Cosmological Parameter Estimation using $H(z)$ measurements for a flat Λ Cold Dark Matter Universe.

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ABSTRACT

The estimation of cosmological parameters has been a topic of discussion since the late 1990s with the acceptance of the standard model of cosmology or standard model, serving as a measure of improvement for our models. In this cosmographic study, we consider observational data measurements of the Hubble parameter $H(z)$, to estimate densities of dark matter Ω_{dm}, baryonic density Ω_b and H_0 in a flat ΛCDM Universe. Using observational data, obtained by different methods, of the Hubble parameter as a function of redshift, we calculate the comoving distance, the angular diameter distance and estimate the best fit for H_0, Ω_{dm}, and Ω_b. For this we performed a χ^2 minimization on the mentioned parameters for 1000 different models. We found the best fit with the data for the parameters $H_0 = 71.4692$, $\Omega_{dm} = 0.2012$ and $\Omega_b = 0.0419$, giving good agreement with the values currently found in the literature.

Key words: cosmological parameters – cosmology: observations – software: data analysis – supernovae: general

1 INTRODUCTION

The history of measurements in cosmology could date back to 1929 with the determination of Hubble's law by E. Hubble in Hubble (1929). At that time it was considered that the universe was static, i.e., there was no expansion. This would change at the beginning of the eighties when cold dark matter (CDM) would be introduced to solve the problems of galaxy formation.

Throughout the 1980s, the introduction of dark matter and its dominance over baryonic matter solved the problems of galaxy formation, but with the discovery of the anisotropies of the Cosmic Microwave Background (CMB), needed the modification of CDM models, including expansion and dark energy, given as a result the ΛCDM model.

This new model resolved: the structure of the CMB, the clustering of galaxies, the expansion off the universe, among other problems. Also it received good support from the community because of its agreement with observations, being 2dFGRS, WMAP and Planck the most notable.

Reaching the present era, known as precision cosmology, the ΛCDM model is one the most accepted, despite several discussion in the community such as the origin and nature of dark energy and dark matter, or the hubble tension, i.e. the discrepancy of Hubble parameter $H(t)$, from different observation methods.

At present we can, thanks to measurements from different sources in the universe, estimate the parameters of different cosmological models, in this way we can evaluate its reliability when compared with the data. ΛCDM give us a Hubble constant $H_0 = 67.74\,\mathrm{kms}^{-1}\,\mathrm{Mpc}^{-1}$, baryon density parameter $\Omega_b = 0.0486$, dark matter density parameter $\Omega_{DM} = 0.2589$, matter density parameter $\Omega_b = 0.3089$, and dark energy density parameter $\Omega_\Lambda = 0.6911$ Planck Collaboration et al. (2016).

In this project, considering an ΛCDM model with a flatness condition, we present the estimation of the co-moving distance and the angular diameter distance, taking the measurements of $H(z)$ obtained with different methods, summarized in Table 1 of Magaña et al. (2018). Once the initial estimation is performed, the variation of the parameters h, Ω_{DM} and Ω_Λ is considered, and a χ^2 fit is performed to obtain the best fit to the data.

This work details the estimation of cosmological parameters based on the data summarized in Magaña et al. (2018). The structure os the present document is as follows: section 2 details the ΛCDM model. The equations used for section 3 describes the method used to estimate the cosmological parameters. Section 4 discusses the results obtained with the literature. Finally, section 5 describes the conclusions of this work.

2 ΛCDM MODEL: COMOVING DISTANCE AND ANGULAR DIAMETER DISTANCE

The ΛCMD or Lambda-CDM model is currently the model that best describes the universe, known as the standard model of cosmology, due to its simplicity and its explanation of properties of the CMB or the formation of the Large Scale Structures (LSS). This model contains information on the expansion of the universe associated with Λ, baryonic matter and dark matter or CDM. In this section we describe the mathematical tools necessary for the estimation of the comoving distance, and angular diameter distance.

