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STAT 488-001
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1 3.3A) Posterior distribution: $\prod_{i=1}^{32} N(y_{ci} \mid \mu_c, \sigma_c^2) \prod_{i=1}^{36} N(y_{ti} \mid \mu_t, \sigma_t^2)$

control distributions Treatment distributions

From Section 3.2, we see the marginal posterior distribution of μ_t is $t_{n_t-1}(\mu_t, \frac{\sigma_t^2}{n_t}) = t_{(36)-1}(1.173, \frac{(0.2)^2}{(36)}) = \boxed{t_{35}(1.173, \frac{1}{900})}$.

3.3B) See next page

2 3.4A) From Section 3.7, we see that a reasonable prior is $Beta(\frac{1}{2}, \frac{1}{2})$ w $\frac{p^{(\frac{1}{2})-1}(1-p)^{(\frac{1}{2})-1}}{\frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2}+\frac{1}{2})}} \propto p^{-\frac{1}{2}}(1-p)^{-\frac{1}{2}}$. The posterior for sampling would then be $\propto$

$p_0^{39}(1-p_0)^{674-39} p_1^{22}(1-p_1)^{680-22}, p^{-\frac{1}{2}}(1-p)^{-\frac{1}{2}} \propto$ α = deaths

$p(p_0, p_1 \mid y) \propto p(y \mid p_0, p_1)\, p(p_0, p_1) \propto \boxed{p_0^{38.5}(1-p_0)^{634.5} p_1^{21.5}(1-p_1)^{657.5}}$. β = total − deaths

3.4B) We can see the distribution is right-skew with a mean of 0.56.

3.4C) We can see from the contour plots in problem 3.4A that a lot of the simulations fall outside the outermost contour level. Although the choice of prior appears to be good, better choices of the α and β parameters could improve the contour plot.

3 3.6A) This noninformative prior distribution is easier to work with. It is improper because $\int_0^\infty \frac{1}{\lambda}\, d\lambda$ diverges. $p(N, \theta) = \int_0^\infty p(N, \lambda, \theta)\, d\lambda =$
$\int_0^\infty p(N \mid \lambda, \theta)\, p(\lambda, \theta)\, d\lambda =$
$\int_0^\infty \frac{(\frac{\lambda}{\theta})^N e^{-\frac{\lambda}{\theta}}}{N!} \cdot \frac{1}{\lambda}\, d\lambda =$
$\frac{1}{\theta^N \cdot N!} \int_0^\infty \lambda^{N-1} e^{-\frac{1}{\theta}\lambda}\, d\lambda =$
$\frac{1}{\theta^N \cdot N!}(\Gamma(N)\theta^N) = \frac{(N-1)!}{N!} = \boxed{\frac{1}{N}}$

3.6B) $p(N, \theta \mid y) = p(y \mid N, \theta)\, p(N, \theta) =$
$\prod_{i=1}^{5} \binom{N}{y_i} \theta^{y_i} (1-\theta)^{N-y_i} \cdot \frac{1}{N} =$
(for $n=5$) $\left(\prod_{i=1}^{5} \binom{N}{y_i}\right) \theta^{\Sigma y_i} (1-\theta)^{\Sigma N - y_i} \cdot \frac{1}{N} \propto$
$\theta^{\Sigma y_i} (1-\theta)^{5N - \Sigma y_i} \sim$
$Beta\left(\sum_{i=1}^{5} y_i + 1, 5N - \sum_{i=1}^{5} y_i + 1\right)$

$p(N \mid y) = \frac{1}{N} \sum_{i=1}^{n} \binom{N}{y_i} \int_0^1 \theta^{\Sigma y_i} (1-\theta)^{nN - \Sigma y_i}\, d\theta =$
$\boxed{\frac{1}{N} \sum_{i=1}^{N} \binom{N}{y_i} \cdot Beta\left(\sum_{i=1}^{n} y_i + 1, nN - \sum_{i=1}^{n} + 1\right)}$ (for any size n)

3.6C) We could not use a Poisson distribution with fixed μ as a prior because we don't know what μ is and don't have any reasonable estimate.

