Numerical Methods Homework-6

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1.

Find the directional derivative of:

$$f(x,y) = 2x^2 + y^2$$

at $x = 2$ and $y = 4$ in the direction of $h = 3i + 2j$.

ANS :

1.Code_main :

```
close all
clear all
format long

f =@(x,y) -(2.*x.^2+y.^2)
[xx,yy]=meshgrid(-1:0.1:7);
mesh(xx,yy,f(xx,yy))
%% gradient
delta = 1e-6;
x_p = 2;
y_p = 4;
fx = (f(x_p+delta,y_p)-f(x_p,y_p))/delta;
fy = (f(x_p,y_p+delta)-f(x_p,y_p))/delta;
d_f = [fx,fy];
%% directional derivative
dir_v = [3,2]';
dir_v = dir_v./(dir_v'*dir_v)    ;
d_d = d_f*dir_v;
v_x = linspace(x_p,x_p+3,50);
v_y = linspace(y_p,y_p+2,50);
hold on
quiver3(x_p,y_p,0,3,2,0,'r','filled','LineWidth',2);
fprintf('directional derivative = %f\n',d_d)
%% tangent line
lambda=linspace(-1,1);
m = sqrt(d_f*d_f');
fz = f(x_p,y_p);
x_t = x_p+lambda*(sqrt(4/((fy/fx)^2+1)))/2;
y_t = y_p+lambda*(sqrt(4/((fy/fx)^2+1)))/2*fy/fx;
if(fx <= 0)
    z_t = fz +lambda*-m;
else
    z_t = fz +lambda*m;
end
plot3(x_t,y_t,z_t,'k-','LineWidth',2,'MarkerSize',15)
plot3(x_p,y_p,fz,'o')
xlabel('x')
ylabel('y')
zlabel('z')
```

2.Result:

```
directional derivative = -3.076924
```

2.

Given:

$$f(x,y) = 2.25xy + 1.75y - 1.5x^2 - 2y^2$$

Construct and solve a system of linear algebraic equations that maximizes $f(x,y)$. Notice that this is done by setting the partial derivative of f with respect to both x and y to zero.

ANS :

1.Code_main :

```
close all
clear all
format long

f =@(x,y) (2.25.*x.*y+1.75.*y-1.5.*x.^2-2*y.^2)
[xx,yy]=meshgrid(-4:0.1:4);
mesh(xx,yy,f(xx,yy))
%% gradient
error = 1e-8;
fx = 1;
fy = 1;
lr = 1e-4;
delta = 1e-6;
x_p = 7;
y_p = 7;
while(abs(fx)>error || abs(fy)>error)
    fx = (f(x_p+delta,y_p)-f(x_p,y_p))/delta
    fy = (f(x_p,y_p+delta)-f(x_p,y_p))/delta
    x_p = x_p + lr*fx;
    y_p = y_p + lr*fy;
end
z_p = f(x_p,y_p)
hold on
plot3(x_p,y_p,z_p,'o')
xlabel('x')
ylabel('y')
zlabel('z')
fprintf("max point is (%f,%f,%f)\n",x_p,y_p,z_p)
```

2.Result:

```
max point is (0.567566,0.756755,0.662162)
```

3.

Consider a system with 2 frictionless masses connected to walls by 3 linear elastic springs, where $M_1 = 5kg$, $M_2 = 10kg$, $k_a = 0.1N/m$, $k_b = 1N/m$, and $k_c = 0.2N/m$. Plot the positions of both carts by modeling it as a 2×2 matrix and then find the roots of the characteristic polynomial.

