)))/length(x1),
    sum((x2 - mean(x2)) * (x6 - mean(x6)))/length(x2), sum((x3 - mean(x3)) * (x6 -
        mean(x6)))/length(x3), sum((x4 - mean(x4)) * (x6 - mean(x6)))/length(x4),
    sum((x5 - mean(x5)) * (x6 - mean(x6)))/length(x5), sum((x6 - mean(x6))^2)/length(x6)),
    nrow = 6, ncol = 6)
Sn
```

```
##           [,1]       [,2]       [,3]       [,4]        [,5]         [,6]
## [1,] 4.6072534  0.9218418 0.58368175 0.27408964  1.063917438  0.156537068
## [2,] 0.9218418  0.6065679 0.10980144 0.11726018  0.384918205 -0.024598397
## [3,] 0.5836818  0.1098014 0.56559795 0.08611715  0.344438992  0.109007601
## [4,] 0.2740896  0.1172602 0.08611715 0.10928245  0.215187224  0.021591837
## [5,] 1.0639174  0.3849182 0.34443899 0.21518722  0.853374694 -0.008727718
## [6,] 0.1565371 -0.0245984 0.10900760 0.02159184 -0.008727718  0.852665556
```

```r
R <- matrix(c(cor(x1, x1), cor(x1, x2), cor(x1, x3), cor(x1, x4), cor(x1, x5),
    cor(x1, x6), cor(x2, x1), cor(x2, x2), cor(x2, x3), cor(x2, x4), cor(x2, x5),
    cor(x2, x6), cor(x3, x1), cor(x3, x2), cor(x3, x3), cor(x3, x4), cor(x3, x5),
    cor(x3, x6), cor(x4, x1), cor(x4, x2), cor(x4, x3), cor(x4, x4), cor(x4, x5),
    cor(x4, x6), cor(x5, x1), cor(x5, x2), cor(x5, x3), cor(x5, x4), cor(x5, x5),
    cor(x5, x6), cor(x6, x1), cor(x6, x2), cor(x6, x3), cor(x6, x4), cor(x6, x5),
    cor(x6, x6)), nrow = 6, ncol = 6)
R
```

```
##            [,1]        [,2]      [,3]       [,4]        [,5]        [,6]
## [1,] 1.00000000  0.55143669 0.3615773 0.38627479  0.53655840  0.07897812
## [2,] 0.55143669  1.00000000 0.1874625 0.45544470  0.53500626 -0.03420407
## [3,] 0.36157729  0.18746250 1.0000000 0.34638617  0.49577944  0.15696886
## [4,] 0.38627479  0.45544470 0.3463862 1.00000000  0.70464665  0.07073348
## [5,] 0.53655840  0.53500626 0.4957794 0.70464665  1.00000000 -0.01023155
## [6,] 0.07897812 -0.03420407 0.1569689 0.07073348 -0.01023155  1.00000000
```

We can see the correlations between most but not all pairs of variables are positive and vary from no to moderate strength. The correlation between amount of food eaten and appetite ($r = 0.7046467$) is the strongest.

Problem 1.17

