

# **Design of a holonomic five legged robot**

Final Report

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## Part 1. Preamble

This report is a description of the work I completed during the year on my final year project, Design of a holonomic five legged robot.

This report contains a copy of my approved project proposal and documentation on the technical parts of my project. These can be found in parts 3 and 4 respectively. The technical documentation contains a detailed recording of the steps taken to overcome design challenges. This includes circuit diagrams, algorithm flowcharts and test results. This section appears on the CD that accompanies this printed report.

This project does not build on any previous project. Instead it is a completely different approach to the holonomic exploration robot problem that was also addressed in earlier years. Although this project has a similar goal to that of previous years, it does not build on these as the locomotion is completely different.

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## **LIST OF ABBREVIATIONS**

<b>LED</b>	light emitting diode
<b>PWM</b>	pulse width modulation

## **Part 2. Summary**

This report documents the development of a robot intended for exploration in unknown terrain by moving holonomically and using legs for locomotion.

***What has been done***

***What has been achieved***

***Findings***

## **Part 3. Project identification: approved Project Proposal**

This section contains the problem identification in the form of the complete approved Project Proposal, unchanged from the final approved version.

## 1. Problem statement

**Motivation.** Robots used for exploration need to be highly manoeuvrable to cross terrain not possible for humans and cars. The motivation for this project is to develop a robot using five legs, capable of crossing rough terrain, that can move holonomically. A robot like this can be used to explore autonomously and can also be used to carry supplies or equipment to remote places.

**Context.** A vehicle is considered holonomic if it does not suffer from a phenomenon called the parallel parking problem [1]. This phenomenon restricts the vehicle to forward and backward movements only, while slightly turning the front wheels. The vehicle is therefore capable of moving in arcs but never sideways. One approach to achieve holonomic movement in vehicles is to swap the cylindrical wheels found on most vehicles for spherical wheels [2]. While this solution solves the parallel parking problem, it introduces a new restriction of only functioning properly on relatively flat surfaces, unless very complicated suspension mechanisms are implemented. This solution is therefore not suitable for any off-road application. This project will extend on this to solve the problem of terrain without sacrificing holonomic movement. LS3 [3] and BigDog [4] are examples of existing legged robots that function in a way that mimics the way four legged animals walk. These robots were both developed by Boston Dynamics to help humans carry loads across terrain that is not accessible by car. They can therefore follow someone on a foot trail as well as being able to navigate to a given location using GPS.

**Technical challenge.** The aim of this project is to build a five legged holonomic robot which is capable of moving in any direction from a standing position as well as being able to rotate about its own axis. The control system of the robot will consist of a processor calculating all the required joint angles of all the legs to move the legs in a way that the robot moves in the desired direction. The engineering challenge in this project is the development of a control system algorithm that is fast enough to make the robot react in real time while still being thorough enough to ensure that the robot does not damage itself or its surroundings while moving.

**Limitations.** One of the technical limitations that makes this project challenging is the trade-off considering the length of the robot legs. Longer legs will mean more manoeuvrability for the robot and faster movements but will also mean increased torque required by the servo motors. This causes a larger power consumption and therefore a shorter battery life. It is also easier to construct a larger robot but this will again eventually lead to bigger power requirements.



## 2. Project requirements

### ELO 3: Design part of the project

## Mission requirements of the product

A five legged holonomic robot will be built in this project. The requirements of the robot that would determine whether the project is successful can be summarized in the list below.

- The robot should be able to move in any direction from a stationary position.
- The robot should be able to rotate about its own axis while remaining in the same position.
- The robot should be controlled remotely by using a smartphone application.
- The robot should use five legs to execute any of the required movements.
- The robot should be able to move on both smooth and coarse surfaces.
- The robot should be able to move on both flat and slanted surfaces.

## Student tasks: design

The tasks that are vital to ensuring that the product meets the mission critical requirements are listed below.

- The mathematical analysis and design for the movement of the legs should be done on paper.
- The design should then be implemented in a graphical mathematics package such as Python for further refinement.
- Once the algorithm design is sound, electronic design can commence in a simulation environment such as LTSpice.
- The design can then be implemented in electronics using a microcontroller and support electronics together with some driving circuitry.
- A smartphone application should be developed to remotely control the robot.
- Each subsystem should be tested for isolated functionality as well as interaction with other subsystems to make sure all requirements are met.

**ELO 4: Investigative part of the project****Research questions**

The investigative section on this project will focus on the amount of legs of a legged robot and the effect on stability. Do five legs provide more stability when walking on slippery surfaces? Can the robot still work without some of its legs?

**Student tasks: experimental work**

The experiments that will be conducted to address the research questions above are listed below.

An experimental setup to test the ability of the robot to move on a variety of surfaces is required.

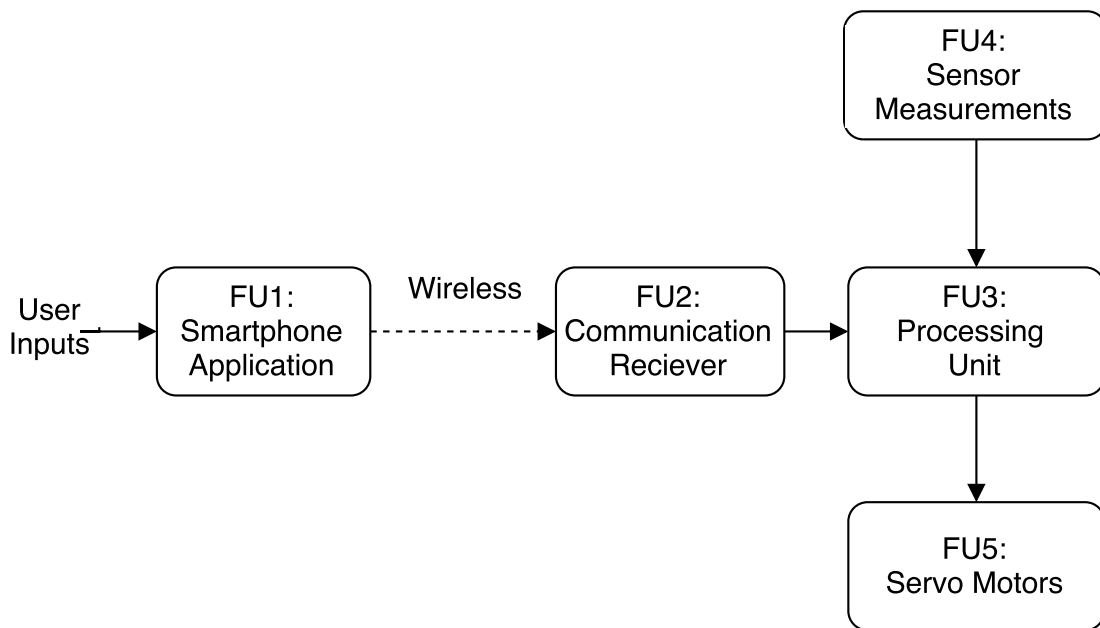
- The robot will be placed on both smooth and coarse flat surfaces to perform the same manoeuvres.
- The robot should be able to perform these manoeuvres with a similar degree of difficulty and in similar time.

