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Actions

Worksheet 2 Problem 2

2. Show that if $X \sim Geom(p)$ then $P(X = n + k | X > n) = P(X = k)$ for $n, k \geq 1$.

Using Bayes' Formula, we write that $P(X = n + k | X > n) = \frac{P(X=n+k \bigcap X>n)}{P(X>n)}$. Observe that the event $X = n + k$ is contained inside the event $X > n$. Therefore, $P(X = n + k \bigcap X > n) = P(X = n + k)$, and thus $\frac{P(X=n+k \bigcap X>n)}{P(X>n)} = \frac{P(X=n+k)}{P(X>n)}$. By the definition of the geometric distribution, $P(X = n + k) = (1 - p)^{(n+k)-1}p$, or more easily written for reducing later, $(1 - p)^n(1 - p)^{k-1}p$.

In the denominator, the probability $P(X > n)$ (which is the same as $P(X \geq n + 1)$) can be rewritten as $(1 - p)^n p + (1 - p)^{n+1}p + \cdots$, or $\sum_{i=n+1}^{\infty}(1 - p)^{i-1}p = p\sum_{i=n+1}^{\infty}(1 - p)^{i-1}$. We can change the index of the summation by saying that some new index $i' = i - (n + 1)$ (and, inside the summation, $i = i' + (n + 1)$), making the summation $p\sum_{i'=0}^{\infty}(1 - p)^{(i'+n+1)-1}$. The purpose of this is to change the index to 0 while still preserving the value of the summation. Then, the exponent can be rewritten: $p\sum_{i'=0}^{\infty}(1 - p)^{i'+n} = p\sum_{i'=0}^{\infty}(1 - p)^{i'}(1 - p)^n$. Additionally, the term $(1 - p)^n$, unrelated to i', can be brought outside of the summation: $p(1 - p)^n \sum_{i'=0}^{\infty}(1 - p)^{i'}$. Because $0 \leq p \leq 1$ (and thus $0 \leq 1 - p \leq 1$), it turns out that $\sum_{i'=0}^{\infty}(1 - p)^{i'}$ is the geometric series that sums to $\frac{1}{1-(1-p)} = \frac{1}{p}$. Therefore, $p(1 - p)^n \sum_{i'=0}^{\infty}(1 - p)^{i'} = p(1 - p)^n \frac{1}{p} = (1 - p)^n$. We now know that $P(X > n) = (1 - p)^n$.

Returning to the original fraction, $\frac{(1-p)^n(1-p)^{k-1}p}{(1-p)^n} = (1 - p)^{k-1}p$, which, by the definition of the geometric distribution, is $P(x = k)$.

hw2

Edit good note | 0

Updated 5 years ago by Charles Hwang

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