```r
m1 <- c(11.57, 11.12, 11.15, 11.14, 11.46, 11.17, 10.98, 11.65, 10.79, 11.31,
    12.52, 11.72, 11.09, 11.42, 11.63, 11.13, 10.73, 10.81, 11.1, 10.83,
    11.92, 11.41, 11.56, 11.38, 11.43, 11.45, 11.14, 11.36, 11.62, 11.49,
    11.8, 11.76, 11.5, 11.72, 11.09, 11.66, 11.08, 11.32, 11.41, 11.96,
    11.28, 10.93, 11.3, 11.3, 10.77, 12.38, 12.13, 11.06, 11.16, 11.34,
    11.22, 11.33, 11.25, 10.49)
m2 <- c(22.94, 22.23, 22.7, 22.48, 23.05, 22.6, 22.62, 23.84, 22.01, 22.92,
    25.91, 23.92, 21.97, 23.36, 23.91, 22.39, 21.99, 21.71, 22.1, 22.67,
    24.5, 23.06, 23.86, 22.82, 23.02, 23.15, 22.6, 23.33, 23.37, 23.8, 25.1,
    23.96, 23.37, 23.83, 23.13, 23.69, 22.81, 23.13, 23.31, 24.68, 23.35,
    22.13, 22.88, 22.35, 21.87, 25.45, 24.54, 22.38, 22.82, 22.88, 22.56,
    23.3, 22.71, 21.34)
m4 <- c(52.5, 48.63, 50.62, 51.45, 53.3, 50.62, 49.91, 53.68, 49.81, 49.64,
    61.65, 52.57, 47.99, 52.92, 53.02, 50.14, 48.25, 47.6, 49.43, 50.56,
    55.64, 51.5, 55.08, 51.05, 51.07, 52.06, 51.31, 51.93, 51.56, 53.67,
    56.23, 56.07, 52.56, 54.62, 48.89, 52.96, 51.35, 51.6, 52.45, 55.18,
    54.75, 49.28, 51.92, 49.88, 49.11, 56.32, 55.08, 49.67, 51.69, 51.32,
    52.74, 52.6, 53.15, 48.83)
m8 <- c(2.05, 1.98, 1.94, 1.97, 2.07, 1.97, 1.97, 2, 1.93, 2.04, 2.28, 2.1,
    1.89, 2.02, 2.09, 2.01, 1.94, 1.92, 1.94, 2, 2.15, 1.99, 2.1, 2, 2.01,
    2.07, 1.96, 2.01, 1.97, 2.09, 1.97, 2.07, 2.12, 2.06, 2.02, 2.03, 1.93,
    1.97, 2.03, 2.24, 2.12, 1.95, 1.98, 1.92, 1.91, 2.29, 2.12, 1.96, 1.99,
    1.98, 2.08, 2.06, 2.01, 1.94)
m15 <- c(4.25, 4.02, 4.05, 4.08, 4.29, 4.17, 4, 4.22, 3.84, 4.34, 4.82,
    4.52, 4.03, 4.12, 4.54, 4.1, 4.03, 3.96, 3.97, 4.09, 4.48, 4.02, 4.36,
    4.1, 3.98, 4.24, 3.98, 4.16, 3.96, 4.24, 4.25, 4.35, 4.39, 4.33, 4.19,
    4.2, 4.06, 4.1, 4.01, 4.62, 4.41, 3.99, 3.96, 3.9, 3.87, 5.42, 4.52,
    4.01, 4.09, 3.97, 4.38, 4.38, 3.92, 3.95)
m30 <- c(9.19, 8.63, 8.78, 8.82, 9.81, 9.04, 8.54, 9.26, 8.1, 9.37, 11.1,
    9.84, 8.87, 8.71, 9.89, 8.69, 8.64, 8.51, 8.37, 8.96, 9.71, 8.55, 9.5,
    9.11, 8.36, 9.33, 8.59, 8.74, 8.39, 9.01, 8.96, 9.21, 9.31, 9.24, 8.89,
    9.08, 8.57, 8.76, 8.53, 10.21, 9.81, 8.53, 8.5, 8.36, 8.38, 13.12, 9.94,
    8.48, 8.81, 8.6, 9.63, 10.07, 8.53, 8.43)
mt <- c(150.32, 143.51, 154.35, 143.05, 174.18, 147.41, 148.36, 152.23,
    139.39, 155.19, 212.33, 164.33, 145.19, 149.34, 166.46, 148, 148.27,
    141.45, 135.25, 153.4, 171.33, 148.5, 154.29, 158.1, 142.23, 156.36,
    143.47, 139.41, 138.47, 146.12, 145.31, 149.23, 169.28, 167.09, 144.06,
    158.42, 143.43, 146.46, 141.06, 221.14, 165.48, 144.18, 143.29, 142.5,
    141.31, 191.58, 154.41, 146.51, 150.39, 145.51, 159.53, 162.39, 151.43,
    141.16)
x <- matrix(c(mean(m1), mean(m2), mean(m4), mean(m8), mean(m15), mean(m30),
    mean(mt)))
x
```