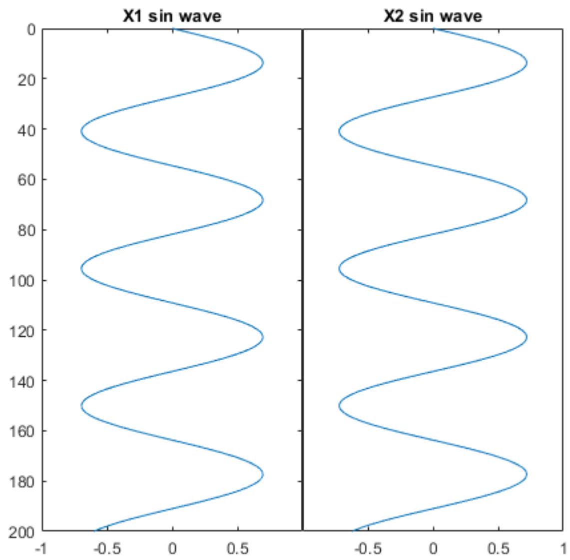
ANS :

1. Code_main:

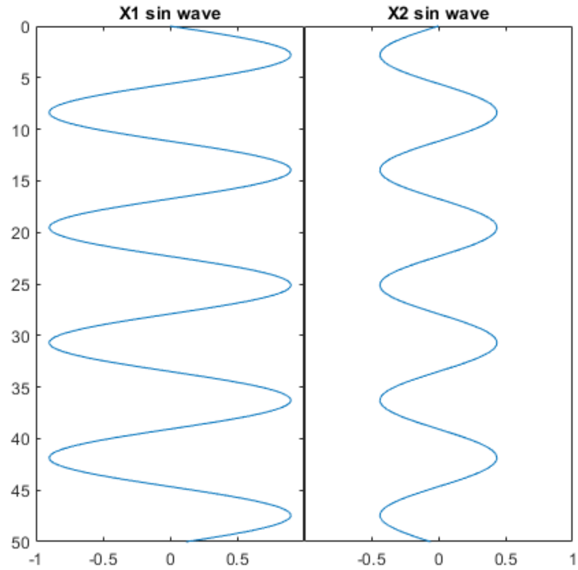
```
close all
clear all
format long

m1 = 5;
m2 = 10;
ka = 0.1;
kb = 1;
kc = 0.2;
syms w
A = [ka+kb-m1*w^2, -kb;
        -kb,ka+kb-m2*w^2]
f = det(A)
root = double(solve(f));
w1 = root(1)
w2 = root(2)
B = [ka+kb-m1*w2^2, -kb;
        -kb,ka+kb-m2*w2^2];
n_v = null(B);
t = 0:0.1:50;
alpha = 1;
X = alpha .* n_v'
X1 = X(1)*sin(w2.*t);
X2 = X(2)*sin(w2.*t);

subplot('Position',[0.05 0.05 0.35 0.9]);
plot(X1,t)
set(gca,'xtick',[-1,-0.5,0,0.5])
set(gca,'YDir','reverse')
title('X1 sin wave')
subplot('Position',[0.4 0.05 0.35 0.9]);
plot(X2,t)
set(gca,'ytick',[])
set(gca,'YDir','reverse')
xlim([-1,1])
set(gca,'xtick',[-0.5,0,0.5,1])
title('X2 sin wave')

fprintf("w1 = %f , X1 = %f , X2 = %f \n",w1,X(1),X(2))

fprintf("w2 = %f , X1 = %f , X2 = %f \n",w2,X(1),X(2))
```

2. Result:

w1 = 0.115152 , X1 = 0.695294 , X2 = 0.718725   w2 = 0.562797 , X1 = 0.900220 , X2 = -0.435436



ω1



ω2

4.

An inverse investigator has reported the data tabulated below. It is known that such data can be modeled by the following equation:

$$x = e^{(y-b)/a}$$

where a and b are parameters. Use a transformation to linearize this equation and then employ linear regression to determine a and b. Based on your analysis predict y at x=2.6.

x	1	2	3	4	5
y	0.5	2	2.9	3.5	4

ANS:

