Homework 5

Charles Hwang

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Charles Hwang

Dr. Matthews

STAT 488-001

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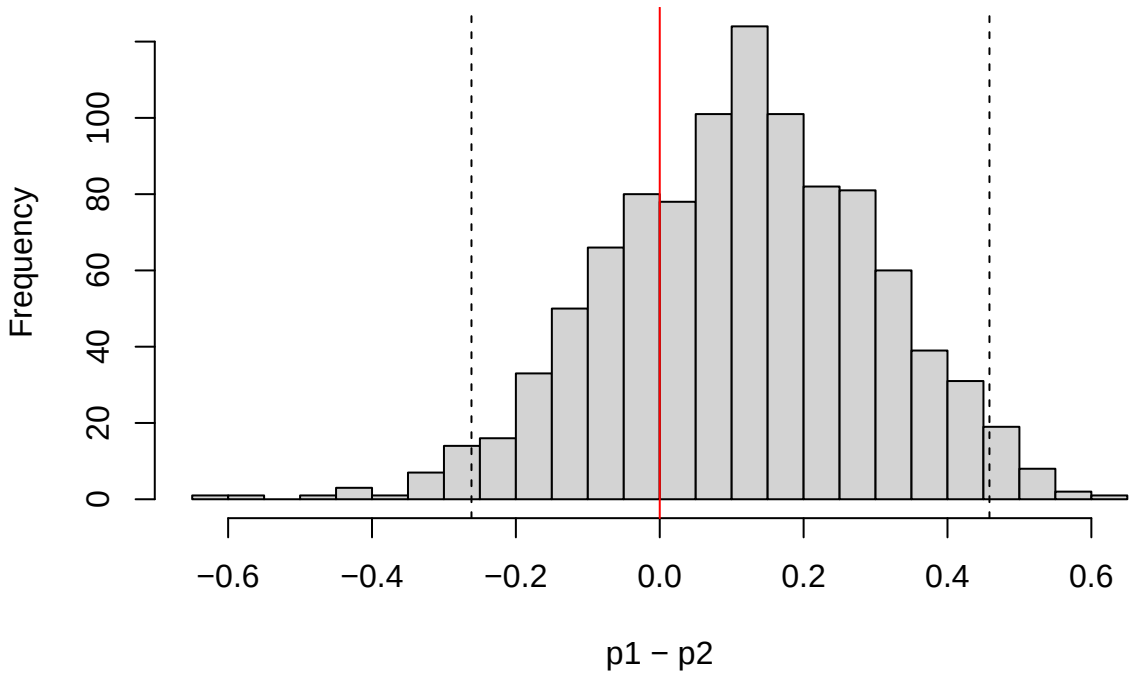
Problem 10.3

Problem 10.3(a)

```
rm(list=ls())
set.seed(306)
p1<-rbeta(1000,6,10-6)
p2<-rbeta(1000,10,20-10)
quantile(p1-p2,c(0.05/2,1-0.05/2))
```

```
##       2.5%      97.5%
## -0.2617284  0.4583608
```

```
hist(p1-p2,breaks=20,main="Problem 10.3(a) - Histogram of 1,000 Draws from p_1-p_2")
abline(v=0,col="red")
abline(v=quantile(p1-p2,c(0.05/2,1-0.05/2)),lty=2)
```

