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1. A_1: basic model A_2: deluxe model B extended warranty

$$P(A_1|B) = \frac{P(A_1 \cap B)}{P_B} = \frac{0.6 \times 0.3}{0.6\times 0.3 + 0.4 \times 0.5} = \frac{0.18}{0.38} \approx 0.474$$

2. a. $F(x) = \int_{-\infty}^{\infty} f(x)dx = 0 + \int_0^{\infty} \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(\frac{x}{\beta})^\alpha} dx$ ##

claim $\mu = (\frac{x}{\beta})^\alpha \Rightarrow d\mu = d(\frac{x}{\beta})^\alpha = \frac{\alpha}{\beta^\alpha} x^{\alpha-1}$ $(\mu \in (0, +\infty))$

$\therefore F(x) = \int_0^{\infty} \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(\frac{x}{\beta})^\alpha} dx = \int_0^{\infty} e^{-\mu} d\mu = -e^{-\mu} \Big|_0^{\infty} = -e^{-\infty} - (-e^{0}) = 1$

So, this is a legitimate pdf.

b. when $x<0$, $F(x) = 0$

when $x \geq 0$, $F(x) = \int_0^x \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(\frac{x}{\beta})^\alpha} dx$. (If $\mu = (\frac{x}{\beta})^\alpha$)

$\therefore F(x) = \int_0^{(\frac{x}{\beta})^\alpha} e^{-\mu} d\mu = -e^{-\mu} \Big|_0^{(\frac{x}{\beta})^\alpha} = -e^{-(\frac{x}{\beta})^\alpha} - (-e^{0}) = 1 - e^{-(\frac{x}{\beta})^\alpha}$

$$\therefore F(x) = \begin{cases} 0, & \text{for } x<0, \\ 1-e^{-(\frac{x}{\beta})^\alpha}, & \text{for } x \geq 0, \end{cases}$$

c. $F(x) = 1 - e^{-(\frac{x}{0.5})^2}$ $(x \geq 0)$.

$$P(1<x<5) = P(5) - P(1) = \cancel{1} - e^{-(2\times 5)^2} - (\cancel{1} - e^{-(2\times1)^2}) = e^{-4} - e^{-100}$$

3. a. $P(Y<4) = \int_0^{\infty}\int_0^4 f(x,y)dy\,dx = \int_0^{\infty}\int_0^4 xe^{-x(1+y)}dy\,dx = \int_0^{\infty} xe^{-x}\int_0^4 \cancel{x}e^{-xy}dy\,dx$

$= \int_0^{\infty} xe^{-x} \frac{e^{-xy}}{-x}\Big|_0^4 dx = \int_0^{\infty} \cancel{x}e^{-x}(\frac{e^{-4x}}{-\cancel{x}} - \frac{e^0}{-\cancel{x}})dx = \int_0^{\infty} e^{-x} - e^{-5x}\,dx = -e^{-x} + \frac{e^{-5x}}{5}\Big|_0^{\infty}$

$= -(-1+\frac{1}{5}) = \frac{4}{5}$

b. $f(x) = \int_0^{\infty} f(x,y)dy = \int_0^{\infty} xe^{-x(1+y)}dy = xe^{-x}\int_0^{\infty} e^{-xy}dy = \cancel{x}e^{-x}\frac{e^{-xy}}{-\cancel{x}}\Big|_0^{\infty} = e^{-x}(0+1) = e^{-x}$ $(x \geq 0)$

$f(y) = \int_0^{\infty} f(x,y)dx = \cancel{\int_0^{\infty} xe^{-x}e^{-xy}dx} = \int_0^{\infty} xe^{-x(1+y)}dy = \int_0^{\infty} \frac{x}{-(1+y)} d(e^{-x(1+y)})$

$= \frac{x}{-(1+y)}e^{-x(1+y)} - \int \frac{e^{-x(1+y)}}{-(1+y)}dx = -\frac{x}{(1+y)}e^{-x(1+y)} - \frac{e^{-x(1+y)}}{(1+y)^2}\Big|_0^{\infty} = \frac{1}{(1+y)^2}$ $(y \geq 0)$

(C). $\because f(x,y) \neq f(x) \cdot f(y)$.

$\therefore$ X and Y are not independent.

4.

(a)

86% CI: $(\bar{x} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \bar{x} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}})$

$Z_{\frac{\alpha}{2}} = Z_{0.07} = 1.48$

$147.6 \pm 1.48 \frac{4.5}{\sqrt{78}} = (146.846, 148.354)$

(b) $n = (2Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{w})^2 = (2 \cdot 2.58 \cdot \frac{4.5}{1})^2 = 539.168 \approx 540$

5.

P_1 : population proportion of all elementary school teachers who are satisfied.

