Numerical Methods Homework-2

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1 Find the Taylor series for f (x) = x3 −10x2 +6 about x =5.

ANS :

1.Result :

在 x=5 處展開

f(5) = 3*125 – 10*25 + 6 = -119

$f^{(1)}(5)$ = 3*25 – 20*5 = -25

$f^{(2)}(5)$ = 6 *5 – 20 = 10

$f^{(3)}(5)$ = 6

f(x) = -119 + (-25)*(x-5) + (10/2!)*(x-5)^2 + (6/3!)*(x-5)^3

2 Show that $\frac{de^x}{dx} = e^x$ from the viewpoint of Taylor expansion.

ANS :

1.Result :

$e^x = 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \ldots + \frac{1}{n!}x^n$ ， $n \to \infty$

$\frac{de^x}{dx} = 0 + 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \ldots + \frac{1}{n-1!}x^{n-1}$ ， $n \to \infty$

因此 $e^x = \frac{de^x}{dx}$

3(a) Find the Taylor series of sin(x) , and then plot the error as a function of the number of iteration.

ANS :

1.Code_function :

Sin_taylor.m

```
function [result] = sin_taylor(x,n)
    sign = 1;
    result = 0;
    fac = 1;
    x_m = x;
    for i = 1:n
        result = result + sign.*x_m./fac;
        fac = fac*(2*i)*(2*(i)+1);
        sign = sign*-1;
        x_m = x_m .* x .* x;
    end
end
```

2.Code_main :

```
close all
clear all
```

```
format long
iter_n = 30;
posi_x = 1;
error = [];
for i = 1:iter_n
        result = sin_taylor(posi_x,i);
        real_r = sin(1);
        error = [error,real_r - result]
end

plot(1:30,error,'x-')
hold on
set(gca,'xtick',-1:2:30)
set(gca,'ytick',-1:0.1:+1)
ylim([-0.5,0.5])
```

3.Result :

圖(3-a)

3(b) Also find the Taylor series for the even function cos(x) , and then plot it as family curves ranging from n=0 to n=4.

ANS :

1.Code_function :

cos_taylor.m

```
function [result] = cos_taylor(x,n)
        sign = 1;
        result = 0;
        fac = 1;
        x_m = x.^0;
        for i = 1:n
             result = result + sign.*x_m./fac;
             fac = fac*(2*i)*(2*(i)-1);
             sign = sign*-1;
             x_m = x_m .* x .* x;
        end
end
```

2.Code_main :

```
close all
clear all
format long
iter_n =4;
posi_x = -2*pi:0.1:2*pi
M = [];
for i = 1:iter_n
        result = cos_taylor(posi_x,i);
```

```
        plot(posi_x,result,'-')
        M = [M;'iter = '+string(i)]
        hold on
    end
    real_r = cos(posi_x);
    plot(posi_x,real_r,'k--')
    M = [M;'real cos']
    legend(M)

    y_axis = 5;
    x_axis = 7;
    set(gca,'xtick',-x_axis:1:x_axis)
    xlim([-x_axis,x_axis])
    set(gca,'ytick',-y_axis:0.5:+y_axis)
    ylim([-y_axis,y_axis])
    xlabel('x')
    ylabel('cos(x)')
```

3. Result :

圖(3-b)

3(c) Plot the real part and imaginary part of the complex function f (x, y) = e^{x+iy}.

ANS :

1. Code_main :

```
close all
clear all
i = sqrt(-1)
func =@(x,y) exp(x+i.*y);

x_range = -pi:0.1:pi;
y_range = -2*pi:0.1:2*pi;
[xx,yy] = meshgrid(x_range,y_range);
zz = func(xx,yy);
z_r = real(zz);
z_i = imag(zz);
figure(1)
mesh(xx,yy,z_r)
xlabel('x')
ylabel('y')
zlabel('z_r')
figure(2)
mesh(xx,yy,z_i)
xlabel('x')
ylabel('y')
zlabel('z_i')
figure(3)
mesh(xx,yy,sqrt(z_r.^2+z_i.^2))
```

```
xlabel('x')
ylabel('y')
zlabel('radius')
figure(4)
mesh(xx,yy,atan(z_i./z_r))
xlabel('x')
ylabel('y')
zlabel('agnle')
```

