Exercise 1

We show that the multiplicative identity, $1 \bigotimes 1$, is equal to the additive identity.
Since co prime, by chinese remainder theorem, $xm + yn = 1$ for some x, y.

$$1 \bigotimes 1 = (xm + yn \bigotimes 1) = (xm \bigotimes 1) + (y \bigotimes n) = 0$$

Exercise 2

Using hint and see that $a \bigotimes M \cong aM$ and $A \bigotimes M = M$.

Exercise 3

A is local so if $M_k = k \bigotimes_A M = M/mM = 0 \to M = 0$.
$M \bigotimes_A N = 0 \to M_k \bigotimes_A N_k = 0$. As vector spaces their dimension must be 0 so M_k or $N_k = 0$ hence M or $N = 0$.

Exercise 4

$$0 \to N' \to N \to N'' \to 0$$

is exact so

$$0 \to N' \bigotimes (\bigoplus_i M_i) \to N \bigotimes (\bigoplus_i) \to N'' \bigotimes (\bigoplus_i M_i) \to 0$$

Follow by expanding it and using the exactness properties.

Exercise 5

Exercise 6

Exercise 7

$(A/a)[x] \cong A[x]/a[x]$. If a was prime, LHS a domain so $a[x]$ is prime. if a was max, A/a a field but $A/a[x]$ may not be a field so $a[x]$ is not max.

Exercise 8

(i) Apply the exactness property of M then N and then that tensoring associates.

(ii)