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STAT 307-001

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Homework 7 – Exercise 11.1

Source	DF	SS	MS
Cars	3	15	5
Error	8	16	2

$n = 3; v_1 = 3; v_2 = 8;$
$\varepsilon = .05; \varepsilon/2 = .025,$
$1 - \varepsilon/2 = 1 - .025 = .975$

$$\sigma_\alpha^2 = \frac{MS_A - MS_E}{n} = \frac{5-2}{3} = \frac{3}{3} = \mathbf{1}; \quad \sigma^2 = MS_E = \mathbf{2}; \quad \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma^2} = \frac{1}{1+2} = \mathbf{\frac{1}{3}}$$

$$L = \frac{1}{n}\left(\frac{MS_{Trt}/MS_E}{F_{0.025,3,8}} - 1\right) = \frac{1}{3}\left(\frac{5/2}{5.41496234} - 1\right) = \frac{1}{3}(0.461683728) = 0.153894576$$

$$U = \frac{1}{n}\left(\frac{MS_{Trt}/MS_E}{F_{0.975,3,8}} - 1\right) = \frac{1}{3}\left(\frac{5/2}{0.06877633} - 1\right) = \frac{1}{3}(36.34971508) = 12.11657169$$

$$\left(\frac{L}{L+1}, \frac{U}{U+1}\right) = \left(\frac{0.1538946}{0.1538946+1}, \frac{12.1165717}{12.1165717+1}\right) = \mathbf{(0.1333697, 0.9237606)}$$

$$\sigma_\alpha^2 = \mathbf{1}; \quad \sigma^2 = \mathbf{2}; \quad \mathbf{0.13336970} \le \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma^2} \le \mathbf{0.92376056} \text{ (CI)}; \quad \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma^2} = \mathbf{\frac{1}{3}} \text{ (PE)}$$

Homework 7 – Exercise 11.4

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ $\quad \alpha = .05$
H_A: At least one μ_i is different

$a = 5, N = 16$

$$n' = \frac{1}{a-1}\left[N - \frac{1}{N}\Sigma_{i=1}^{a} n_i^2\right] =$$

$$n' = \frac{1}{5-1}\left[16 - \frac{1}{16}\Sigma_{i=1}^{5} n_i^2\right] =$$

$$n' = \frac{1}{4}\left[16 - \frac{1}{16}(4^2 + 2^2 + 5^2 + 3^2 + 2^2)\right] =$$

$$n' = 4 - \frac{1}{64}(16 + 4 + 25 + 9 + 4) =$$

$$n' = 4 - \frac{(58)}{64} =$$

$$n' = \frac{99}{32} = 3.09375$$

Source	DF	SS	MS	F	P-Value
Group	4	0.0553	0.0138	6.4674	0.0063
Error	11	0.0235	0.0021		
Total	15	0.0788			

We reject H_0 at $\alpha = .05$. There is sufficient evidence that at least one μ_i is different and that there exists head to head variability.

$$\sigma_\alpha^2 = \frac{MS_{Trt} - MS_E}{n'} = \frac{0.0138 - 0.0021}{3.09375} = \frac{0.0117}{3.09375} = \mathbf{0.00378182}; \quad \sigma^2 = MS_E = \mathbf{0.0021};$$

$$\frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma^2} = \frac{0.00378181818}{0.0038 + 0.0021} = \frac{0.003782}{0.005882} = \mathbf{0.64296754239} \text{ (ICC PE)}$$

Approximately 64.296754239 percent of the variance in the observations is a result of differences between treatments.