


## Chapter 10, Question 10b-c

I don't understand what the question is trying to convey when it says "Put  $f_n(x) = \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{(2k+1)!}$ ", and I'm also having trouble with writing the function. I tried using a `while` loop (not sure if this is even the correct way) but it wouldn't accept `k` as an index; however, when I tried to use `i`, the factorial function treated `i` as the **imaginary number** and would either return an error or a vector of complex numbers. Subsequently, I cannot start on 10.10c. My code is below but I've changed it several times:

```
estpi <- function(x = 3,n = 9){  
  f <- rep(0,n)  
  i <- 0  
  while(i <= n) {  
    f[i+1] <- (-1)^i*(x^(2i+1))/factorial(2i+1)  
    i <- i+1  
  }  
  print(f)  
}  
estpi()
```

run code snippet

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10. How do we know  $\pi = 3.1415926$  (to 7 decimal places)? One way of finding  $\pi$  is to solve  $\sin(x) = 0$ . By definition the solutions to  $\sin(x) = 0$  are  $k\pi$  for  $k = 0, \pm 1, \pm 2, \dots$ , so the root closest to 3 should be  $\pi$ .

- (a). Use a root-finding algorithm, such as the Newton-Raphson algorithm, to find the root of  $\sin(x)$  near 3. How close can you get to  $\pi$ ? (You may use the function `sin(x)` provided by R.)

The function  $\sin(x)$  is *transcendental*, which means that it cannot be written as a rational function of  $x$ . Instead we have to write it as an infinite sum:

$$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}.$$

(This is the infinite order Taylor expansion of  $\sin(x)$  about 0.) In practice, to

calculate  $\sin(x)$  numerically we have to truncate this sum, so any numerical calculation of  $\sin(x)$  is an approximation. In particular the function `sin(x)` provided by R is only an approximation of  $\sin(x)$  (though a very good one).

- (b). Put

$$f_n(x) = \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{(2k+1)!}.$$

Write a function in R to calculate  $f_n(x)$ . Plot  $f_n(x)$  over the range  $[0, 7]$  for a number of values of  $n$ , and verify that it looks like  $\sin(x)$  for large  $n$ .

- (c). Choose a large value of  $n$ , then find an approximation to  $\pi$  by solving

$f_n(x) = 0$  near 3. Can you get an approximation that is correct up to 6 decimal places? Can you think of a better way of calculating  $\pi$ ?

[Edit](#)

good question | 0

Updated 4 years ago by Charles Hwang

**S** the students' answer, where students collectively construct a single answer

I think part of your issue is that you have 2i in your exponent and factorial, and it should be 2\*i (you need to have the \* in there).

[Actions](#)[Edit](#)

thanks! | 0

Updated 4 years ago by Holly Michalak

**followup discussions** for lingering questions and comments



Resolved



Unresolved

@25\_f1

[Actions](#)

**Charles Hwang** 4 years ago

Good point--I've fixed that and am now back in the real numbers but I don't know if the output is correct. The output is

```
[1] 3.000000e+00 -4.500000e+00 2.025000e+00 -4.339286e-01 5.424107e-02
[6] -4.437906e-03 2.560330e-04 -1.097284e-05 3.630720e-07 -9.554528e-09
```

helpful! | 0

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