

Exercise 6

$\{x_1, \dots, x_n\}$ is the generating set of K , Assume $\text{Char} K = 0$ so K is finitely generated over \mathbb{Z} (sums of products of x_i and inject \mathbb{Z}), and hence \mathbb{Q} .

(7.9) says that K is finite algebraic over \mathbb{Q} , (7.8) says that \mathbb{Q} is finite \mathbb{Z} -Algebra, which \mathbb{Q} cannot be.

So assuming $\text{char} K = 0$ leads to a contradiction.

Assume $\text{char} K \neq 0$, $\text{char} K = p$, so a $\mathbb{Z}/p\mathbb{Z}$ -Algebra, (7.9) says K is finite extension over $\mathbb{Z}/p\mathbb{Z}$

$[K:\mathbb{Z}/p\mathbb{Z}] = n$, $[K(a):\mathbb{Z}/p\mathbb{Z}] < n$ and multiply to n so K is finite field.

Exercise 8

$A[x]$ contains A . Infinite chain in A would mean an infinite chain in $A[x]$ $\varphi: A \rightarrow A[x], a \mapsto a$ is an injection.

Exercise 9

Exercise 10

$M[x]$ becomes an $A[x]$ -Module in the natural way. $M[x] \cong A[x] \otimes_A M$.

Exercise 11

A_p is noetherian for all p , primes (max ideals are also prime). Hence all maximals, all ideals are contained in a maximal ideal, $\{A_i\}_i \leq 0$ be an infinite chain in A , will be contained in some max (but never reach it) Localize A at this max, this chain will be infinite in the local ring, a contradiction to having an infinite chain in A .