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STAT 307-001

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Take-Home Exam – Exercise 1

a. $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$

$n = 8, v_1 = 4, v_2 = 35, \varepsilon = .1, \varepsilon/2 = .05, 1 - \varepsilon/2 = 1 - .05 = .95$

y_{ij} — predicted weight μ — mean α_i — treatment effect ε_{ij} — error term

Source	DF	SS	MS	F	P-Value
Group	4	5581.15	1395.2875	3.01551	.030807
Error	35	16194.625	462.7036		
Total	39	21775.775			

$\sigma^2 + n\sigma_\alpha^2 = (462.7036) + (8)(116.573) = (462.7036) + 932.584 = \mathbf{1395.2876}$
$\sigma^2 = \mathbf{462.7036}$

b.

$$\sigma_\alpha^2 = \frac{MS_A - MS_E}{n} = \frac{1395.2875 - 462.7036}{8} = \frac{932.584}{8} = \mathbf{116.573}; \quad \sigma^2 = MS_E = \mathbf{462.704}$$

c.

$H_0: \mu_A = \mu_B = \mu_C = \mu_D = \mu_E$ $\alpha = .05$
H_A: At least one μ_i is different
$v_1 = 4$ $v_2 = 35$ $F = 3.01551$ $p = .030807$

We reject H_0 at $\alpha = .05$. There is sufficient evidence that there exists sire-to-sire variability.

d.

$$\frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma^2} = \frac{116.573}{116.573 + 462.7036} = \frac{116.573}{579.2766} = \mathbf{0.20123892454}$$

$$L = \frac{1}{n}\left(\frac{MS_{Trt}/MS_E}{F_{0.05,4,35}} - 1\right) = \frac{1}{8}\left(\frac{1395.2875 - 462.7036}{2.64146519} - 1\right) = 44.0069415987$$

$$U = \frac{1}{n}\left(\frac{MS_{Trt}/MS_E}{F_{0.95,4,35}} - 1\right) = \frac{1}{8}\left(\frac{1395.2875 - 462.7036}{0.17453772} - 1\right) = 667.895670346$$

$$\left(\frac{L}{L+1}, \frac{U}{U+1}\right) = \left(\frac{44.00694}{44.00694 + 1}, \frac{667.89567}{667.89567 + 1}\right) = \mathbf{(0.9777812, 0.9985045)}$$

We are 90 percent confident that between 97.7781205 and 99.8504999 percent of the variance of a given observation is the result of differences between treatments.

Take-Home Exam – Exercise 2

a.

x = log(dose + 1) — Transformation of Data

0.0	0.3	1.0	3.0	10.0
2.48490665	3.688879454	4.48863637	5.407171771	6.333279628
2.708050201	3.784189634	4.532599493	5.529429088	6.405228458
2.772588722	3.850147602	4.65396035	5.560681631	6.536691598
2.890371758	3.931825633	4.736198448	5.648974238	6.555356892
2.944438979	3.970291914	4.787491743	5.703782475	6.56667243
3.091042453	4.127134385	4.795790546	5.746203191	6.606650186
3.258096538	4.219507705	4.875197323	5.823045895	6.668228248

Source	DF	SS	MS	F	P-Value
Group	4	56.6155	14.1539	459.4223	< .00001
Error	30	0.9242	0.0308		
Total	34	57.5397			

A	B	C	D	E
2.878499329	3.938853761	4.695696325	5.631326898	6.524586777

The means of all five dosages are different.

b.

H_0: $\mu_A = \mu_B = \mu_C = \mu_D = \mu_E$ $\alpha = .05$

H_A: At least one μ_i is different

Take-Home Exam – Exercise 3

a. There will be at minimum 30 visitors required for the study.
b. There will be at minimum three replications per campsite.
c.
d.
e.

Take-Home Exam – Exercise 4

a.

$$\bar{y}_{N\cdot\cdot} = \frac{\bar{y}_{NB\cdot} + \bar{y}_{NM\cdot} + \bar{y}_{NT\cdot}}{n_{N\cdot\cdot}} = \frac{40.333 + 68.667 + 53}{3} = \frac{162}{3} = 54$$

$$\bar{y}_{F\cdot\cdot} = \frac{\bar{y}_{FB\cdot} + \bar{y}_{FM\cdot} + \bar{y}_{FT\cdot}}{n_{F\cdot\cdot}} = \frac{41 + 35 + 39}{3} = \frac{115}{3} = 38.333$$

$$\bar{y}_{\cdot B\cdot} = \frac{\bar{y}_{NB\cdot} + \bar{y}_{FB\cdot}}{n_{\cdot B\cdot}} = \frac{40.333 + 41}{2} = \frac{81.333}{2} = 40.667$$

