

Milisecond Pulsars and Gravitational Radiation

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Overview

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- 2 Millisecond Pulsars
- 3 Gravitational Waves
- 4 Summary

Introduction

- 1967 First observed **pulsating** radio source by Jocelyn Bell.
- periods between 0.25 s and 1.3 s
- Emits beams of electromagnetic radiation out of its magnetic poles. [▶ Link](#)

Up until 1982, most of the ~ 300 known pulsars had similar periods.
But ...

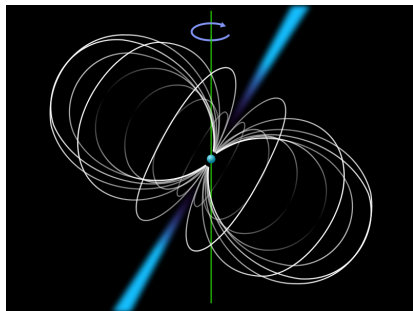


Figure 1: Schematic view of a pulsar

Pulsars

Exceptions:

- Crab pulsar (1054 A.D). period 33 ms
- Vela pulsar. period 89 ms
- Hulse-Taylor binary pulsar. period 59 ms

Crab and Vela pulsars had rapid slow-down rates.

Pulsars are rotating neutron stars, born in supernova explosions with periods 10-20 ms and gradually slowing down to periods of 1 s over million years.

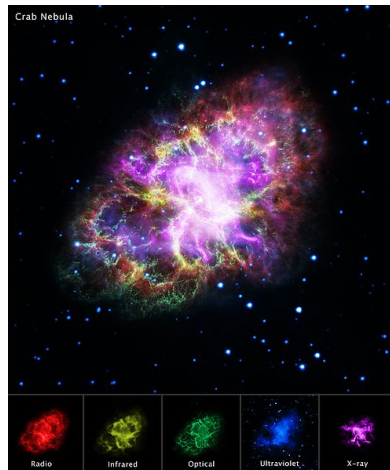


Figure 2: The Crab Nebula - five observatories (2017)

Milisecond Pulsars

Milisecond Pulsar (MSP) 1937+21:

- period 1.558 ms (642 times per second)
- resulted from "recycling" of an old, slowly rotating neutron star through accretion from a low-mass companion.
- Mass transferred from the companion also carries angular momentum from the orbit to the neutron star, spinning it up and reactivating the pulsar emission process.

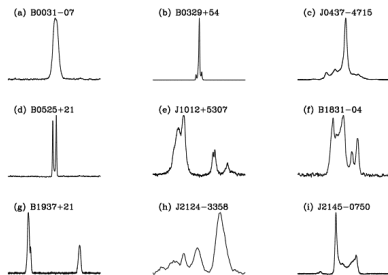


Figure 3: pulse profiles. Lorimer 2005

Absence of binary companion:

- Disruption of the binary by asymmetric mass loss in accretion-induced collapse of likely WD remnant of companion star → Ablation of companion
- “Black widow” pulsar PSR B1957+20

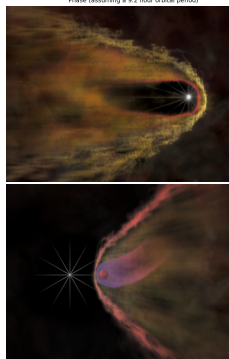
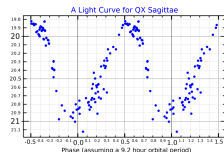


Figure 4: PSR B1957+20

$P - \dot{P}$ diagram

- over 1500 pulsars
- differences in magnetic fields and ages.
- magnetic field strengths
 $B \propto (P\dot{P})^{0.5}$
- characteristic age $\tau_c = P/(2\dot{P})$