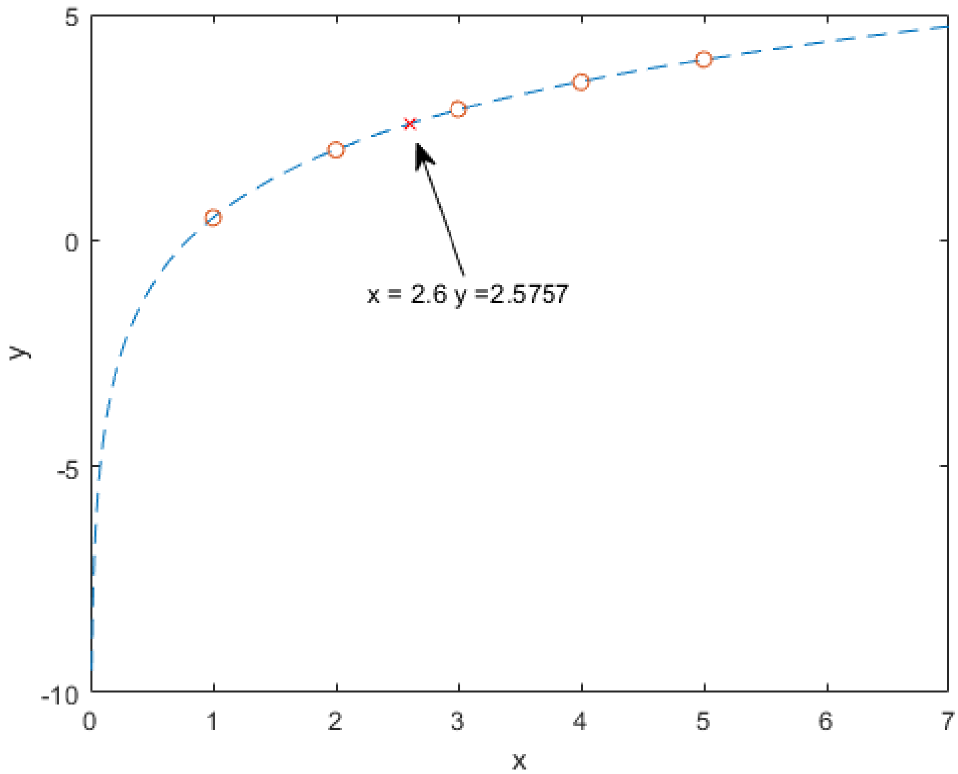
1. Code_main:

```
close all
clear all
format long

f = @(x,b,a) b+a.*log(x)
f_inv = @(y,b,a) exp((y-b)./a)

x = [1,2,3,4,5]
y = [0.5,2,2.9,3.5,4]'
A = [ones(size(x))',log(x)'] %*[b;a]
cof = inv(A'*A)*A'*y % b ; a
n = 0:0.01:7;
f_y = f(n,cof(1),cof(2));
```

```
plot(n,f_y,'--')
hold on
plot(x,y','o')
x_p = 2.6;
y_p = f(x_p,cof(1),cof(2))
plot(x_p,y_p,'rx')
xlabel('x')
ylabel('y')
fprintf("a = %f , b = %f\n",cof(2),cof(1))
fprintf("x=2.6 , y_predict = %f\n",y_p)
```

2.Result:

```
a = 2.172917 , b = 0.499436
x=2.6 , y_predict = 2.575683
```



5.

The following data are provided:

x	1	2	3	4	5
y	2.2	2.8	3.6	4.5	5.5

You have to use the least-squares regression to fit this data with the following model:

$$y = a + bx + \frac{c}{x}$$

ANS :

1. Code_main:

```
close all
clear all
format long


f = @(x,a,b,c) a+b.*x+c./x

x = [1,2,3,4,5]
y = [2.2;2.8;3.6;4.5;5.5];
A = [ones(size(x))',x',1./x'] %*[b;a]
cof = inv(A'*A)*A'*y % b ; a
n = 0.1:0.01:7;
f_y = f(n,cof(1),cof(2),cof(3));
plot(n,f_y,'--')
hold on
plot(x,y','o')
xlabel('x')
ylabel('y')
fprintf("a = %f , b = %f , c = %f\n",cof(1),cof(2),cof(3))
```

2. Result:

```
a = 0.374497 , b = 0.986443 , c = 0.845638
```