Another experimental setup is required to test the robot's ability to handle small obstacles and bumps.

- The robot will be placed on a surface with bumps and small obstacles such as rocks.
- It should be able to execute the same set of manoeuvres mentioned in the experimental setup above in similar time.

### 3. Functional analysis

The functional analysis of the system can be shown best in a flow diagram. This can be seen in Figure 1 below.



**Figure 1. Functional block diagram of the product.**

In Figure 1 above, the process is shown to begin with the input from the user into a smartphone application. This consists of a user interface on the screen of the smartphone that the user can interact with. The user interface will allow the user to make the robot translate as well as rotate. This information is sent to the microcontroller on the robot via wireless technology. The robot analyses this combination of translation and rotation commands and breaks it into a separate vector for each of the legs. The final position of each leg as well as the route to each individual leg destination is computed next. This makes use of a process called inverse kinematics. The manoeuvres requested by the user can be realized through a series of movements with servo motors installed on the joints of the robot legs. In order to make a limb move, the exact required angle of each joint is calculated and communicated to the servo. This process is repeated continuously to make the robot react to the varying inputs of the user.

#### 4. Specifications

### Mission-critical system specifications

<b>SPECIFICATION (IN MEASURABLE TERMS)</b>	<b>ORIGIN OR MOTIVATION OF THIS SPECIFICATION</b>	<b>HOW WILL YOU CONFIRM THAT YOUR SYSTEM COMPLIES WITH THIS SPECIFICATION?</b>
The robot should be able to move in a given direction in 30 degree increments from a stationary position.	Proving that the robot can move straight forward, left, backwards and right (directions spaced 90 degrees apart), will prove that the robot is holonomic.	The robot will be placed on a grid on the floor and instructed to move in one of the chosen directions at a time.
The robot should be able to rotate 90 degrees without translating the centre of the body by more than 5% of the body diameter.	Rotating 90 degrees proves that the robot does not suffer from the parallel parking problem. 5% of the robot diameter can still be considered negligible and prove that no translation is required to rotate.	The robot will be placed on the grid on the floor and instructed to rotate. The translation of the centre of the body will be noted.
The time it requires to complete a manoeuvre should not vary more than 25% between smooth and rough surfaces.	A variation of less than 25% in time can still be considered to be a similar time. This proves that the robot can move over different surfaces with similar difficulty.	The robot will be instructed to complete a specific set of movements on a flat smooth surface and the time to completion will be recorded. Various obstacles will be placed in the way of the robot (small blocks, sand, etc.), and the robot will be instructed to repeat the specific set of instructions and the time difference will be noted.
The robot should be able to walk at a speed of at least 100mm/s.	100mm/s is a reasonable speed for the scale of the robot. This proves that all of the subsystems function together well.	The robot will be instructed to walk at maximum speed in a straight line for one meter. The time to completion will be measured.

The robot should be able to walk on a surface with a 10% incline without falling over.	An incline of 10% proves that the robot platform is stable while not putting too much strain on the servo motors.	The robot will be placed on a slanted surface and instructed to translate and rotate. It should not fall over while performing these manoeuvres.
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**Table 1. Mission-critical system specification**

## Field conditions

REQUIREMENT	SPECIFICATION (IN MEASURABLE TERMS)
The robot should be in range of the wireless smartphone controller.	For wireless communication to work reliably, the user with the smartphone should be less than 10m from the robot.
To prevent falling over, the robot should walk on surfaces close to horizontal.	The robot should not be operated on surfaces with an incline of more than 10%
The robot should stay dry to protect electronics.	The robot should never be operated near water or wet surfaces.
The robot should work in normal temperature conditions for South Africa.	The robot should be operated in the temperature range 10°C to 40°C.

**Table 2. Field conditions**

## Functional unit specifications

SPECIFICATION	ORIGIN OR MOTIVATION
FU1. The smartphone application should communicate commands from the user interface to the robot at a frequency of at least 10 Hz.	The robot should be in constant communication with the smartphone application to update the trajectory vectors. An update frequency of 10 Hz will update the robot fast enough to make the robot feel responsive.

FU2. The inverse kinematics calculator of the robot should be able to calculate the joint positions correctly to move each leg to the desired location.	The joints should all be calculated correctly in order for the robot to make the correct set of movements to walk.
FU3. The servo motor angles should all be within 5 degrees from the calculated values.	The servo angles have to be accurate in order to make the robot walk predictably. A tolerance of 5 degrees can still be considered accurate for hobbyist servo motors.

**Table 3. Functional unit specifications**

## 5. Deliverables

### Technical deliverables

DELIVERABLE	DESIGNED AND IMPLEMENTED BY STUDENT	OFF-THE-SHELF
Microcontroller for control of the robot.		X
Control code for inverse kinematics and servo control.	X	
Android application with a user interface for control of the robot.	X	
Wireless module for communication between the smartphone interface and the robot		X
Circuits implemented on PCB for interfacing all hardware with the microcontroller.	X	
Servo motors for movement of the joints.		X
Robot body and legs.	X	
Simulations on all implemented software and analogue design	X	

**Table 4. Deliverables**

### Demonstration at the examination

1. The robot will be placed on a grid on the floor and the examiners will be shown that the robot is capable of holonomic movement.
2. The centre of the robot will be noted on the grid and the examiners will see that the robot is capable of rotating around its centre without translating.
3. The robot will execute a specific set of movements while the time to completion is being recorded.
4. Small obstacles will be placed in the way of the robot to make movement more challenging. This includes small blocks to step over as well as a change in surface such as sand.
5. The robot will repeat the set of movements over the obstacles while the time to completion is measured again.

6. The examiners will see that the robot is capable of moving with similar effort over various surfaces.



**6. References**

- [1] J. Reeds and L. Shepp, “Optimal paths for a car that goes both forwards and backwards”, *Pacific Journal of Mathematics*, vol. 145, 1990.
- [2] K. Tadakuma, R. Tadakuma and J. Berengeres, “Development of holonomic omnidirectional vehicle with omni-ball: Spherical wheels”, *Intellegent Robots and Systems*, vol. 2007, 2007.
- [3] B. Dynamics. (2012). Ls3 - legged squad support systems. Accessed 2017-04-12, [Online]. Available: <http://www.bostondynamics.com/robot-ls3.html>.
- [4] M. Raibert, K. Bankespoor, G. Nelson and R. Playter. (2008). Bigdog, the rough-terrain quaduped robot. Accessed 2017-04-12, [Online]. Available: <http://www.bostondynamics.com/img/BigDog-IFAC-Apr-8-2008.pdf>.

