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In-Class Assignment 1

8.43.

a. $f(y) = \dfrac{1}{\theta}$ Uniform distribution for $0 \le y \le \theta$

$f_{y(n)}(y) = nF(y)^{n-1}f(y) =$

$f_{y(n)}(y) = n(y/\theta)^{n-1}(1/\theta) =$ $F(y) = \int f(y)\, dy = \int 1/\theta\, dy = y/\theta$

$f_{y(n)}(y) = n\ \dfrac{y^{n-1}}{\theta^{n}}$ for $0 \le y \le \theta$

$f_u(u) = f_n(y_{(n)})\ \dfrac{|d_{y(n)}|}{|du|} =$

$f_u(u) = f_n(U\theta)\ \dfrac{|d_{y(n)}|}{|du|} =$ $U = y_{(n)}/\theta \rightarrow y_{(n)} = U\theta$

$f_u(u) = n\ \dfrac{(U\theta)^{n-1}|\theta|}{\theta^{n}} =$ $\dfrac{d_{y(n)}}{du} = \theta$

$f_u(u) = nU^{n-1}$ for $0 \le u \le 1$

$F_u(u) = U^n \rightarrow$ $F_u(u) = \int f_u(u) = \int nU^{n-1}\, du = U^n$

$$\boldsymbol{F_u(u) = \begin{cases} 1 & \text{for } u < 1 \\ U^n & \text{for } 0 \le u \le 1 \\ 0 & \text{for } u > 0 \end{cases}}$$

b. $P(U \le c) = 0.95$ Arbitrary choice of variable c

$c = 0.95^{1/n}$ $P(U \le c) = c^n = 0.95 \rightarrow c = 0.95^{1/n}$

$P(y_{(n)}/\theta \le 0.95^{1/n}) = 0.95$

$P(\theta \ge (y_{(n)})0.95^{-1/n}) = 0.95$ Multiplying both sides by $0.95^{-1/n}\theta$

Lower bound: $(y_{(n)})0.95^{-1/n}$

8.60.

a. $\bar{y} \pm (z^*)s / \sqrt{n} =$

$98.25 \pm (2.576)(0.73)/\sqrt{(130)} =$ $\bar{y}=98.25$, $z^*=2.576$, $s=0.73$, $n=130$

(98.085, 98.415)

b. The confidence interval obtained in problem 8.60(a) does not contain 98.6. This suggests the mean body temperature may be different than 98.6 degrees.

8.70.

a. $(z^*)\sqrt{(p(1\text{-}p)/n)} \le 0.05$

$(1.96)\sqrt{((0.9)(1\text{-}(0.9))/n)} \le 0.05$ $z^* = 1.96$ (95% confidence interval)

$n \approx 138.2976 \rightarrow$ **139**

b. $(z^*)\sqrt{(p(1\text{-}p)/n)} \le 0.05$

$(1.96)\sqrt{((0.5)(1\text{-}(0.5))/n)} \le 0.05$ $z^* = 1.96$ (95% confidence interval)

$n \approx 384.16 \rightarrow$ **385**

8.85.	$\bar{y}_2 - \bar{y}_1 \pm (t^*) *$ $\sqrt{\{[(n_1-1)s_1^2+(n_2-1)s_2^2][1/n_1+1/n_2] / [n_1+n_2-2]\}} =$ $(12) - (11) \pm (2.032) *$ $\sqrt{\{[(16-1)(6)^2 + (20-1)(8)^2][1/16+1/20]/34\}} =$ **(-3.8986, 5.8986)**	Using t-statistic instead of z-statistic because $n_1 < 30$ and $n_2 < 30$ $\bar{y}_1$=11, $\bar{y}_2$=12, n_1=16, n_2=20, $df=n_1+n_2-2$=34, t^*_{34}=2.032 (95%), s_1=6, s_2=8