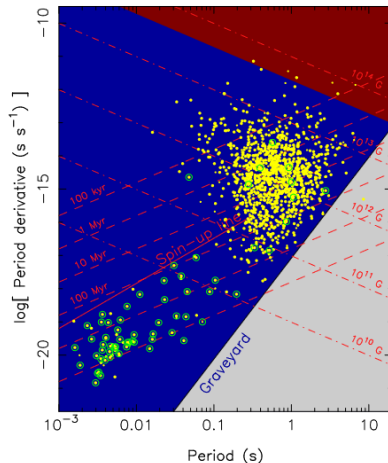
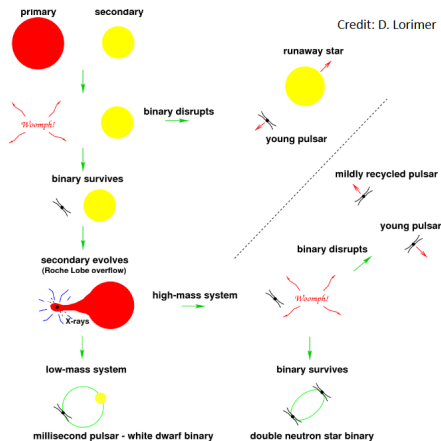


Figure 5: $P - \dot{P}$ diagram, M. Kramer

Formation and evolution



Pulsars:

- 1 Mildly recycled pulsars ($P > 20$ ms) with heavy companion (neutron star).
- 2 Fully recycled pulsars ($P < 20$ ms). MSPs, with light companion (white dwarf)
- 3 Pulsar-black hole possible

Companion of binary pulsars found so far: white dwarf, neutron star, pulsar, main-sequence star, planet.

Figure 6: evolutionary scenarios

Binary evolution tree

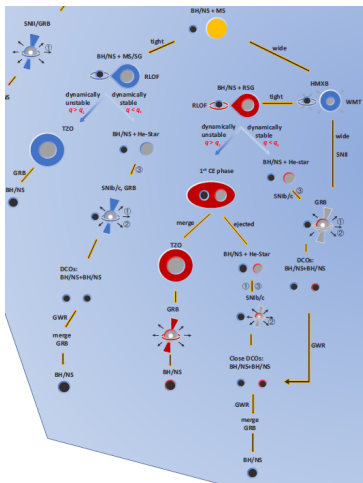


Figure 7: Binary evolution tree. Han, et al.

Timing binary Pulsars

5 post-Keplerian (PK) parameters. (P_b , x , e , w and T_0)

Constraints on the masses of the pulsar m_p and the orbiting companion m_c can be placed by combining x and P_b to obtain the mass function

$$f_{\text{mass}} = \frac{4\pi^2 x^3}{G P_b^2} = \frac{(m_c \sin i)^3}{(m_p + m_c)^2}, \quad (11)$$

relativity (GR) the PK formalism gives the relativistic advance of periastron

$$\dot{\omega} = 3 \left(\frac{P_b}{2\pi} \right)^{-5/3} (T_\odot M)^{2/3} (1 - e^2)^{-1}, \quad (12)$$

the time dilation and gravitational redshift parameter

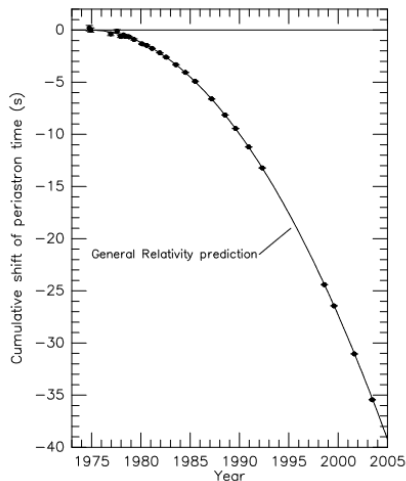
$$\gamma = e \left(\frac{P_b}{2\pi} \right)^{1/3} T_\odot^{2/3} M^{-4/3} m_c (m_p + 2m_c), \quad (13)$$

the rate of orbital decay due to gravitational radiation

$$\dot{P}_b = -\frac{192\pi}{5} \left(\frac{P_b}{2\pi} \right)^{-5/3} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4 \right) (1 - e^2)^{-7/2} T_\odot^{5/3} m_p m_c M^{-1/3} \quad (14)$$

key point: given precisely measured Keplerian parameter, the only two unknowns are the masses of pulsar and companion.