## **Part 4. Main report**

# 1. Literature study

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## 1.1 Background and context

## 1.2 Application of background to this project

In this section of the report, a brief overview of the existing research that is relevant to this project will be given, as well as a short discussion on how this existing research will be used to aid in the design of a five legged holonomic robot during the course of this project.

Legged vehicles are preferred to wheeled vehicles for exploration and rescue in remote areas because of their ability to cross rough terrain quickly with a severely reduced risk of getting stuck. The price to pay for this superior drive-train is far more complex electronics and movement algorithms [1]. Instead of just driving the wheel motors and steering, each limb actuator of each leg has to be controlled to move to a calculated position.

Inverse kinematics is the mathematical process used to calculate the required angles of the limbs of a structure to reach a specific set of coordinates. This is used extensively in this project because of its ability to easily transform desired Cartesian coordinates into the required actuator angles. In the paper [2], a method is discussed for solving the inverse kinematic equations involved in a seven degrees of freedom robotic arm. This method takes into account specified minimum and maximum values for each actuator and degree of freedom in order to avoid self-collision. This method can be applied to any system with a specified degrees of freedom and can therefore be simplified to solve the inverse kinematic equations for the system designed in this project.

To obtain the Cartesian coordinates desired at any given time, the method described in [1], namely Sine pattern methods is used. In [1], the method was applied to a quadruped robot. In the case of this project, the method is expanded to make it suitable for use on a robot with five legs. Adaptation is required because the method relies on the symmetry of a quadruped robot to schedule the lifting of the legs. Since the same symmetry does not exist in a robot with five legs, a different scheduling technique is investigated. This method will rely on information from the current position of the legs, the limits of leg movement, as well as the current horizontal tilt of the robot.

In the conference paper [3], motion planning of omnidirectional robots is discussed and a sophisticated yet simple and efficient method of route planning is proposed. This method is based on vehicles that use three omnidirectional wheels in combination to form a resultant force vector in the desired direction. These type of vehicles differ largely in terms of locomotion when compared to the holonomic legged design used in this project, but there are a few key similarities. Both these designs can move holonomically, therefore they have the ability to move in a straight line while rotating, move in an arc without rotating, or any combination of the two. These similarities mean that some of the methods discussed and applied in [3] can be used to aid in the design of the algorithm used in this project.

A team from Instituto Tecnológico de la Laguna in Mexico designed a hexapod robot [4] similar to the robot designed in this project. The purpose of the hexapod was to investigate its potential for use in exploration of areas that are hard to reach by any commonly used means of transportation. Legged vehicles are more suited to cross rough terrain, but rough terrain complicates the design of the drive-train. In applications where the surface is smooth or close to smooth, open loop control can be applied where the leg is simply moved to the desired position and it is assumed by the designer that the robot foot is making contact with the ground at this point, and therefore supporting part of the distributed robot weight. When the robot is crossing rough terrain, where the surface consists of mainly bumps and holes, this assumption could be false. In such a scenario, the robot could lift another leg while under the impression that its weight is being supported by the other legs. If this is not the case, the robot could fall over and possibly damage itself or be unable to rectify itself. To avoid this problem, closed loop control should be used in the height positioning of the legs. This involves having a sensor in the system that could provide feedback on the state of the foot. The hexapod in [4] used miniature resistive force sensors attached to the bottom of each foot of the robot. The robot therefore has the ability to take analogue measurements from these sensors and determine the weight distribution of the individual feet of the robot. This data is used to confirm that all robot feet are making contact with the surface and correct the situation if this is not the case. This type of feedback is also useful when the robot is operated on slanted surfaces because of the effect that the center of gravity has on a slanted surface. A feedback sensor will be included in the design of the five legged holonomic robot in this project.

If the robot is operating on a slanted surface without it being aware of this and the centre of gravity shifts over the lowest foot making contact, the robot could fall over even when all of its legs are making contact with the surface. In a paper on reactive robot navigation [5], it is proposed that the use of a digital inclinometer can aid in solving this problem. The sensor provides information on the current tilt of the robot in two dimensions. This sensor in combination with the feedback sensors on the robot feet can be used to ensure that the robot levels itself automatically to avoid tipping over. The data collected from this sensor can also be used to collect information on the terrain. In the journal article [5], this data is used for hill climbing as well as finding valleys in unexplored areas. In order to enable the robot designed in this project to walk on slanted surfaces, a digital inclinometer will be used. Some of the

reactive navigation techniques discussed in [5] will also be implemented to aid the robot in navigating on slanted planes.

In a paper on the effects of slippery surfaces on biped robots [6], methods on avoiding falling over of a biped robot is investigated. Since these robots have to balance themselves to stay upright, an unforeseen slippery patch on a surface could be fatal for the robot. If it were possible to foresee a slippery surface, slowing down the walking gait and increasing foot surface would help increase the traction of the robot, and therefore lower the risk of slipping. Since a five legged robot is inherently stable and there is no balancing required, slippery surfaces may influence the traction of the robot but there is very low risk of falling over on level surfaces that are slippery. It is therefore suitable to just slow down movements on slippery surfaces to increase the traction where possible.

## 2. Approach

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This section outlines the initial approach to solve the functions shown in the functional block diagram. The functional block diagram of the project can be found in Figure 1 in Part 3 of this document.

The core of the functional block diagram is the processing unit which is implemented in the form of a 32 bit microcontroller with an algorithm embedded in firmware. The microcontroller that will be implemented in the final design will be determined by the amount digital inputs and outputs (IO) required as well as the amount of mathematical equations required per second.

