Reconstructing the Dimensionality of Cantor Numbers: A Systematic Hypothesis of 3.5D Topological Existence

Abstract

Cantor’s diagonalization method is traditionally understood as a static technique for demonstrating the uncountability of the real number set within set theory. This paper presents a new interpretation: the Cantor number can be regarded not as a fixed counterexample, but as a dynamic product generated by an input-output system whose evolution along the time axis forms a topological disturbance. We define the notion of an 'instantaneous Cantor number'—a number generated at each time step by perturbing a diagonal element corresponding to the input’s row and column in a hypothetical list of decimals. Each perturbation yields a slice, and the accumulation of such slices over time forms a non-closable structure in 3D space. We propose that only through embedding time as a half-dimension—yielding a 3.5-dimensional structure—can this structure be properly closed and understood. To support this claim, we introduce an analogy: a rotating circle in 2D becomes a 2.5D object and can be viewed as a sphere in 3D; similarly, a dynamic Cantor structure in 3D becomes observable only when interpreted within a time-embedded dimensional framework. In 4D, Cantor numbers appear as static and well-defined entities. This model provides a topological metaphor to unify interpretations of Gödelian incompleteness, Turing undecidability, and symbolic overflow in AI systems. This work presents a conceptual hypothesis developed independently by the author, an undergraduate student, during a process of theoretical reflection and dialogue with AI systems. It is intended as a structural thought experiment, and has not been subjected to empirical validation.

1. Introduction

Cantor’s diagonal method, introduced in 1891, established a foundational result in set theory by proving the uncountability of real numbers. It has since influenced logic, computability theory, and philosophy, particularly in the works of Gödel and Turing.

Yet, it is often treated as a static counterexample—a fixed logical construct. This paper offers a restructured view: Cantor numbers are not merely the result of contradiction but are generated dynamically through input-driven perturbations in a symbolic system over time. Each construction should be viewed as a 'disturbance slice' existing only at a particular moment. The accumulation of these disturbances over continuous time yields a higher-order geometric structure.

2. The Input-Disturbance-Output System

Let S be a hypothetical infinite list of real numbers between 0 and 1, indexed by natural numbers. For any input row x, the decimal $s_x = 0.a_1a_2a_3...$ is perturbed by modifying the x-th digit from the x-th entry. The output is constructed as $C(x) = 0.b_1b_2...b_n$, where b_x is the perturbed diagonal digit and the rest are identical to the input. This output, C(x), does not appear in the original list S and represents a local structural deviation—a disturbance slice. Each C(x) exists only at the instant of computation and forms a temporal, not static, entity. We refer to this as an 'instantaneous Cantor number'.

3. 3.5-Dimensional Topological Model

When the system generates a new Cantor number at each time step, the accumulation of such slices forms a spiral-like structure. In 3D space, this spiral cannot be closed due to non-aligned orientations. However, if we embed a 'time axis' as a geometric factor, this structure gains closure. We define this hybrid structure as 3.5-dimensional: the usual three spatial dimensions plus a half-dimension representing continuous perturbation over time. Only in this 3.5D space can the dynamic structure of Cantor numbers be fully represented.

4. Dimensional Analogy and the Concept of Half-Dimension

To validate the 3.5D structure, we propose a dimensional analogy: a circle in 2D rotates to become a dynamic object with internal evolution—effectively a 2.5D object. When observed in 3D, this becomes a sphere. Similarly, Cantor numbers constructed through time-based perturbations in 3D cannot be fully understood without introducing time as a formal dimension. Thus, they exist in 3.5D, and become static, observable objects only in 4D space.

5. Symbolic Systems and Structural Incompleteness

This model implies that incompleteness in logic and AI systems can be visualized as topological consequences of internal symbolic distortion. Gödel’s undecidable propositions, Turing’s halting problems, and AI semantic overflow can all be seen as results of persistent internal perturbations that resist closure. These are geometric echoes of systems that cannot fully contain themselves.

6. Conclusion

We have proposed a new dimensional interpretation of Cantor numbers, treating them as dynamic outputs of an input-driven perturbation system. Each construction forms a disturbance slice that can only be closed in a 3.5-dimensional topological space. This approach provides structural insight into logical incompleteness and symbolic overflow, presenting Cantor numbers not just as counterexamples, but as dynamic topological traces in time.

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This paper was written by a final-year undergraduate student from an ordinary university in China. It is the author's first attempt to share an original structural hypothesis on this platform. I sincerely thank the arXiv moderators and the broader academic community for offering the opportunity to express and preserve this personal moment of inspiration.

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