Stat 301
Final Exam (A)
2022

Name:—————————————

Exam Rules:

- There is no collaborating on this exam. You must work on it independently.

- Calculators or other computational software may be used only for simple arithmetic on this exam.

- You may use your course notes and the textbook on this exam.

- Write and submit your answers to each question according to university policy.

Note: You must use correct mathematical notation, show all of your work, and present your work in a neatly organized manner in order to get full credit for your answer.

Problem 1. [5 points] An electronics store sells brand of television is made in a basic model or a deluxe model. Over the past year, 40% of the televisions sold have been the deluxe model. Of those buying the basic model, 30% purchased an extended warranty, whereas 50% of all deluxe purchasers do so. If you know that a randomly selected purchaser has an extended warranty, how likely is it that they have the basic model?

Problem 2. Consider the following function of the random variable X with paramteters $\alpha > 0$ and $\beta > 0$:

$$f(x; \alpha, \beta) = \begin{cases} \frac{\alpha}{\beta^{\alpha}} x^{\alpha-1} e^{-(x/\beta)^{\alpha}} & x \geq 0 \\ 0 & x < 0 \end{cases}.$$

(a) [5 points] Verify that this is a legitimate pdf.

(b) [5 points] What is the cdf of this distribution?

(b) [5 points] If $\alpha = 2$ and $\beta = .5$, compute $P(1 < X < 5)$.
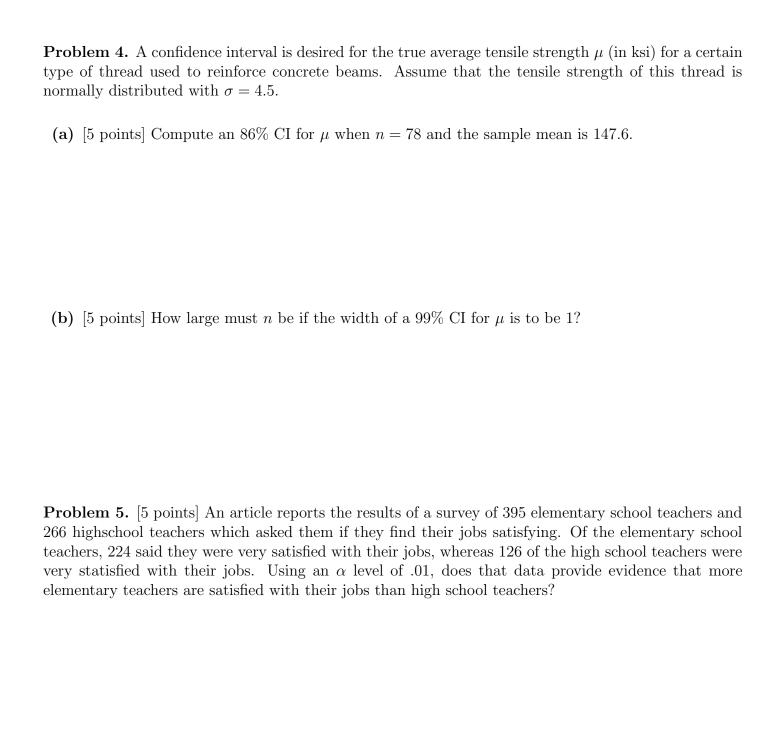
Problem 3. Two coupled electrical parts for an MRI machine have the following joint pdf for their useful lifetimes X and Y:

$$f(x, y) = \begin{cases} xe^{-x(1+y)} & x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) [10 points] What is the probability that the lifetime Y of the second componant is less than 4?

(b) [5 points] What are the marginal pdf's of X and Y?

(c) [5 points] Are X and Y independent? Explain.

Problem 4. A confidence interval is desired for the true average tensile strength μ (in ksi) for a certain type of thread used to reinforce concrete beams. Assume that the tensile strength of this thread is normally distributed with $\sigma = 4.5$.

(a) [5 points] Compute an 86% CI for μ when $n = 78$ and the sample mean is 147.6.

(b) [5 points] How large must n be if the width of a 99% CI for μ is to be 1?

Problem 5. [5 points] An article reports the results of a survey of 395 elementary school teachers and 266 highschool teachers which asked them if they find their jobs satisfying. Of the elementary school teachers, 224 said they were very satisfied with their jobs, whereas 126 of the high school teachers were very statisfied with their jobs. Using an α level of .01, does that data provide evidence that more elementary teachers are satisfied with their jobs than high school teachers?

Problem 6. Concrete with excellent drainage properties is beneficial in areas that receive a lot of rainfall. A study reported the following data related to $x =$ unit weight (pcf) and $y =$ porosity (%) is various concrete specimens:

$$n = 15, \sum x_i = 1640.1, \sum x_i^2 = 179,849.73, \sum y_i = 299.8, \sum y_i^2 = 6,430.06, \sum x_i y_i = 32,308.59.$$

A regression analysis is desired with independent variable x and dependent variable y.

(a) [5 points] Compute S_{xx}, S_{yy}, and S_{xy}.

(b) [5 points] Compute and test an estimate of the slope parameter β_1.

(c) [5 points] Write a simple linear regression model with x as the indepednend variable and y as the dependent variable.

(d) [5 points] What is the proportion of variablity in the data that is explained by this model?

(e) [5 points] Suppose you know that in the data, the range for x is 99 to 120. Would you use the regression model to estimate y when $x = 80$? Why or why not?

Problem 7. [5 points] Suppose that μ_1 and μ_2 are the true average densities for two types of slate. Assuming normality of the two density distributions, use a two-tailed test against $H_0 : \mu_1 - \mu_2 = 0$. The relevent data are as follows:

$$m = 7, \ \bar{x} = 22.73, \ s_1 = .16, \ n = 6, \ \bar{y} = 21.9, \ s_2 = .24.$$

Problem 8. [5 points] A brand of lightbulb is advertised to last 700 hours on average. To test this claim, a random sample of 60 lightbulbs was selected and the lifetime of each was determined. The folowing was computed from the sample data:

$$\bar{x} = 674.3, \ s = 24.35.$$

Use a test of hypothesis to determine if there is sufficient evidence at the $\alpha = .05$ level that the advertised value is incorrect.

Problem 9. A random sample of eight flame-retardant fabrics were selected and data on $x =$ stiffness (mg-cm) and $y =$ thickness (mm) were collected, and the relevant innformation is below.

$$\sum x_i = 143.05, \sum x_i^2 = 3326.6, \sum y_i = 3.91, \sum y_i^2 = 2.34, \sum x_i y_i = 84.09.$$

(a) [5 points] Compute the sample correlation coefficient for the data.

(b) [5 points] Use the appropriate statistical test to see if there is sufficiant evidence that a linear relationship exists at the $\alpha = .05$ level.