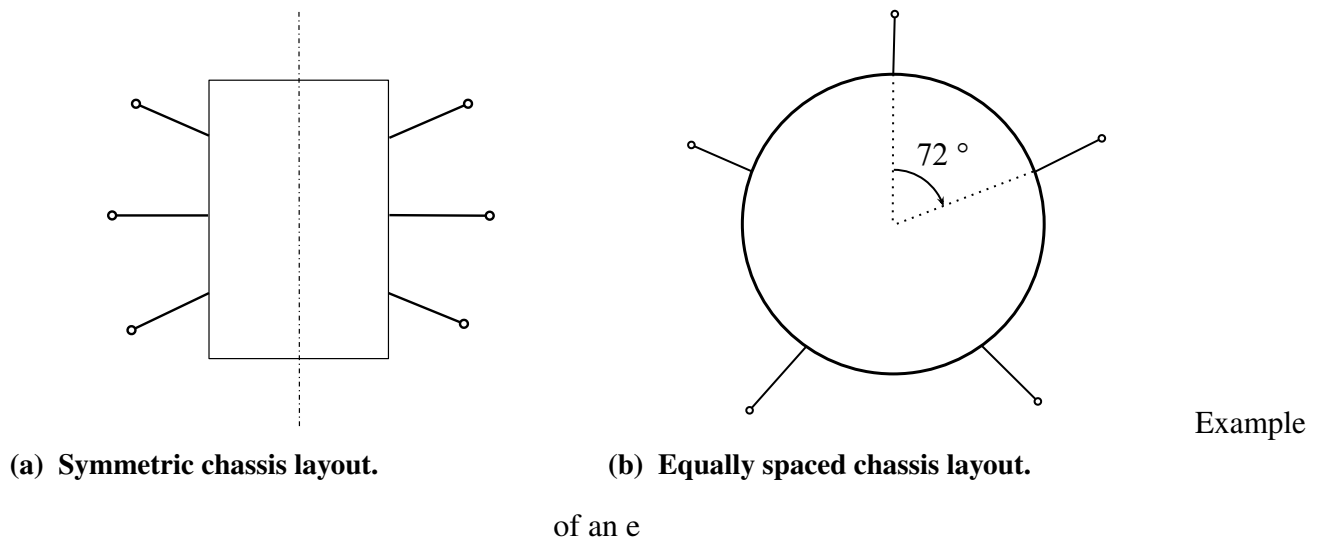
The microcontroller will act on instructions received through a wireless communication channel that originate from a smartphone application. The application will only act as a user interface for the robot and will output basic instructions for the robot to follow.

### 2.1 Design alternatives

The two wireless technologies built into most modern smartphones that best suit the needs of the communication channel is Wi-Fi and Bluetooth. Journal article [7] makes some comparisons between these two similar but different technologies. Wi-Fi has a much higher bandwidth than Bluetooth as well as far superior range. Wi-Fi networks are generally much more complex than Bluetooth networks. Bluetooth is also better suited for peer to peer communication whereas Wi-Fi is mostly intended to be used with a router. Network security for Wi-Fi is much more advanced and therefore much more secure than that of Bluetooth.

After the commands are passed from the smartphone to the microcontroller, the microcontroller interfaces to actuators to execute the movements desired by the user. When considering possible actuators that can be used, three possible solutions come to mind. These are servo motors, geared DC motors and stepper motors. Geared DC motors are small, light and easy to implement but lack feedback on shaft position. Such feedback needs to be implemented manually to be able to make controlled movements. Stepper motors have fixed step resolution so making controlled movements is easier once the current position is known. The drawback of these motors is that a homing mechanism and routine needs to be implemented to find a reference. Stepper motors are also heavy and require high current even when stationary. Servo motors are basically geared DC motors with a positional feedback control system built-in. The motor only receives power and a desired position and it will attempt to reach the position. The disadvantage is that these have a limited range of motion.

The overall shape of the robot and, more specifically, the placement of legs around the robot will greatly influence the robot's performance when moving forward in a straight line, moving



**Figure 2. Examples of two different chassis layout options for legged robots.**

sideways and moving over obstacles.

A popular design in hexapod robots is placing the legs in groups of three on opposite sides. This means that the legs are all aligned in a similar direction while the body is usually a long rectangular shape, similar to that found on insects. The advantage of this design lies mainly in the ability to move forward very quickly since the legs are placed optimally for forward motion. While holonomic movement is possible, it is usually much slower than forward or backward motion. This design is usually more suited to robots with an even number of legs due to the symmetry of the design.

An alternative design approach is to space the legs evenly around the robot chassis. The robot chassis would normally be round or a polygon with the same number of sides as legs. In the case of a five legged robot, the legs would be positioned  $\frac{360^\circ}{5} = 72^\circ$  apart. This equally spaced design approach has the disadvantage of not being particularly fast in any given direction but the advantage that it can manage the maximum speed in any given direction. Figure 2 shows an examples of both layouts.

## 2.2 Preferred solution

Since the data transfer in the communication channel between the smartphone and the robot consists only of simple commands and the required bandwidth is therefore low, Bluetooth will be used instead of Wi-Fi. Bluetooth has the advantage of being less expensive and far less complicated to implement. The added security that Wi-Fi offers is not required for this application. The disadvantage of using Bluetooth instead of Wi-Fi is the very limited range that Bluetooth offers.

The commands received via Bluetooth will be actuated with the use of servo motors. These compact units are ideal for this scenario as they have all of the advantages of geared DC motors with all of the required control systems built-in. Servo motors have the advantage that they are usually light for the torque they can provide, which is ideal for a robot where weight is often a problem.

This will all be implemented in a circular shaped chassis with equally spaced legs as this design is better suited for holonomic movements than the symmetric design.



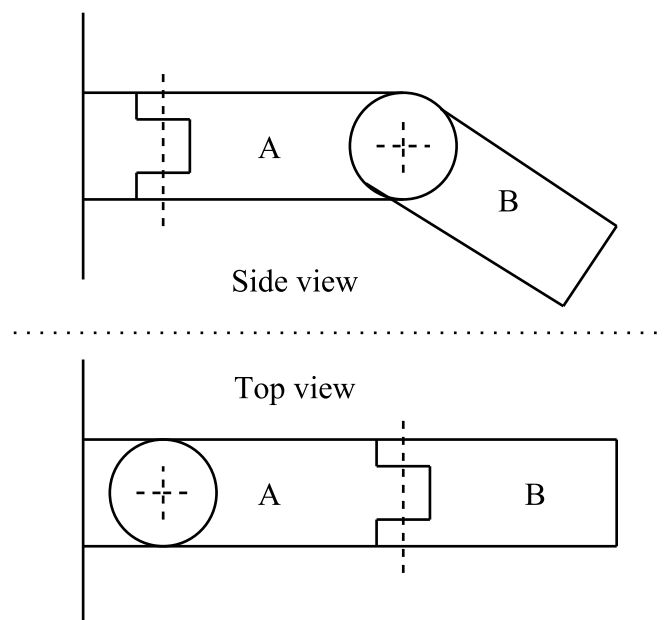
### 3. Design and implementation

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#### 3.1 Background

A large part of the design of a legged robot depends on the design of a simple leg - more specifically the degrees of freedom that a single leg has. The degrees of freedom that a single leg has is determined by the amount of dimensions that the leg can make a controlled movement in, independently from any other dimensions. There are two common underlying designs in robot leg design - these are for two and three degrees of freedom respectively.

