Numerical Methods Homework-2

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1. Consider the following equation:



(a) Iterate parabolic interpolation 10 times to locate the minimum.

ANS :

1. Code\_function :

function [xe]= parabolic(f,x1,x2,x3,iter)

x\_m = [x1;x2;x3];

for n = 1:iter

n

f\_m = f(x\_m);

ai\_2 = f\_m(1)\*(x\_m(2)^2-x\_m(3)^2)+f\_m(2)\*(x\_m(3)^2-x\_m(1)^2)+f\_m(3)\*(x\_m(1)^2-x\_m(2)^2)

ai\_3 = f\_m(1)\*(x\_m(2)-x\_m(3))+f\_m(2)\*(x\_m(3)-x\_m(1))+f\_m(3)\*(x\_m(1)-x\_m(2))

xe = ai\_2/(2\*ai\_3)

if(ismember(xe,x\_m))

break;

end

[f\_max,pos]= max(f\_m,[],1);

x\_m(pos) = xe;

end

end

1. Code\_main :

close all

clear all

format long

x1 = 0.1

x2 = 1

x3 = 5

f = @(x) 2.\*x+3./x;

iter = 10;

[x\_min]= parabolic(f,x1,x2,x3,iter)

x=-5:0.01:5;

plot(x,f(x),'-k')

hold on

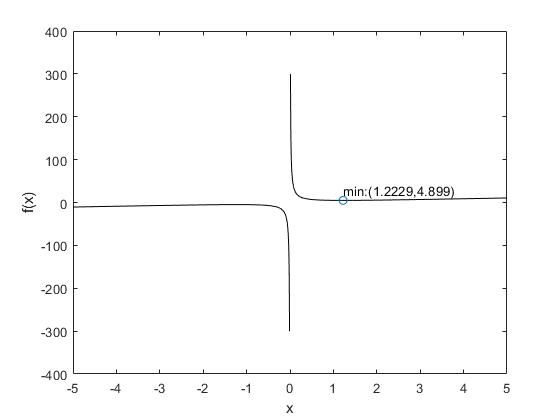
plot(x\_min,f(x\_min),'o')

text(x\_min,f(x\_min)+25,'min:('+string(x\_min)+','+string(f(x\_min))+')')

xlabel('x')

ylabel('f(x)')

1. Result :



使用0.1、1、5作為初始值，迭代10-次的結果。

在x=1.2229時有區域最小值4.899

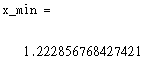


圖4-1-a local minimum

(b) Comment on the convergence of your results ( x1 =0.1, x2 =0.5, x3 =5).

ANS :

1. Result :

在0到點最小值之間為單調下降曲線，而點最小值至5之間為單調上升 曲線，符合二次曲線特徵，因此可以使用拋物線插值找出最小值，但經過插 值得出的頂點必須也落在0至5之間，若落在負區間則結果會發散。

2. Find the local minima and maxima of the following function ranging from 0 to 2.



(a) Use golden-section search.

ANS :

1. Code\_function :

function [x]= golden\_section(f,x1,x2,iter)

gr = (sqrt(5) + 1) / 2;

x3 = x1+(x2-x1)/gr

x4 = x2-(x2-x1)/gr

for n=1:iter

n

if(f(x3)>f(x4))

x2 = x3;

else

x1 = x4;

end

x3 = x1+(x2-x1)/gr

x4 = x2-(x2-x1)/gr

end

x = (x3+x4)/2

end

1. Code\_main :

close all

clear all

format long

x1 = 0

x2 = 2

f = @(x) (9\*exp(-x).\*sin(2\*pi\*x)-3.5);

inv\_f = @(x) -(9\*exp(-x).\*sin(2\*pi\*x)-3.5);

iter = 50;

[x\_min]= golden\_section(f,x1,x2,iter)

