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STAT 307-001

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Homework 8 – Exercise 13.3

H_0: $\mu_A = \mu_B = \mu_C = \mu_D$	$\alpha = .05$
H_A: At least one μ_i is different	$\mu_A = 70.375$; $\mu_B = 58.125$; $\mu_C = 64.125$; $\mu_D = 66.625$

Source	DF	SS	MS	F	P-Value
Trt	3	635.375	211.7917	0.8381	0.4844
Error	28	7075.50	252.6964		
Total	31	7710.88			

B	C	D	A
58.125	64.125	66.625	70.375

We fail to reject H_0 at $\alpha = .05$. The differences between the means are not significant.

Homework 8 – Problem 2

	Day 1	Day 2	Day 3	Day 4
Method A	143.15	140.6	152.55	151.7
Method B	145.15	126.75	137.2	142.6

$$\bar{y}_{A\bullet\bullet} = \frac{\bar{y}_{A1\bullet} + \bar{y}_{A2\bullet} + \bar{y}_{A3\bullet} + \bar{y}_{A4\bullet}}{n_{A\bullet\bullet} / n_{Ai\bullet}} = \frac{143.15 + 140.6 + 152.55 + 151.7}{8 / 2} = \frac{588}{4} = 147$$

$$\bar{y}_{B\bullet\bullet} = \frac{\bar{y}_{B1\bullet} + \bar{y}_{B2\bullet} + \bar{y}_{B3\bullet} + \bar{y}_{B4\bullet}}{n_{B\bullet\bullet} / n_{Bi\bullet}} = \frac{145.15 + 126.75 + 137.2 + 142.6}{8 / 2} = \frac{551.7}{4} = 137.925$$

$$\bar{y}_{\bullet1\bullet} = \frac{\bar{y}_{A1\bullet} + \bar{y}_{B1\bullet}}{n_{\bullet1\bullet} / n_{i1\bullet}} = \frac{143.15 + 145.15}{4 / 2} = \frac{288.3}{2} = 144.15$$

$$\bar{y}_{\bullet2\bullet} = \frac{\bar{y}_{A2\bullet} + \bar{y}_{B2\bullet}}{n_{\bullet2\bullet} / n_{i2\bullet}} = \frac{140.6 + 126.75}{4 / 2} = \frac{267.35}{2} = 133.675$$

$$\bar{y}_{\bullet3\bullet} = \frac{\bar{y}_{A3\bullet} + \bar{y}_{B3\bullet}}{n_{\bullet3\bullet} / n_{i3\bullet}} = \frac{152.55 + 137.2}{4 / 2} = \frac{289.75}{2} = 144.875$$

$$\bar{y}_{\bullet4\bullet} = \frac{\bar{y}_{A4\bullet} + \bar{y}_{B4\bullet}}{n_{\bullet4\bullet} / n_{i4\bullet}} = \frac{151.7 + 142.6}{4 / 2} = \frac{294.3}{2} = 147.15$$

$$\bar{y}_{\bullet\bullet\bullet} = \frac{\bar{y}_{A\bullet\bullet} + \bar{y}_{B\bullet\bullet}}{n_{i\bullet\bullet}} = \frac{147 + 137.925}{2} = \frac{284.925}{2} = 142.4625$$

$\alpha_A = \bar{y}_{A\bullet\bullet} - \bar{y}_{\bullet\bullet\bullet} = 147 - 142.4625 = 4.5375$

$\alpha_B = \bar{y}_{B\bullet\bullet} - \bar{y}_{\bullet\bullet\bullet} = 137.925 - 142.4625 = -4.5375$

$\beta_1 = \bar{y}_{\bullet1\bullet} - \bar{y}_{\bullet\bullet\bullet} = 144.15 - 142.4625 = 1.6875$

$\beta_2 = \bar{y}_{\bullet2\bullet} - \bar{y}_{\bullet\bullet\bullet} = 133.675 - 142.4625 = -8.7875$

$\beta_3 = \bar{y}_{\bullet3\bullet} - \bar{y}_{\bullet\bullet\bullet} = 144.875 - 142.4625 = 2.4125$

$\beta_4 = \bar{y}_{\bullet4\bullet} - \bar{y}_{\bullet\bullet\bullet} = 147.15 - 142.4625 = 4.6875$

