Inferring community characteristics in labelled networks

IIB Project

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Overview

Introduction

Motivation

Preliminaries

The stochastic block model (SBM)

Initial parameters:

- N – number of vertices
- B – number of blocks

SBM parameters:

- b – block membership vector
- e – block connectivity matrix
- k – degree sequence

$$A \sim \text{DC-SBM}_{\text{MC}}(b, e, k) \qquad (1)$$

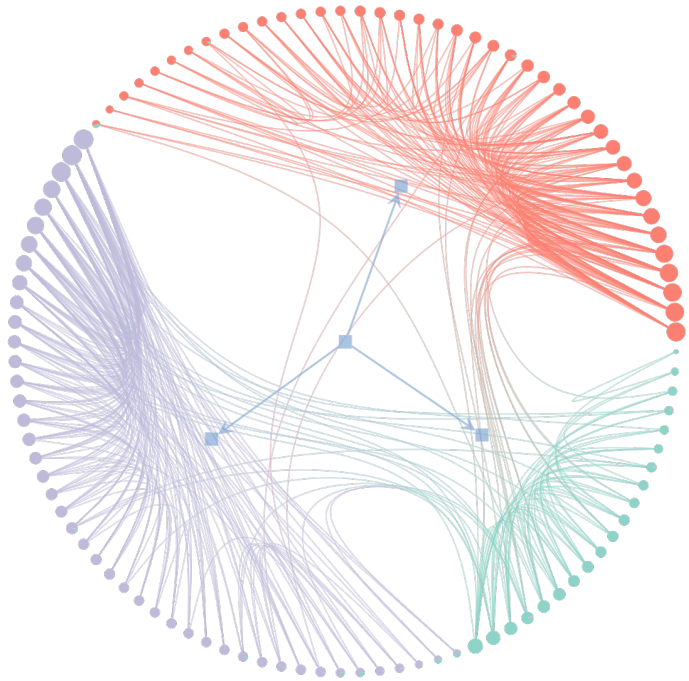

Figure: Typical SBM

The feature-first block model

The feature-first block model (FFBM)

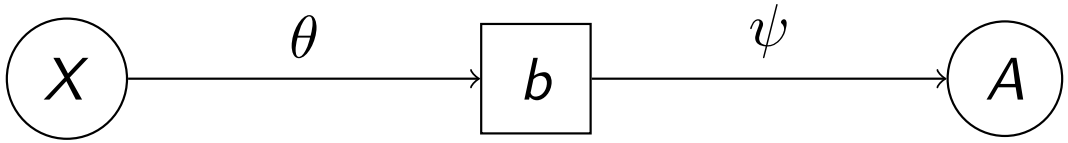

Figure: The feature-first block model (FFBM)

$$p(b|X;\theta) = \prod_{i\in[N]} \phi_{b_i}(x_i;\theta) = \prod_{i\in[N]} \frac{\exp(w_{b_i}^T x_i)}{\sum_{k\in[B]} \exp(w_k^T x_i)} \tag{2}$$

$$p(A|b;\psi) \sim \text{DC-SBM}_{\text{MC}}(b, \psi_e, \psi_k) \tag{3}$$

Inference

Inference procedure

We want to draw:

$$\theta^{(t)} \sim p(\theta|A, X). \tag{4}$$

We achieve this by:

$$b^{(t)} \sim p\Big(b|A, X\Big) \tag{5}$$

$$\theta^{(t)} \sim p\Big(\theta|X, b^{(t)}\Big) \tag{6}$$

Metropolis-Hastings (reference) [1]

We want to draw samples for $\left\{x^{(t)}\right\}$ from some distribution,

$$\pi^*(x) \propto \pi(x). \tag{7}$$

Just need to be able to evaluate $\pi(x)$ point-wise and simulate from a proposal $q(x, x')$. If we accept each proposal with probability,

$$\alpha(x, x') = \min\left(\frac{\pi(x')q(x', x)}{\pi(x)q(x, x')}, 1\right), \tag{8}$$

then the resulting Markov chain is in detailed balance with $\pi(x)$.

