Numerical Methods Homework-5

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1. Solve the following system:

$$\begin{bmatrix} 3+2i & 4 \\ -i & 1 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 2+i \\ 3 \end{bmatrix}$$

ANS :

1.Code_function :

```
function [X]=Cramer(A,B)
    row = size(A,1);
    col = size(A,2);
    if(row ~= col)
       fprintf('row != col')
       X = [];
       return
    end
    X = [];
    d_A = det(A);
    for j = 1:col
        A_temp = A;
        for i = 1:row
            A_temp(i,j) = B(i,1);
        end
        x_ = det(A_temp)/d_A;
        X = [X;x_];
    end
end
```

2.Code_main :

```
close all
clear all
format long
i = sqrt(-1)
A = [3+2i,4;-i,1]
b = [2+i;3]

x_ans = Cramer(A,b)
```

3.Result:

```
x_ans =

 -0.533333333333334 + 1.400000000000000i
  1.600000000000000 - 0.533333333333333i
```

2. Solve the following tridiagonal system:

$$\begin{bmatrix} 0.8 & -0.4 & 0 \\ -0.4 & 0.8 & -0.4 \\ 0 & -0.4 & 0.8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 41 \\ 25 \\ 105 \end{bmatrix}$$

ANS :

1.Code_function :

```
function [X]=Tridiagonal(e,f,g,r)
    n = size(f,2)
    X = []
    for k = 2:n
        factor = e(k)/f(k-1);
        f(k) = f(k) - factor*g(k-1);
        r(k) = r(k) - factor*r(k-1);
    end
    X(n) = r(n)/f(n);
    for k = n-1:-1:1
       X(k) = (r(k)-g(k)*X(k+1))/f(k)
    end
end
```

2.Code_main :

```
close all
clear all
format long
i = sqrt(-1)
e = [0,-0.4,-0.4]
f = [0.8,0.8,0.8]
g = [-0.4,-0.4,0]
b = [41,25,105]

[X]=Tridiagonal(e,f,g,b)
```

3.Result:

```
X =

   1.0e+02 *

   1.737500000000000   2.450000000000000   2.537500000000000
```

3. (a)Determine the LU factorization without pivoting by hand for the following matrix and check your results by validating that [L][U]=[A].

$$\begin{bmatrix} 8 & 5 & 1 \\ 3 & 7 & 4 \\ 2 & 3 & 9 \end{bmatrix}$$

ANS :

(b). Employ the result of (a) to compute the determinant.

ANS:

(c). To verify your answer, repeat (a) and (b) using MATLAB.

ANS:

1.Code_main:

```
close all
clear all
format long

A = [8,5,1;
    3,7,4;
    2,3,9]
[L,U] = lu(A)
det_A = det(L)*det(U)
```

2.Result:

```
L =

   1.000000000000000                   0                   0
   0.375000000000000   1.000000000000000                   0
   0.250000000000000   0.341463414634146   1.000000000000000

U =

   8.000000000000000   5.000000000000000   1.000000000000000
                   0   5.125000000000000   3.625000000000000
                   0                   0   7.512195121951219

det_A =

   308
```

4. One of the ill-conditioned matrices is the Vandermonde matrix, which has the following form:

$$\begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{bmatrix}$$

(a). Determine the condition number based on the row-sum norm for the case where x1 = 4 , x2 = 2 , and x3 = 7 .

ANS:

1.Code_main:

```
close all
clear all
```

```
format long

A = [4^2,4,1;
    2^2,2,1;
    7^2,7,1]
inv_A = inv(A)
row_sum_A = norm(A,inf)
row_sum_inv_A = norm(inv_A,inf)
cond_row_sum = row_sum_A*row_sum_inv_A
```

2.Result:

```
cond_row_sum =

   323
```

(b). Find the spectral and Frobenius condition numbers.

ANS:

1.Code_main:

```
close all
clear all
format long

A = [4^2,4,1;
    2^2,2,1;
    7^2,7,1]
inv_A = inv(A)
%spectral
sp_A = norm(A,2)
sp_inv_A = norm(inv_A,2)
cond_spe = sp_A*sp_inv_A
cond_2 = cond(A,2)
%Fro
Fro_A = norm(A,'fro')
Fro_inv_A = norm(inv_A,'fro')
cond_Fro = Fro_A*Fro_inv_A
cond_Fr = cond(A,'fro')
```

2.Result:

```
cond_spe =

     2.161293589504254e+02
```

```
cond_Fro =

     2.174842523034714e+02
```