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The expansion of the universe is parameterized by the scale factor $a(t)$, which is commonly normalized to the present epoch t_0, i.e. $a_0 = a(t_0) = 1$. It is related to the redshift in the following way

$$a(t) = \frac{1}{1+z}, \tag{1}$$

and its ratio is described by the Hubble parameter defined as

$$H(t) = \frac{\dot{a}}{a}. \tag{2}$$

The evolution of the scale factor, i.e., the expansion of the universe is described by the Friedmann equation for an isotropic and homogeneous universe

$$\left(\frac{\dot{a}}{a}\right) = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}. \tag{3}$$

In this study, we will use $k = 0$, i.e. a flat universe. In this particular case the universe has a critical density

$$\rho_{\rm cr} = \frac{3H_0^2}{8\pi G} = 1.8 \times 10^{-29}\,{\rm h^2\,g\,cm^{-3}}. \tag{4}$$

It is useful to describe the density of matter in the universe at the present time, as the dimensionless quantity

$$\Omega_i = \frac{\rho_i}{\rho_{\rm cr}}, \tag{5}$$

where i is baryons (b), radiation (rad), dark matter (DM), or dark energy (Λ). Such that the total density $\Omega_0 = \Omega_b + \Omega_{\rm DM} + \Omega_{\rm rad} + \Omega_\Lambda$. In addition, for dominant matter universe, the density varies as $\rho_m \propto a^{-3}$, and for radiation dominated universes $\rho_{\rm rad} \propto a^{-4}$

Finally, the Friedmann equation can be rewritten in terms of the density parameters and the redshift as

$$H(z)^2 = H_0^2[\Omega_m(1+z)^3 + \Omega_\Lambda] \tag{6}$$

where $E(z) = H(z)/H_0 = \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$ is the dimensionless Hubble function.

2.1 Comoving distance (line of sight)

There are two comoving distances, the one measured at the line of sight and the transverse distance, or angular separation. The line-of-sight comoving distance is the measurement between two objects separated by a distance defined at the current epoch. The total line-of-sight comoving distance of an observer is computed with the integral

$$D_{\rm C} = D_{\rm H}\int_0^z \frac{dz'}{E(z')}, \tag{7}$$

where $D_{\rm H}$ is defined as $D_H = \frac{c}{H_0} = 3000\,h^{-1}\,{\rm Mpc} = 9.26 \times 10^{25}\,{\rm h^{-1}\,m}$

The comoving distance is the exact measurement of two events if they were stationary in the Hubble flow.

2.2 Angular diameter distance

The angular diameter distance DA is defined as the ratio of an object's physical transverse size to its angular size. It is related to the comoving distance by

$$D_{\rm A} = \frac{D_{\rm M}}{1+z}, \tag{8}$$

where $D_{\rm M} = D_{\rm C}$ if $k = 0$

3 χ^2 FITTING

A statistical technique called the chi-square (χ^2) test is used to compare actual outcomes to predictions. This test aims to determine whether a discrepancy between actual and observed data is caused by chance or by a connection between the variables being examined. The χ^2 test is a great option for comprehend and evaluate the relationship between our two category variables as a result.

It is defined as the sum of the squared difference between the actual (model) and observed data, divided by the squared errors. For the observational data of $H(z)$ it is defined as

$$\chi^2_{\rm OHD} = \sum_{i=1}^{N_{\rm OHD}} \frac{[H(z_i) - H_{obs}(z_i)]^2}{\sigma^2_{H_i}} \tag{9}$$

Our goal with this definition is find the minimum, using python scripts, in this way finding the best fit for H_0, Ω_{dm} and Ω_b. The data used as inputs are summarized in Table 1, taken from Magaña et al. (2018).

4 RESULTS

In this work, we consider the flat ΛCDM model of Equation 6 where $\Omega_M = \Omega_{dm} + \Omega_b$. Assuming density contributions $\Omega_{rad} = 9.237 \times 10^{-5}$, $\Omega_{dm} = 0.25$, $\Omega_b = 0.05$ ($\Omega_M = 0.3$), the comoving distance is plotted in Figure 2. The angular diameter distances is shown in Figure 3. Taking the measurements of the Hubble parameter $H(z)$ listed in Table 1 the data and the model are displayed in Figure 4. This sections describes the results and the steps we follow to obtain them.

To estimate the cosmological parameters, we first started with the observational data from Magaña et al. (2018), which show the $H(z)$ measurements, obtained with different observational methods.

These were compared with an ΛCDM model with flatness condition ($\Omega_\Lambda = 1 - \Omega_{dm} - \Omega_b - \Omega_{rad}$ with $\Omega_{rad} = 9.237 \times 10^{-5}$) and cosmological parameters taken from Hogg (1999). The results of this comparison are shown in Figure 1 where it can be clearly observed a deviation mostly due to the choice of H_0.