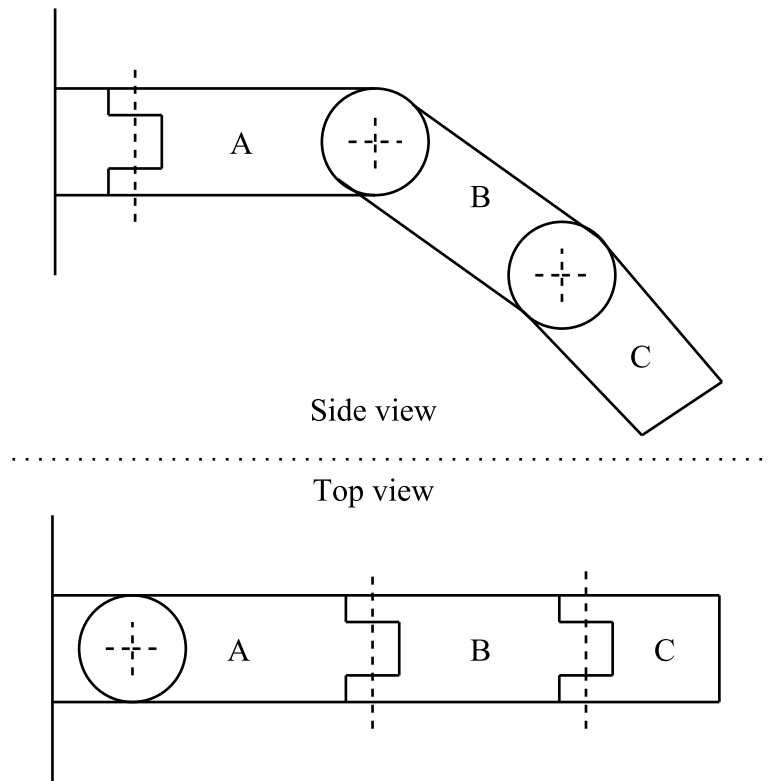
Figure 3 shows an example of a common design that has two degrees of freedom. The design therefore contains two joints in one leg. In this example, the first joint is positioned vertically to form the hip of the robot leg. This allows for the side to side motion required for walking. The second joint is positioned horizontally to allow the lowest limb to move up and down. The two degrees of freedom controlled by this leg design is therefore the horizontal angle of the leg and the height of the foot. These two (and only these two) parameters therefore can be controlled completely independently.



**Figure 3. Example of a leg design with two degrees of freedom.**

An example of a robotic leg with three degrees of freedom can be seen in Figure 4. This

design is similar to that of the leg with two degrees of freedom seen in Figure 3, with the only addition being a second horizontal joint further down from the first. This additional limb means that the horizontal distance from the foot to the hip can be controlled as well as the height of the foot. These two controlled parameters together with the horizontal leg angle which can also be controlled independently as in the case of the two degrees of freedom design means that this design has a total of three degrees of freedom.



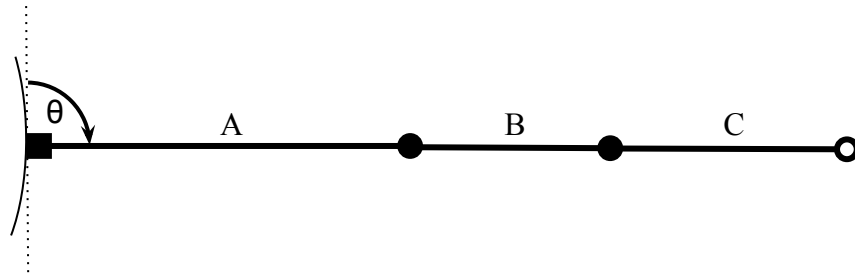
**Figure 4. Example of a leg design with three degrees of freedom.**

The design with three degrees of freedom has the advantage of being able to lift a leg without altering the horizontal position of the leg. This means that the robot is able to walk without altering the height of the body of the robot. This means that the robot is able to much better cross rougher terrain because of the ability to alter the height the robot feet as the terrain requires. With the design that has two degrees of freedom, the foot height is a function of the horizontal extension of the leg. With this design the main advantage is the simplicity - both mechanically and in software. The cost and power consumption will also be much lower because of the reduced amount of actuators.

Due to the much greater flexibility of the design with three degrees of freedom and the ability to cross rougher terrain, this will be the platform implemented in the final design.

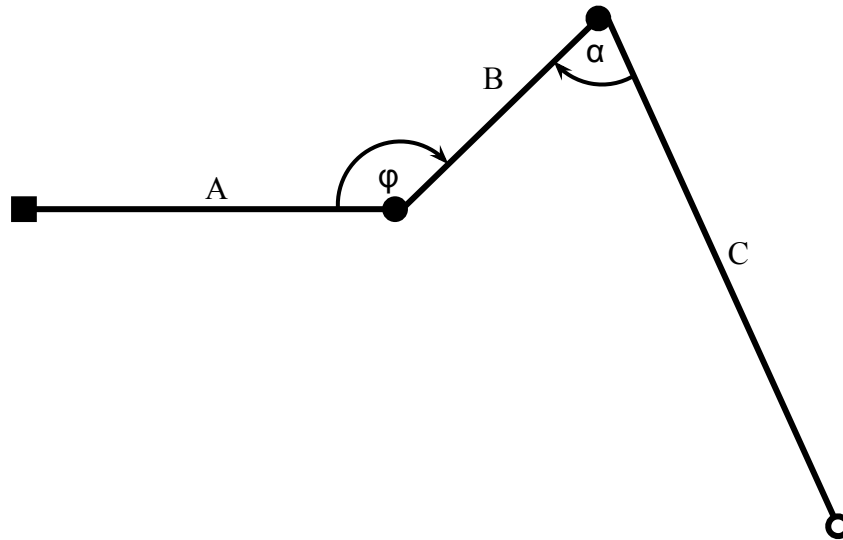
### 3.2 Theoretical analysis and modelling

Since each leg has three degrees of freedom, there are three angles that need to be controlled in order to move the leg to a desired position. These will be referred to as theta ( $\theta$ ), phi ( $\phi$ ) and alpha ( $\alpha$ ) throughout this document. Theta describes the horizontal angle between the tangent of the body and leg segment A as shown in Figure 5.



**Figure 5. Top view of the leg design model for mathematical analysis.**

Phi describes the angle between leg segments A and B viewed directly from the side. In the same plane, alpha describes the angle between leg segments B and C. Both of these angles can be seen in Figure 6.