Testing General Relativity



- Hulse & Taylor 1974.
- comparing third PK parameter with predicted value based on masses of other two.
- Measurements within 0.2% of GR prediction.
- First indirect evidence of gravitational waves.

Figure 8: Orbital decay in binary pulsar B1913+16

EM waves and GW contrasted

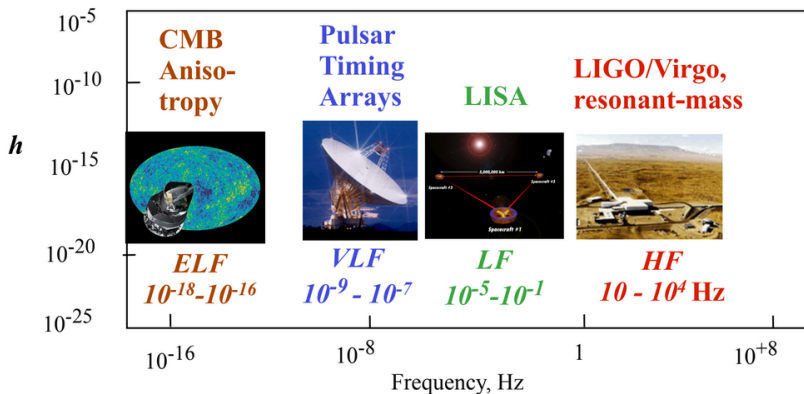
Electromagnetic waves

- Oscillations of EM field propagating through spacetime
- Incoherent superposition of waves
- easily absorbed and scattered
- Most detectors very large compared to wavelength. Narrow field of view. Good angular resolution

Gravitational waves

- Oscillations of “fabric” of spacetime itself.
- Coherent emission by bulk motion of matter.
- Never significantly absorbed or scattered.
- Most detectors small compared to wavelength . See entire sky at once. Poor angular resolution.

Frequency Bands and detectors



Gravitational Waves

- The gravitational-wave field, h_{jk}^{GW}
Symmetric, transverse, traceless (TT);
two polarizations: +, x

- + Polarization

$$h_{xx}^{\text{GW}} = h_+(t - z/c) = h_+(t - z)$$

$$h_{yy}^{\text{GW}} = -h_+(t - z)$$

Lines of force

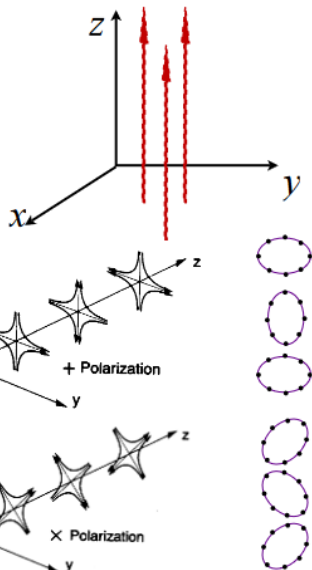
$$\ddot{x}_j = \frac{1}{2} \ddot{h}_{jk}^{\text{GW}} x_k$$

$$\ddot{x} = \ddot{h}_+ x$$

$$\ddot{y} = -\ddot{h}_+ y$$

- x Polarization

$$h_{xy}^{\text{GW}} = h_{yx}^{\text{GW}} = h_x(t - z)$$



GW Generation

$$\Phi = -\frac{M}{r} - \frac{3}{2} \frac{\mathcal{I}_{jk} n_j n_k}{r^3} + \dots$$

quadrupole moment

$c = 1 = 3 \times 10^5 \text{ km/s} = 1,$
 $G/c^2 = 1 = 1.48 \text{ km}/M_\odot = 0.742 \times 10^{-27} \text{ cm/g}$

for Newtonian source $\mathcal{I}_{jk}^{\text{GW}} = \int \rho \left(x^j x^k - \frac{1}{3} r^2 \delta_{jk} \right) d^3 x$