We also calculated the comoving distance and the angular diameter distance, as shown in Figure 2 which clearly show a flat universe trend, and Figure 3 the peculiar and apparent increase in size at $z \sim 1.5$.

To correct, and obtain a model more in agreement with the H(z) data, we made variations in H_0, Ω_d and Ω_b. We took a range of values for the parameters, from 60 to 95 km s^{-1} Mpc^{-1} for H_0, and from 0.1 to 0.2 for Ω_{dm} and Ω_b simultaneously, so that $\Omega_{dm} + \Omega_b$ are always equal to 0.3, in order to find agreement with the value of $\Omega_m = \Omega_d + \Omega_b \sim 0.3$ found in the literature review.

Figure 4 shows different models as an example compared to the $H(z)$ data, where a growing trend is observed due to the growth of H_0, thus finding an upper and lower bound on the range of parameters, which was applied in our search for the best model.

To find the best fit of the many models evaluated, we used the χ^2 minimization method, defined for observational data in Equation 9. We write a short Jupyter Notebook and took 1000 models between the range of parameters described above and performed a χ^2 minimization on each one. The best fit was found with the model that obtained the lowest value of χ^2 in the Equation 9.

The best model found obtained a minimization of $\chi^2 = 32.89768854505845$, with values for the cosmological parameters of $H_0 = 71.4692$, $\Omega_{dm} = 0.2012$ and $\Omega_b = 0.0419$. This model is

Table 1. 52 Hubble parameter measurements $H(z)$ an their errors σ_H. Taken and adapted from Magaña et al. (2018)

z	$H(z)$ ($\mathrm{km\,s^{-1}\,Mpc^{-1}}$)	σ_H ($\mathrm{km\,s^{-1}\,Mpc^{-1}}$)
0	73.24	1.74
0.07	69	19.6
0.1	69	12
0.12	68.6	26.2
0.17	83	8
0.1791	75	4
0.1993	75	5
0.2	72.9	29.6
0.24	79.69	2.65
0.27	77	14
0.28	88.8	36.6
0.3	81.7	6.22
0.31	78.17	4.74
0.35	82.7	8.4
0.3519	83	14
0.36	79.93	3.39
0.38	81.5	1.9
0.3802	83	13.5
0.4	95	17
0.4004	77	10.2
0.4247	87.1	11.2
0.43	86.45	3.68
0.44	82.6	7.8
0.4497	92.8	12.9
0.47	89	34
0.4783	80.9	9
0.48	97	62
0.51	90.4	1.9
0.52	94.35	2.65
0.56	93.33	2.32
0.57	92.9	7.8
0.59	98.48	3.19
0.5929	104	13
0.6	87.9	6.1
0.61	97.3	2.1
0.64	98.82	2.99
0.6797	92	8
0.73	97.3	2.1
0.7812	105	12
0.8754	125	17
0.88	90	40
0.9	117	23
1.037	154	20
1.3	168	17
1.363	160	33.6
1.43	177	18
1.53	140	14
1.75	202	40
1.965	186.5	50.4
2.33	224	8
2.34	222	7
2.36	226	8

plotted and compared with the data and models obtained previously by the same method in Figure 5.

To discuss in possible sources of errors we compare the best fit result with those obtained in Planck Collaboration et al. (2016) shown in Table 2.

First of all the sensitivity of the χ^2 minimization to the initial guesses may be important to consider for the differences in the estimations compared to the literature. The errors in the measurements

-	Best fit	Planck2016
H_0	71.46	67.74
Ω_{DM}	0.2	0.2589
Ω_b	0.04	0.0486

Table 2. Best fit parameters result vs Planck Collaboration et al. (2016)

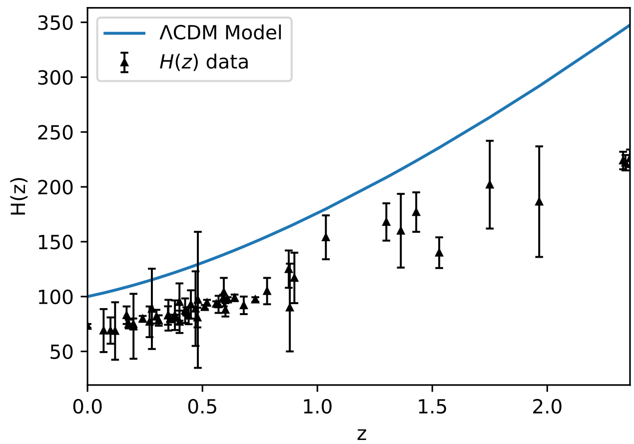

Figure 1. Comparison of data from Table 1 and ΛCDM model with parameters $H_0 = 100\,h\,\mathrm{kms^{-1}\,Mpc^{-1}}$, $\Omega_b = 0.05$ and $\Omega_d = 0.25$

