6.)

a.) $\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$

$(\nabla(\phi\psi))_i = \partial^i(\phi\psi)$

$(\nabla(\phi\psi))_i = (\underbrace{\partial^i\phi}_{\nabla\phi}\psi + \underbrace{\partial^i\psi}_{\nabla\psi}\phi)$

$\nabla(\phi\psi) = \psi\nabla\phi + \phi\nabla\psi$

b.) $\nabla\cdot(\nabla\times a) = \partial^i(\varepsilon_{ijk}\,\partial^j a^k)$

$\varepsilon_{ijk}\,\partial^i\partial^j a^k$

Antisimetrico

f.) $\nabla\times(\nabla\times a) = \nabla\cdot(\nabla\cdot a) - \nabla^2 a$

$\nabla\times(\nabla\times a) = \varepsilon_{ijk}\frac{\partial}{\partial x_j}(\nabla\times a)_k$

$= \varepsilon_{ijk}\frac{\partial}{\partial x_j}\varepsilon_{klm}\frac{\partial a_k}{\partial x_l}$

$= \varepsilon_{ijk}\varepsilon_{klm}\frac{\partial^2 a_m}{\partial x_j\partial x_l}$

$= (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})\,\partial_j\partial_l a_m$

$= \partial_j\partial_i a_j - \partial_j\partial_j a_i$

$= \nabla(\nabla\cdot a) - \nabla^2 a$