2.)

$$\operatorname{sen}(3\alpha) = 3\cos^2\alpha \operatorname{sen}\alpha - \operatorname{sen}^3\alpha$$

$$\cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\operatorname{sen}^2(\alpha)$$

utilizamos la formula de Moivre

$$(\cos(\theta) + i\operatorname{sen}(\theta))^n = (\cos(n\theta) + i\operatorname{sen}(n\theta))$$

$$\cos(3\alpha) + i\operatorname{sen}(3\alpha) = (\cos\theta + i\operatorname{sen}\theta)^3$$

$$= \cos^3\alpha - 3\cos\alpha\operatorname{sen}^2\alpha + i\left[3\cos^2\alpha\operatorname{sen}\alpha - \operatorname{sen}^3\alpha\right]$$

igualamos la parte real y imaginario

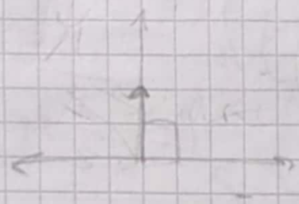
$$\cos 3\alpha = \cos^3\alpha - 3\cos\alpha\operatorname{sen}^2\alpha$$

$$i\operatorname{sen}(3\alpha) = i\left[3\cos^2\alpha\operatorname{sen}\alpha - \operatorname{sen}^3\alpha\right]$$

5.

a). $(2i)^{1/2} \longrightarrow |z| = \sqrt{2^2} = 2$

$$\arg(z) = \frac{\pi}{2}$$

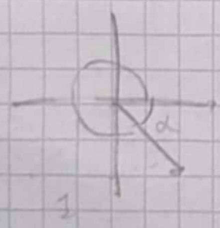


$$2i = 2\left(\cos\frac{\pi}{2} + i\operatorname{sen}\frac{\pi}{2}\right)$$

$$\sqrt{2\left(\cos\frac{\pi}{2} + i\operatorname{sen}\frac{\pi}{2}\right)} = \sqrt{2}\left(\cos\frac{\pi}{2} + i\operatorname{sen}\frac{\pi}{2}\right)^{\frac{1}{2}}$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\operatorname{sen}\frac{\pi}{4}\right)$$

b.) $(1 - \sqrt{3}i)^{1/2} = \sqrt{2}\left(\cos\frac{\pi}{6} + i\operatorname{sen}\frac{\pi}{6}\right)$



$$\tan\alpha = \frac{\sqrt{3}}{1}$$

$$\alpha = \frac{1}{3}\pi$$

c.) $(-1)^{\frac{1}{3}} = -1$

d.) $8^{1/6} = 1.4142$

e.) $(-8 - 8\sqrt{3}i)^{1/4} = 16^{\frac{1}{4}}\left(\cos\frac{\pi}{3} + i\operatorname{sen}\frac{\pi}{3}\right)^{\frac{1}{4}}$

$= 2\left(\cos\frac{\pi}{12} + i\operatorname{sen}\frac{\pi}{12}\right)$

6. a) $\log(-ie) =$ $\qquad$ $\operatorname{Log}(z) = \ln|z| + i\operatorname{Arg} z$

$\ln e - i\frac{\pi}{2} = 1 - i\frac{\pi}{2}$

b.) $\log(1-i) = \ln\sqrt{2} - i\frac{\pi}{4}$ $\qquad$ $\theta = \tan^{-1}(-1)$

$\theta = -\frac{\pi}{4}$

c.) $\log(e)$

partimos de $\log(-ie) = \log e + \log(-i) = 1 - i\frac{\pi}{2}$

$\log e + \pi i\left(\frac{1}{2} + 2n\right) = 1 - i\frac{\pi}{2}$

$\log e = 1 + i\pi\left(-\frac{1}{2} + \frac{1}{2} + 2n\right)$

$\log e = 1 + i\pi\left(\frac{1}{2} + 2n\right)$

d.) $\log(-i) = \log i + \log(-1)$

$\ln(1) + i\frac{\pi}{2} + \ln(1) + i(\pi + 2\pi n)$

$= \pi i\left(\frac{1}{2} + 1 + 2n\right)$