Assignment: Linear Regression

Import the required packages

In [143…

```python
import numpy as np
from matplotlib import pyplot as plt
```

Function to calculate the Gradient descent

In [144…

```python
def gradient_descent(x, y, theta, alpha, m, max_steps):
    # HERE YOU HAVE TO IMPLEMENT THE UPDATE OF THE PARAMETERS
    thetaHist = np.empty([max_steps, 2])
    for i in range(0, max_steps):
        cost, loss = cost_function(x, y, theta)
        theta = theta - (1 / m) * alpha * cost
        thetaHist[i] = theta
    return theta, thetaHist
```

Function to calculate the cost function

In [145…

```python
def cost_function(x, y, theta):
    # HERE YOU HAVE TO IMPLEMENT THE COST FUNCTION
    loss = np.dot(x, theta) - y
    cost = np.dot(x.transpose(), loss)
    return cost, loss
```

Define some training data

In [146…

```python
def init():
    x = np.array([[1, 0], [1, 0.5], [1, 1], [1, 1.5], [1, 2], [1, 2.5], [1, 3], [1, 4], [1, 5]])
    y = np.array([0, 0.5, 1, 1.5, 2, 2.5, 3, 4, 5])
    return x, y


def init_coefficients(a1, b1, a2, b2):
    x = b1 + a1 * np.random.rand(100, )
    x_ = np.c_[np.ones((100, 1)), x]  # add x0 = 1 to each instance
    y = b2 + a2 * x + np.random.rand(100, )
    return x_, y
```

Cost function

In [147…

```python
def plot_cost_function(x, y, theta0, theta1, m, J):

    for i in range(0, len(theta0)):
        for j in range(0, len(theta1)):
            c, loss = cost_function(x, y, [theta0[i], theta1[j]])
            J[i, j] = c.sum() / m

    theta0, theta1 = np.meshgrid(theta0, theta1)
    fig2 = plt.figure(2)
    ax = fig2.add_subplot(121, projection="3d")
    ax.plot_surface(theta0, theta1, np.transpose(J))
    ax.set_xlabel('theta 0')
    ax.set_ylabel('theta 1')
    ax.set_zlabel('Cost J')
    ax.set_title('Cost function Surface plot')
    ax = fig2.add_subplot(122)
    ax.contour(theta0, theta1, np.transpose(J))
    ax.set_xlabel('theta 0')
    ax.set_ylabel('theta 1')
    ax.set_title('Cost function Contour plot')
    fig2.subplots_adjust(bottom=0.1, right=1.5, top=0.9)
    plt.show()
    return J, theta0, theta1
```

Gradient descent implementation

Here we implement Gradient Descent

In [148…

```python
def plot_hypothesis_function(t, x, y, alpha, m, max_steps, theta0, theta1, J, hist):
    t, thetaHist = gradient_descent(x, y, t, alpha, m, max_steps)
    if hist:
        plt.figure(3)
        plt.contour(theta0, theta1, np.transpose(J))
        plt.plot(thetaHist[:, 0], thetaHist[:, 1], 'x')
        plt.show()
    xs = np.array([x.min(), x.max()])
    h = np.array([[t[1] * xs[0] + t[0]], [t[1] * xs[1] + t[0]]])
    plt.figure(1)
    plt.plot(x[:, 1], y, 'x')  # Data
    plt.plot(xs, h, '-o')  # hypothesis function
    plt.show()
    return
```

