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Homework 2

8.6.

a. $E(\theta\hat{}_3) = aE(\theta\hat{}_1) + (1 - a)E(\theta\hat{}_2) =$
$E(\theta\hat{}_3) = a\theta + (1 - a)\theta = \boldsymbol{\theta}$

$\theta\hat{}_3 = a\theta\hat{}_1 + (1 - a)\theta\hat{}_2$, $E(\theta\hat{}_1) = E(\theta\hat{}_2) = \theta$
Unbiased estimator: $E(\theta\hat{}) = \theta$

b. $\text{Var}(\theta\hat{}_3) = a^2\text{Var}(\theta\hat{}_1) + (1 - a)^2\text{Var}(\theta\hat{}_2) =$
$\text{Var}(\theta\hat{}_3) = a^2\sigma_1^2 + (1 - a)^2\sigma_2^2$
$\text{d}(\text{Var}(\theta\hat{}_3))/\text{d}a = 2\sigma_1^2 a + 2\sigma_2^2 a - 2\sigma_2^2 = 0$
$\boldsymbol{a = \sigma_2^2 / (\sigma_1^2 + \sigma_2^2)}$

$\text{Var}(\theta\hat{}_1) = \sigma_1^2$, $\text{Var}(\theta\hat{}_2) = \sigma_2^2$
Set $\text{d}(\text{Var}(\theta\hat{}_3))/\text{d}a$ equal to 0 to find minimum

8.7.

$\text{Var}(\theta_3) =$
$a^2\text{Var}(\theta_1)+(1-a)^2\text{Var}(\theta_2)+2a(1-a)\text{Cov}(\theta_1,\theta_2) =$
$\text{Var}(\theta_3) = a^2\sigma_1^2 + (1 - a)^2\sigma_2^2 + 2a(1 - a)c$
$\text{d}(\text{Var}(\theta\hat{}_3))/\text{d}a=2\sigma_1^2a+2\sigma_2^2a-2\sigma_2^2-4ca+2c = 0$
$\boldsymbol{a = (\sigma_2^2 - c) / (\sigma_1^2 + \sigma_2^2 - 2c)}$

$\text{Cov}(\theta\hat{}_1, \theta\hat{}_2) = c \neq 0$
Set $\text{d}(\text{Var}(\theta\hat{}_3))/\text{d}a$ equal to 0 to find minimum

8.8.

a. $E(\theta\hat{}_1) = Y_1$
$E(\theta\hat{}_2) = (Y_1 + Y_2) / 2$
$E(\theta\hat{}_3) = (Y_1 + 2Y_2) / 3$
$E(\theta\hat{}_4) = \min(Y_1, Y_2, Y_3)$
$E(\theta\hat{}_5) = \bar{y} = (Y_1 + Y_2 + Y_3) / 3$

b.

8.17.

a. $\text{Bias}(\hat{p}_2) = E(\hat{p}_2) - \hat{p}_2 =$
$\text{Bias}(\hat{p}_2) = (np + 1) / (n + 2) - p =$
$\mathbf{Bias}(\hat{p}_2) \mathbf{= (1 - 2}p\mathbf{) / (}n\mathbf{ + 2)}$

$\text{Bias}(\theta\hat{}) = E(\theta\hat{}) - \theta$
$\hat{p}_2 = (Y+1)/(n+2)$
$E(Y) = np$, $E(\hat{p}_2) = (np + 1) / (n + 2)$
$\text{Var}(\hat{p}_2) = np(1 - p)/(n - 2)^2$

b. $\text{MSE}(\hat{p}_1) = \text{Var}(\hat{p}_1) + (\text{Bias}(\hat{p}_1))^2 =$
$\mathbf{MSE}(\hat{p}_1) \mathbf{=} p\mathbf{(1 -} p\mathbf{)/}n \mathbf{+ 0}$
$\text{MSE}(\hat{p}_2) = \text{Var}(\hat{p}_2) + (\text{Bias}(\hat{p}_2))^2 =$
$\text{MSE}(\hat{p}_2)=(np(1-p)/(n+2))^2+((1-2p)/(n+2))^2 =$
$\mathbf{MSE}(\hat{p}_2) \mathbf{= (}np\mathbf{(1-}p\mathbf{) + (1-2}p\mathbf{)^2)/(}n\mathbf{+2)^2}$

c. $\text{MSE}(\hat{p}_1) < \text{MSE}(\hat{p}_2)$
$p(1 - p) / n < (np(1 - p) + (1 - 2p)^2) / (n + 2)^2$
$(-8n - 4)p^2 + (8n + 4)p + n < 0$
$p = (-b \pm \sqrt{}(b^2 - 4ac)) / 2a =$

Quadratic equation

$$p = \frac{1}{2} \pm \frac{\sqrt{(3n + 1)}}{2\sqrt{(2n + 1)}}$$

$$\boldsymbol{p = \left(0, .5 - \frac{\sqrt{(3n+1)}}{2\sqrt{(2n+1)}}\right), \left(.5 + \frac{\sqrt{(3n+1)}}{2\sqrt{(2n+1)}}, 1\right)}$$

Verify using Desmos

8.21. $\bar{y} = 11.5$ hours

$s = 3.5$ hours, $z^* = 1.96$, $n = 50$

$s(z^*) / \sqrt{n} = (3.5)(1.96) / \sqrt{(50)} \approx 0.97$

$\bar{y} \pm s(z^*) / \sqrt{n} \approx$ **(10.53, 12.47)**

8.28. $\hat{p}_1 - \hat{p}_2 = 126/180 - 54/100 = .70 - .54 = .16$

$\hat{p}_1 = 126/180 = .7$, $\hat{p}_2 = 54/100 = .54$

$1\text{-}\hat{p}_1 = 54/180 = .3$, $1\text{-}\hat{p}_2 = 46/100 = .46$

$z^* = 1.96$, $n_1 = 180$, $n_2 = 100$

$(z^*)\sqrt{(\hat{p}_1(1 - \hat{p}_1) / n_1 + \hat{p}_2(1 - \hat{p}_2) / n_2)} =$

$(1.96)\sqrt{(.3(1\text{-}.3) / 180 + .46(1\text{-}.46) / 100)} =$

0.118424664

$\hat{p}_1 - \hat{p}_2 \pm 0.118424664 =$ **(.0416, .2784)**