Abstract We look at a particular semigroup of matrices and examine the idempotents, nilpotents, and zero divisors.

1. Introduction

Definition 1.1. A *semigroup* S is a set with a single binary associative operation.

The set of $n \times n$ matrices over the integers forms a semigroup under matrix multiplication. In this paper, we consider semigroups of non-square matrices. Let S be the set of all $n \times m$ matrices over the integers. Let P be a fixed $m \times n$ matrix over the integers. We then define

$$M * N = MPN$$

In this paper we consider non-square matrices containing only 0 and 1. The matrices in the semigroup S contain at most one non-zero entry. The sandwich matrix P contains at least one non-zero entry in each row and column. Such a semigroup S is an example of a *completely 0-simple* (C0S) semigroup. This type of semigroup has been studied extensively; see [1, 2].

In Section 2 we present the main results of this thesis. In Section 3 we present the computer program which we wrote to verify the results.

2. Semigroup Preliminaries

IN this section wse introduce several basic ideas which we will work with throughout this paper.

Definition 2.1. An element in $s \in S$ is *idempotent* if $s^2 = s$.

Definition 2.2. An element in $s \in S$ is nilpotent if $s^n = 0$ for some positive integer n.

In this paper we consider only nilpotent elements s satisfying $s^2 = 0$.

Definition 2.3. An element $0 \neq s \in S$ is a *left zero divisor* if $st = 0$ for some $0 \neq t \in S$. integer n.

Definition 2.4. An element $0 \neq s \in S$ is a *right zero divisor* if $ts = 0$ for some $0 \neq t \in S$. integer n.

3. Main Results

In this section we consider the semigroup in Section ?/??. That is, S be the set of all $n \times m$ matrices over $\{0, 1\}$ with at most one non-zero entry, and P is a matrix over $\{0, 1\}$ whichi contains at least one non-zero entry in each row and in each column. Of course, that different sandwich matrices yield different semigroups.

If M is a matrix with 1 in the (i, j) position and 0 elsewhere, we denote M by $E_{i,j}$.

Lemma 3.1. *The matrix $E_{i,j}$. is nilpotent if and only if the (j, i) entry in P is 0.*
The The matrix $E_{i,j}$. is idempotent if and only if the (j, i) entry in P is 1.

Proof. $E_{i,j} * E_{i,j} = E_{i,j}PE_{i,j} = E_{i,j}E_{j,i}E_{i,j}$. This element is zero iff the (j, i) entry in P is zero, and this element is 1 iff (j, i) entry in P is 1. $\square$

Theorem 3.2. *Every element of S is either idempotent or nilpotent of index 2.*
The number of idempotents in S equals the number of $1's$ in P.
The number of nilpotents in S equals the number of $0's$ in P.

Proof. This follows directly from Lemma 3.1. $\square$

4. Program

References

[1] A. H. Clifford and G. B. Preston, The Algebraic Theiry of Semigroups,

[2] Howie, Introduction to the Theore of Semigroups