M6.4

ITERATIVE IDENTIFICATION AND RESTORATION OF IMAGES

R.L. Lagendijk, A.K. Katsaggelos+ and J. Biemond

Delft University of Technology, Dept. of EE, Information Theory Group,
Delft, The Netherlands
+ Northwestern University, Dept. of EECS, The Technological Institute,
Evanston IL 60201, USA

ABSTRACT

In order to restore distorted images, the unknown blurs have to be identified from the blurred images themselves. We formulate the blur identification problem as a constrained maximum likelihood problem. The constraints directly incorporate a priori known relations between the blur (and image model) coefficients, such as symmetry properties, into the identification procedure. The resulting nonlinear minimization problem is solved iteratively, yielding a very general identification algorithm. An example of blur identification on synthetic data is given.

I. INTRODUCTION

The first step towards the restoration of degraded images is the identification of the kind of degradation the image has suffered. Modeling a blurred image as the output of a 2-dimensional linear system, the identification problem is the problem of estimating the unknown characterizing point-spread function (PSF) of this system. One approach to blur identification is to obtain a model of the blurring system from the physical nature of the problem. Unfortunately, one has hardly ever enough a priori knowledge to determine the PSF in this way. Therefore, the information about the blurring process has to be determined from the blurred image itself.

The earliest work on blur identification concentrated on identifying PSFs that have zeros only on the unit bi-circle [1]. One of the shortcomings of this method is that PSFs which do not satisfy this requirement, such as a properly truncated Gaussian PSF, cannot be identified. In more recent work [2,3] the original image is first modeled as a 2-D autoregressive (AR) process. Then, if the observed blurred image is assumed noiseless, the image and blur model identification problem is specified as a 2-D autoregressive moving-average (ARMA) identification problem, where the AR coefficients are related only to the image model, and the MA coefficients only to the blur model (PSF).

Tekalp et al. [2] derived maximum likelihood estimates for these ARMA parameters, and computed them by first decomposing the PSF into four (causal) quarterplane convolutional factors, each of which is stable in its direction of recursion, and next identifying each of these factors recursively. This approach assumes that the unknown PSF is real, symmetric (i.e. zero phase) and has a positive Fourier transform. Biemond et al. [3] showed that the 2-D ARMA identification can be done in parallel, where each of the parallel channels requires the identification of a 1-D complex ARMA process. An intermediate high-order AR approximation step is used to compute these ARMA coefficients.

In this paper we formulate the blur identification problem as a constrained maximum likelihood (ML) problem. The linear constraints incorporated in the formulation represent a priori known relations between the blur (or image model) coefficients. The resulting nonlinear minimization problem is solved by employing an iterative gradient based minimization procedure. It is conceptually advantageous to use iterative methods, since they offer the possibility of incorporating a priori knowledge about the original blur and image model into the identification procedure. Furthermore, since they act upon one complete image they are free from the causality restrictions imposed by recursive techniques.

In Section II we describe the mathematical (probabilistic) models for the image and degradation. Next, in Section III, we formulate the identification problem as a ML problem. In this section we also describe the iterative algorithm for minimizing the resulting ML index. Some preliminary experimental results are presented in Section IV. Finally, Section V summarizes relevant conclusions and discusses areas of further research.

II. IMAGE AND DEGRADATION MODELS

Basic Model Development

It is assumed that the original image $f(i,j)$ (of the size NxN pixels) can be represented by the output of a 2-D AR system

$$f(i,j) = \sum_{k,\ell \in W_A} a(k,\ell) f(i-k,j-\ell) + v(i,j), \quad (1)$$

where $a(k,\ell)$ are the image model coefficients, and W_A the support of the image model, which is not necessarily causal.

By lexicographically ordering of the image data [1] we can use the more compact matrix-vector notation

$$f = Af + v. \quad (2)$$

+A.K. Katsaggelos was partially supported by the National Science Foundation under grant number MIP - 8614217.

CH2561-9/88/0000-0992 $1.00 © 1988 IEEE

Assuming that a circular convolution is used in (1) to solve the boundary value problem, the matrix A has a block-circulant structure [1]. This property will be made use of in the identification algorithm.