**Figure 6. Side view of the leg design model for mathematical analysis.**

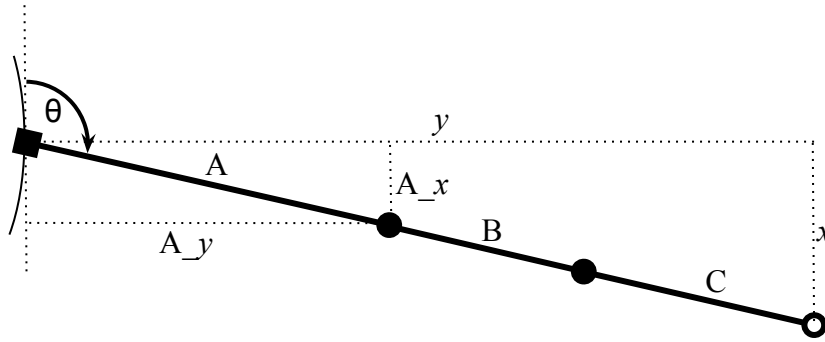
In order to control the robot in a Cartesian world it is necessary to find a transfer function for a leg that can translate a coordinate vector in the domain  $[\theta, \phi, \alpha]$  to a vector in the domain

$[x, y, z]$ . This is done through mathematical analysis of a single leg.

By using Figure 5 and some simple trigonometry, theta can be found to be

$$\theta = \arctan\left(\frac{x}{y}\right) \quad (1)$$

where  $x$ ,  $y$  and  $z$  are the desired coordinates for the foot of the relevant leg. It is important to note that the hip of the leg where section A meets the chassis is the origin of the Cartesian system for this analysis. This can be seen in Figure 7

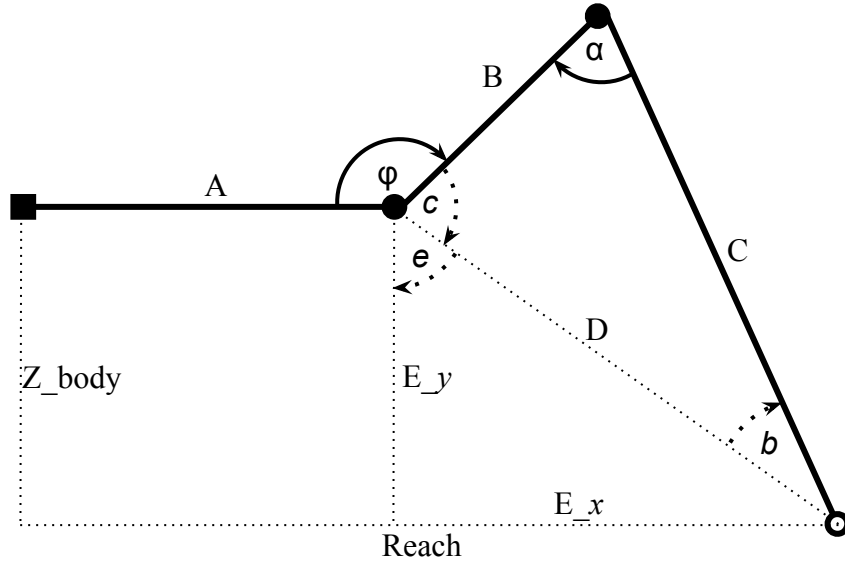


**Figure 7. Annotated top view of the leg design model for mathematical analysis.**

In order to do an analysis on phi and alpha, it is necessary to add a few construction lines to the generic model in Figure 6. The additions can be seen in Figure 8 and the additions are explained below.

From Figure 8:

- $Z_{body}$  is the nominal height difference between the robot chassis and ground. This value is a constant.
- $Reach$  is the horizontal distance between the hip and the foot.
- $D$  is an imaginary line between joint 2 and the foot that completes the triangle  $BCD$  that will form the basis of this analysis.
- $E_x$  is the horizontal distance between joint 2 and the foot.
- $E_y$  is the horizontal line that completes triangle  $DE_xE_y$ . Quantitatively  $E_y = Z_{body}$  since the ground is assumed to be level in this analysis.
- $c$  is the angle between  $B$  and  $D$ .
- $b$  is the angle between  $C$  and  $D$ .



**Figure 8. Annotated side view of the leg design model for mathematical analysis.**

- $e$  is the angle between  $E_y$  and  $D$ .

$$\therefore Reach = \sqrt{x^2 + y^2} \quad (2)$$

By using triangle  $BCD$  and the Cosine rule,

$$\cos(\alpha) = \frac{B^2 + C^2 - D^2}{2 \times B \times C} \quad (3)$$

$$\therefore \alpha = \cos^{-1} \left( \frac{B^2 + C^2 - D^2}{2 \times B \times C} \right). \quad (4)$$

The value of  $E_x$  can be calculated similar to that of reach, but first finding the  $x$  and  $y$  component lengths of segment A as shown in Figure 7.

$$A_x = A \times \sin(\theta) \quad (5)$$

$$A_y = A \times \cos(\theta) \quad (6)$$

$$E_x = \sqrt{(x - A_x)^2 + (y - A_y)^2} \quad (7)$$

The Sine rule can then be used on triangle  $DE_xE_y$  to find,

$$\frac{\sin(c)}{C} = \frac{\sin(\alpha)}{D} \quad (8)$$

$$\therefore \sin(c) = C \times \frac{\sin(\alpha)}{D} \quad (9)$$

$$\therefore c = \sin^{-1}\left(C \times \frac{\sin(\alpha)}{D}\right). \quad (10)$$

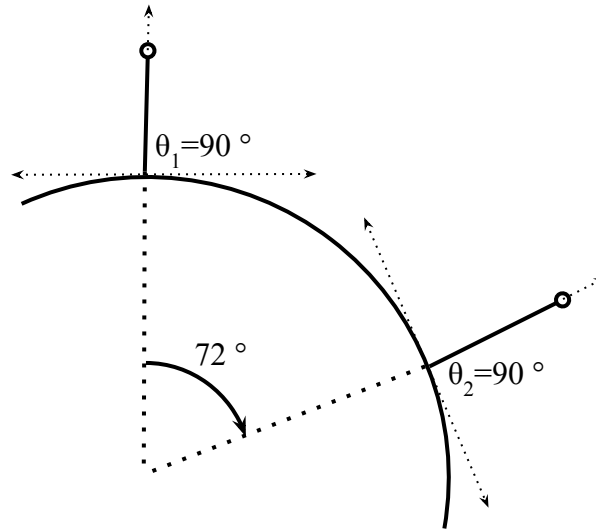
Angle  $e$  can easily be found as

$$e = \tan^{-1}\left(\frac{E_x}{E_y}\right). \quad (11)$$

With all of this calculated, phi can be calculated as

$$\phi = 270^\circ - c - e. \quad (12)$$

The analysis above is valid for the generic 3 segment design shown in Figures 5 and 6 where the hip is at the origin of the Cartesian system. The real robot has five legs that need to be calculated for. In order to avoid using five different Cartesian systems, it is necessary to be able to rotate and translate this generic model to suit the requirements of any of the legs.