```
##            [,1]
## [1,]  11.357778
## [2,]  23.118519
## [3,]  51.989074
## [4,]   2.022407
## [5,]   4.189444
## [6,]   9.080741
## [7,] 153.619259
```

```r
Sn <- round(matrix(c(sum((m1 - mean(m1))^2)/length(m1), sum((m1 - mean(m1)) *
    (m2 - mean(m2)))/length(m1), sum((m1 - mean(m1)) * (m4 - mean(m4)))/length(m1),
    sum((m1 - mean(m1)) * (m8 - mean(m8)))/length(m1), sum((m1 - mean(m1)) *
        (m15 - mean(m15)))/length(m1), sum((m1 - mean(m1)) * (m30 - mean(m30)))/length(m1),
    sum((m1 - mean(m1)) * (mt - mean(mt)))/length(m1), sum((m1 - mean(m1)) *
        (m2 - mean(m2)))/length(m1), sum((m2 - mean(m2))^2)/length(m2),
    sum((m2 - mean(m2)) * (m4 - mean(m4)))/length(m2), sum((m2 - mean(m2)) *
        (m8 - mean(m8)))/length(m2), sum((m2 - mean(m2)) * (m15 - mean(m15)))/length(m2),
    sum((m2 - mean(m2)) * (m30 - mean(m30)))/length(m2), sum((m2 - mean(m2)) *
        (mt - mean(mt)))/length(m2), sum((m1 - mean(m1)) * (m4 - mean(m4)))/length(m1),
    sum((m2 - mean(m2)) * (m4 - mean(m4)))/length(m2), sum((m4 - mean(m4))^2)/length(m4),
    sum((m4 - mean(m4)) * (m8 - mean(m8)))/length(m4), sum((m4 - mean(m4)) *
        (m15 - mean(m15)))/length(m4), sum((m4 - mean(m4)) * (m30 - mean(m30)))/length(m4),
    sum((m4 - mean(m4)) * (mt - mean(mt)))/length(m4), sum((m1 - mean(m1)) *
        (m8 - mean(m8)))/length(m1), sum((m2 - mean(m2)) * (m8 - mean(m8)))/length(m2),
    sum((m4 - mean(m4)) * (m8 - mean(m8)))/length(m4), sum((m8 - mean(m8))^2)/length(m8),
    sum((m8 - mean(m8)) * (m15 - mean(m15)))/length(m8), sum((m8 - mean(m8)) *
        (m30 - mean(m30)))/length(m8), sum((m8 - mean(m8)) * (mt - mean(mt)))/length(m8),
    sum((m1 - mean(m1)) * (m15 - mean(m15)))/length(m1), sum((m2 - mean(m2)) *
        (m15 - mean(m15)))/length(m2), sum((m4 - mean(m4)) * (m15 - mean(m15)))/length(m4),
    sum((m8 - mean(m8)) * (m15 - mean(m15)))/length(m8), sum((m15 - mean(m15))^2)/length(m15),
    sum((m15 - mean(m15)) * (m30 - mean(m30)))/length(m15), sum((m15 - mean(m15)) *
        (mt - mean(mt)))/length(m15), sum((m1 - mean(m1)) * (m30 - mean(m30)))/length(m1),
    sum((m2 - mean(m2)) * (m30 - mean(m30)))/length(m2), sum((m4 - mean(m4)) *
        (m30 - mean(m30)))/length(m4), sum((m8 - mean(m8)) * (m30 - mean(m30)))/length(m8),
    sum((m15 - mean(m15)) * (m30 - mean(m30)))/length(m15), sum((m30 - mean(m30))^2)/length(m30),
    sum((m30 - mean(m30)) * (mt - mean(mt)))/length(m30), sum((m1 - mean(m1)) *
        (mt - mean(mt)))/length(m1), sum((m2 - mean(m2)) * (mt - mean(mt)))/length(m2),
    sum((m4 - mean(m4)) * (mt - mean(mt)))/length(m4), sum((m8 - mean(m8)) *
        (mt - mean(mt)))/length(m8), sum((m15 - mean(m15)) * (mt - mean(mt)))/length(m15),
    sum((m30 - mean(m30)) * (mt - mean(mt)))/length(m30), sum((mt - mean(mt))^2)/length(mt)),
    nrow = 7, ncol = 7), 7)
Sn
```