[x\_max]= golden\_section(inv\_f,x1,x2,iter)

figure(1)

x=-0.5:0.01:2.5;

plot(x,f(x),'-k')

hold on

text(x\_min,f(x\_min)-0.6,'min:('+string(x\_min)+','+string(f(x\_min))+')')

text(x\_max+0.1,f(x\_max),'max:('+string(x\_max)+','+string(f(x\_max))+')')

plot(x\_min,f(x\_min),'ro')

plot(x\_max,f(x\_max),'gx')

xlabel('x')

ylabel('f(x)')

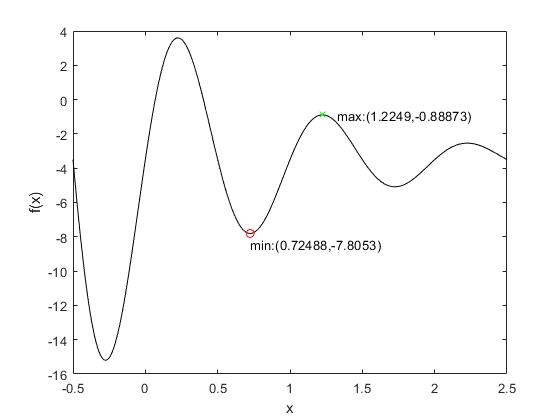
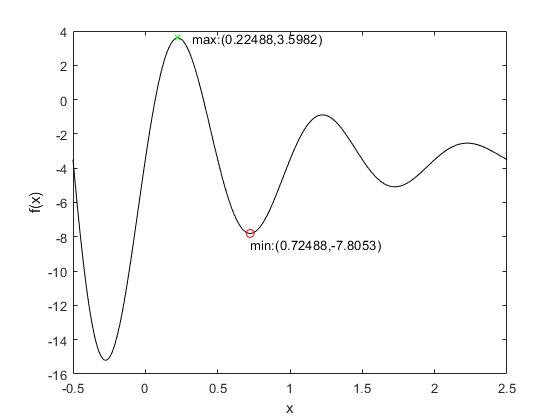
1. Result :

圖4-2-a-2 local minimum and maxima

with initial value 0、1

圖4-2-a-1 local minimum and maxima

with initial value 0、2

若初始值使用0、2，則會因為計算得出的步幅過大而直接跨過最大值所在的位置，而找到錯誤的值。

(b) Use the brute force stepwise.

ANS :

1. Code\_function :

function [x\_min,x\_max]= brute(f,x1,x2,precise)

x\_range = x1:precise:x2;

f\_m = f(x\_range);

[f\_min,pos\_min] = min(f\_m);

[f\_max,pos\_max] = max(f\_m);

x\_min = x\_range(pos\_min);

x\_max = x\_range(pos\_max);

end

1. Code\_main :

close all

clear all

format long

x1 = 0

x2 = 2

f = @(x) (9\*exp(-x).\*sin(2\*pi\*x)-3.5);

precise = 0.0001;

x=-0.5:0.01:2.5;

[x\_min,x\_max]=brute(f,x1,x2,precise)

plot(x,f(x),'-k')

hold on

text(x\_min,f(x\_min)-0.6,'min:('+string(x\_min)+','+string(f(x\_min))+')')

text(x\_max+0.1,f(x\_max),'max:('+string(x\_max)+','+string(f(x\_max))+')')

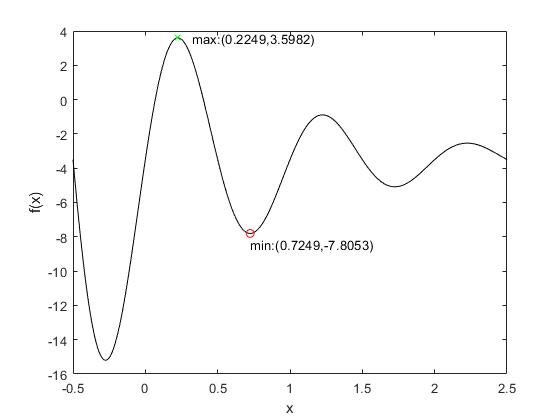
plot(x\_min,f(x\_min),'ro')

plot(x\_max,f(x\_max),'gx')

xlabel('x')

ylabel('f(x)')

1. Result :



使用精度為0.0001，範圍從0到2。

圖4-2-b local minimum and maxima

3. Given:



Use a root-location technique to determine the maximum of this function.