Homework 2

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6/14/2022

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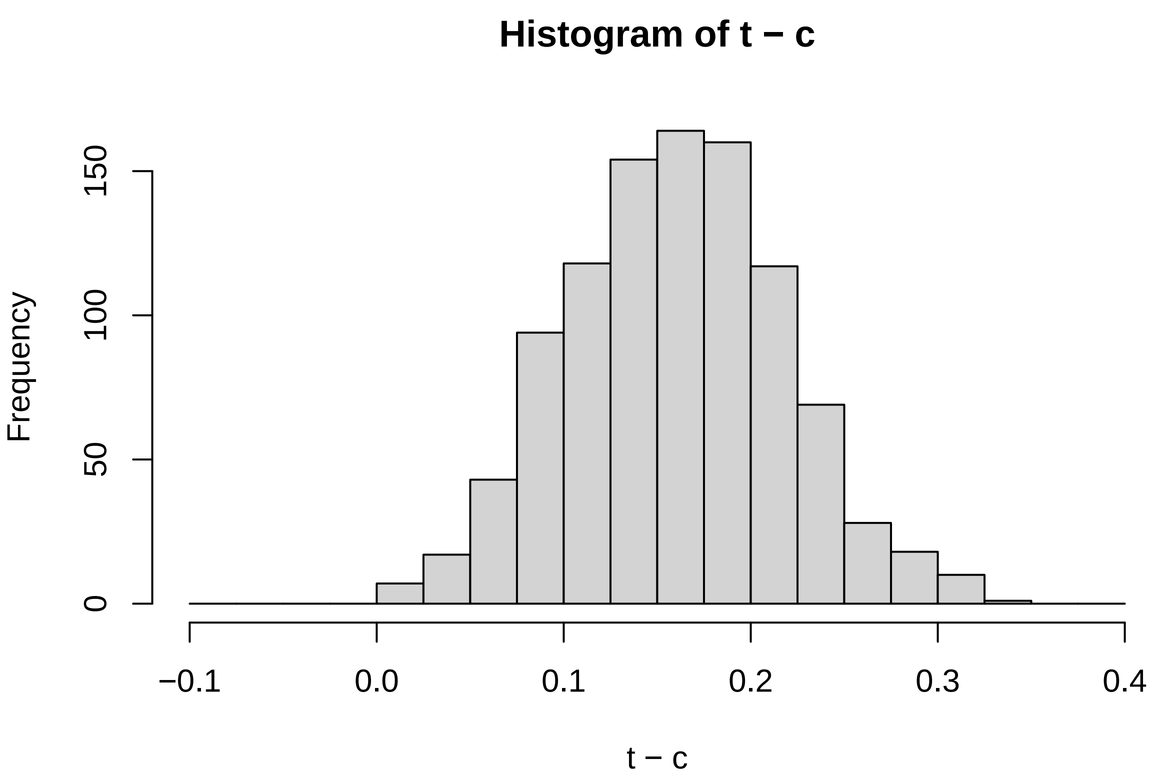
Problem 3.3

Problem 3.3(b)

```
rm(list=ls())
set.seed(1406)
c<-1.013+0.24/sqrt(32)*rt(1000,32-1)
t<-1.173+0.20/sqrt(36)*rt(1000,36-1)
hist(t-c,breaks=seq(-0.1,0.4,0.025))
```