```
##            [,1]       [,2]       [,3]      [,4]      [,5]       [,6]       [,7]
## [1,] 0.1524395  0.3381800  0.8747905 0.0271905 0.0823377  0.2295516   4.253915
## [2,] 0.3381800  0.8471052  2.1522283 0.0649406 0.1990084  0.5440844  10.192673
## [3,] 0.8747905  2.1522283  6.6205417 0.1784411 0.4997476  1.4003933  28.368477
## [4,] 0.0271905  0.0649406  0.1784411 0.0074072 0.0210180  0.0602427   1.197068
## [5,] 0.0823377  0.1990084  0.4997476 0.0210180 0.0728090  0.2121523   3.474285
## [6,] 0.2295516  0.5440844  1.4003933 0.0602427 0.2121523  0.6524476  10.507830
## [7,] 4.2539150 10.1926730 28.3684771 1.1970684 3.4742848 10.5078302 265.265148
```

```r
R <- matrix(c(cor(m1, m1), cor(m1, m2), cor(m1, m4), cor(m1, m8), cor(m1,
    m15), cor(m1, m30), cor(m1, mt), cor(m2, m1), cor(m2, m2), cor(m2, m4),
    cor(m2, m8), cor(m2, m15), cor(m2, m30), cor(m2, mt), cor(m4, m1), cor(m4,
        m2), cor(m4, m4), cor(m4, m8), cor(m4, m15), cor(m4, m30), cor(m4,
        mt), cor(m8, m1), cor(m8, m2), cor(m8, m4), cor(m8, m8), cor(m8,
        m15), cor(m8, m30), cor(m8, mt), cor(m15, m1), cor(m15, m2), cor(m15,
        m4), cor(m15, m8), cor(m15, m15), cor(m15, m30), cor(m15, mt), cor(m30,
        m1), cor(m30, m2), cor(m30, m4), cor(m30, m8), cor(m30, m15), cor(m30,
        m30), cor(m30, mt), cor(mt, m1), cor(mt, m2), cor(mt, m4), cor(mt,
        m8), cor(mt, m15), cor(mt, m30), cor(mt, mt)), nrow = 7, ncol = 7)
R
```

```
##            [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 1.0000000 0.9410886 0.8707802 0.8091758 0.7815510 0.7278784 0.6689597
## [2,] 0.9410886 1.0000000 0.9088096 0.8198258 0.8013282 0.7318546 0.6799537
## [3,] 0.8707802 0.9088096 1.0000000 0.8057904 0.7197996 0.6737991 0.6769384
## [4,] 0.8091758 0.8198258 0.8057904 1.0000000 0.9050509 0.8665732 0.8539900
## [5,] 0.7815510 0.8013282 0.7197996 0.9050509 1.0000000 0.9733801 0.7905565
## [6,] 0.7278784 0.7318546 0.6737991 0.8665732 0.9733801 1.0000000 0.7987302
## [7,] 0.6689597 0.6799537 0.6769384 0.8539900 0.7905565 0.7987302 1.0000000
```

The correlations between all pairs of races are positive and at least moderately strong ($r \geq 0.6689597$), but we can see the correlation is stronger between races with similar distances.

Problem 1.18

```
ms1 <- 100/m1
ms2 <- 200/m2
ms4 <- 400/m4
ms8 <- 800/(m8 * 60)
ms15 <- 1500/(m15 * 60)
ms30 <- 3000/(m30 * 60)
mts <- 42195/(mt * 60)
x <- matrix(c(mean(ms1), mean(ms2), mean(ms4), mean(ms8), mean(ms15),
    mean(ms30), mean(mts)))
x
```

```
##          [,1]
## [1,] 8.814772
## [2,] 8.664408
## [3,] 7.712067
## [4,] 6.604214
## [5,] 5.989687
## [6,] 5.542701
## [7,] 4.620264
```

