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STAT 405-00[illegible]
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10.67A) $H_0: \mu = 280$, $H_A: \mu > 280$, $\alpha = 0.01$ $n=10$, $\bar{y}=358$, $s=54$
Because $n = 10 > 30$, we should use a t-test with $10-1=9$ df.
$t = \frac{\bar{y}-\mu}{\frac{s}{\sqrt{n}}} = \frac{358-280}{\frac{54}{\sqrt{10}}} = \frac{78\sqrt{10}}{54} = \frac{13\sqrt{10}}{9} \approx 4.5677$ $|4.5677| > 2.821 = t_{0.01,9}$
We ~~reject~~ H_0 at the $\alpha = 0.01$ level. There is $p = 0.00068$
sufficient evidence that the mean calories is greater than 280.

10.67B) $\bar{y} - t_{0.01,9}\left(\frac{s}{\sqrt{n}}\right) = 358 - (2.821)\left(\frac{(54)}{\sqrt{(10)}}\right) = 358 - \frac{152.358\sqrt{10}}{10} \approx \boxed{309.8 \text{ calories}}$

10.86) $H_0: \sigma^2 = 100$, $H_A: \sigma^2 > 100$, $\alpha = 0.01$ $n=20$, $\sigma=10$, $s=12$, $s^2=144$
We can conduct a χ^2-test for population variance with $20-1=19$ df.
$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(20-1)(144)}{(100)} = \frac{(19)(144)}{(100)} = 27.36$ $27.36 < 36.1908 = \chi^2_{0.01,19}$
We fail to reject H_0 at the $\alpha = 0.01$ level. There is $p = 0.09654$
insufficient evidence that the new test has higher variability.

10.70A) $H_0: \mu_J = \mu_N$, $H_A: \mu_J > \mu_N$, $\alpha = 0.05$ $n_J = 10$ $\bar{y}_J = 0.041$ $s_J = 0.017$ $n_N = 13$, $\bar{y}_N = 0.026$, $s_N = 0.006$
Because $n_J = 10 > 30$ and $n_N = 13 > 30$, we should use a t-test with $10+13-2 = 21$ df.
$t = \frac{\bar{y}_J - \bar{y}_N - 0}{\sqrt{\frac{s_J^2}{n_J} + \frac{s_N^2}{n_N}}} = \frac{0.041 - 0.026 - 0}{\sqrt{\frac{(0.017)^2}{10} + \frac{(0.006)^2}{13}}} \approx 2.665$ $|2.665| > 1.721 = t_{0.05,21}$
We reject H_0 at the $p = 0.00724$
$\alpha = 0.05$ level. There is sufficient evidence that the mean DDT is higher in juveniles than in nestlings.

10.70B) $H_0: \mu_J - \mu_N = 0.01$, $H_A: \mu_J - \mu_N > 0.01$, $\alpha = 0.05$
$t = \frac{\bar{y}_J - \bar{y}_N - 0.01}{\sqrt{\frac{s_J^2}{n_J} + \frac{s_N^2}{n_N}}} = \frac{0.041 - 0.026 - 0.01}{\sqrt{\frac{(0.017)^2}{10} + \frac{(0.006)^2}{13}}} \approx 0.888$ $|0.888| < 1.721 = t_{0.05,21}$
We fail to reject H_0 at $p = 0.19217$
the $\alpha = 0.05$ level. There is insufficient evidence that the mean DDT in juveniles is more than 0.01 greater than the mean DDT in nestlings.

10.87) $H_0: \sigma_J^2 = \sigma_N^2$, $H_A: \sigma_J^2 > \sigma_N^2$, $\alpha = 0.05$ $s_J^2 = 0.000289$, $s_N^2 = 0.000036$
$F = \frac{s_J^2}{s_N^2} = \frac{(0.000289)}{(0.000036)} = \frac{289}{36} = 8.027$ $8.028 > 2.796 = F_{0.05,9,12}$
We can conduct a one-tailed F-test for equality of two variances with $n_J - 1 = 10 - 1 = 9$ and $n_N - 1 = 13 - 1 = 12$ degrees of freedom. $\boxed{p = 0.00072}$
We reject H_0 at the $\alpha = 0.05$ level. There is sufficient evidence that the variance in DDT in juveniles is greater than the variance in DDT in nestlings.