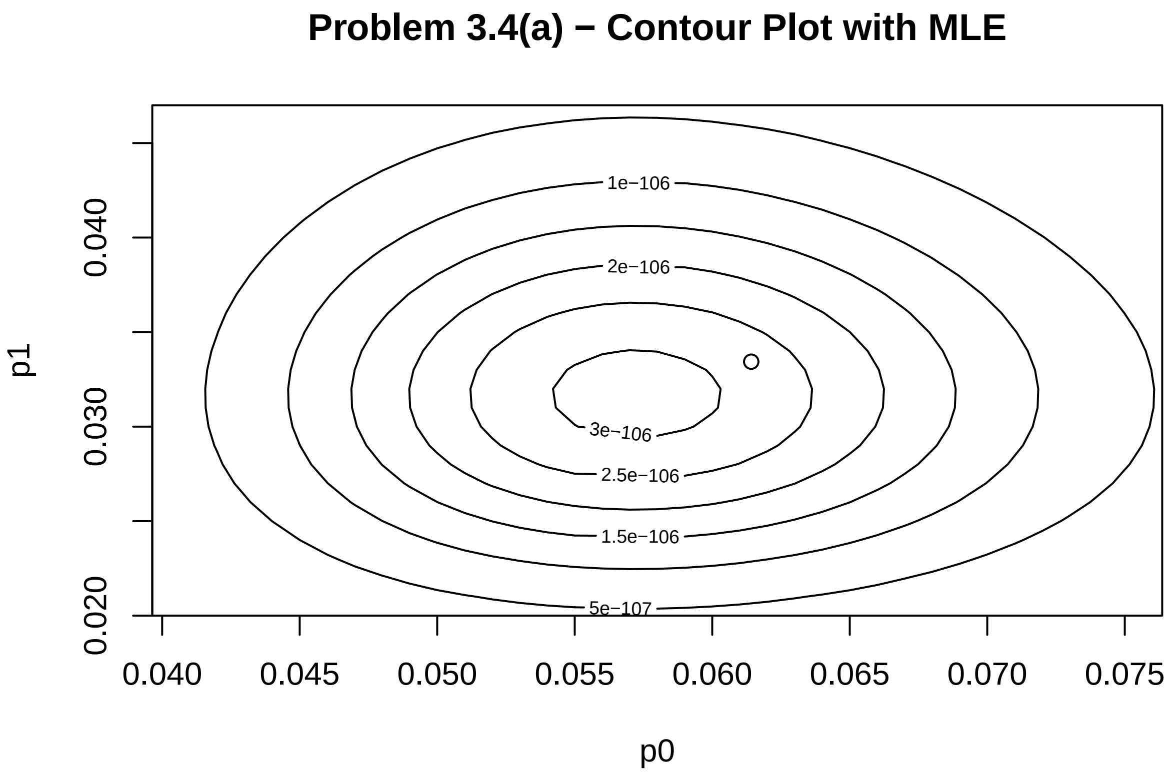

```
quantile(t-c,c(0.05/2,1-0.05/2))
```

```
##          2.5%       97.5%
## 0.05193314 0.27704710
```