```
Sn <- round(matrix(c(sum((ms1 - mean(ms1))^2)/length(ms1), sum((ms1 -
    mean(ms1)) * (ms2 - mean(ms2)))/length(ms1), sum((ms1 - mean(ms1)) *
    (ms4 - mean(ms4)))/length(ms1), sum((ms1 - mean(ms1)) * (ms8 - mean(ms8)))/length(ms1),
    sum((ms1 - mean(ms1)) * (ms15 - mean(ms15)))/length(ms1), sum((ms1 -
        mean(ms1)) * (ms30 - mean(ms30)))/length(ms1), sum((ms1 - mean(ms1)) *
        (mts - mean(mts)))/length(ms1), sum((ms1 - mean(ms1)) * (ms2 -
        mean(ms2)))/length(ms1), sum((ms2 - mean(ms2))^2)/length(ms2),
    sum((ms2 - mean(ms2)) * (ms4 - mean(ms4)))/length(ms2), sum((ms2 -
        mean(ms2)) * (ms8 - mean(ms8)))/length(ms2), sum((ms2 - mean(ms2)) *
        (ms15 - mean(ms15)))/length(ms2), sum((ms2 - mean(ms2)) * (ms30 -
        mean(ms30)))/length(ms2), sum((ms2 - mean(ms2)) * (mts - mean(mts)))/length(ms2),
    sum((ms1 - mean(ms1)) * (ms4 - mean(ms4)))/length(ms1), sum((ms2 -
        mean(ms2)) * (ms4 - mean(ms4)))/length(ms2), sum((ms4 - mean(ms4))^2)/length(ms4),
    sum((ms4 - mean(ms4)) * (ms8 - mean(ms8)))/length(ms4), sum((ms4 -
        mean(ms4)) * (ms15 - mean(ms15)))/length(ms4), sum((ms4 - mean(ms4)) *
        (ms30 - mean(ms30)))/length(ms4), sum((ms4 - mean(ms4)) * (mts -
        mean(mts)))/length(ms4), sum((ms1 - mean(ms1)) * (ms8 - mean(ms8)))/length(ms1),
    sum((ms2 - mean(ms2)) * (ms8 - mean(ms8)))/length(ms2), sum((ms4 -
        mean(ms4)) * (ms8 - mean(ms8)))/length(ms4), sum((ms8 - mean(ms8))^2)/length(ms8),
    sum((ms8 - mean(ms8)) * (ms15 - mean(ms15)))/length(ms8), sum((ms8 -
        mean(ms8)) * (ms30 - mean(ms30)))/length(ms8), sum((ms8 - mean(ms8)) *
```

