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STAT 405-001

16 April 2021

10.2A) A Type I error is concluding that the drug induces sleep in less than 80 percent of insomniacs when it actually induces sleep in 80 percent of insomniacs.

10.2B) $\alpha = P(Y \leq 12 | p = 0.8) = \boxed{0.032}$ $(n=20)$ Appendix III, Table 1D

10.2C) A Type II error is concluding that the drug induces sleep in 80 percent of insomniacs when it actually induces sleep in less than 80 percent of insomniacs.

10.2D) $\beta_{p=0.6} = P(Y > 12 | p = 0.6) = 1 - P(Y \leq 12 |_{p=0.6}) = 1 - 0.584 = \boxed{0.416}$

$n = 20$, $a = 12$, $p = 0.6$ Appendix III, Table 1D

10.2E) $\beta_{p=0.4} = P(Y > 12 |_{p=0.4}) = 1 - P(Y \leq 12 |_{p=0.4}) = 1 - 0.979 = \boxed{0.021}$

$n = 20$, $a = 12$, $p = 0.4$ Appendix III, Table 1D

10.3A) Since $n = 20$ and $p = 0.8$, we can check Appendix III, Table 1D to see that $a = \boxed{11 = c}$ when $\alpha = 0.01$.

10.3B) $\beta_{p=0.6} = P(Y > 11 | p = 0.6) = 1 - P(Y \leq 11 |_{p=0.6}) = 1 - 0.404 = \boxed{0.596}$

$n = 20$, $a = 11$, $p = 0.6$ Appendix III, Table 1D

10.3C) $\beta_{p=0.4} = P(Y > 11 |_{p=0.4}) = 1 - P(Y \leq 11 |_{p=0.4}) = 1 - 0.943 = \boxed{0.057}$

$n = 20$, $a = 11$, $p = 0.4$ Appendix III, Table 1D

10.19) $H_0: \mu = 130$, $H_A: \mu < 130$, $\alpha = 0.05$ $n = 40$, $\bar{Y} = 128.6$, $s = 2.1$

Because $n = 40 > 30$, the Central Limit Theorem dictates that we can use the normal distribution

$z = \frac{\bar{Y} - \mu}{\frac{s}{\sqrt{n}}} = \frac{128.6 - 130}{\frac{2.1}{\sqrt{40}}} = \frac{-1.4\sqrt{40}}{2.1} = \frac{-4\sqrt{10}}{3} \approx -4.216$ $|-4.22| > z_{0.05} = 1.645$

We reject H_0 at the $\alpha = 0.05$ level. There is sufficient evidence that the mean output voltage is less than 130.

10.37) $Z_{0.05} = \frac{\bar{Y}-\mu_0}{\frac{\sigma}{\sqrt{n}}} \le 1.645 \rightarrow \bar{Y}-\mu_0 \le 1.645\left(\frac{\sigma}{\sqrt{n}}\right) \rightarrow \bar{Y} < 130 - 1.645\left(\frac{2.1}{\sqrt{40}}\right) \rightarrow \bar{Y} < 129.454$

$\beta_{\mu=128} = P(\bar{Y} \ge 129.454 \mid \mu=128) = P\left(\frac{\bar{Y}-\mu_0}{\frac{\sigma}{\sqrt{n}}} \ge \frac{129.454-128}{\frac{2.1}{\sqrt{40}}}\right) = P(Z \ge 4.3784)\ \boxed{<0.0001}$

10.49A) $\bar{y} + Z_{0.025}\left(\frac{s}{\sqrt{n}}\right) = 128.6 + (1.96)\left(\frac{2.1}{\sqrt{40}}\right) = 128.6 + \frac{1029\sqrt{10}}{5000} \approx 129.25 < \mu = 130$

The value $\mu = 130$ is greater than the 95 percent upper confidence bound.

10.49B) Yes 10.49C) No 10.24) $H_0: p = 0.15,\ H_A: p < 0.15,\ \alpha = 0.05$ $n = 100$, $\hat{p} = 0.15$; $q_0 = 1 - p_0 = 0.85$

$Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.13-0.15}{\sqrt{\frac{(0.15)(0.85)}{100}}} \approx -0.5601$ $|-0.5601| < Z_{0.05} = 1.645$

We fail to reject H_0 at the $\alpha = 0.05$ level. There is insufficient evidence that the percentage reported by Children's Hospital is too high.