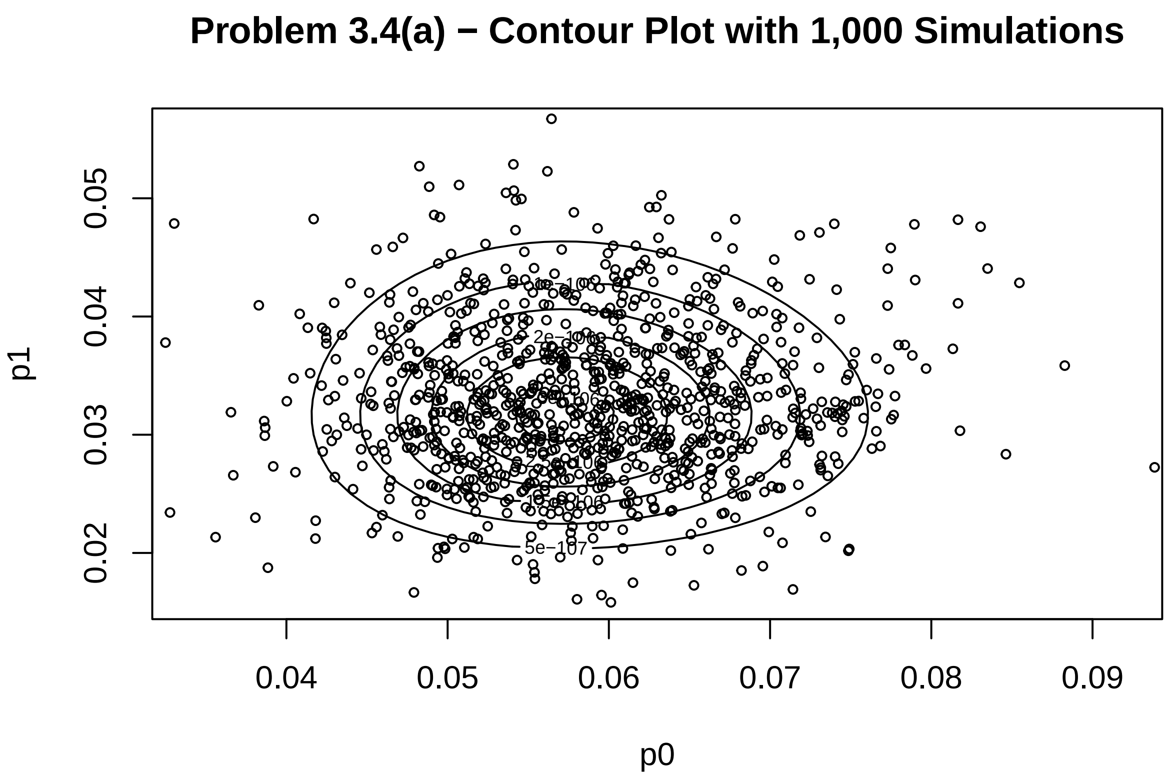
Problem 3.4

Problem 3.4(a)

```
library(ggplot2)
library(metR)
p0<-p1<-seq(0,1,0.001)
p0p1<-expand.grid(p0,p1)
zim<-function(p0,p1){return(p0^38.5*(1-p0)^634.5*p1^21.5*(1-p1)^657.5)}
z<-vector(mode="numeric",length=nrow(p0p1))
for(i in 1:nrow(p0p1)){z[i]<-zim(p0p1[i,"Var1"],p0p1[i,"Var2"])}
contour(p0,p1,matrix(z,1001,1001),xlim=c(0.041,0.075),ylim=c(0.021,0.046),xlab="p0",ylab="p1",main="Pro
points(39/(674-39),22/(680-22))
```

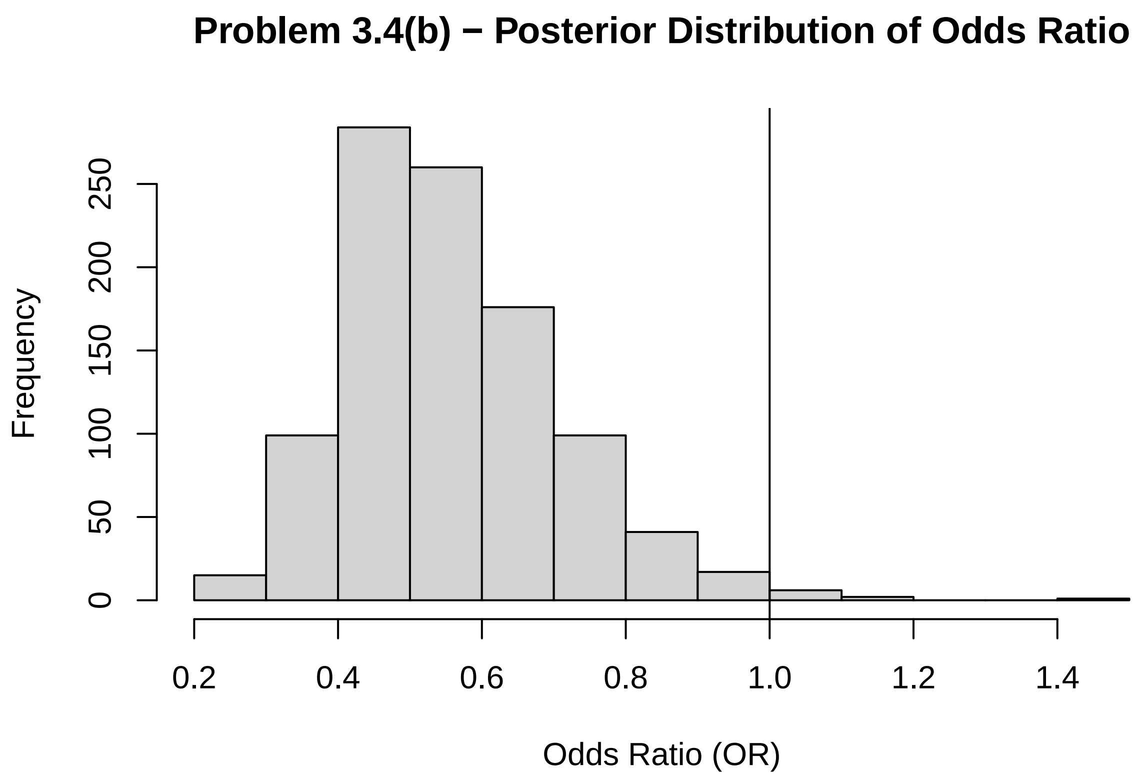



```
set.seed(1406)
x<-rbeta(1000,39.5,635.5)
y<-rbeta(1000,22.5,658.5)
contour(p0,p1,matrix(z,1001,1001),xlim=c(0.034,0.092),ylim=c(0.016,0.056),xlab="p0",ylab="p1",main="Pro
points(x,y,cex=.6)
```