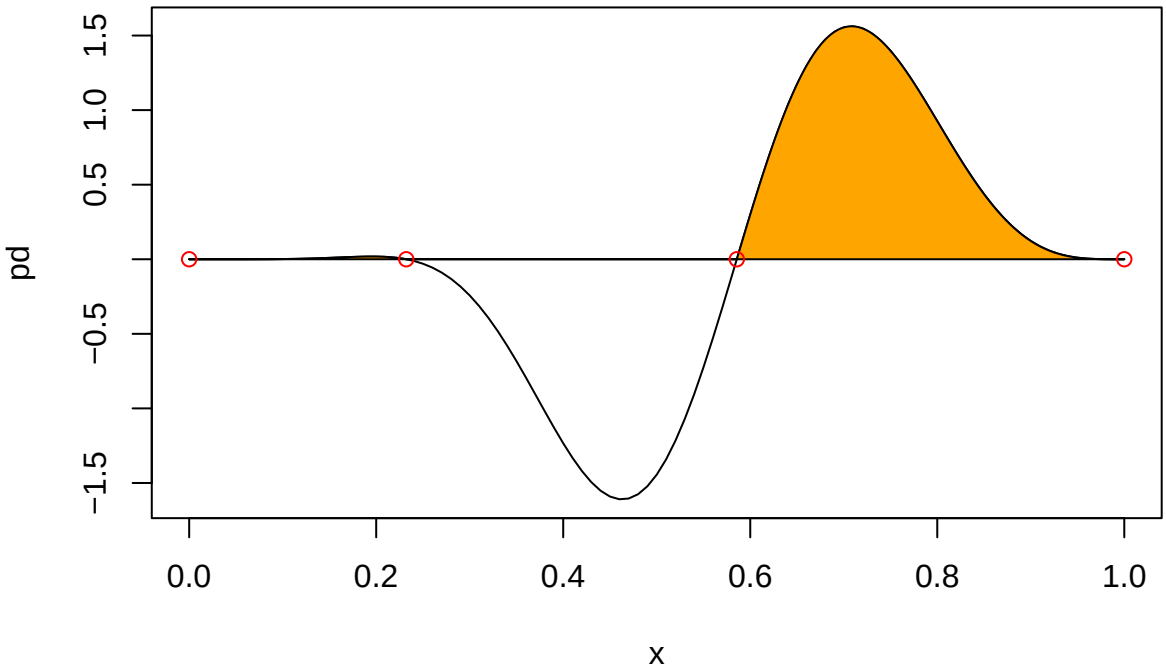



```
mean(p1>p2)
```

```
## [1] 0.727
```

Problem 10.3(b)

```
pd<-\(x) x^(7-1)*(1-x)^(5-1)/beta(7,5)-x^(11-1)*(1-x)^(11-1)/beta(11,11)
x<-c(seq(0,0.232165,0.0001),seq(0.5856,1,0.0001)) # Finding points via Desmos
plot(pd,main="Problem 10.3(b) - Plot of Beta(7,5)-Beta(11,11)")
polygon(x,c(0,pd(seq(0,1,0.0001))[pd(seq(0,1,0.0001))>=0]),col="orange")
points(c(0,0.232165,0.58564,1),rep(0,4),col="red") # Intersection points with x-axis
```




```
length(x)/length(seq(0,1,0.0001)) # Length of [0,1] for which the function is positive
```

```
## [1] 0.6466353
```

```
0.58564/0.232165*(integrate(pd,0,0.232165)$value+integrate(pd,0.58564,1)$value) # C * E[f(x)]
```

```
## [1] 0.7476288
```

Problem 2.

```
y<-c(144.8,113.2,82.6,46.1,117.6)
mean(y)            # mean = alpha / beta
```

