

? question @34

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Actions

# 4.23

Has anyone figured out a solution to Chapter 4 #23 on page 174?

hw4

Edit undo good question 2

Updated 5 years ago by Sophie Hecht

**S** the students' answer, where students collectively construct a single answer

Actions

Here is a solution I found online: <https://www.coursehero.com/file/p2fcrqko/Whether-you-want-to-use-the-continuity-correction-is-up-to-you-Solution-Let-X>

Edit - The website is a bit funky, it may be easier if I screenshot the solution:

4.2 Suppose that the distribution of the lifetime of a car battery, produced by a certain car company, is well approximated by a normal distribution with a mean of  $1.2 \times 10^3$  hours and variance  $10^4$ . What is the approximate probability that a batch of 100 car batteries will contain at least 20 whose lifetimes are less than 1,100 hours?

Solution. Let  $X$  be the lifetime of a single car battery. Then, letting  $Z \sim N(0, 1)$  we have

$$\begin{aligned} P(X \leq 1,100) &= P(1,200 + 10^2 Z \leq 1,100) \\ &= P(Z \leq -1) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587. \end{aligned}$$

Now let  $W$  be the number of car batteries, in a batch of 100, whose lifetimes are less than 1,100 hours. Note that  $W \sim \text{Binomial}(100, 0.1587)$ . Using a normal approximation, we have

$$\begin{aligned} P(W \geq 20) &= P\left(\frac{W - 100 \cdot 0.1587}{\sqrt{100 \cdot 0.1587 \cdot 0.8413}} \geq \frac{20 - 100 \cdot 0.1587}{\sqrt{100 \cdot 0.1587 \cdot 0.8413}}\right) \approx P(Z \geq 1.13) \\ &= 1 - \Phi(1.13) = 1 - 0.8708 \\ &= 0.1292. \end{aligned}$$

Edit 2 - The first part is  $\text{normalcdf}(0, 1100, 1200, \sqrt{10^4})$ . I'm not entirely sure what the second part is because of the formatting, although it appears to be something like  $P(W \geq 20) = P\left(\frac{W - np}{\sqrt{np(1-p)}} \geq \frac{20 - np}{\sqrt{np(1-p)}}\right)$  which seems like a complicated form of the binomial distribution.

Edit thanks! 1

Updated 5 years ago by Charles Hwang

**i** the instructors' answer, where instructors collectively construct a single answer

You are given  $X \sim N(1200, 100)$  and you are asked to find the probability that in 100 batteries at least 20 have a lifetime less than 1100 hours. This looks like Binomial with  $n = 100$  and  $p = P(X < 1100)$ . You have to find  $p$  first.

$$p = P(X < 1100) = \text{normalcdf}(-\infty, 1100, 1200, 100) = 0.1586$$

Then if  $Y$  is the number of successes in 100 trials we have

$$P(Y \geq 20) = 1 - P(Y \leq 19) = 1 - \text{binomcdf}(100, 0.1586, 19) = 0.159$$

If we use the normal approximation with continuity correction to  $Y$  we get  $Y \sim N(15.86, \sqrt{100(0.1586)(0.8414)})$  and so

$$P(Y \geq 20) = \text{normalcdf}(19.5, 999999, 15.86, 3.65) = 0.1593 \text{ which is pretty close to the exact solution.}$$

[undo thanks](#) | 1

Updated 5 years ago by E.N. Barron

#### followup discussions for lingering questions and comments

☒ Resolved ☐ Unresolved @34\_f1 



**E.N. Barron** 5 years ago

I'll answer tomorrow afternoon if no one posts the solution by 1.

[good comment](#) | 0

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