2.Result :

實部圖(3-c)、虛部圖(3-d)、絕對值長度圖(3-e)、角度圖(3-f)

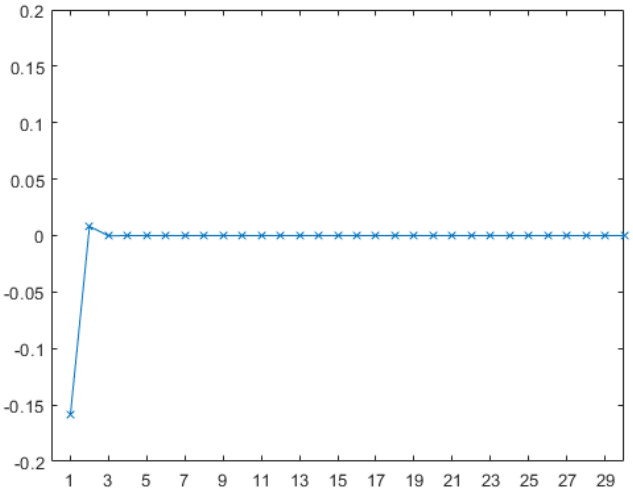

圖 3-a Taylor_sin error of sin(1)

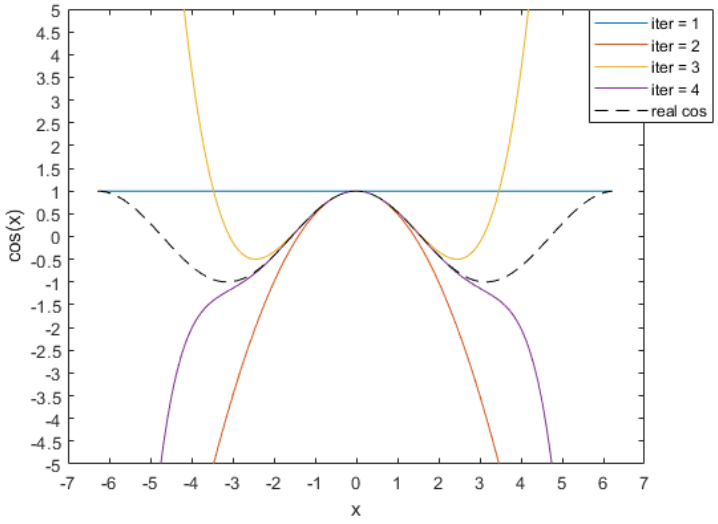

圖 3-b Taylor_cos

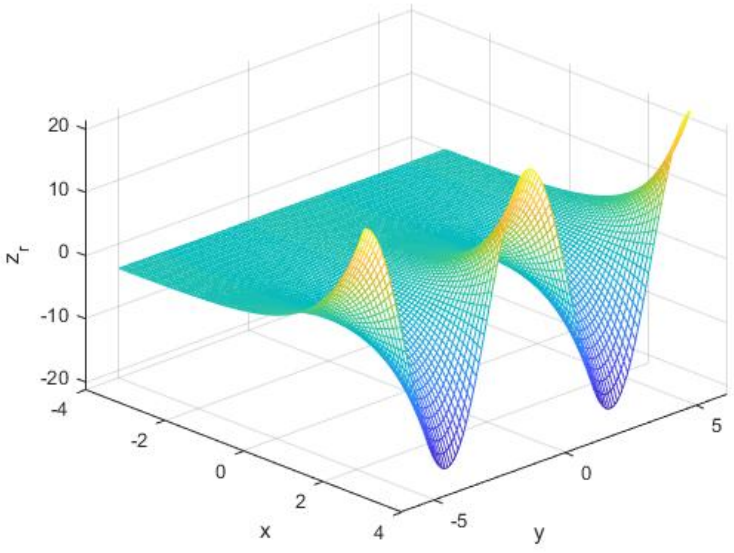