Sampling sequence

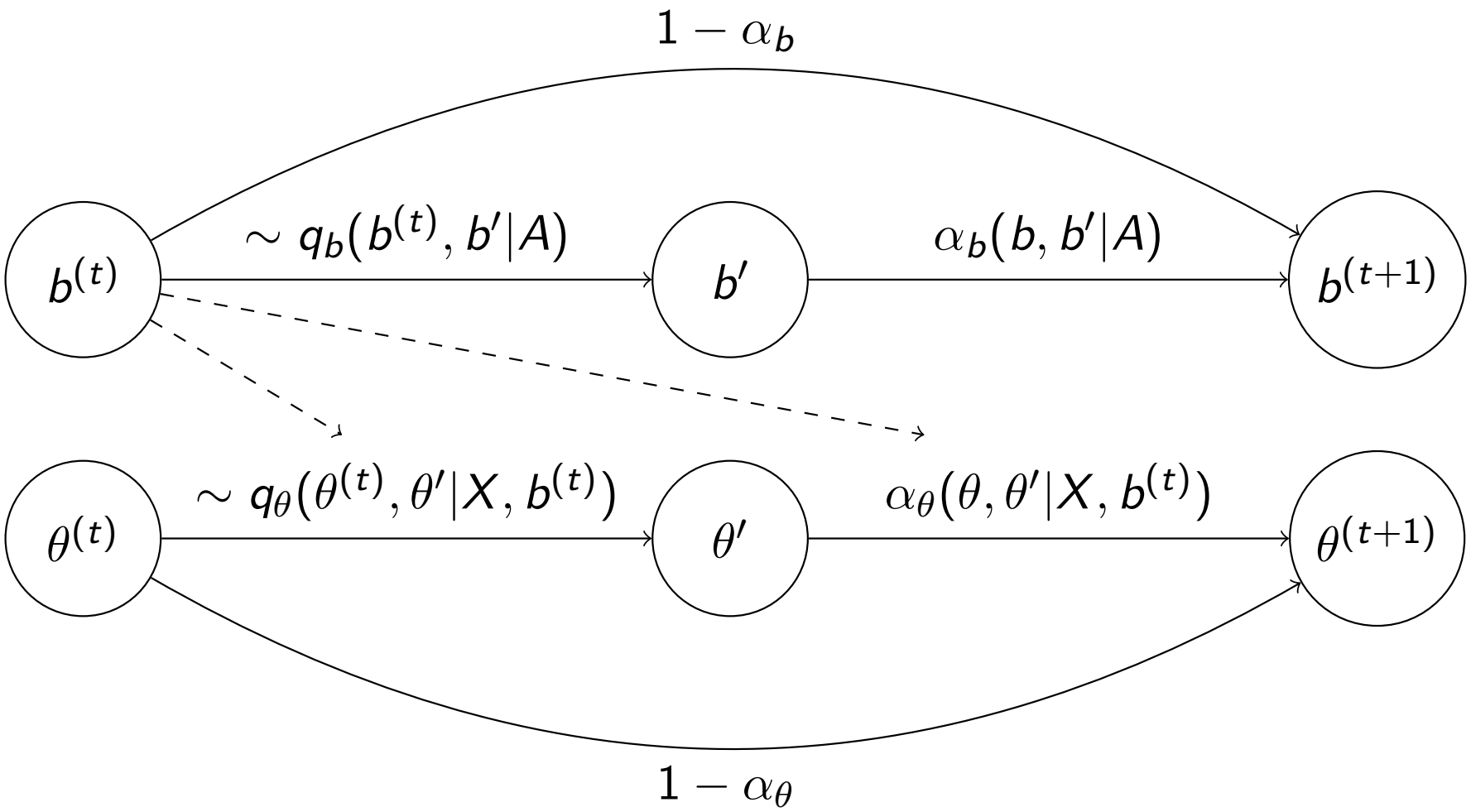

Figure: Sampling sequence.

Speed up computation

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Dimensionality Reduction

Experiments

Political Books

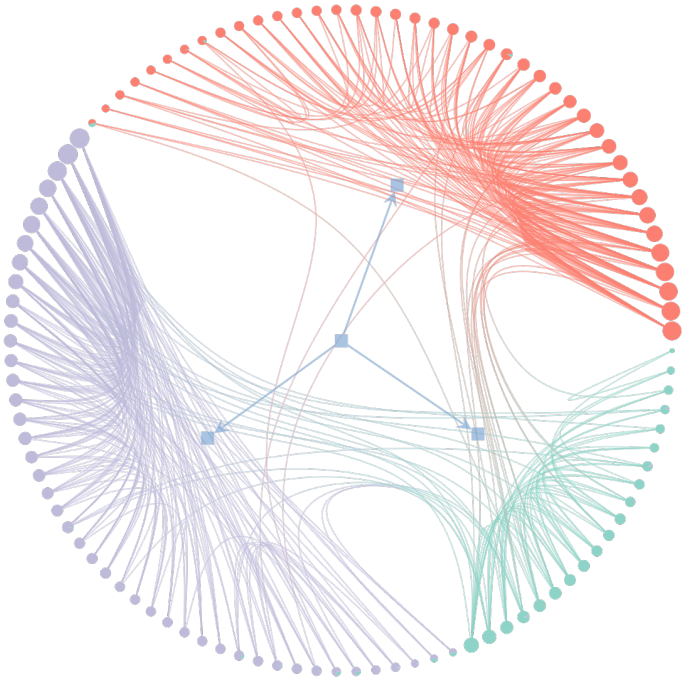

Figure: Polbooks

References

[1] W. K. Hastings. Monte carlo sampling methods using markov chains and their applications. *Biometrika*, 57(1):97–109, 1970. ISSN 00063444. URL `http://www.jstor.org/stable/2334940`.