10.91A) $N(\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}$, $L(y_1, \dots, y_n | \mu) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y_i-\mu}{\sigma}\right)^2} = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-\sum_{i=1}^{n}\frac{(y_i-\mu)^2}{2\sigma^2}}$

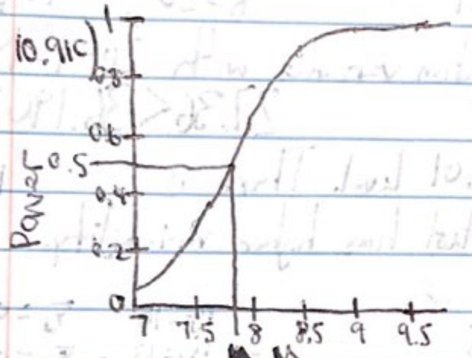
$\frac{L(\mu_0)}{L(\mu_A)} < k \rightarrow \bar{Y} > \frac{-2\sigma^2 \ln k - n\mu_0^2 + n\mu_A^2}{2n(\mu_A - \mu_0)} = k'$ From Example 10.23 in the textbook and slides 57-58, we can see that the one-sample z-test as described in Section 10.3 and Example 10.5, is the uniformly most powerful test

10.91B) $\bar{Y} > k' = \mu_0 + z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right) = (7) + (1.645)\left(\frac{\sqrt{5}}{\sqrt{20}}\right) = (7) + (1.645)\left(\frac{1}{2}\right) = 7 + 0.822 = 7.822$

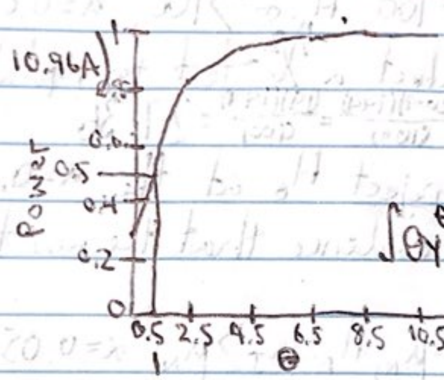
$Z = \frac{\bar{Y} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{7.822 - \mu}{\frac{\sqrt{5}}{\sqrt{20}}} = \frac{7.822 - \mu}{0.5} = \frac{7.822 - (7.5)}{0.5} \approx 0.645 \rightarrow \boxed{0.259}$ $\frac{7.822 - (8)}{0.5} \approx 0.355 \rightarrow \boxed{0.639}$

$\frac{7.822 - (8.5)}{0.5} \approx 1.355 \rightarrow \boxed{0.912}$ $\frac{7.822 - (9)}{0.5} \approx 2.355 \rightarrow \boxed{0.991}$

10.91C)



10.96A)



10.96B) $f(y|\theta) = L(y|\theta) = \theta y^{\theta-1}$

$\frac{L(\theta_0)}{L(\theta_A)} = \frac{L(1)}{L(\theta_A)} = \frac{(1)y^{(1)-1}}{\theta_A y^{\theta-1}} < k \rightarrow$

$\frac{y^0}{y^{\theta-1}} < \theta_A k \rightarrow$

$\int \theta y^{\theta-1} dy = y^\theta$ $\frac{1}{\theta_A k} < y^{\theta_A - 1} \rightarrow$

$y > \left(\frac{1}{\theta_A k}\right)^{\frac{1}{\theta-1}} = k'$

$P(Y < k') = 1 - \alpha$

$\rightarrow k'^{\theta} = 1 - \alpha \rightarrow k' = 1 - \alpha$

Because k' does not depend on θ_A, this is the uniformly most powerful test.

16.6) $f(y, p) = f(y|p)f(p) = \binom{n}{y} p^y (1-p)^{n-y} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1} = \binom{n}{y}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{y+\alpha-1}(1-p)^{n-y+\beta-1}$

$f(y) = \int f(y, p)\,dp = \int \binom{n}{y}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{y+\alpha-1}(1-p)^{n-y+\beta-1}\,dp = \binom{n}{y}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\frac{\Gamma(y+\alpha)\Gamma(n-y+\beta)}{\Gamma(n+\alpha+\beta)}$

$f(p|y) = \frac{f(y,p)}{f(y)} = \boxed{\frac{\Gamma(n+\alpha+\beta)}{\Gamma(y+\alpha)\Gamma(n-y+\beta)} p^{y+\alpha-1}(1-p)^{n-y+\beta-1}}$ Check slides 2 and 3

This is the same posterior distribution as the one in Example 16.1.