The input v of the AR-model is assumed to be a zero-mean homogeneous discrete random process with Gaussian probability density distribution

$$p_v(v/Q_v) = (2\pi)^{-N^2/2}|Q_v|^{-\frac{1}{2}}\exp\{-\tfrac{1}{2}\, v^t\, Q_v^{-1}\, v\}, \qquad (3)$$

where $|Q_V|$ is the absolute value of the determinant of the (diagonal) covariance matrix Q_V of v. If the imaging system is linear and spatially invariant, and we assume the noise w(i,j) to be an additive process at the output of the system, then the observed noisy image g(i,j) can be modeled by the following 2D convolutional summation

$$g(i,j) = \sum_{m,n\in W_D} d(m,n)\, f(i-m,j-n) + w(i,j), \qquad (4)$$

where w(i,j) is a zero-mean white noise process uncorrelated with the data, where d(m,n) is the PSF of the blur and where W_D is the support of the PSF. Eq. (4) is written in the more compact matrix-vector form by lexicographically ordering of the data:

$$g = Df + w. \qquad (5)$$

Again, assuming a circular convolution in (4), the matrix D has a block-circulant structure.

Probability Distribution of the Observations

In the next section we will focus on maximum likelihood identification. To determine the ML index it is necessary to have the a priori distribution of the observed data conditioned on the parameters to be identified. By combining (2) and (5) we can eliminate the unknown original image f, and assuming that the effect of the observation noise w is negligible on the estimation of the parameters, we arrive at

$$g = D(I-A)^{-1}\, v. \qquad (6)$$

From (3) and (6) the probability density function (pdf) of g, given the parameters $\{a(k,\ell),d(m,n),Q_v\} \triangleq \theta$, can be obtained:

$$p_g(g/\theta) = (2\pi)^{-N^2/2}\, |Q_v|^{-\frac{1}{2}}|D|^{-1}|I-A|\,\cdot$$

$$\exp\{-\tfrac{1}{2}||(I-A)\, D^{-1}g||^2_{Q_v^{-1}}\} \qquad (7)$$

It is assumed that the inverse of D exists. If D is singular, D^{-1} is replaced by the generalized inverse D^+.

III. MAXIMUM LIKELIHOOD IDENTIFICATION

ML Method

Before observations of the distorted image are available, the probability density function of g, given the parameters θ (Eq. (7)) associates a pdf with every outcome g of an experiment with fixed θ. In the blur identification problem only the observed image g is available, and we wish to find the various values in θ which might have led to the blurred image actually obtained. In the ML method the ML index (the likelihood function $\ell(\theta/g)$, or the log-likelihood function $L(\theta/g)$) is maximized. The reason for choosing the ML method over other methods, such as least-squares, is the generality of the former method [4,5].

Mathematically, the ML method is formulated as

$$\underset{\theta}{\text{Max}}\ L(\theta/g) = \underset{\theta}{\text{Max}}\ \log p_g(g/\theta). \qquad (8)$$

Substituting (7) into (8), and omitting all constant terms yields

$$\underset{\theta}{\text{Max}}\ L(\theta/g) = \underset{\theta}{\text{Min}}\ \{\log|Q_v|+2\log|D|-2\log|I-A| + ||(I-A)D^{-1}g||^2_{Q_v^{-1}}\} \qquad (9)$$

Constraints on the Image and Blur Model

In many practical situations of interest one has some a priori information about the structure of the PSF or the image model to be identified. This knowledge might be due to the physical nature of the blur, to assumed properties of the image model, or may simply be enforced to reduce the complexity of the identification problem. Here we consider the incorporation of the following four types of a priori relations:

(i) The PSF of the blurring system exhibits some kind of symmetry, or in a more general formulation, a number of PSF coefficients have equal values. We define $E_D(r,s)$ as a subset of the blur support W_D for which the related PSF coefficients are constrained to take the same value as d(r,s):

$$E_D(r,s) = \{(t,u)|d(t,u)=d(r,s)\}\ . \qquad (10)$$

(ii) Usually some kind of symmetry is enforced on the image model as well. $E_A(r,s)$ is defined as a subset of the image model support W_A for which the related image model coefficients must have the same value as a(r,s):

$$E_A(r,s) = \{(t,u)|a(t,u)=a(r,s\}. \qquad (11)$$

(iii) The imaging system does not absorb or generate energy:

$$\sum_{m,n\in W_D} d(m,n) = 1.0 \qquad (12)$$

We define C_D as the subset of the blur support for which the related coefficients are chosen to satisfy the above constraint. Hence

$$d(r,s)=1-\frac{1}{|C_D|}\sum_{p,q\in W_D\setminus C_D} d(p,q), \quad \text{for } (r,s)\in C_D, \qquad (13)$$

where $|C_D|$ denotes the number of elements in the set C_D.

