

2.)

$$\sin(3x) = 3\cos^2 x \sin x - \sin^3 x$$

$$\cos(3x) = \cos^3(x) - 3\cos(x)\sin^2(x)$$

Utilizaremos la fórmula de Moivre

$$(\cos(\theta) + i\sin(\theta))^n = (\cos(n\theta) + i\sin(n\theta))$$

$$\cos(3x) + i\sin(3x) = (\cos x + i\sin x)^3$$

$$= \cos^3 x - 3\cos x \sin^2 x + i[3\cos^2 x \sin x - \sin^3 x]$$

igualamos la parte real y la imaginaria

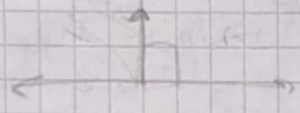
$$\cos 3x = \cos^3 x - 3\cos x \sin^2 x$$

$$i\sin(3x) = i[3\cos^2 x \sin x - \sin^3 x]$$

5.

$$a.) (2i)^{1/2} \Rightarrow |z| = \sqrt{2^2} = 2$$

$$\arg(z) = \frac{\pi}{2}$$

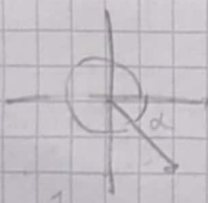


$$2i = 2(\cos \frac{\pi}{2} + i\sin \frac{\pi}{2})$$

$$\sqrt{2}(\cos \frac{\pi}{2} + i\sin \frac{\pi}{2}) = \sqrt{2}(\cos \frac{\pi}{2} + i\sin \frac{\pi}{2})^{1/2}$$

$$= \sqrt{2}(\cos \frac{\pi}{4} + i\sin \frac{\pi}{4})$$

$$b.) (1 - \sqrt{3}i)^{1/2} = \sqrt{2}(\cos \frac{\pi}{6} + i\sin \frac{\pi}{6})$$



$$\tan \alpha = \sqrt{3}$$

$$\alpha = \frac{1}{3}\pi$$

$$c.) (-1)^{\frac{1}{3}} = -1$$

$$d.) 8^{\frac{1}{6}} = 1.4142$$

$$c.) (-8 - 8\sqrt{3}i)^{\frac{1}{4}} = 16^{\frac{1}{4}} \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)^{\frac{1}{4}} \\ = 2 \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)$$

$$6.a) \log(-ie) =$$

$$\log(e) = \ln|e| + i \arg e$$

$$\ln e - i \frac{\pi}{2} = 1 - i \frac{\pi}{2}$$

$$b.) \log(1-i) = \ln \sqrt{2} - i \frac{\pi}{4} \quad \theta = \tan^{-1}(-1) \\ \theta = -\frac{\pi}{4}$$

$$c.) \log(e)$$

$$\text{partimos de } \log(-ie) = \log e + \log(-i) = 1 - i \frac{\pi}{2}$$

$$\log e + \pi i \left(\frac{1}{2} + 2n \right) = 1 - i \frac{\pi}{2} \quad d.) \log(-i) = \log|e| + i \arg(-i)$$

$$\log e = 1 + i \pi \left(-\frac{1}{2} + \frac{1}{2} + 2n \right) = \log |e| e^{i \left(-\frac{\pi}{2} + 2n\pi \right)}$$

$$= \ln(1) + i \left(-\frac{\pi}{2} + 2n\pi \right) \\ \log e = 1 + i \pi \left(\frac{1}{2} + 2n \right)$$