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In-Class Assignment 3

$L(y_1, \ldots, y_n|\theta) = f(y_1, \ldots, y_n|\theta) = \Pi_{i=1}^{n} f(y_i|\theta) =$

$$\Pi_{i=1}^{n} \frac{1}{\sigma\sqrt{(2\pi)}} \exp\left(\frac{-(y_i - (\beta_0 + \beta_1 x_i))^2}{2\sigma^2} \right)$$

$$\ln\left[\Pi_{i=1}^{n} \frac{1}{\sigma\sqrt{(2\pi)}} \exp\left(\frac{-(y_i - (\beta_0 + \beta_1 x_i))^2}{2\sigma^2} \right)\right] =$$ Finding log-likelihood function

$$\Sigma_{i=1}^{n} \ln\left[\frac{1}{\sqrt{(2\pi\sigma^2)}} \exp\left(\frac{-(y_i - (\beta_0 + \beta_1 x_i))^2}{2\sigma^2} \right)\right] =$$ Substitute $\sigma = \sqrt{\sigma^2}$

$$\Sigma_{i=1}^{n}\ln[(\sigma^2)^{-1/2}]+\Sigma_{i=1}^{n}\ln[(2\pi)^{-1/2}]+\Sigma_{i=1}^{n}\ln\left[\exp\left(\frac{-(y_i - (\beta_0 + \beta_1 x_i))^2}{2\sigma^2} \right)\right] =$$

$$\frac{-1}{2} \Sigma_{i=1}^{n} \ln(\sigma^2) - \frac{1}{2} \Sigma_{i=1}^{n} \ln(2\pi) - \Sigma_{i=1}^{n} \frac{(y_i - (\beta_0 + \beta_1 x_i))^2}{2\sigma^2} =$$

$$\frac{-n}{2} \ln(\sigma^2) - \frac{n}{2} \ln(2\pi) - \frac{1}{2\sigma^2} \Sigma_{i=1}^{n} (y_i - (\beta_0 + \beta_1 x_i))^2$$

$$\frac{d(\ln(L)}{d(\beta_0)} = \frac{-1}{2\sigma^2} \Sigma_{i=1}^{n} 2(y_i - (\beta_0 + \beta_1 x_i))(-1) = 0 \rightarrow$$ $$\frac{d(\ln(L)}{d(\beta_0)} [-(\beta_0 + \beta_1 x_i)] = -1$$

$\Sigma_{i=1}^{n} y_i - (\beta_0+\beta_1 x_i) = 0 \rightarrow n\bar{y} - n\beta_0 - n\beta_1\bar{x} = 0 \rightarrow \bar{y} - \beta_0 - \beta_1\bar{x} = 0 \rightarrow$

$\boldsymbol{\beta_0 = \bar{y} - \beta_1\bar{x}}$

$$\frac{d(\ln(L)}{d(\beta_1)} = \frac{-1}{2\sigma^2} \Sigma_{i=1}^{n} 2(y_i - (\beta_0 + \beta_1 x_i))(-x_i) = 0 \rightarrow$$ Substitute $\beta_0 = \bar{y} - \beta_1\bar{x}$

$\Sigma_{i=1}^{n} (y_i - (\beta_0 + \beta_1 x_i))(x_i) = 0 \rightarrow \Sigma_{i=1}^{n} x_i y_i - \beta_0 x_i - \beta_1(x_i^2) = 0 \rightarrow$

$n(\bar{xy}) - n\beta_0\bar{x} - n\beta_1(\bar{x^2}) = 0 \rightarrow (\bar{xy}) - \beta_0\bar{x} - \beta_1(\bar{x^2}) = 0 \rightarrow$

$\beta_1(\bar{x^2}) = (\bar{xy}) - \beta_0\bar{x} \rightarrow \beta_1(\bar{x^2}) = (\bar{xy}) - (\bar{y} - \beta_1\bar{x})\bar{x} \rightarrow$

$\beta_1(\bar{x^2}) = (\bar{xy}) - \bar{x}\bar{y} + \beta_1(\bar{x})^2 \rightarrow \beta_1[(\bar{x^2}) - (\bar{x})^2] = (\bar{xy}) - \bar{x}\bar{y} \rightarrow$

$$\boldsymbol{\beta_1 = \frac{(\bar{xy}) - \bar{x}\bar{y}}{(\bar{x^2}) - (\bar{x})^2}}$$

$(\bar{x^2})$ = mean of x_i^2
$(\bar{x})^2$ = (mean of x_i)^2
$(\bar{xy})$ = mean of $x_i y_i$ = "*xy*-bar"
$\bar{x}\bar{y} = (\bar{x})(\bar{y})$ = (mean of x_i)(mean of y_i)

$$\frac{d(\ln(L)}{d(\sigma^2)} = \frac{1}{2(\sigma^2)^2} \Sigma_{i=1}^{n} (y_i - (\beta_0 + \beta_1 x_i))^2 - \frac{n}{2} \left(\frac{1}{\sigma^2} \right) = 0 \rightarrow$$

$$\frac{1}{\sigma^2} \Sigma_{i=1}^{n} (y_i - (\beta_0 + \beta_1 x_i))^2 - n = 0 \rightarrow$$

$$\frac{1}{\sigma^2} \Sigma_{i=1}^{n} (y_i - (\beta_0 + \beta_1 x_i))^2 = n \rightarrow$$

$$\boldsymbol{\sigma^2 = \frac{1}{n} \Sigma_{i=1}^{n} (y_i - (\beta_0 + \beta_1 x_i))^2}$$