Testing strategy

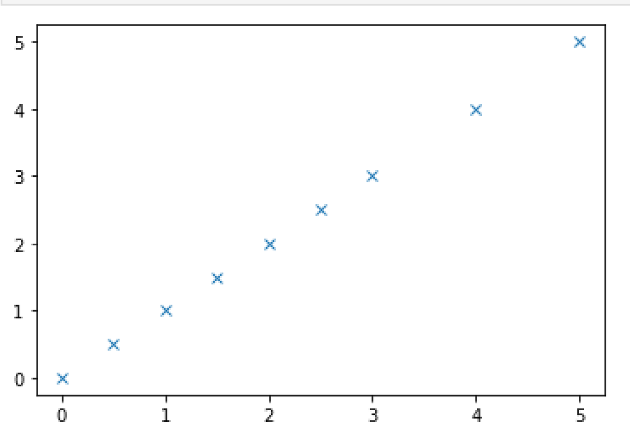
In [149…

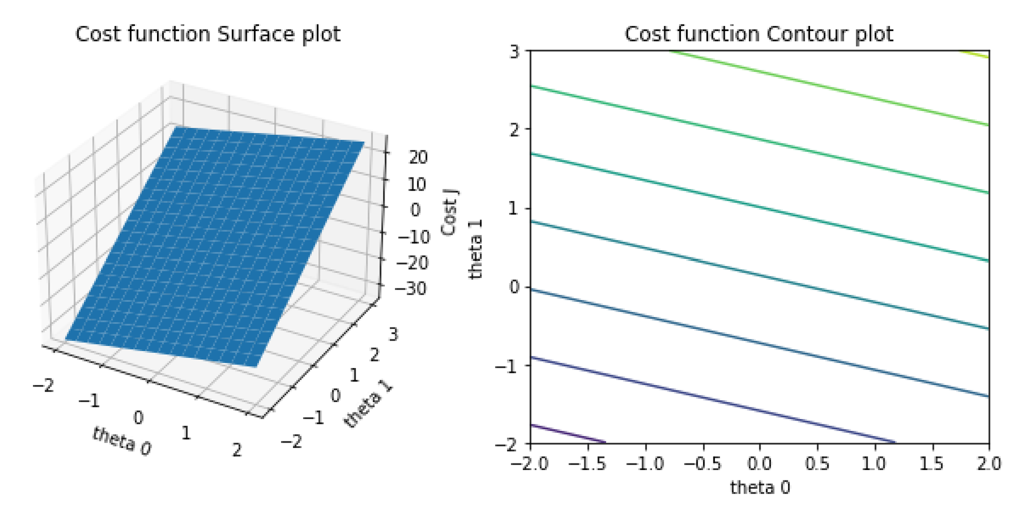
```python
def test(t, x, y, alpha, max_steps, data, cost):
    theta0 = np.arange(-2, 2.01, 0.25)
    theta1 = np.arange(-2, 3.01, 0.25)
    J = np.zeros((len(theta0), len(theta1)))
    m, n = np.shape(x)
    if data:
        plt.figure(1)  # An empty figure with no axes
        plt.plot(x[:, 1], y, 'x')
        plt.show()
    if cost:
        J, theta0, theta1 = plot_cost_function(x, y, theta0, theta1, m, J)
        plot_hypothesis_function(t, x, y, alpha, m, max_steps, theta0, theta1, J, True)
    else :
        plot_hypothesis_function(t, x, y, alpha, m, max_steps, theta0, theta1, J, False)
    plt.show()
    return
```

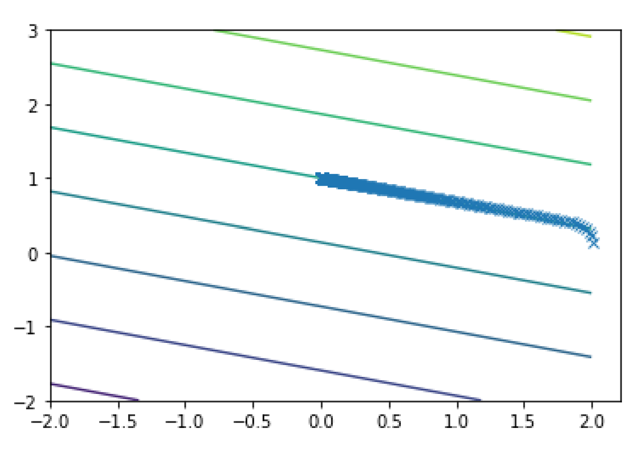
Testing with given parameters

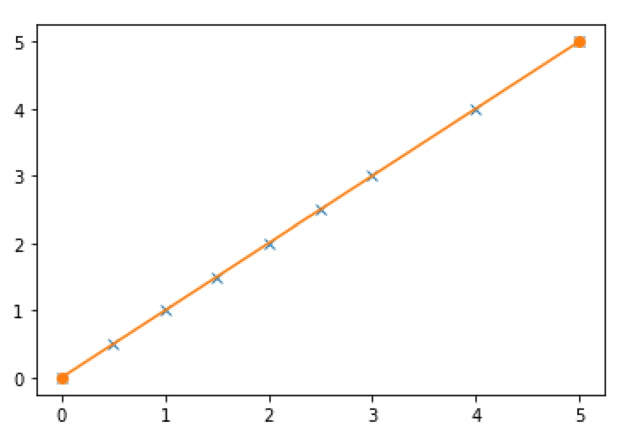
In [150…

```python
X, Y = init()
t = [2, 0]
a = 0.05  # learning parameter
m_steps = 1000  # number of iterations that the algorithm is running
test(t, X, Y, a, m_steps, True, True)
```