$$h_{jk}^{\text{GW}} = 2 \left(\frac{\ddot{\mathcal{I}}_{jk}(t-r)}{r} \right)^{\text{TT}}$$

$$h_{jk}^{\text{GW}} \sim h_+ \sim h_\times \sim \frac{E_{\text{kin}}^{\text{quad}}/c^2}{r} \sim 10^{-21} \left(\frac{E_{\text{kin}}^{\text{quad}}}{M_\odot c^2} \right) \left(\frac{100 \text{ Mpc}}{r} \right)$$

$$h_+ = \frac{\ddot{\mathcal{I}}_{\theta\theta} - \ddot{\mathcal{I}}_{\varphi\varphi}}{r}$$

$$h_\times = \frac{2\ddot{\mathcal{I}}_{\theta\varphi}}{r}$$

$$\frac{dM}{dt} = -\frac{dE_{\text{GW}}}{dt} = -\oint_S T_{\text{GW}}^{tr} dA = -\frac{1}{16\pi} \oint_S \langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle dA$$



$$\frac{dE_{\text{GW}}}{dt} = \frac{1}{5} \ddot{\mathcal{I}}_{jk} \ddot{\mathcal{I}}_{jk}$$

$$\frac{dE_{\text{GW}}}{dt} \sim \mathcal{P}_o \left(\frac{\mathcal{P}^{\text{quad}}}{\mathcal{P}_o} \right)^2$$

Internal power flow
in quadrupolar motions

$$\mathcal{P}_o = \frac{c^5}{G} = 3.6 \times 10^{52} \text{ W} = 3.6 \times 10^{59} \text{ erg/s} \sim 10^{27} L_\odot \sim 10^7 L_{\text{EM universe}}$$

Laboratory sources of GWs

- **Me waving my arms**

$$\mathcal{P}^{\text{quad}} \sim \frac{(10 \text{ kg})(5 \text{ m/s})^2}{1/3 \text{ s}} \sim 100 \text{ W}$$

$$\frac{dE_{\text{GW}}}{dt} \sim 4 \times 10^{52} \text{ W} \left(\frac{100 \text{ W}}{4 \times 10^{52} \text{ W}} \right)^2 \sim 10^{-49} \text{ W}$$

Each graviton carries an energy

$$\hbar\omega = (7 \times 10^{-34} \text{ joule s})(2\text{Hz}) \sim 10^{-33} \text{ joule}$$

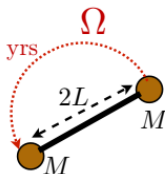
I emit $10^{-16} \text{ gravitons s}^{-1} \sim 3 \text{ gravitons each 1 billion yrs}$

- **A rotating two tonne dumb bell**

$$M = 10^3 \text{ kg}, \quad L = 5 \text{ m}, \quad \Omega = 2\pi \times 10/\text{s}$$

$$\mathcal{P}^{\text{quad}} \sim \Omega M (L\Omega)^2 \sim 10^{10} \text{ W}$$

$$\frac{dE_{\text{GW}}}{dt} \sim 4 \times 10^{52} \text{ W} \left(\frac{10^{10} \text{ W}}{4 \times 10^{52} \text{ W}} \right)^2 \sim 10^{-33} \text{ W} \quad \hbar(2\Omega) \sim 10^{-32} \text{ joule}$$



1 graviton emitted each 10 s At $r = (1 \text{ wavelength}) = 10^4 \text{ km}$, $h_+ \sim h_\times \sim 10^{-43}$

