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Homework 3

9.70.

$\mu'_1 = E(Y) = \mu = \lambda$

$m'_1 = \Sigma_{i=1}^{n} Y_i / n = \bar{y} = \lambda \rightarrow$

$\boldsymbol{\hat{\lambda} = \bar{y}}$

$Y \sim \text{Poisson}(\lambda) \rightarrow E(Y) = \lambda$

9.77.

$\mu'_1 = E(Y) = \mu = (0 + 3\theta) / 2 = 3\theta / 2$

$m'_1 = \Sigma_{i=1}^{n} Y_i / n = \bar{y} = 3\theta / 2 \rightarrow$

$\boldsymbol{\theta\text{^} = 2\bar{y} / 3}$

$Y \sim \text{Uniform}(a,b) \rightarrow$
$E(Y) = (a + b) / 2$

9.78.

$\mu'_1 = E(Y) = \int_0^3 \alpha y^{\alpha-1} / 3^{\alpha}\, dy =$

$\boldsymbol{3\alpha / (\alpha + 1)}$

$m'_1 = \Sigma_{i=1}^{n} Y_i / n = \bar{y} = 3\alpha / (\alpha + 1) \rightarrow$

$\boldsymbol{\alpha\text{^} = \bar{y} / (3 - \bar{y})}$

$f(y|\alpha) = \alpha y^{\alpha-1}/3^{\alpha}$ for $0 \le y \le 3$;
$\int_0^3 \alpha y^{\alpha-1}/3^{\alpha}\, dy = |_0^3\ \alpha y^{\alpha-1}/(3^{\alpha}(\alpha + 1)) =$
$\alpha / (3^{\alpha}(\alpha + 1))(3^{\alpha-1} - 0^{\alpha-1}) =$
$\alpha / (3^{\alpha}(\alpha + 1))(3^{\alpha-1}) = 3\alpha / (\alpha + 1)$

9.80.

a. $\bar{y} = \lambda$

$\boldsymbol{\lambda\text{^} = \bar{y}}$

Invariance property

b. $E(\lambda\text{^}) = E(\bar{y}) = \boldsymbol{\lambda}$

$\text{Var}(\lambda\text{^}) = \text{Var}(\bar{y}) = (s / \sqrt{n})^2 = \boldsymbol{\lambda / n}$

$E(Y) = \text{Var}(Y) = \lambda$

c. From Problem 9.80(b), we can see that λ^ is an unbiased estimator and that Var(λ^) converges to 0 when $n \rightarrow \infty$. Therefore, per Theorem 9.1, λ^ is consistent for λ.

$\lim_{n\rightarrow\infty} \text{Var}(\lambda\text{^}) =$
$\lim_{n\rightarrow\infty} (\lambda / n) = 0$

d. $P(Y = 0) = e^{-\lambda} \rightarrow$

$\text{MLE}(P(Y = 0)) = \boldsymbol{e^{-\lambda\text{^}}}$

Invariance property

9.83.

a. $L(p) = \Pi_{i=1}^{n}\ 1 / (2\theta + 1)^{i} = 1 / (2\theta + 1)^{n}$

$Y_{(n)} = 2\theta + 1 \rightarrow$

$\boldsymbol{\theta\text{^} = (Y_{(n)} - 1) / 2}$

Since we are looking for an MLE, we can substitute $Y_{(n)} = 2\theta + 1$ to minimize the denominator and maximize $L(p)$.

b. $\text{Var}(Y) = (2\theta + 1)^2 / 12 \rightarrow$

$\text{MLE}(\text{Var}(Y)) = \boldsymbol{(Y_{(n)})^2 / 12}$

Invariance property

9.84.

a. $L(\theta) = \Pi_{i=1}^{n} [y_i^{2-1} e^{-yi/\theta}/((\Gamma(2))^{n} \theta^{2n})]$

$\ln(L(\theta)) = \ln(\Pi_{i=1}^{n} Y_i) - \Sigma_{i=1}^{n} Y_i/\theta - n\ln(\Gamma(2)) - 2n\ln(\theta)$

$\ln(L(\theta))' = \Sigma_{i=1}^{n} Y_i / \theta^2 - 2n / \theta = 0 \rightarrow$

α = 2

Take derivative and set equal to 0

$\boldsymbol{\theta\text{^} = \bar{y} / 2}$

b. $E(\theta\text{^}) = E(\bar{y}/2) = E(\bar{y})/2 = \alpha\theta / 2 = (2)\theta / 2 =$
$\boldsymbol{E(\theta\text{^}) = \theta}$
$\text{Var}(\theta\text{^}) = \text{Var}(\bar{y}/2) = \theta^2 / \alpha n =$ — $n = 3$
$\mathbf{Var}(\boldsymbol{\theta}\text{^}) \boldsymbol{= \theta^2 / 6}$

c. $\theta \pm z^*(s / \sqrt{n}) =$
$130 \pm (1.96)((65\sqrt{6} / 3) / \sqrt{3}) \approx$
$130 \pm 60.0569 =$
(69.9431, 190.0569)

$z^* = 1.96$, $n = 3$, $\theta = 130$,
$s = \sqrt{\text{Var}(Y)} = \sqrt{(\theta^2 / 6)} = \theta / \sqrt{6} =$
$130 / \sqrt{6} = 65\sqrt{6} / 3$

d. $\text{Var}(Y) = \alpha\theta^2 \rightarrow$
$\text{MLE}(\text{Var}(Y)) = \boldsymbol{2(\theta\text{^})^2}$

Invariance property