$$\bar{y}_{\cdot M\cdot} = \frac{\bar{y}_{NM\cdot} + \bar{y}_{FM\cdot}}{n_{\cdot M\cdot}} = \frac{68.667 + 35}{2} = \frac{103.667}{2} = 51.833$$

$$\bar{y}_{\cdot T\cdot} = \frac{\bar{y}_{NT\cdot} + \bar{y}_{FT\cdot}}{n_{\cdot T\cdot}} = \frac{53 + 39}{2} = \frac{92}{2} = 46$$

$$\bar{y}_{\cdot\cdot\cdot} = \frac{\bar{y}_{N\cdot\cdot} + \bar{y}_{F\cdot\cdot}}{2} = \frac{54 + 38.333}{2} = \frac{92}{2} = 46.167$$

$\alpha_N = \bar{y}_{N\cdot\cdot} - \bar{y}_{\cdot\cdot\cdot} = 54 - 46.167 = 7.833$

$\alpha_F = \bar{y}_{F\cdot\cdot} - \bar{y}_{\cdot\cdot\cdot} = 38.333 - 46.167 = -7.833$

$\beta_B = \bar{y}_{\cdot B\cdot} - \bar{y}_{\cdot\cdot\cdot} = 40.667 - 46.167 = -5.5$

$\beta_M = \bar{y}_{\cdot M\cdot} - \bar{y}_{\cdot\cdot\cdot} = 51.833 - 46.167 = 5.667$

$\beta_T = \bar{y}_{\cdot T\cdot} - \bar{y}_{\cdot\cdot\cdot} = 46 - 46.167 = -0.167$

$\alpha\beta_{NB} = \bar{y}_{NB\cdot} - \bar{y}_{N\cdot\cdot} - \bar{y}_{\cdot B\cdot} + \bar{y}_{\cdot\cdot\cdot} = 40.333 - 54 - 40.667 + 46.167 = -8.167$

$\alpha\beta_{NM} = \bar{y}_{NM\cdot} - \bar{y}_{N\cdot\cdot} - \bar{y}_{\cdot M\cdot} + \bar{y}_{\cdot\cdot\cdot} = 68.667 - 54 - 51.833 + 46.167 = 9$

$\alpha\beta_{NT} = \bar{y}_{NT\cdot} - \bar{y}_{N\cdot\cdot} - \bar{y}_{\cdot T\cdot} + \bar{y}_{\cdot\cdot\cdot} = 53 - 54 - 46 + 46.167 = -0.833$

$\alpha\beta_{FB} = \bar{y}_{FB\cdot} - \bar{y}_{F\cdot\cdot} - \bar{y}_{\cdot B\cdot} + \bar{y}_{\cdot\cdot\cdot} = 41 - 38.333 - 40.667 + 46.167 = 8.167$

$\alpha\beta_{FM} = \bar{y}_{FM\cdot} - \bar{y}_{F\cdot\cdot} - \bar{y}_{\cdot M\cdot} + \bar{y}_{\cdot\cdot\cdot} = 35 - 38.333 - 51.833 + 46.167 = -9$

$\alpha\beta_{FT} = \bar{y}_{FT\cdot} - \bar{y}_{F\cdot\cdot} - \bar{y}_{\cdot T\cdot} + \bar{y}_{\cdot\cdot\cdot} = 39 - 38.333 - 46 + 46.167 = 0.833$

$a = 2$, $s = 3$, $n = 18$, $a-1 = 2-1 = 1$, $s-1 = 3-1 = 2$, $(a-1)(s-1) = (2)(3) = 6$, $as(n-1) = (2)(3)(17) = 102$

$SS_A = sn\Sigma_{i=1}^{2}(\alpha_i)^2 = (3)(18)((7.833)^2 + (-7.833)^2) = (108)(7.833)^2 = 3(2209) = 6627$

$SS_S = an\Sigma_{i=1}^{3}(\beta_j)^2 = (2)(18)((-5.5)^2 + (5.667)^2 + (-0.167)^2) = (36)(62.3889) = 2246$

$SS_{AS} = n\Sigma_{i=1,j=1}^{2,3}(\alpha\beta_{ij})^2 = (18)(2)((8.167)^2 + (9)^2 + (0.833)^2) = (36)(148.3889) = 5342$

Source	DF	SS	MS	F	P-Value
A	1	6627	6627	17.9431	.02143
S	2	2246	1123	3.04061	.18988
AS	2	5342	2671	**7.23195**	**.07119**
Error	102	369.33	3.6209		
Total	107				

b.

H_0: AS is not significant — $\alpha = .05$

H_A: AS is significant

$a = 2$ — $s = 3$ — $F = 7.23195$ — $p = .07119$

We fail to reject H_0 at $\alpha = .05$. There is insufficient evidence that the interaction term is significant.

c.

d.