**Figure 9. Top view of the robot showing individual Cartesian systems for individual legs.**

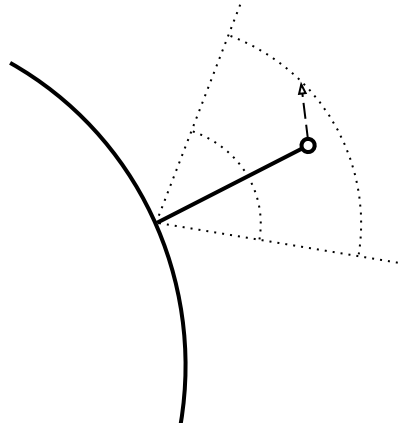
Figure 9 illustrates that both leg 1 and 2 have a theta angle of  $\theta = 90^\circ$ , yet they are clearly not facing the same direction. This is because of this separate coordinate system that each of the legs have in order to do the inverse kinematic calculations. In order to do calculations on the whole robot, the centre of the robot is used as the origin.

If  $leg$  denotes the number of the desired leg, a leg's coordinate system can be rotated by using

$$(x', y') = (x \times \cos(\gamma) - y \times \sin(\gamma), x \times \sin(\gamma) + y \times \cos(\gamma)) \quad (13)$$

$$\text{where } \gamma = (leg - 1) \times 72^\circ. \quad (14)$$

For the robot to be able to walk, all five legs should be working together to move in a specific direction. The use of vectors work well since they can be added easily and contain both direction and magnitude information. The movement of each foot can be constrained for each of the bends ( $\theta, \phi, \alpha$ ) to limit the angle to a realistic value for the servo. These limitations can be seen in Figure 10 as dotted lines. The dashed line represents the vector for movement. The movement vector is limited by the boundary.



**Figure 10. Top view of the robot showing the addition of vectors and leg boundaries.**

All of the legs move together towards a point inside or on the edge of each individual perimeter. The direction of movement of the robot can be calculated as

$$angle = \arctan\left(\frac{y}{x}\right), \quad (15)$$

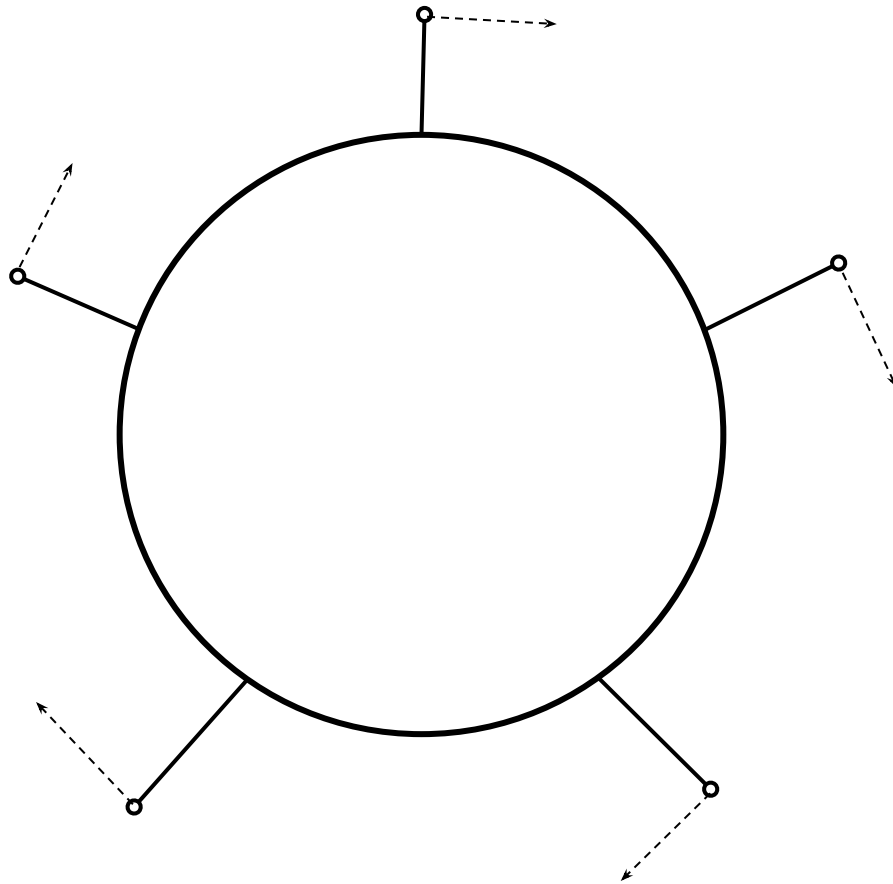
where  $x$  and  $y$  denote the input from the user interface. The magnitude can be found to be

$$magnitude = \sqrt{x^2 + y^2}. \quad (16)$$

The magnitude and direction can be used to check if the vector fits inside the perimeter.