```
        (mts - mean(mts)))/length(ms8), sum((ms1 - mean(ms1)) * (ms15 -
        mean(ms15)))/length(ms1), sum((ms2 - mean(ms2)) * (ms15 - mean(ms15)))/length(ms2),
    sum((ms4 - mean(ms4)) * (ms15 - mean(ms15)))/length(ms4), sum((ms8 -
        mean(ms8)) * (ms15 - mean(ms15)))/length(ms8), sum((ms15 - mean(ms15))^2)/length(ms15),
    sum((ms15 - mean(ms15)) * (ms30 - mean(ms30)))/length(ms15), sum((ms15 -
        mean(ms15)) * (mts - mean(mts)))/length(ms15), sum((ms1 - mean(ms1)) *
        (ms30 - mean(ms30)))/length(ms1), sum((ms2 - mean(ms2)) * (ms30 -
        mean(ms30)))/length(ms2), sum((ms4 - mean(ms4)) * (ms30 - mean(ms30)))/length(ms4),
    sum((ms8 - mean(ms8)) * (ms30 - mean(ms30)))/length(ms8), sum((ms15 -
        mean(ms15)) * (ms30 - mean(ms30)))/length(ms15), sum((ms30 -
        mean(ms30))^2)/length(ms30), sum((ms30 - mean(ms30)) * (mts -
        mean(mts)))/length(ms30), sum((ms1 - mean(ms1)) * (mts - mean(mts)))/length(ms1),
    sum((ms2 - mean(ms2)) * (mts - mean(mts)))/length(ms2), sum((ms4 -
        mean(ms4)) * (mts - mean(mts)))/length(ms4), sum((ms8 - mean(ms8)) *
        (mts - mean(mts)))/length(ms8), sum((ms15 - mean(ms15)) * (mts -
        mean(mts)))/length(ms15), sum((ms30 - mean(ms30)) * (mts - mean(mts)))/length(ms30),
    sum((mts - mean(mts))^2)/length(mts)), nrow = 7, ncol = 7), 7)
Sn
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.0888616 0.0938359 0.0948822 0.0638591 0.0806972 0.0904359 0.0795980
## [2,] 0.0938359 0.1125479 0.1117612 0.0735374 0.0942408 0.1034839 0.0915824
## [3,] 0.0948822 0.1117612 0.1352372 0.0794420 0.0936755 0.1063106 0.0999941
## [4,] 0.0638591 0.0735374 0.0794420 0.0721613 0.0848532 0.0979074 0.0925592
## [5,] 0.0806972 0.0942408 0.0936755 0.0848532 0.1215472 0.1410534 0.1162641
## [6,] 0.0904359 0.1034839 0.1063106 0.0979074 0.1410534 0.1733142 0.1438463
## [7,] 0.0795980 0.0915824 0.0999941 0.0925592 0.1162641 0.1438463 0.1636268
```

```
R <- matrix(c(cor(ms1, ms1), cor(ms1, ms2), cor(ms1, ms4), cor(ms1, ms8),
    cor(ms1, ms15), cor(ms1, ms30), cor(ms1, mts), cor(ms2, ms1), cor(ms2,
        ms2), cor(ms2, ms4), cor(ms2, ms8), cor(ms2, ms15), cor(ms2,
        ms30), cor(ms2, mts), cor(ms4, ms1), cor(ms4, ms2), cor(ms4,
        ms4), cor(ms4, ms8), cor(ms4, ms15), cor(ms4, ms30), cor(ms4,
        mts), cor(ms8, ms1), cor(ms8, ms2), cor(ms8, ms4), cor(ms8, ms8),
    cor(ms8, ms15), cor(ms8, ms30), cor(ms8, mts), cor(ms15, ms1), cor(ms15,
        ms2), cor(ms15, ms4), cor(ms15, ms8), cor(ms15, ms15), cor(ms15,
        ms30), cor(ms15, mts), cor(ms30, ms1), cor(ms30, ms2), cor(ms30,
        ms4), cor(ms30, ms8), cor(ms30, ms15), cor(ms30, ms30), cor(ms30,
        mts), cor(mts, ms1), cor(mts, ms2), cor(mts, ms4), cor(mts, ms8),
    cor(mts, ms15), cor(mts, ms30), cor(mts, mts)), nrow = 7, ncol = 7)
R
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 1.0000000 0.9383028 0.8655248 0.7974687 0.7764777 0.7287297 0.6601124
## [2,] 0.9383028 1.0000000 0.9058875 0.8159945 0.8057456 0.7409469 0.6748635
## [3,] 0.8655248 0.9058875 1.0000000 0.8041737 0.7306437 0.6944025 0.6722005
## [4,] 0.7974687 0.8159945 0.8041737 1.0000000 0.9060324 0.8754795 0.8518052
## [5,] 0.7764777 0.8057456 0.7306437 0.9060324 1.0000000 0.9718385 0.8244153
## [6,] 0.7287297 0.7409469 0.6944025 0.8754795 0.9718385 1.0000000 0.8541900
## [7,] 0.6601124 0.6748635 0.6722005 0.8518052 0.8244153 0.8541900 1.0000000
```

Just like in Problem 1.17, the correlations between all pairs of races are positive and at least moderately strong ($r \geq 0.6601124$) and the correlation is stronger between races with similar distances. We can see this matrix is very similar to the matrix in Problem 1.17.