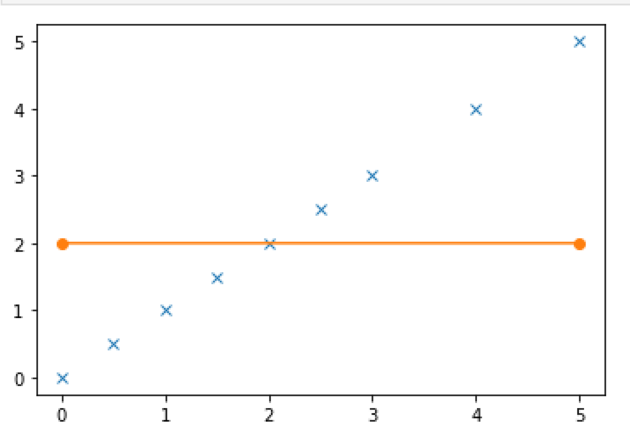


Testing with low alpha

As we can see from the hypothesis function a low alpha value will not get even close to the expected predictions, indeed they are still quite close to the given initial values [2,0]

In [151…

```python
a = 1e-8  # learning parameter
m_steps = 1000  # number of iterations that the algorithm is running
test(t, X, Y, a, m_steps, False, False)
```

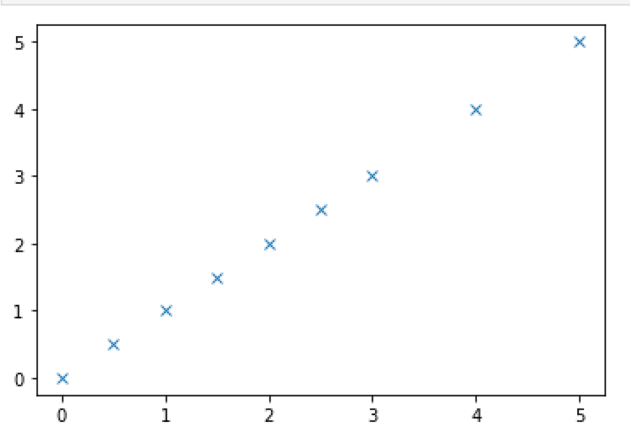


Testing with high alpha

An alpha value too high will not make the method converge because it will tilt around too quick without balancing its slope and rise. Also, the predicted values are quite far from the data points, meaning the method failed.

In [152…

```python
a = 1  # learning parameter
m_steps = 1000  # number of iterations that the algorithm is running
test(t, X, Y , a, m_steps, False, False)
```



Finding absolute global minimum

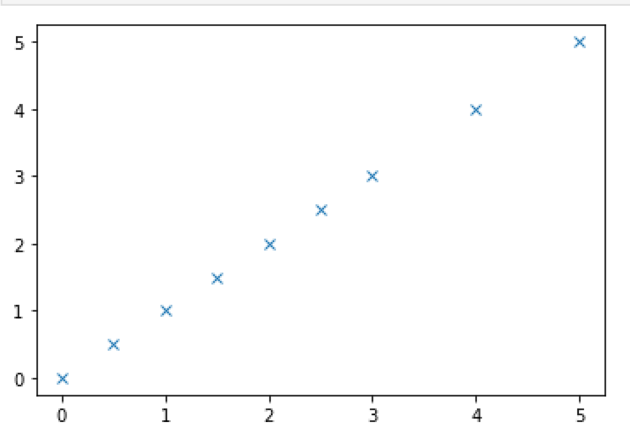
Can Linear Regression really find the absolute global minimum? There is no guarantee that the method can find the absolute global minimum without exceeding overfitting. The reason for this is also the maximum amount of iterations taken and the intent of minimizing the loss function. However, with variations of the heuristic one can eventually escape local minima in search of the global one.