Generation and detection of GWs in lab is hopeless

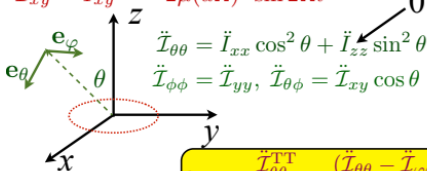
Binary Star System: Circular Orbit

$$I_{jk} = \int \rho x_j x_k d^3x \quad \text{trace} = \mu a^2 = \text{const}$$

$$I_{xx} = \mu a^2 \cos^2 \Omega t, \quad I_{yy} = \mu a^2 \sin^2 \Omega t, \quad I_{xy} = \mu a^2 \cos \Omega t \sin \Omega t$$

$$\ddot{I}_{xx} = \ddot{I}_{xx} = -2\mu(a\Omega)^2 \cos 2\Omega t, \quad \ddot{I}_{yy} = \ddot{I}_{yy} = 2\mu(a\Omega)^2 \cos 2\Omega t$$

$$\ddot{I}_{xy} = \ddot{I}_{xy} = -2\mu(a\Omega)^2 \sin 2\Omega t$$

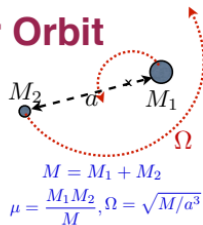


$$\ddot{I}_{\theta\theta} = \ddot{I}_{xx} \cos^2 \theta + \ddot{I}_{zz} \sin^2 \theta$$

$$\ddot{I}_{\phi\phi} = \ddot{I}_{yy}, \quad \ddot{I}_{\theta\phi} = \ddot{I}_{xy} \cos \theta$$

$$f = \frac{2\Omega}{2\pi} = \frac{1}{\pi} \sqrt{\frac{M}{a^3}} = 200 \text{ Hz} \left(\frac{10 M_\odot}{M} \right) \left(\frac{10 \text{ M}}{a} \right)^{3/2}$$

$= 10^{-4} \text{ to } 10^{-3} \text{ Hz for EM-observed compact binaries}$
 $= 30 \text{ to } 3000 \text{ Hz for final stages of NS/NS, BH/BH}$



$$h_+ = 2 \frac{\ddot{I}_{\theta\theta}^{\text{TT}}}{r} = \frac{(\ddot{I}_{\theta\theta} - \ddot{I}_{\phi\phi})}{r} = -2(1 + \cos^2 \theta) \frac{\mu(a\Omega)^2}{r} \cos 2\Omega(t - r)$$

$$h_\times = 2 \frac{\ddot{I}_{\theta\phi}^{\text{TT}}}{r} = -4 \cos \theta \frac{\mu(a\Omega)^2}{r} \sin 2\Omega(t - r)$$

- Angular dependence comes from TT projection
- As seen from above, $\theta=0$, circular polarized: $h_+ = A \cos 2\Omega t$, $h_\times = A \sin 2\Omega t$
- As seen edge on, $\theta=\pi/2$, linear polarized: $h_+ = A \cos 2\Omega t$, $h_\times = 0$



- **Energy Loss \Rightarrow Inspiral; frequency increase: “Chirp”**

$$\frac{dE_{\text{GW}}}{dt} = \frac{1}{5} \left[(\ddot{I}_{xx})^2 + (\ddot{I}_{yy})^2 + 2(\ddot{I}_{xy})^2 \right] = \frac{32}{5} \mu^2 a^4 \Omega^6 = \frac{32}{5} \frac{\mu^2 M^3}{a^5}$$

$$\frac{dE_{\text{binary}}}{dt} = \frac{d}{dt} \left(\frac{-\mu M}{2a} \right) = -\frac{32}{5} \frac{\mu^2 M^3}{a^5}$$

$$a = a_o(1 - t/\tau_o)^{1/4}, \quad \tau_o = \frac{5}{256} \frac{a_o^4}{\mu M^2} = \frac{5\mathcal{M}}{256(\mathcal{M}\Omega)^{8/3}} \quad \boxed{\mathcal{M} = \text{chirp mass} = \mu^{3/5} M^{2/5}}$$

- **Observables:**

- from h_+ & $h_\times \sim \cos(2\Omega t + \text{phase})$: **GW frequency $f = \Omega/\pi$**