$\alpha\beta_{A1} = \bar{y}_{A1\bullet} - \bar{y}_{A\bullet\bullet} - \bar{y}_{\bullet 1\bullet} + \bar{y}_{\bullet\bullet\bullet} = 143.15 - 147 - 144.15 + 142.4625 = -5.5375$
$\alpha\beta_{A2} = \bar{y}_{A2\bullet} - \bar{y}_{A\bullet\bullet} - \bar{y}_{\bullet 2\bullet} + \bar{y}_{\bullet\bullet\bullet} = 140.6 - 147 - 133.675 + 142.4625 = 2.3875$
$\alpha\beta_{A3} = \bar{y}_{A3\bullet} - \bar{y}_{A\bullet\bullet} - \bar{y}_{\bullet 3\bullet} + \bar{y}_{\bullet\bullet\bullet} = 152.55 - 147 - 144.875 + 142.4625 = 3.1375$
$\alpha\beta_{A4} = \bar{y}_{A4\bullet} - \bar{y}_{A\bullet\bullet} - \bar{y}_{\bullet 4\bullet} + \bar{y}_{\bullet\bullet\bullet} = 151.7 - 147 - 147.15 + 142.4625 = 0.0125$
$\alpha\beta_{B1} = \bar{y}_{B1\bullet} - \bar{y}_{B\bullet\bullet} - \bar{y}_{\bullet 1\bullet} + \bar{y}_{\bullet\bullet\bullet} = 145.15 - 137.925 - 144.15 + 142.4625 = 5.5375$
$\alpha\beta_{B1} = \bar{y}_{B1\bullet} - \bar{y}_{B\bullet\bullet} - \bar{y}_{\bullet 2\bullet} + \bar{y}_{\bullet\bullet\bullet} = 126.75 - 137.925 - 133.675 + 142.4625 = -2.3875$
$\alpha\beta_{B1} = \bar{y}_{B1\bullet} - \bar{y}_{B\bullet\bullet} - \bar{y}_{\bullet 1\bullet} + \bar{y}_{\bullet\bullet\bullet} = 137.2 - 137.925 - 144.875 + 142.4625 = -3.1375$
$\alpha\beta_{B1} = \bar{y}_{B1\bullet} - \bar{y}_{B\bullet\bullet} - \bar{y}_{\bullet 1\bullet} + \bar{y}_{\bullet\bullet\bullet} = 142.6 - 137.925 - 147.15 + 142.4625 = -0.0125$

$a = 2$ $b = 4$ $n = 16$ $a - 1 = 2 - 1 = 1$ $b - 1 = 4 - 1 = 3$ $n - 1 = 16 - 1 = 15$
$(a - 1)(b - 1) = (1)(3) = 3$ $ab(n - 1) = (2)(4)(15) = 120$

$SS_A = bn\sum_{i=1}^{2}(\alpha_i)^2 = (4)(16)((4.5375)^2 + (-4.5375)^2) = (64)(41.1778125) = \mathbf{2635.38}$
$SS_B = an\sum_{i=1}^{4}(\beta_j)^2 = (2)(16)((1.6875)^2 + (-8.7875)^2 + (2.4125)^2 + (4.6875)^2) = (32)(107.86) = \mathbf{3451.54}$
$SS_{AB} = n\sum_{i=1,j=1}^{2,4}(\alpha\beta_{ij})^2 = (16)((-5.5375)^2 + (2.3875)^2 + (3.1375)^2 + (0.0125)^2 + (5.5375)^2 + (-2.3875)^2 + (-3.1375)^2 + (-0.0125)^2) = (16)(92.41625) = \mathbf{1478.66}$
$SS_E = \sum_{i=1,j=1,k=1}^{2,4,2}(y_{ijk} - \bar{y}_{ij\bullet})^2 = 2((144 - 143.15)^2 + (146.3 - 140.6)^2 + (156.5 - 152.55)^2 + (152 - 151.7)^2 + (147.4 - 145.15)^2 + (127.6 - 126.75)^2 + (138.9 - 137.2)^2 + (142.9 - 142.6)^2) = 2(57.67) = \mathbf{115.34}$

$MS_A = SS_A / (a - 1) = 2635.38 / 1 = \mathbf{2635.38}$
$MS_B = SS_B / (b - 1) = 3451.54 / 3 = \mathbf{1150.51333333}$
$MS_{AB} = SS_{AB} / [(a - 1)(b - 1)] = 1478.66 / [(1)(3)] = 1478.66 / [(3)] = \mathbf{492.88666667}$
$MS_E = SS_E / [ab(n-1)] = 115.34 / 120 = \mathbf{0.9611666667} = 5767 / 6000$

$F_A = MS_A / MS_E = 2635.38 / 0.9611667 = \mathbf{2741.85538408}$
$F_B = MS_B / MS_E = 1150.51333 / 0.9611667 = \mathbf{1196.99670539}$
$F_{AB} = MS_{AB} / MS_E = 492.886667 / 0.9611667 = \mathbf{512.800416161}$

Source	DF	SS	MS	F	P-Value
A	1	2635.38	2635.38	2741.855	< .00001
B	3	3451.54	1150.5133	1196.997	< .00001
AB	3	1478.66	492.88667	512.8004	< .00001
Error	120	115.34	0.9611667		
Total	127	7680.92			

We reject H_0 at $\alpha = .05$. The parameters A and B and the interaction term AB are significant.

$\sigma^2 = MS_E = 0.9611666667$ $n = 16$

$$\sigma_{\alpha\beta}^2 = \frac{MS_{AB} - MS_E}{n} = \frac{492.8866667 - 0.961166667}{16} = \frac{491.9255}{16} = \mathbf{30.7453437}$$

$$\sigma_{\alpha}^2 = \frac{MS_A - MS_{AB}}{nb} = \frac{2635.38 - 492.8866667}{(16)(4)} = \frac{2142.4933333}{64} = \mathbf{33.4764583}$$

$$\sigma_{\beta}^2 = \frac{MS_B - MS_{AB}}{na} = \frac{1150.5133333 - 492.8866667}{(16)(1)} = \frac{657.6266667}{16} = \mathbf{41.101666667}$$