Topics for Final Exam

Math 212 Fall Semester 2017

The final exam will contain *four* sections: (I) **Definitions**, (II) **T/F**, (III) **Multiple Choice**, and (IV) **Problems**. The following topics might appear on the final exam.

- **Definitions:**

  1. *linear equation*
  2. *particular solution to a linear equation*
  3. *general solution to a linear equation*
  4. *$m \times n$ linear system of equations*
  5. *particular solution a linear system of equations*
  6. *general solution to a linear system of equations*
  7. *consistent linear system of equations*
  8. *augmented matrix of a linear system of equations*
  9. *three types of row operations*
  10. *row echelon form*
  11. *pivot entry*
  12. *pivot positions*
  13. *pivot columns*
  14. *Fundamental Theorem of Linear Systems*
  15. *reduced row echelon form*
  16. *free variable*
  17. *matrix equality*
  18. *matrix addition/subtraction*
  19. *matrix-scalar multiplication*
  20. *matrix-matrix multiplication*
  21. *indentity matrix I_n*
  22. *matrix transpose*
  23. *matrix trace*
  24. *matrix inverse*
  25. *nonsigular matrix*
  26. *singular matrix*
  27. *matrix power A^n*
  28. a *linear transformation* $T : \mathbb{R}^n \to \mathbb{R}^m$
  29. the two *linearity properties* of a linear transformation
  30. *linear combination* of vectors u_1, u_2, ..., u_k in $\mathbb{R}^n$
  31. *null space* of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$
  32. *unit vector* e_j in $\mathbb{R}^n$
  33. *standard matrix* A of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$
  34. *determinant* of a square matrix A in terms of ref and factors
  35. *minor* of an entry a_{ij} in a matrix A
  36. *cofactor* of an entry a_{ij} in a matrix A
  37. *adjoint* of a matrix A

38. *closure* of $\oplus$ and $\odot$ in a vector space
39. *commutativity* of $\oplus$
40. *associativity* of $\oplus$
41. *distributivity* of $\odot$ across $\oplus$
42. the vector spaces $\mathbb{R}^n$, $\mathcal{M}_{mxn}$, $\mathbb{P}$, $\mathbb{P}_n$, $\mathbb{R}^+$ (with $x \oplus y = xy$ and $\alpha \oplus x = x^\alpha$), $\mathbb{R}$ (with $x \oplus y = x - y - b$ and $\alpha \oplus x = \alpha(x - b) + b$)
43. a *subspace* of an abstract vector space
44. the *span*$\{v_1, v_2, ..., v_k\}$
45. a nontrivial *linear combination* of the vectors $\{v_1, v_2, ..., v_k\}$
46. *linearly dependent* vectors $\{v_1, v_2, ..., v_k\}$
47. *linearly independent* vectors $\{v_1, v_2, ..., v_k\}$
48. *basis* of a vector space V
49. *finite-dimensional* vector space V
50. $dim(V)$
51. *standard basis* for $\mathbb{R}^n$
52. *coordinates* of a vector $(v)_B$ with respect to a basis B
53. *transition matrix* $P_{B \to B'}$ where B and B' are bases of the same vector space
54. *eigenvalue* λ of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$
55. *eigenvector* v of an eigenvalue λ
56. *characteristic polynomial* $p(\lambda) = \det(\lambda I - A)$
57. *eigenspace* of an eigenvalue λ
58. $Row(A)$, $Col(A)$, $Null(A)$
59. *nullity* of A
60. *rank* of A
61. *one-to-one* function
62. *onto* function
63. *injection*, *surjection*, and *bijection*
64. *identity* linear transformation

- **Problems:** Be able to do the following types of problems.

1. Form the *augmented matrix* of a linear system of equation, put it into *row echelon form*, and find the *general solution* of the system.
2. Put a matrix in row echelon form into *reduced row echelon form.*
3. Perform varied types of computations with matrices: add, subtract, scalar-multiply, multiply, find transposes, find traces, and find powers of matrices.
4. Prove simple facts about matrices, their inverses, transposes, powers, traces, etc.
5. Find the inverse of a matrix by solving a system of equations.
6. Know the *seven equivalent statements* (so far) that can be made about a matrix A: A is invertible $\Leftrightarrow$ $Ax = 0$ has only the trivial solution $\Leftrightarrow$ the reduced row echelon form of A is the identity I_n $\Leftrightarrow$ A is the product of elementary matrices $\Leftrightarrow$ $Ax = b$ is consistent for any choice of rhs b $\Leftrightarrow$ $Ax = b$ has a unique solution for every choice of rhs b $\Leftrightarrow$ $\det(A) \neq 0$.
7. Know how to *prove* that a function $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation from the definition of a linear transformation.
8. Know the *two commuting diagrams* presented in class for a linear transformation.