- from $df/dt = -3f/8\tau_o$: **Time to merger τ_o and chirp mass \mathcal{M}**

- from GW amplitudes $h_+^{\text{amp}} = -2(1 + \cos\theta) \frac{\mu(a\Omega)^2}{r} = -2(1 + \cos\theta) \frac{\mathcal{M}(\pi\mathcal{M}f)^{2/3}}{r}$
and $h_\times^{\text{amp}} = -4\cos\theta \frac{\mathcal{M}(\pi\mathcal{M}f)^{2/3}}{r}$: **Orbital inclination angle θ and distance to source r**

- **At Cosmological Distances: $\mathcal{M}(1+z)$, Luminosity distance**

- complementary to EM astronomy, where z is the observable

Inspiral Waveforms

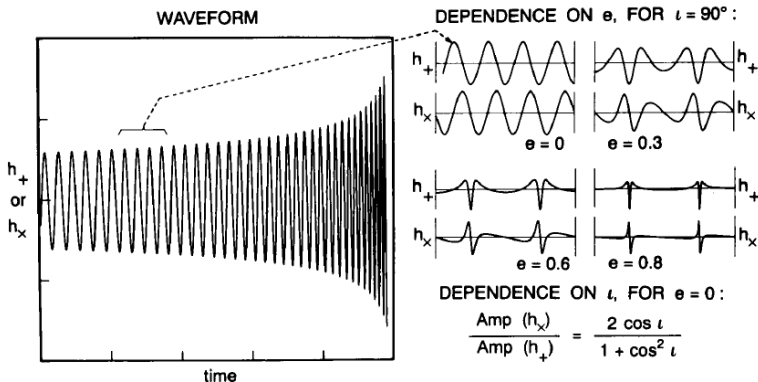


Figure 9: Waveforms from the inspiral of compact binary. Newtonian Gravity for orbital evolution. Quadpole-moment approximation for wave generation.

High-Spin High-Mass Xray Binaries

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Do high-spin high mass X-ray binaries contribute to the population of merging binary black holes?

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ABSTRACT

Gravitational-wave observations of **binary black hole (BBH)** systems point to black hole spin magnitudes being relatively low. These measurements appear in tension with high spin measurements for **high-mass X-ray binaries (HMXBs)**. We use grids of **MESA** simulations combined with the rapid population-synthesis code **COSMIC** to examine the origin of these two binary populations. **It has been suggested that Case-A mass transfer while both stars are on the main sequence can form high-spin BHs in HMXBs.** Assuming this formation channel, we show that depending on critical mass ratios for the stability of mass transfer, 48–100% of these Case-A HMXBs merge during the common-envelope phase and up to 42% result in binaries too wide to merge within a Hubble time. **Both MESA and COSMIC show that high-spin HMXBs formed through Case-A mass transfer can only form merging BBHs within a small parameter space where mass transfer can lead to enough orbital shrinkage to merge within a Hubble time.** We find that only up to 11% of these Case-A HMXBs result in BBH mergers, and at most 20% of BBH mergers came from Case-A HMXBs. Therefore, it is not surprising that these two spin distributions are observed to be different.

High-Spin High-Mass Xray Binaries

Our main conclusions are:

1. Case-A HMXBs do not tend to form BBHs. When using only COSMIC simulations to model the binary evolution, we find that at most 2% of Case-A HMXBs result in BBHs. When combining the COSMIC population with grids of BH-H-rich star MESA simulations, we find at most 12% form BBHs.
2. Case-A HMXBs contribute only a small fraction to the total merging BBH population. When considering all the BBHs for the range of masses investigated here, only 7% had a Case-A HMXB progenitor. When considering the individual mass ranges, the most massive H-rich donor, $M_{\text{donor}} = (45 \pm 2.5)M_{\odot}$, had the largest fraction with at most 20% of BBHs having a Case-A HMXB progenitor.
3. The scenario of Case-A MT while both stars are on the MS allows for the formation of high-spin HMXBs while forming a minority of BBHs, such that the expected population of GW sources would contain primarily low-spin BHs.



Duncan Lorimer

Binary and Milisecond Pulsars

Max Planck Institute for Gravitational Physics



Kip S. Thorne

Gravitational Radiation: A New Window Onto the Universe

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Thanks for your attention