P_2 : population proportion of all high school teachers who are satisfied.

hypothesis : H_0: $P_1 - P_2 > 0$

H_1: $P_1 - P_2 \leq 0$

$\hat{p}_1 = \frac{224}{395} = 0.567$ $\quad \hat{q}_1 = 1 - \hat{p}_1 = 0.433$

$\hat{p}_2 = \frac{126}{266} = 0.474$ $\quad \hat{q}_2 = 1 - \hat{p}_2 = 0.526$

$\alpha = 0.01$ $\quad Z_{\frac{\alpha}{2}} = Z_{0.005} = 2.58$

CI : $\hat{p}_1 - \hat{p}_2 \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{m} + \frac{\hat{p}_2 \hat{q}_2}{n}} = 0.567 - 0.474 \pm 2.58 \sqrt{\frac{0.567 \times 0.433}{395} + \frac{0.474 \times 0.526}{266}}$

$= (-0.0086, 0.195)$

$\because 0.195 > 0$

$\therefore$ It don't reject H_0.

So the data provide evidence that more elementary teachers are satisfied with their jobs than high school teachers.

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6. (a). $S_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \cancel{1640.1}\ 179849.73 - \frac{(1640.1)^2}{15} = 521.196$

$S_{yy} = \sum y_i^2 - \frac{(\sum y_i)^2}{n} = 6430.06 - \frac{(299.8)^2}{15} = 438.057$

$S_{xy} = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n} = 32308.59 - \frac{1640.1 \times 299.8}{15} = -471.542$

(b) $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{-471.542}{521.196} \approx -0.905$

(c) $\hat{\beta}_0 = \frac{\sum y_i - \hat{\beta}_1 \sum x_i}{n} = \frac{299.8 - (-0.905)\,1640.1}{15} = 118.939.$

$y = -0.905x + 118.939.$

(d). $SSE = \sum y_i^2 - \hat{\beta}_0 \sum y_i - \hat{\beta}_1 \sum x_i y_i = 6430.06 - 118.939 \times 299.8 - (-0.905) \times 32308.59 = 11.422$

$SST = S_{yy} = \sum y_i^2 - \frac{(\sum y_i)^2}{n} = 438.057.$

$r^2 = 1 - \frac{SSE}{SST} = 1 - \frac{11.422}{438.057} \approx 0.974.$

(e) No. the value 80 is outside the range of x values for which observations was available.

7. $H_0: \mu_1 - \mu_2 = 0.$

$H_a: \mu_1 - \mu_2 \neq 0$, we will reject H_0 if the P-value is less than a.

$V = \frac{\left(\frac{s_1^2}{m} + \frac{s_2^2}{n}\right)^2}{\frac{(s_1^2/m)^2}{m-1} + \frac{(s_2^2/n)^2}{n-1}} = \frac{\left(\frac{0.16^2}{7} + \frac{0.24^2}{6}\right)^2}{\frac{(0.16^2/7)^2}{7-1} + \frac{(0.24^2/6)^2}{6-1}} \approx 8.506 \approx 8$

$t = \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}} = \frac{22.73 - 21.9 - (0)}{\sqrt{\frac{0.16^2}{7} + \frac{0.24^2}{6}}} = \cancel{11.845 \approx 7.048}\ 7.208$

P-value: $2[P(t > 11.845)] < 2(0.0005) = 0.001.$

$\therefore$ We reject H_0. and conclude that there is a difference in the densities of the two slate types

8. $H_0: \mu = 700$

$H_a: \mu \neq 700$

$$T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{674.3 - 700}{\frac{24.35}{\sqrt{60}}} = -8.175$$

$Z_\alpha = 1.645$

Since test statistic is smaller than the critical value, we reject the null hypothesis and conclude that the ~~advertesting~~ advertising is overestimating.

9 a. $$r = \frac{S_{xy}}{\sqrt{S_{xx}}\sqrt{S_{yy}}} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}\sqrt{\sum y_i^2 - \frac{(\sum y_i)^2}{n}}} = \frac{84.09 - \frac{143.05 \times 3.91}{8}}{\sqrt{\underset{3326.6}{(\cancel{\ })^2} - \frac{(143.05)^2}{8}}\sqrt{2.34 - \frac{(3.91)^2}{8}}}$$

$$= \cancel{0.699}\ 0.781$$

b. $H_0: \rho = 0$ $\quad t_{0.05, 8} = 1.781$

$H_a: \rho > 0$

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \frac{0.699\sqrt{8-2}}{\sqrt{1-0.699^2}} = \overset{2.675}{\cancel{2.394}} > t_{0.05, 8}$$

So H_0 ~~is not~~ should be rejected.

∴ There is a sufficient evidence that a linear relationship exists at the $\alpha = 0.05$ level.