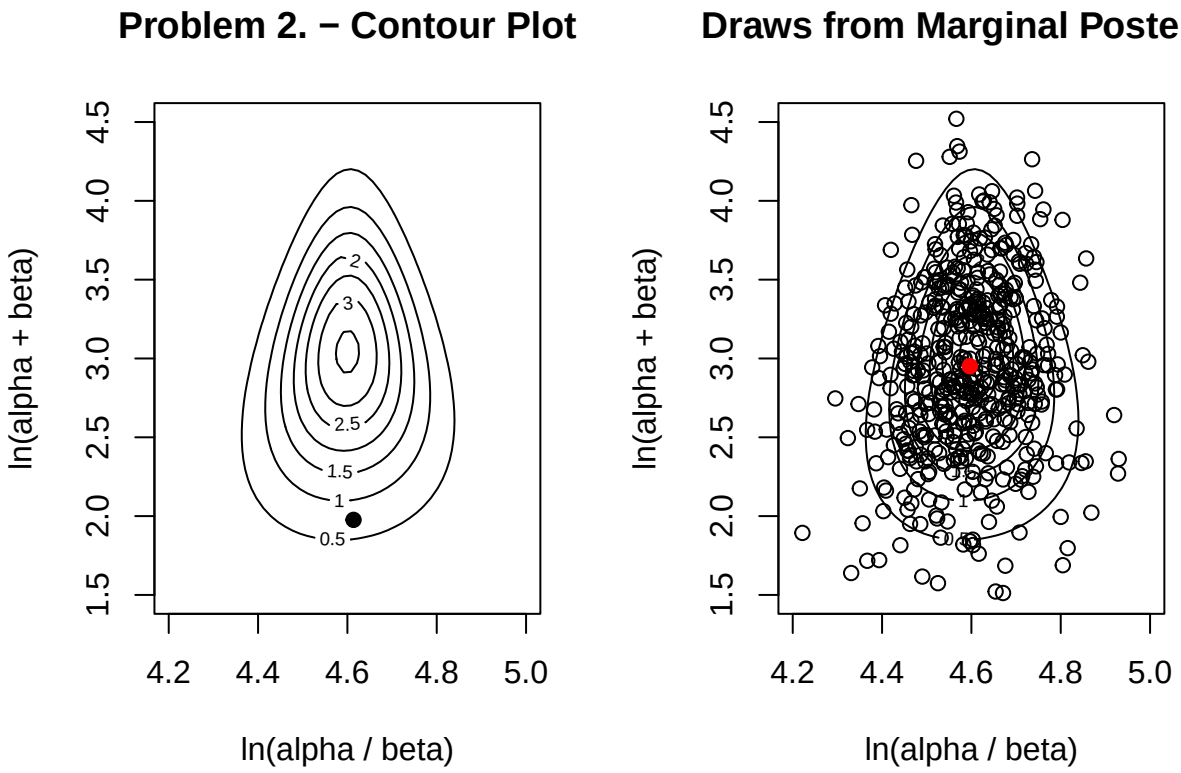
```
## [1] 100.86
```

```
var(y)         # variance = alpha / beta^2
```

```
## [1] 1423.828
```

```
b<-mean(y)/var(y) # beta = mean / variance
a<-b*mean(y)      # alpha = beta * mean
Xm<-Ym<-rep(NA,500)
mpd<-function(X,Y){a<-exp(Y)*exp(X)/(exp(X)+1)
  b<-exp(Y)/(exp(X)+1)
  exp(-1845+log(a)/2+log(b+1)+sum(a*log(b)-(y+a)*log(b+1)+lgamma(y)-lbeta(a,y))+2*Y+X-2*log(exp(X)+1))}
X<-seq(4.2,5,length=50) # Setting bounds of contour plot via trial-and-error
Y<-seq(1.5,4.5,length=50)
z<-matrix(NA,length(Y),length(X))
for (i in 1:length(Y)){for (j in 1:length(X)){z[j,i]<-mpd(X[j],Y[i])}}
par(mfrow=c(1,2))
contour(X,Y,z,xlab="ln(alpha / beta)",ylab="ln(alpha + beta)",main="Problem 2. - Contour Plot")
points(log(a/b),log(a+b),pch=19) # The original estimate appears to be a bit off.
```

```
set.seed(306)
for (k in 1:length(Xm)){Xm[k]<-sample(X,1,TRUE,apply(z,1,sum)/sum(apply(z,1,sum)))
  Ym[k]<-sample(Y,1,TRUE,z[which(Xm[k]==X),]/sum(z[which(Xm[k]==X),]))}
contour(X,Y,z,xlab="ln(alpha / beta)",ylab="ln(alpha + beta)",main="Draws from Marginal Posterior")
Xm<-Xm+runif(length(Xm),-diff(X)/2,diff(X)/2)
Ym<-Ym+runif(length(Ym),-diff(Y)/2,diff(Y)/2)
points(Xm,Ym)
points(mean(Xm),mean(Ym),pch=19,col="red")
```




```
alpha<-exp(Ym)*(exp(Xm)/(exp(Xm)+1))
beta<-exp(Ym)/(exp(Xm)+1)
quantile(alpha,c(0.05/2,1-0.05/2))      # 95 percent credible interval of alpha
```

```
##      2.5%     97.5%
##  6.098151 53.717306
```

```
a       # We can see this interval contains our original estimate of alpha.
```

```
## [1] 7.144641
```

```
quantile(beta,c(0.05/2,1-0.05/2))       # 95 percent credible interval of beta
```

```
##       2.5%      97.5%
## 0.06269434 0.52769513
```

```
b       # We can see this interval contains our original estimate of beta.
```

```
## [1] 0.07083721
```

```
quantile(alpha/beta,c(0.05/2,1-0.05/2)) # 95 percent credible interval of y
```

```
##      2.5%     97.5%
##  80.57112 123.02468
```

```r
sort(y)
```

```
## [1]  46.1  82.6 113.2 117.6 144.8
```

```r
# We can see this interval contains the middle three values of y. It is reasonable
# that the minimum and maximum values are not included as they appear to be outliers.
```