

Exercise 1

We show that the multiplicative identity, $1 \otimes 1$, is equal to the additive identity. Since co prime, by chinese remainder theorem, $xm + yn = 1$ for some x, y .

$$1 \otimes 1 = (xm + yn) \otimes 1 = (xm \otimes 1) + (y \otimes n) = 0$$

Exercise 2

Using hint and see that $a \otimes M \cong aM$ and $A \otimes M = M$.

Exercise 3

A is local so if $M_k = k \otimes_A M = M/mM = 0 \rightarrow M = 0$.

$M \otimes_A N = 0 \rightarrow M_k \otimes_A N_k = 0$. As vector spaces their dimension must be 0 so M_k or $N_k = 0$ hence M or $N = 0$.

Exercise 4

$$0 \rightarrow N' \rightarrow N \rightarrow N'' \rightarrow 0$$

is exact so

$$0 \rightarrow N' \otimes \left(\bigoplus_i M_i \right) \rightarrow N \otimes \left(\bigoplus_i M_i \right) \rightarrow N'' \otimes \left(\bigoplus_i M_i \right) \rightarrow 0$$

Follow by expanding it and using the exactness properties.

Exercise 5**Exercise 6****Exercise 7**

$(A/a)[x] \cong A[x]/a[x]$. If a was prime, LHS a domain so $a[x]$ is prime. if a was max, A/a a field but $A/a[x]$ may not be a field so $a[x]$ is not max.