Problem 3.4(b)

```
OR<-function(p0,p1){return(p1/(1-p1)/(p0/(1-p0)))}
pOR<-apply(data.frame(p0=x,p1=y),1,function(row) OR(row[1],row[2]))
hist(pOR,xlab="Odds Ratio (OR)",main="Problem 3.4(b) - Posterior Distribution of Odds Ratio")
abline(v=1)
```



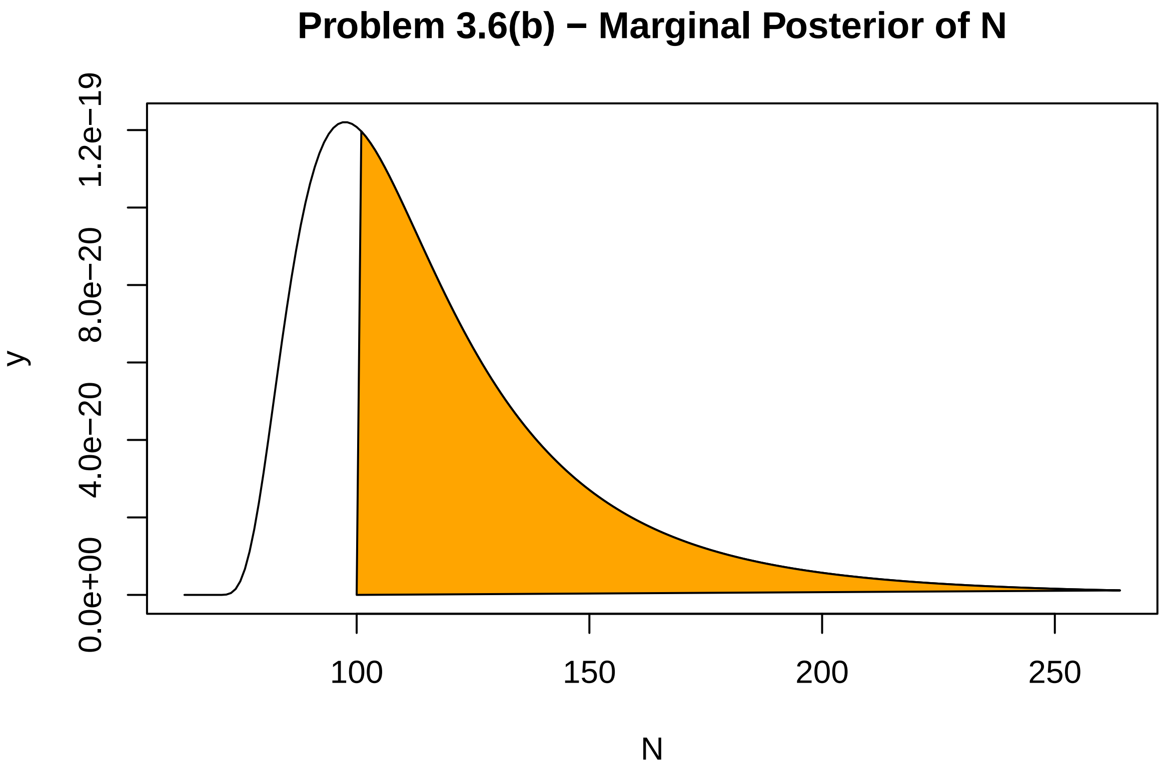

```
summary(pOR)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.2237  0.4544  0.5359  0.5589  0.6384  1.4712
```

We can see the distribution is right-skew with a mean of 0.5589333.

Problem 3.6(b)

```
w<-c(53,57,66,67,72)
b<-function(N,w){return(prod(sapply(w,function(x) choose(N,x)/N))*beta(sum(w)+1,5*N-sum(w)+1))}
N<-seq(63,264,1) # Testing different bounds
y<-sapply(N,function(x) b(x,w))
plot(N,y,type="l",main="Problem 3.6(b) - Marginal Posterior of N")
polygon(N[N>=100],c(0,y[(100-min(N)+1+1):length(y)]),col="orange")
```




```
sum(y[(N>100)])/sum(y)
```

```
## [1] 0.6729019
```