ANS :

1. Code\_function :

function[root] = newton(f,x0,c,iter,error\_std)

eps = 1e-6;

for n = 1:iter

n;

m = (f(x0+eps,c)-f(x0,c))/eps;

x1 = x0-f(x0,c)/m;

error = abs(x1-x0)/x0;

x0 = x1;

end

root = x1;

end

1. Code\_main :

close all

clear all

format long

f = @(x,c) -2.\*x.^6-1.5.\*x.^4+10.\*x+2+c

%f\_b = @(x) -2.\*x.^6-1.5.\*x.^4+10.\*x+2

x0 = 5;

c = 0;

iter = 20

error\_std = 0.00

c\_iter = 0

x= -0.5:0.01:1;

plot(x,f(x,0))

hold on

root0 = 0;

root1 = 0;

root = [];

while(1)

x0 = 5;

root0 = newton(f,x0,c,iter,error\_std);

x0 = -5;

root1 = newton(f,x0,c,iter,error\_std);

if(root1 > root0)

max\_range = root

break

end

root = [];

root = [root,root0,root1];

f\_root = f(root,0);

plot(root(1),f\_root(1),'ro')

hold on

plot(root(2),f\_root(2),'bx')

c = c - 0.005;

c\_iter = c\_iter + 1

end

xlabel('x')

ylabel('f(x)')

figure(2)

x= max\_range(2)-0.01:0.0001:max\_range(1)+0.01;

plot(x,f(x,0))

hold on

plot(max\_range(2),f(max\_range(2),0),'bx')

hold on

plot(max\_range(1),f(max\_range(1),0),'ro')

xlabel('x')

ylabel('f(x)')

%[x\_min,x\_max]=brute(f\_b,0.87,0.89,0.0000001)

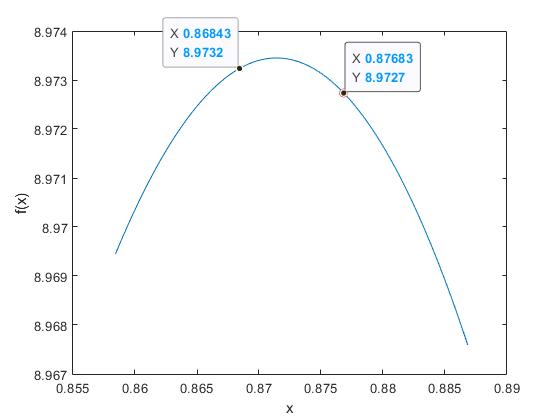
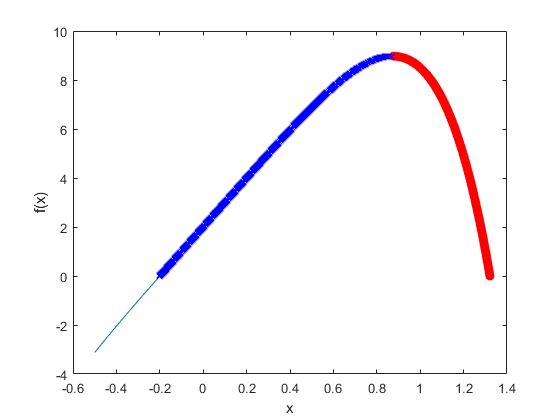
1. Result :

圖4-3-a find roots per f(x)-c

圖4-2-b maxima range

使用牛頓法找根，先從左右兩邊各找出一個根，接下來將函數減去0.005的偏移量，使f(x)圖形下降，再找出兩個根，漸漸逼近最大值，直到兩根交錯而停止，就可得到最大值的區間範圍。使用牛頓法的原因是因為函數為連續的，且在最大值附近的斜率變化較為平緩，沒有劇烈的變化，圖形凹口一致向下，使用牛頓法可以較穩定的找到根。