(iv) The AR-model for the original image is assumed not to absorb or generate energy:

$$\sum_{k,\ell \in W_A} a(k,\ell) = 1.0. \tag{14}$$

The coefficients defined by the subset C_A of the model support W_A are thus given by

$$a(r,s) = 1 - \frac{1}{|C_A|} \sum_{p,q \in W_A \setminus C_A} a(p,q), \quad \text{for } (r,s) \in C_A. \tag{15}$$

Formulation of the algorithm

To minimize Eq. (9) with respect to d(m,n) and $a(k,\ell)$ subject to the constraints described above, we use an iterative algorithm of the following form

$$d^{k+1}(r,s) = d^{k}(r,s) - \beta_1 \frac{\partial}{\partial d(r,s)} L(\theta^k/g), \tag{16a}$$

$$a^{k+1}(r,s) = a^{k}(r,s) - \beta_2 \frac{\partial}{\partial a(r,s)} L(\theta^k/g). \tag{16b}$$

At each iteration step (k) the restoration result is given by $\hat{f}^k = (D^k)^{-1}g$. The evaluation of the partial derivative of log $L(\theta/g)$ with respect to d(r,s) is outlined in Appendix B (making use of the theorems in Appendix A), while a similar derivation holds for the partial derivative with respect to a(r,s). The resulting equations are:

$$\frac{\partial}{\partial d(r,s)} L(\theta/g) = 2. \sum_{p,q \in E_D(r,s)} \{ \sum_{m,n=0}^{N-1} [c_{-p,-q}(m,n)]^{-1} - \frac{1}{Q_v} h^t_{pq}[D^{-1}]^t h\} - 2 \frac{|E_D(r,s)|}{|C_D|} \sum_{k,\ell \in C_D} \{ \sum_{m,n=0}^{N-1} [c_{-k,-\ell}(m,n)]^{-1} - \frac{1}{Q_v} h^t_{k\ell}[D^{-1}]^t h\} \tag{17a}$$

where $h = (I-A)D^{-1}g$; h_{pq} the image h shifted circularly over the vector (p,q) and $c_{-p,-q}(m,n)$ the (m,n)-th DFT coefficient of d(m,n) shifted circularly over the vector (-p,-q).

$$\frac{\partial}{\partial a(r,s)} L(\theta/g) = 2. \sum_{p,q \in E_A(r,s)} \{ \sum_{m,n=0}^{N-1} [b_{-p,-q}(m,n)]^{-1} - \frac{1}{Q_v} g^t_{pq}[D^{-1}]^t h\} - 2 \frac{|E_A(r,s)|}{|C_A|} \sum_{k,\ell \in C_A} \{ \sum_{m,n=0}^{N-1} [b_{-k,-\ell}(m,n)]^{-1} - \frac{1}{Q_v} g^t_{k\ell}[D^{-1}]^t h\} \tag{17b}$$

where $b_{-p,-q}(m,n)$ is the (m,n)-th DFT coefficient of $1-a(k,\ell)$ shifted circularly over the vector (-p,-q).

IV. EXPERIMENTAL RESULTS

A synthetic image (64x64 pixels) was generated using an AR model with NSHP support. The following coefficients were used

$$a(k,\ell) = \begin{array}{|c|c|c|} \hline -0.449 & 0.630 & 0.127 \\ \hline 0.671 & x & \\ \hline \end{array} \quad Q_v = 200 \tag{18}$$

This image was blurred by a 3x1 PSF (d(0,-1),d(0,0), d(0,1)) with d(0,-1)=d(0,0)=d(0,1)=0.333. We used two different fixed image models to identify the blur only, namely the exact model in Eq. (18), and a heuristic image model which is related to the discrete Laplace operator:

$$a(k,\ell) = \begin{array}{ccc} & \boxed{0.25} & \\ \boxed{0.25} & \boxed{x} & \boxed{0.25} \\ & \boxed{0.25} & \end{array} \tag{19}$$

Noise was added at the levels of 40, 50 and 60 dB. Table 1 summarizes the identification results.