9. Know how to *disprove* that a function $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation.
10. Know that $T(0_n) = 0_m$ where 0_n and 0_m are the zero column vectors in $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively.
11. Know that $T(\alpha u + \beta v) = \alpha T(u) + \beta T(v)$ where $u, v \in \mathbb{R}^n$ and $\alpha, \beta \in \mathbb{R}$. In general, a linear transformation of a linear combination of vectors in $\mathbb{R}^n$ is a linear combination of the functional values of those vectors in $\mathbb{R}^m$.
12. Know how to obtain the *standard matrix* of a linear transformation.
13. Know that the standard matrix of a linear transformation is *unique.*
14. Know how to prove that a vector is in the *range* of a linear transformation by setting up a linear system and proving that it is consistent.
15. Know how to find the *null space* of a linear transformation as the general solution to a linear system of the form $Ax = 0$ where A is the standard matrix of the linear transformation.
16. Know how to show that a linear transformation is *one-to-one*, *onto*, or *both* using its standard matrix.
17. Know what the *factors* are for each type of row operation.
18. Know how to find the *determinant* of a matrix A by using only row operations.
19. Know how to compute a 2×2 determinant.
20. Know how to compute a 3×3 determinant using *Sarrus's Rule.*
21. Know what the *determinants of the elementary matrices* are.
22. Know that a matrix A is invertible $\Leftrightarrow \det(A) \neq 0$.
23. Know that $\det(AB) = \det(A)\det(B)$.
24. Know that $\det(A^{-1}) = 1/\det(A)$.
25. Know that $\det(A^T) = \det(A)$.
26. Know how to compute the determinant of a matrix by *expansion by cofactors.*
27. Know how to compute the *adjoint*, $Adj(A)$, of a matrix A.
28. Know that if A is invertible, then $A^{-1} = 1/\det(A) \cdot Adj(A)$.
29. Know how verify that a given set V equipped with operations $\oplus$ and $\odot$ satisfies the vector space axioms.
30. Know how to do computations in and generally be able to work with the vector spaces given in class: $\mathbb{R}^n$, $\mathcal{M}_{m\times n}$, $\mathbb{P}$, $\mathbb{P}_n$, $\mathbb{R}^+$ (with $x \oplus y = xy$ and $\alpha \oplus x = x^\alpha$), $\mathbb{R}$ (with $x \oplus y = x - y - b$ and $\alpha \oplus x = \alpha(x - b) + b$).
31. Know how do a simple proof using only the vector space axioms.
32. Know how to show that a given subset S of V is a subspace (or not) of a vector space V by checking closure of addition and scalar multiplication.
33. Know how to show that a given vector is in the span of vectors $\{v_1, v_2, ..., v_k\}$.
34. Know how to show that a set of vectors $\{v_1, v_2, ..., v_k\}$ is linearly independent.
35. Know how to show that a set of vectors $\{v_1, v_2, ..., v_k\}$ is linearly dependent.
36. Know an example of an infinite-dimensional vector space.
37. Know examples of bases for $\mathcal{M}_{m\times n}$, $\mathbb{P}_n$, and $\mathbb{P}$.
38. Know how to find the coordinates of a vector with respect to a given basis in $\mathbb{R}^n$, $\mathcal{M}_{m\times n}$, $\mathbb{P}_n$, and $\mathbb{P}$.
39. Know how to change the coordinates of a vector from one basis to another by solving equations.
40. Know how to change the coordinates of a vector from one basis to another by using transition matrices.
41. Know how to find the transition matirx $P_{B\to B'}$, and find the transition matrix $P_{B'\to B}$ by the inverse method.

42. Know how to find all the eigenvalues of a linear transformation $T = T_A$ where A is the standard matrix of T.
43. Know how to find a basis for the eigenspace E_λ of an eigenvalue λ.
44. Know how to diagonalize a matrix A by finding matrices P and P^{-1} such that $P^{-1}AP = D$ where D is a diagonal matrix.
45. Know how to solve a system of first-order, linear differential equations by diagonalization.
46. Know how to find bases for $Row(A)$, $Col(A)$, and $Null(A)$, and $Null(A^T)$.
47. Know what the *Dimension Theorem* is.
48. Know how to find the *standard matrix of the inverse of a one-to-one linear transformation.*