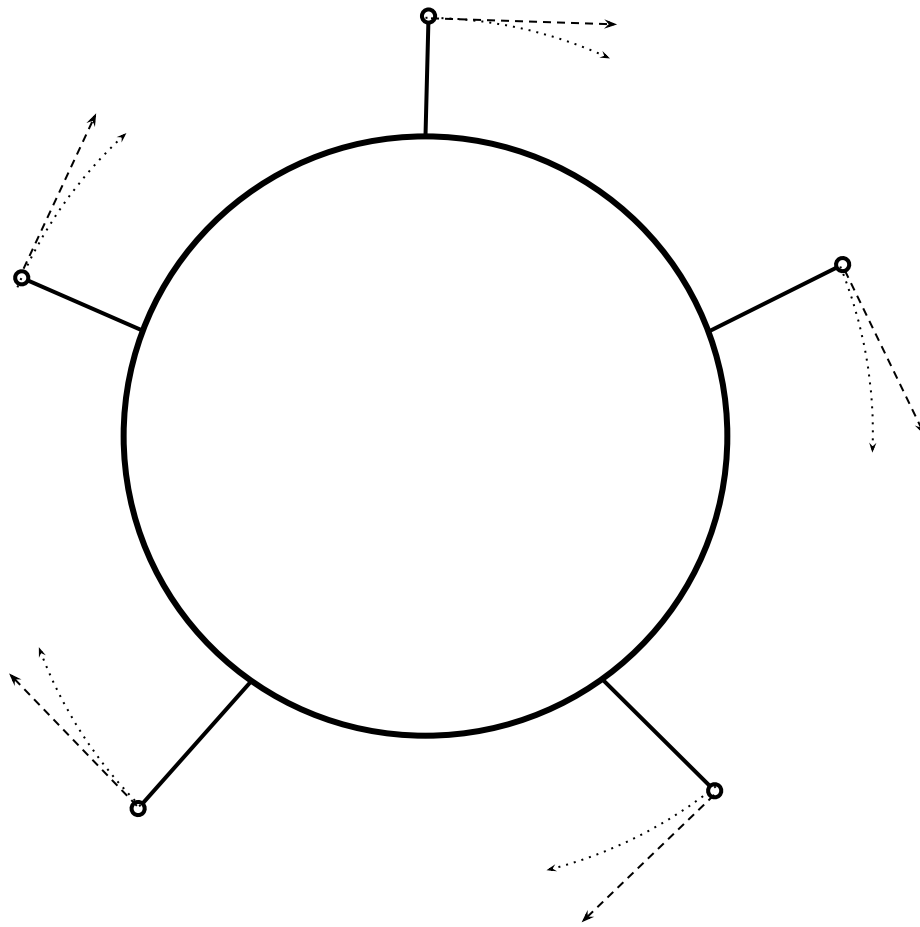
All of the calculations above only account for translation, that is moving forward, backward, left, right or a combination of the above. Being able to move truly holonomically means being able to rotate with or without translating. The vectors used to calculate the trajectory of each leg works well for representing translation but does not work so well for rotation. Rotation can instead be expressed as a scalar value, which has magnitude and a sign indicating the direction of rotation. A positive value indicates a clockwise rotation. When these rotation vectors are applied to the individual legs, it results in a different vector being added to each leg indicating the trajectory required by each leg to rotate the robot body. Figure 11 shows the effect of adding a rotation scalar to the movement. The dashed arrows indicate the trajectory of each individual leg.





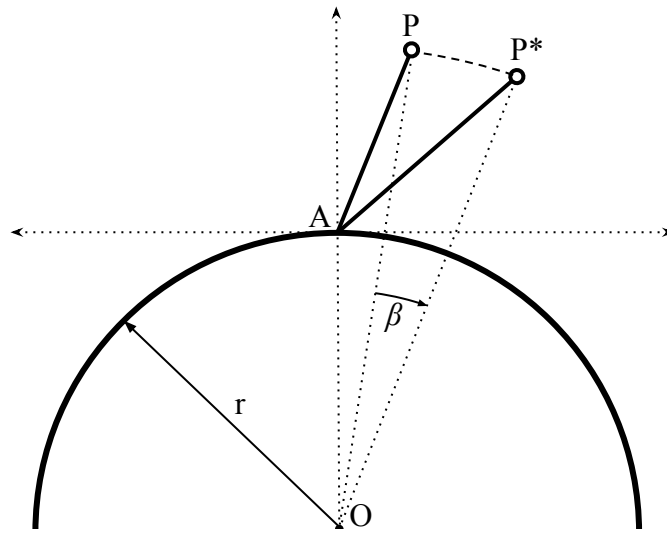
**Figure 11. Top view of the robot showing the addition of vectors for rotation about the origin.**

If each leg were instructed to follow the dashed arrows shown in Figure 11, the legs would fight each other for grip since they vectors will force them to spread out. Since this is not a controlled movement. Simply using the instantaneous trajectory as the movement vector will not work like it did in the case of translational movement. Instead the trajectory will constantly be changing to keep rotate the body while keeping the feet in their positions. This change can be seen in Figure 12. The dotted lines represent this constantly changing vector. This can be calculated by rotating the foot position around the origin of the robot. It is important to note that although the legs will be spreading out as the robot body rotates, the feet should stay exactly where they are until the foot is out of the perimeter that the system allows. At this point the robot will lift up the leg and place it in the neutral position to allow further rotation.



**Figure 12. Top view of the robot showing the addition of curved approximation for rotation about the origin.**

Since a curve can not be added to the vector of translation, the curve will need to be broken up into small segments of straight pieces approximating the curved line. These straight pieces can then be added to the translation as vectors. Rotating the coordinates around the robot is not a simple task because of the shifting Cartesian systems. In order to rotate about a point that is not the origin of the plane, the coordinate system needs to be translated to move the origin, rotated around this origin and then shifted back to the original origin.



**Figure 13. Illustration showing the procedure of rotation about the origin.**

Figure 13 attempts to explain the robot rotation procedure. The following features appear on the sketch:

- Point  $O$  is the origin, the centre of the robot.
- Point  $A$  is the shoulder of the relevant leg.
- Point  $P$  is the position coordinate of the foot before any rotation.
- Point  $P^*$  is the position coordinate of the foot after rotation.
- $\beta$  is the rotation angle.
- $r$  is the robot radius.
- The dashed line is the rotation path of the foot.

When the robot needs to rotate, the procedure is therefore as follows.

1. Start with coordinates of point  $P$ , relative to point  $A$  which is the origin of the Cartesian system at this point.
2. Add the coordinates of  $P$  relative to  $A$  to the coordinates of  $A$  relative to  $O$ . The result is the coordinates of  $P$  relative to  $O$ .
3. Rotate through angle  $\beta$  using the rotation formula used in Equation 13.
4. Move the origin back to  $A$  by subtracting  $A$  relative to  $O$ .

The result can be used to perform the Inverse Kinematic calculations as described in Equations 1 through 12

### **3.3 Simulation**

In order to test the equations and design outlined in the Theory section above, the mathematics was implemented in a Python program consisting of a GUI to enter elementary commands similar to that implemented in the Android application, as well as a window for plotting the result in a 3-Dimensional Cartesian system. Some results showing the validity of the design equations in the previous section can be found below. All of the source code used to build this simulation can be found in

### **3.4 Optimisation**

### **3.5 Hardware design**

### **3.6 Hardware implementation**

### **3.7 Software design**

### **3.8 Software implementation**

### **3.9 Design summary**

## 4. Results

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### 4.1 Summary of results achieved

Description of requirement or specification (intended outcome)	Actual outcome	Location in report
<b>Mission requirements of the product</b>		
1	2	3
<b>Field conditions</b>		
1	2	3
<b>Specifications</b>		
1	2	3
<b>Deliverables</b>		
1	2	3

### 4.2 Qualification tests

## 5. References

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