圖 3-c real part of e^{x+iy}

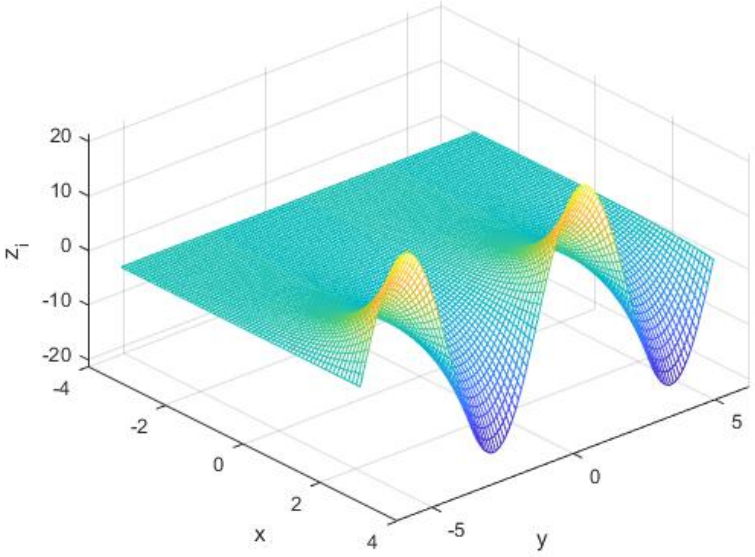

圖 3-d imaginary part of e^{x+iy}

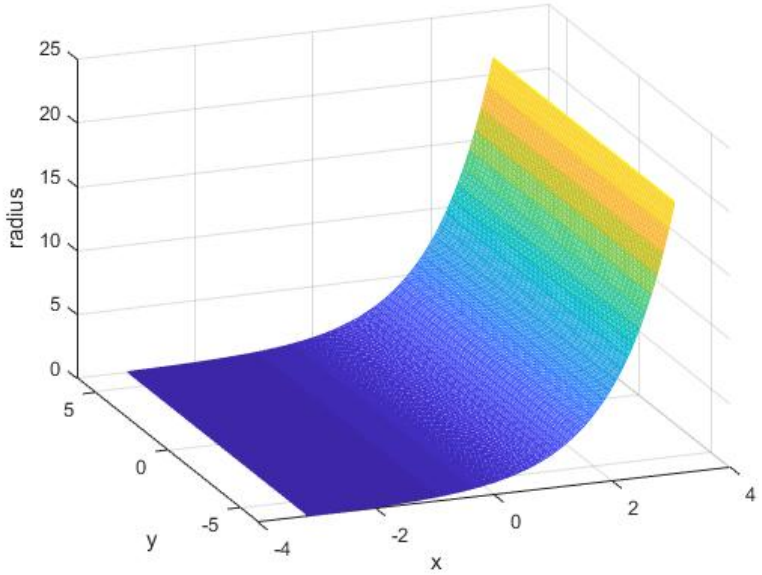

圖 3-e absolute value of e^{x+iy}

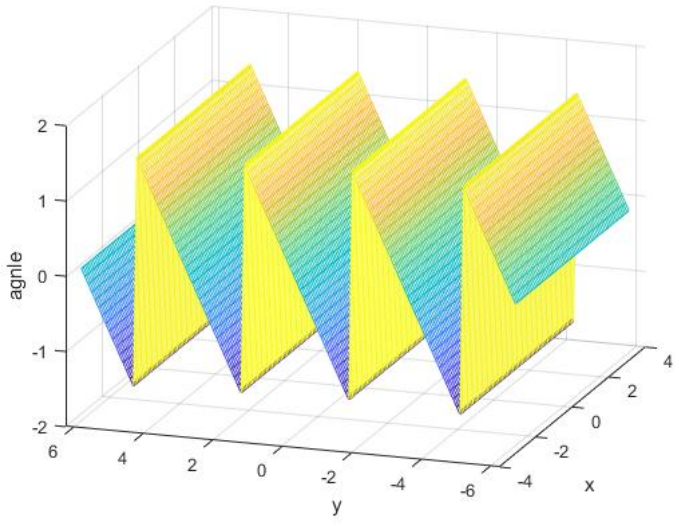

圖 3-e angle of e^{x+iy}