Charles Hwang
Dr. Kong
STAT 405-001
3 February 2021

Homework 1

7.15.

a. $E(\bar{X} - \bar{y}) =$
$E(\bar{X}) - E(\bar{y}) =$
$\boldsymbol{\mu_1 - \mu_2}$

$X \sim N(\mu_1, \sigma^2_1)$, $Y \sim N(\mu_2, \sigma^2_2)$

$E(\bar{X}) = \mu_1$, $E(\bar{y}) = \mu_2$

b. $Var(\bar{X} - \bar{y}) =$
$Var(\bar{X}) + Var(\bar{y}) =$
$$\boldsymbol{\frac{\sigma_1^2}{m} + \frac{\sigma_1^2}{n}}$$

$Var(\bar{X}) = \sigma_1^2 / m$
$Var(\bar{y}) = \sigma_2^2 / n$

c. $P(|\bar{X} - \bar{y} - (\mu_1 - \mu_2)| \leq 1) = .95 =$
$P(-1 \leq \bar{X} - \bar{y} - (\mu_1 - \mu_2) \leq 1) = .95$
$$\frac{(\bar{y} - \mu)}{\sigma_P / \sqrt{n}} = \frac{1}{(3\sqrt{2} / 2) / \sqrt{n}} = 1.96$$
$m = n = 17.2872 \approx \mathbf{18}$

$\sigma_P = \sqrt{(\sigma_1^2 + \sigma_2^2)} = \sqrt{(2 + 2.5)} = \sqrt{(4.5)} = (3\sqrt{2}) / 2$

7.20.

a. $E(U) = \boldsymbol{v}$, $Var(U) = \boldsymbol{2v}$

$U \sim \chi^2_v$ distribution

7.29. $Y \sim F = \dfrac{W_1 / v_1}{W_2 / v_2}$

$$U = \frac{1}{Y} \sim \frac{1}{(W_1 / v_1) / (W_2 / v_2)} = \boldsymbol{\frac{W_2 / v_2}{W_1 / v_1}}$$

F-distribution with v_2 and v_1 degrees of freedom

7.32. *(Answers calculated in* R*)*

a. **2.01505**
b. **0.10**
c. **4.06042**
d. $4.06042 = (2.01505)^2$

F-value = $(\text{t-value})^2$

7.42.

a. $P(\bar{y} > 14.5) \rightarrow P(Z > 2.5) = \mathbf{0.0062}$
$$\frac{(\bar{y} - \mu)}{\sigma / \sqrt{n}} = \frac{(14 - 14.5)}{2 / \sqrt{100}} = 2.5$$

$Y \sim N(14, 2^2)$
$\bar{y} = 14$, $\sigma = 2$, $n = 100$

b. $0.95 = P(-1.96 < Z < 1.96)$
$\pm 1.96 = (14 - u) / 0.2 =$
(13.9608, 14.0392)

7.43. $P(|\bar{y} - \mu| \leq 0.5) = P(-0.5 \leq \bar{y} - \mu \leq 0.5) \rightarrow$
$P(-2 \leq Z \leq 2) = \mathbf{0.9544}$
$$\frac{(\bar{y} - \mu)}{\sigma / \sqrt{n}} = \frac{0.5}{2.5 / \sqrt{100}} = 2$$

$\sigma = 2.5$, $n = 100$

7.44. $P(|\bar{y} - \mu| < 0.4) = P(-0.4 < \bar{y} - \mu < 0.4)$

$$\frac{(\bar{y} - \mu)}{\sigma / \sqrt{n}} = \frac{0.4}{2.5 / \sqrt{n}} = 1.96$$

$0.95 = P(-1.96 < Z < 1.96)$

$n = 150.0625 \approx$ **151 people**