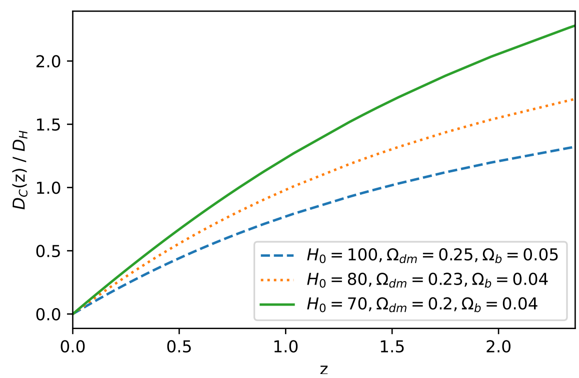

Figure 2. Comoving Distance D_C as a function of redshift z for three different combinations of parameters.

of $H(z)$ may be too large in the minimization of χ^2 and may lead to the underestimation of the values of the densities. Another possible cause for differences observed in Table 2, is that measurements of errors were not Gaussian distributed or the data is correlated in the variables of interest, and those correlations were ignored in the fit. To get an insight of this last point, an MCMC simulation was carried out with 2000 chains and 10 walkers for the 3-dimensional model. The two-dimensional posterior distributions of the parameters are shown in Figure 6. We can see that not all the parameters are distributed Gaussian and may be correlated to each other with the possibility of the matter density parameter $\Omega_M = \Omega_{dm} + \Omega_b$ to be degenerate. Finally, although the results show a Universe apparently flat, we do not discard the possibility of a non-zero curvature and thus a modification of the model considered here.

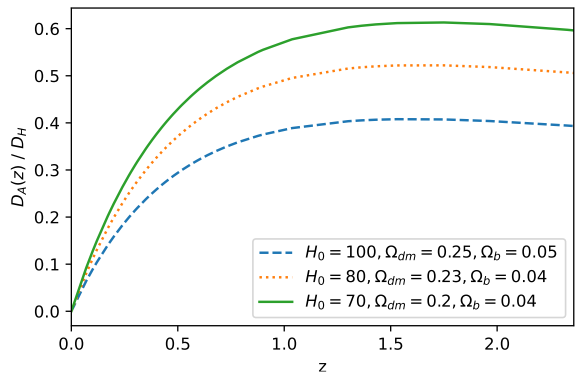

Figure 3. Angular Diameter distance as a function of redshift for three different combinations of parameters.

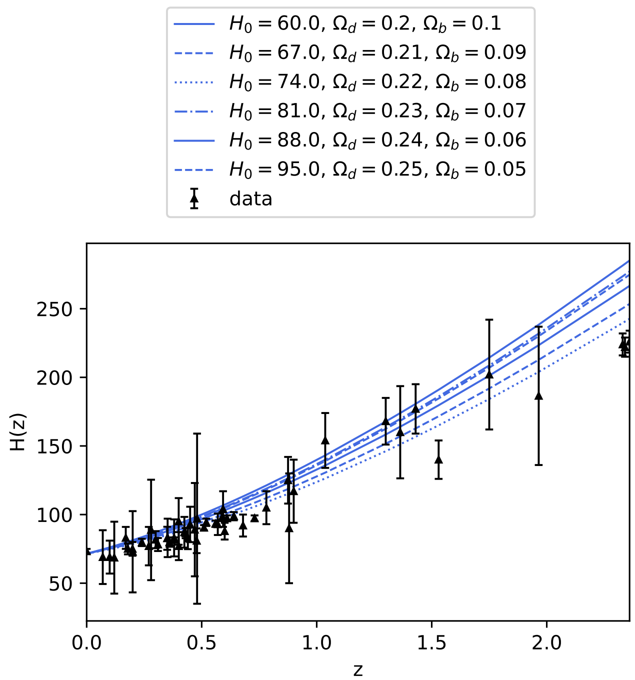

Figure 4. Hubble parameter measurements $H(z)$ (in $\mathrm{km\,s^{-1}\,Mpc^{-1}}$) with their errors at redshift z (black). ΛCDM models (blue)

H_0	Ω_{DM}	Ω_b
71.396 ± 0.95	0.202 ± 0.07	0.042 ± 0.07

Table 3. MCMC estimation of the Hubble parameter together with Dark Matter and Baryonic densities.

