Exercise 15

Exercise 17

(i)

$$X_f \cap X_g = V(f)^c \cap V(g)^c = (V(f) \cup V(g))^c = (V(fg))^c = X_{fg}$$

(ii)

$$X_f = \emptyset \Rightarrow V(f) = X \Rightarrow f^n = 0$$

f is in all primes, (nilradical) so was nilpotent.
f is nilpotent so in the nilradical so in all primes, so no primes do not contain f.

(iii)

$$X_f = X \Rightarrow V(f) = \emptyset$$

no primes contain the ideal that f generates. $yf = 1$
no primes can contain a unit. so every prime will not contain f.

(iv) by equality in Exercise 15

(v)

(vi)

(vii)

Exercise 19

If $R =$ nilradical, if $f, g \in R$, $X_f, X_g \neq \emptyset$, $X_f \cap X_g = X_{fg}$.
Since R is prime, $X_{fg} \neq \emptyset$, every open set can be written as basic open sets.
Spec(A) is irreducible, non empty open sets intersect non trivially, $X_{fg} \neq \emptyset$ so $fg \in R$ so R is prime.

Exercise 20