10.27) $H_0: P_R - P_{PS} = 0$ $H_A: P_R - P_{PS} \ne 0$, $\alpha = 0.01$ $n_R = 6124$, $n_{PS} = 5512$, $\hat{P}_R = 0.4$, $\hat{P}_{PS} = 0.37$

$\hat{P}_P = \frac{n_R\hat{P}_R + n_{PS}\hat{P}_{PS}}{n_R + n_{PS}} = \frac{(6124)(0.40) + (5512)(0.37)}{(6124) + (5512)} \approx 0.386$ (Pooled samples) $\hat{q}_P = 1 - \hat{P}_P \approx 0.614$

$Z = \frac{\hat{P}_R - \hat{P}_{PS} - 0}{\sqrt{\hat{P}_P\hat{q}_P\left(\frac{1}{n_R}+\frac{1}{n_{PS}}\right)}} = \frac{0.4-0.37}{\sqrt{(0.386)(0.614)\left(\frac{1}{6124}+\frac{1}{5512}\right)}} \approx 3.319$ $|3.319| > Z_{0.005} = 2.579$

We reject H_0 at the $\alpha = 0.01$ level. There is sufficient evidence that there is a difference between the two proportions.

10.43A) $H_0: \mu_S - \mu_{NS} = 0$, $H_A: \mu_S - \mu_{NS} > 0$, $\alpha = 0.05$ $\bar{y}_S = 32.19$ $s_S = 4.34$ $\bar{y}_{NS} = 31.68$, $s_{NS} = 4.56$, $n_S = n_{NS} = 37$

$Z = \frac{\bar{y}_S - \bar{y}_{NS} - 0}{\sqrt{\frac{s_S^2}{n_S} + \frac{s_{NS}^2}{n_{NS}}}} = \frac{32.19 - 31.68 - 0}{\sqrt{\frac{(4.34)^2}{37} + \frac{(4.56)^2}{37}}} \approx 0.4928$ $|0.4928| < Z_{0.05} = 1.645$

We fail to reject H_0 at the $\alpha = 0.05$ level. There is insufficient evidence that second graders who participate in sports have a higher mean dexterity score.

10.43B) $\bar{Y}_S - \bar{Y}_{NS} > 1.645\sqrt{\frac{(4.34)^2}{(37)} + \frac{(4.56)^2}{(37)}} \rightarrow \bar{Y}_S - \bar{Y}_{NS} > 1.702$

$\beta_{\mu_S-\mu_{NS}=3} = P(\bar{Y}_S - \bar{Y}_{NS} \le 1.702 \mid \mu_S - \mu_{NS} = 3) = P\left(Z \le \frac{1.702-3}{\sqrt{\frac{(4.34)^2}{(37)} + \frac{(4.56)^2}{(37)}}}\right) = P(Z \le -1.25) \approx \boxed{0.1056}$

10.44) $\alpha = \beta = 0.05 = P(Z \le Z_{0.05} \mid \mu_S - \mu_{NS} = 3) = P\left(\frac{\bar{Y}_S - \bar{Y}_{NS} - 0}{U} \le Z_{0.05} \mid \mu_S - \mu_{NS} = 3\right) =$ $\boxed{U = \sqrt{\frac{4.34^2}{n} + \frac{4.56^2}{n}}}$

$P(\bar{Y}_S - \bar{Y}_{NS} \le Z_{0.05}U \mid \mu_S - \mu_{NS} = 3) \rightarrow P\left(\frac{\bar{Y}_S - \bar{Y}_{NS} - 3}{U} \le \frac{Z_{0.05}U - 3}{U}\right) = P\left(Z \le Z_{0.05} - \frac{3}{U}\right) \rightarrow$

$Z_{0.05} - \frac{3}{U} = -Z_{0.05} \rightarrow \frac{3}{U} = 2Z_{0.05} \rightarrow U = \frac{3}{2Z_{0.05}} \rightarrow \sqrt{\frac{4.34^2}{n} + \frac{4.56^2}{n}} = \frac{3}{2Z_{0.05}} \rightarrow$

$\frac{9}{4(1.645)^2} = \frac{4.34^2 + 4.56^2}{n} \rightarrow n = \frac{4(1.645)^2(4.34^2 + 4.56^2)}{9} \approx 47.66 \rightarrow \boxed{n = 48}$