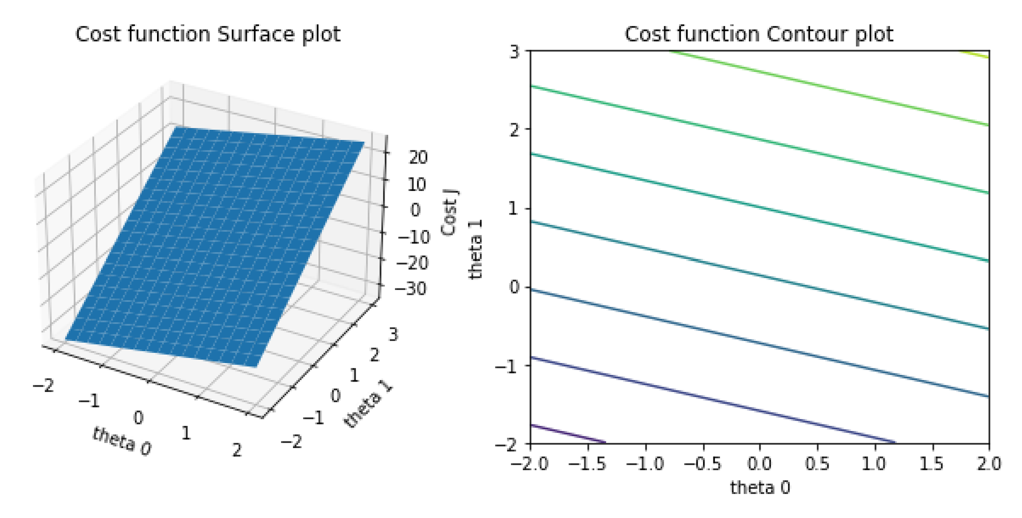
Changing initial theta prediction

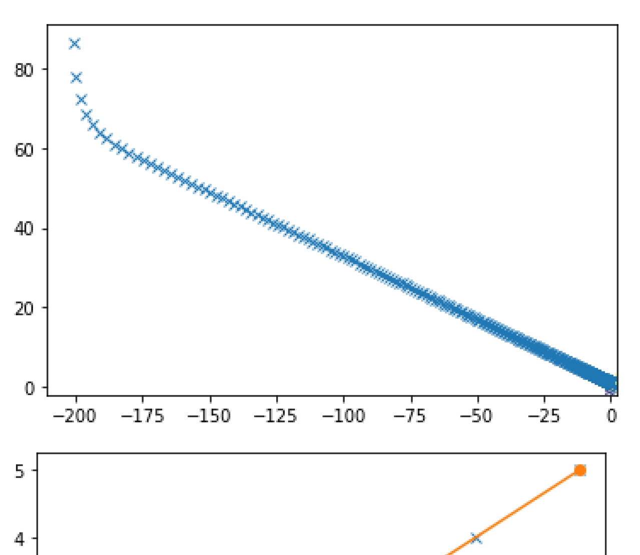
As we can see, with a good alpha value and enough iterations, the method will also eventually converge to the correct theta values, and as we can see from the theta history the convergence is quite fast, too.

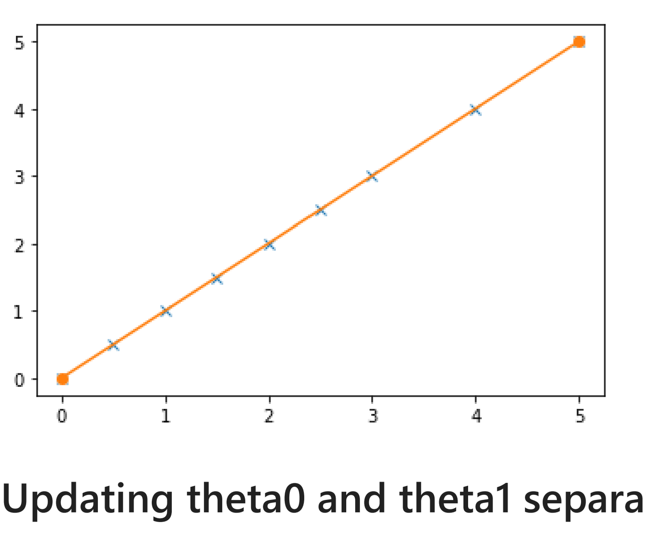
In [154…

```python
t = [-200, 100]
a = 0.05  # learning parameter
m_steps = 1000  # number of iterations that the algorithm is running
test(t, X, Y, a, m_steps, True, True)
```









Updating theta0 and theta1 separately

All theta values needs to be updated together at the same time in order to "move" the function line towards the right direction. As we know the prediction of the trained model will return a linear function between x and theta, as following :

, for every in

How many iterations to compute exact theta

To get exact theta values from the method a huge number of iterations with the most suitable alpha is needed

Testing with noise

As we can see the method performs good with noised data and random initial theta

In [156…

```python
X, Y = init_coefficients(2,1,-2,-3)
t = np.random.rand(2,)
a = 0.05  # learning parameter
m_steps = 1000  # number of iterations that the algorithm is running
test(t, X, Y, a, m_steps, True, True)
```