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Midterm 1

1. $P(X < 5.4)$
   $z = (x - \mu) / (\sigma/\sqrt{n})$
   $z = (5.4 - 5) / (\sqrt{5}/\sqrt{80})$
   $z = 1.6 \rightarrow$
   0.9452

   $\mu = 5$, $\sigma = \sqrt{\text{Var}} = \sqrt{\mu} = \sqrt{5}$, $n = 80$, $x = 5.4$
   Using Central Limit Theorem and normal distribution because $n > 30$

2.
   a. $\bar{y} \pm (z^*)s / \sqrt{n} =$
      $24.05 \pm (1.96)(2.68)/\sqrt{(20)} =$
      (23.1384, 24.9616)

      $\bar{y} = 24.05$, $z^* = 1.96$ (95%), $s = 2.68$, $n = 20$

   b. $P((n\text{-}1)s^2/\chi_{\alpha/2,df}^2 < \sigma^2 < (n\text{-}1)s^2/\chi_{1-\alpha/2,df}^2) = 0.95$
      $((20\text{-}1)(2.68)^2/\chi_{1-0.025,19}^2, (20\text{-}1)(2.68)^2/\chi_{0.025,19}^2) \approx$
      $((19)(7.1824)/32.8523, (19)(7.1824)/8.9065) =$
      (4.1539, 15.3220)

      $s^2 = 2.68^2 = 7.1824$, $\alpha = 1 - 0.95 = 0.05$, $\alpha/2 = 0.05/2 = 0.025$, $df = n - 1 = 20 - 1 = 19$

3. $\bar{y}_1 - \bar{y}_2 \pm (z^*)$ *
   $\sqrt{\{[(n_1\text{-}1)s_1^2+(n_2\text{-}1)s_2^2][1/n_1+1/n_2] / [n_1+n_2\text{-}2]\}} =$
   $(13.8) - (12.9) \pm (1.75)$ *
   $\sqrt{\{[(12\text{-}1)(1.2)^2+(15\text{-}1)(1.5)^2][1/12+1/15]/[12+15\text{-}2]\}} =$
   (-0.0327, 1.8327)

   $\bar{y}_1 = 13.8$, $\bar{y}_2 = 12.9$, $s_1 = 1.2$, $s_2 = 1.5$, $n_1 = 12$, $n_2 = 15$, $z^* \approx 1.75$ (92%)
   Using z-statistic because normality is assumed

4. $p \pm (z^*) \sqrt{(p(1\text{-}p)/n)} =$
   $0.76 \pm (2.33)\sqrt{((0.76)(1\text{-}0.76)/250)} =$
   (0.6971, 0.8229)

   $p = 190/250 = 0.76$, $n = 250$, $z^* \approx 2.33$ (98%)

5.
   a. $U = 2 \sum_{i=1}^{n} Y_i/\theta =$
      $U = 2 \sum_{i=1}^{n} Y_i/(\sum_{i=1}^{n} Y_i/n) =$
      $U = 2n$

      By definition, $\theta = \sum_{i=1}^{n} Y_i/n$

      This is a pivotal quantity because it does not depend on θ.

   b. $P(U < c) = 0.90$

      $P(U < c) = 2c$

6.
   a. $E(\theta\text{^}_1) = 3\bar{y}/2$

      From previous exercise, it was shown that $\bar{y}$ is an unbiased estimator.

      $\text{MSE}(\theta\text{^}_1) = \text{E}[(\theta\text{^}_1 - \theta)^2] =$
      $\text{MSE}(\theta\text{^}_1) = 0$

      Because $\theta\text{^}_1$ is unbiased, $\text{MSE}(\theta\text{^}_1) = 0$ by definition.

   b. $E(\theta\text{^}_2) = E(Y(n)) =$

$E(\theta\hat{}_2) = \boldsymbol{\theta / n \neq \theta}$

$\text{MSE}(\theta\hat{}_2) = \text{E}[(\theta\hat{}_1 - \theta)^2] =$

$\text{MSE}(\theta\hat{}_2) = \text{E}[(\theta\hat{}_1 - \theta)^2] =$

From previous exercise

c. $\theta\hat{}_1$ is a better estimator for θ because it is unbiased.

d. $U = Y_{(n)}/\theta =$

$U = (\theta/n) / \theta =$

$U = 1 / n$

$f_{y(n)}(y) = nF(y)^{n-1}f(y) =$

$f_{y(n)}(y) = n(y^2/\theta^3)^{n-1}(2y/\theta^2) =$

$f_{y(n)}(y) = 2ny^{2n-1}/\theta^{3n} =$

This is a pivotal quantity because it does not depend on θ.

$F(y|\theta) = y^2/\theta^2/\theta = y^2/\theta^3$

e. $P(U < c) = 0.95$

$P(U < c) = 0.95 =$