16.7A) $\hat{p}_B = \frac{\alpha^*}{\alpha^* + \beta^*} = \frac{(y+\alpha)}{(y+\alpha)+(n-y+\beta)} = \frac{y+(1)}{n+\alpha+\beta} = \frac{y+(1)}{n+(1)+(3)} = \boxed{\frac{y+1}{n+4}}$ $\alpha = 1$, $\beta = 3$, $\alpha^* = y + \alpha$, $\beta^* = n - y + \beta$

16.7B) $E(\hat{p}_B) = E\left(\frac{Y+1}{n+4}\right) = \frac{E(Y+1)}{n+4} = \frac{E(Y)+1}{n+4} = \boxed{\frac{np+1}{n+4}}$, $Var(\hat{p}_B) = Var\left(\frac{Y+1}{n+4}\right) = \frac{Var(Y+1)}{(n+4)^2} = \frac{Var(Y)+0}{(n+4)^2} = \frac{Var(Y)}{(n+4)^2} = \boxed{\frac{np(1-p)}{(n+4)^2}}$

16.8A) $\hat{p}_B = \frac{\alpha^*}{\alpha^* + \beta^*} = \frac{Y+(1)}{n+(1)+(1)} = \boxed{\frac{Y+1}{n+2}}$ 16.8B) $\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1} = \frac{\Gamma(1+1)}{\Gamma(1)\Gamma(1)} p^{(1)-1}(1-p)^{(1)-(1)} = \Gamma(2) = \boxed{\text{Uniform}(0,1)}$

16.8C) $MSE(\hat{p}_B) = Var(\hat{p}_B) + Bias(\hat{p}_B)^2 = \frac{np(1-p)}{(n+2)^2} + \left(E(\hat{p}_B) - p\right)^2 = \frac{np(1-p)}{(n+2)^2} + \left(\frac{np+1}{n+2} - p\right)^2 = \boxed{\frac{np(1-p)+(1-2p)^2}{(n+2)^2}}$

16.8D) $\frac{np(1-p)+(1-2p)^2}{(n+2)^2} < \frac{p(1-p)}{n} \rightarrow p = \frac{1}{2} \pm \frac{\sqrt{n+1}}{2\sqrt{2n+1}} \rightarrow p = \left(\frac{1}{2} - \frac{\sqrt{n+1}}{2\sqrt{2n+1}}, \frac{1}{2} + \frac{\sqrt{n+1}}{2\sqrt{2n+1}}\right)$ See Problem 8.17B

See Problem 8.17C

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STAT 405-001

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10.99 A) $P_{oisson}(\lambda) = \frac{e^{-\lambda}\lambda^y}{y!}$

$L(Y_1, \ldots Y_n \mid \lambda) = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{y_i}}{y_i!} = \frac{e^{-n\lambda}\lambda^{\sum_{i=1}^{n} y_i}}{\prod_i (y_i)!}$

$\frac{L(\lambda_0)}{L(\lambda_a)} < k \rightarrow \frac{e^{-n\lambda_0}\lambda_0^{\sum Y_i}}{e^{-n\lambda_a}\lambda_a^{\sum Y_i}} < k \rightarrow$

$e^{-n(\lambda_0-\lambda_a)}\left(\frac{\lambda_0}{\lambda_a}\right)^{\sum_{i=1}^{n} Y_i} < k \rightarrow$

B) $P_{oisson}(k) = \frac{e^{-\lambda}\lambda^k}{k!}$

$\left(\frac{\lambda_0}{\lambda_a}\right)^{\sum_{i=1}^{n} Y_i} < k\, e^{n(\lambda_0-\lambda_a)} \rightarrow$

$\ln\left(\frac{\lambda_0}{\lambda_a}\right)^{\sum_{i=1}^{n} Y_i} < \ln\left(k\, e^{n(\lambda_0-\lambda_a)}\right) \rightarrow$

$\sum_{i=1}^{n} Y_i \ln\left(\frac{\lambda_0}{\lambda_a}\right) < \ln k + n(\lambda_0 - \lambda_a) \leftrightarrow$

$\boxed{\sum_{i=1}^{n} Y_i > \frac{\ln k + n(\lambda_0-\lambda_a)}{\ln\lambda_0 - \ln\lambda_a}}$ $\lambda_a > \lambda_0 \rightarrow \ln\left(\frac{\lambda_0}{\lambda_a}\right) < 0$

$\sum_{i=1}^{n} Y_i > k' \rightarrow n\bar{Y} > k' \rightarrow \underline{n > \frac{k'}{\bar{Y}}}$

$\alpha = P\left(\sum_{i=1}^{n} Y_i > k' \mid H_0 \text{ is true}\right) \rightarrow$

$\alpha = 1 - P\left(\sum_{i=1}^{n} Y_i \leq k'\right) \rightarrow$

$\alpha = 1 - \sum_{y=0}^{k'} \frac{e^{-n\lambda}(n\lambda)^y}{y!} \rightarrow \beta = 1 - \alpha = \sum_{y=0}^{k'} \frac{e^{-n\lambda}(n\lambda)^y}{y!}$

c) Yes, because of Theorem 10.1 and the fact that the same rejection region would have been obtained for any λ_0 and $\lambda_a \ni \lambda_a > \lambda_0$.