5 CONCLUSIONS

In this study we performed the estimation of H_0, Ω_d, and Ω_b, based on the Hubble parameter data measured with different observational methods. A χ^2 fit was performed to obtain the best set of parameters, which were $H_0 = 71.4692$, $\Omega_d = 0.2012$ and $\Omega_b = 0.0419$. These are close to those estimated in the most recent literature.

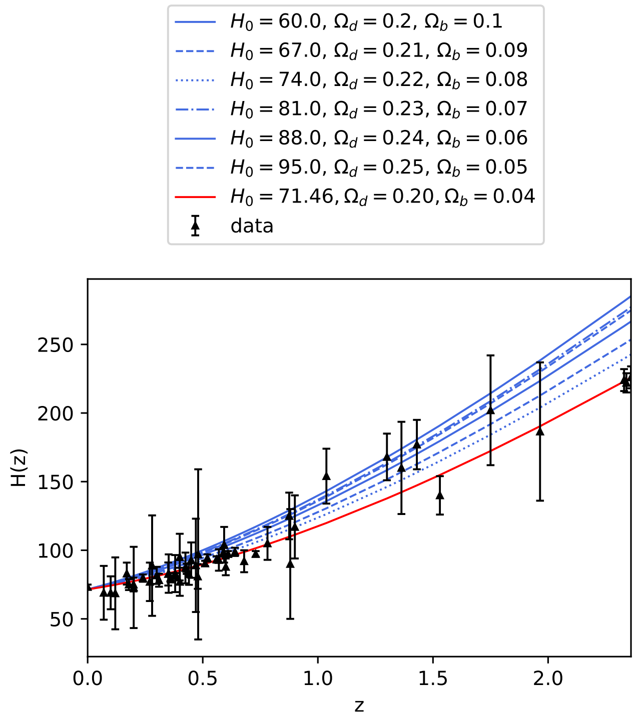

Figure 5. Hubble parameter measurements $H(z)$ (in $\mathrm{km\,s^{-1}\,Mpc^{-1}}$) with their errors at redshift z (black). Best fit ΛCDM model (red)

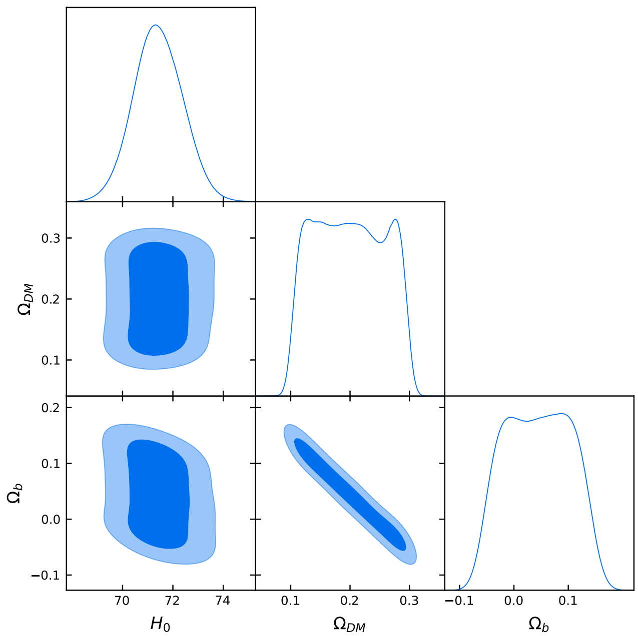

Figure 6. Posterior Distributions for MCMC simulation for the 3 parameters of interest.

REFERENCES

Hogg D. W., 1999, Distance measures in cosmology, doi:10.48550/ARXIV.ASTRO-PH/9905116, https://arxiv.org/abs/astro-ph/9905116
Hubble E., 1929, Proceedings of the National Academy of Sciences, 15, 168
Magaña J., Amante M. H., Garcia-Aspeitia M. A., Motta V., 2018, Monthly Notices of the Royal Astronomical Society, 476, 1036
Planck Collaboration et al., 2016, A&A, 594, A13

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