D) We can see from Problem 10.99A that all steps except the last would be the same, and the resulting rejection region is:

$\sum_{i=1}^{n} Y_i \ln\left(\frac{\lambda_0}{\lambda_a}\right) < \ln k + n(\lambda_0 - \lambda_a) \rightarrow$

$\boxed{\sum_{i=1}^{n} < \frac{\ln k + n(\lambda_0-\lambda_a)}{\ln\lambda_0 - \ln\lambda_a}}$, $\lambda_a < \lambda_0 \rightarrow \ln\left(\frac{\lambda_0}{\lambda_a}\right) > 0$

16.11A) $f(v, \lambda) = \left(\frac{(n\lambda)^v e^{-n\lambda}}{v!}\right)\left(\frac{\lambda^{\alpha-1} e^{-\frac{\lambda}{\beta}}}{\Gamma(\alpha)\beta^\alpha}\right) = \frac{n^v \lambda^{v+\alpha-1}}{v!\beta^\alpha\Gamma(\alpha)} e^{-n\lambda-\frac{\lambda}{\beta}} = \boxed{\frac{n^v\lambda^{v+\alpha-1}}{v!\beta^\alpha\Gamma(\alpha)}\exp\left(\frac{-\lambda}{\left(\frac{\beta}{n\beta+1}\right)}\right)}$

16.11B) $m(v) = \int \frac{n^v}{v!}\left(\frac{\lambda^{v+\alpha-1}}{\Gamma(\alpha)\beta^\alpha}\exp\left(\frac{-\lambda}{\left(\frac{\beta}{n\beta+1}\right)}\right)\right) d\lambda = \boxed{\frac{n^v}{v!\Gamma(\alpha)\beta^\alpha}\left(\Gamma(v+\alpha)\left(\frac{\beta}{n\beta+1}\right)^{v+\alpha}\right)}$

16.11C) $Gamma(\alpha^*, \beta^*)$, $\boxed{\alpha^* = v+\alpha,\ \beta^* = \frac{\beta}{n\beta+1}}$

16.11D) $\hat{\lambda}_B = \alpha^*\beta^* = \boxed{(v+\alpha)\left(\frac{\beta}{n\beta+1}\right)}$

16.11E) $E(\hat{\lambda}_B) = E\left((v+\alpha)\left(\frac{\beta}{n\beta+1}\right)\right) = E(n\bar{Y}+\alpha)\left(\frac{\beta}{n\beta+1}\right) = \boxed{\bar{Y}\left(\frac{n\beta}{n\beta+1}\right) + \alpha\beta\left(\frac{1}{n\beta+1}\right)}$

16.11F) $Bias(\hat{\lambda}_B) = E(\hat{\lambda}_B) - \lambda = \frac{\lambda n\beta}{n\beta+1} + \frac{\alpha\beta}{n\beta+1} - \frac{\lambda(n\beta+1)}{n\beta+1} = \frac{\lambda n\beta + \alpha\beta - \lambda n\beta - \lambda}{n\beta+1} = \frac{\alpha\beta-\lambda}{n\beta+1} \neq 0$

$Var(\hat{\lambda}_B) = Var\left((v+\alpha)\left(\frac{\beta}{n\beta+1}\right)\right) = Var(v+\alpha)\left(\frac{\beta}{n\beta+1}\right)^2 = Var(n\bar{Y})\left(\frac{\beta}{n\beta+1}\right)^2 = \frac{\lambda}{n}\left(\frac{n\beta}{n\beta+1}\right)^2 = \frac{\lambda n\beta^2}{(n\beta+1)^2}$

We can see that $\lim_{n\to\infty} \frac{\lambda n\beta^2}{(n\beta+1)^2} = 0$ and that $\hat{\lambda}_B$ is consistent.

16.19) $\alpha^* = \sum_{i=1}^{n} Y_i + \alpha = (174) + (2) = 176$, $\beta^* = \frac{\beta}{n\beta+1} = \frac{(3)}{(25)(3)+1} = \frac{3}{75+1} = \frac{3}{76} \approx 0.0395$, $n = 25$, $\sum_{i=1}^{n} = 174$

$(\Gamma_{0.025, 176, 0.0395}, \Gamma_{0.975, 176, 0.0395}) \approx \boxed{(5.9589, 8.4107)}$ $\alpha = 2$, $\beta = 3$

16.25) $H_0: \lambda > 6$, $H_A: \lambda \leq 6$, $P^*(\lambda > 6) = Gamma(6, 176, 0.0395) \approx 0.97$, $P^*(\lambda \leq 6) = 1 - 0.97 = 0.03$

We fail to reject H_0. There is no evidence that $\lambda \leq 6$. $P^*(\lambda > 6) \not< P^*(\lambda \leq 6)$