SNR	Image model	d(0,-1)	d(0,0)	d(0,1)	Q_v
60	Eq. (18)	0.334	0.332	0.334	195.1
	Eq. (19)	0.336	0.329	0.335	143.3
50	Eq. (18)	0.333	0.332	0.335	196.4
	Eq. (19)	0.335	0.329	0.336	143.9
40	Eq. (18)	0.333	0.332	0.335	204.8
	Eq. (19)	0.335	0.330	0.335	149.1

Table 1. Identification Results

V. DISCUSSION

We have presented an iterative algorithm to identify the image and blur model from a distorted image. The algorithm is based on a constrained maximum likelihood method, where the constraints directly incorporate linear relations between the image and blur model coefficients into the identification procedure. Preliminary experiments on synthetic data produce stimulating results. In some cases, however, the algorithm converges to a sub-optimal solution due to local extrema. Current research concentrates on developing reject mechanisms for the erroneous solutions, and on reducing the complexity of the identification for example by adopting a one-parameter family description for the PSF (e.g. the extent of the blur) and the image model (e.g. the correlation). Further, the presented ML method can be extended to include noisy data. Initial work in this area will be presented in [6].

Appendix A

Theorem 1

Let A be a block-circulant matrix of the dimension $N^2 x N^2$ with defining bi-sequence {a(i,j)} = {a(0,0),a(0,1),...,a(0,N-1),a(1,0),...,a(N-1,N-1)}. Then the determinant of A, denoted by |A|, is given by

$$|A| = \prod_{i,j=0}^{N-1} c(i,j) \tag{A1}$$

where {c(i,j)} is the 2D DFT of {a(i,j)}.

Proof

Since A has a block-circulant structure, its eigenvalues are equal to the 2D DFT coefficients of the defining bi-sequence $\{a(i,j)\}$ [1]:

$$c(i,j)=\sum_{k,\ell=0}^{N-1} a(k,\ell)\, e^{\frac{-2\pi j}{N^2}(km+\ell n)} \qquad (A2)$$

The determinant of a matrix is the product of its eigenvalues, which proves (A1).

Theorem 2

Let A be a block-circulant matrix of the dimension N^2xN^2 with defining bi-sequence $\{a(i,j)\}$. Then the partial derivative of $|A|$ with respect to a coefficient $a(r,s)$ is

$$\frac{\partial}{\partial a(r,s)} |A| = |A| \cdot \sum_{m,n=0}^{N-1} \frac{1}{c_{-r,-s}(m,n)}, \qquad (A3)$$

where $c_{-r,-s}(m,n)$ are the 2D DFT coefficients of $\{a(i,j)\}$ shifted circularly over the vector $(-r,-s)$.

Proof

Starting out with (A1) and applying the chain rule for derivatives yields

$$\frac{\partial}{\partial a(r,s)} |A| = \sum_{m,n=0}^{N-1} \Big[\prod_{\substack{i,j=0 \\ (i,j)\neq(m,n)}}^{N-1} c(i,j)\Big] \frac{\partial c(m,n)}{\partial a(r,s)}$$

$$= |A| \cdot \sum_{m,n=0}^{N-1} \frac{1}{c(m,n)} \frac{\partial c(m,n)}{\partial a(r,s)}. \qquad (A4)$$

From (A2) it follows that

$$\frac{\partial c(m,n)}{\partial a(r,s)} = e^{\frac{-2\pi j}{N^2}(rm+sn)}. \qquad (A5)$$

Substituting (A5,A2) into (A4) yields

$$\frac{\partial}{\partial a(r,s)} |A| = |A| \sum_{m,n=0}^{N-1} \Big\{ \sum_{k,\ell=0}^{N-1} a(k,\ell) e^{\frac{-2\pi j}{N^2}(km+\ell n)} \Big\}^{-1} . e^{\frac{-2\pi j}{N^2}(rm+sn)}$$

$$= |A| \sum_{m,n=0}^{N-1} \Big\{ \sum_{p,q=0}^{N-1} a(p+r,q+s) e^{\frac{-2\pi j}{N^2}(pm+qn)} \Big\}^{-1} \qquad (A6)$$

The denominator in (A6) is recognized as the (m,n)-th coefficient of the 2D DFT of $\{a(i,j)\}$ shifted circularly over the vector $(-r,-s)$, which proves theorem 2.

Appendix B

We briefly outline the derivation of Eq. (17a). We need to evaluate

$$\frac{\partial}{\partial d(r,s)} L(\theta/g) = \frac{\partial}{\partial d(r,s)} ||(I-A)D^{-1}g||^2_{Q_v^{-1}}$$

$$+ 2 \frac{\partial}{\partial d(r,s)} \log |D| \qquad (B1)$$

We consider the first term on the right hand side in (B1). Defining $h=(I-A)D^{-1}g$ we have

$$\frac{\partial}{\partial d(r,s)} ||h||^2_{Q_v^{-1}} = -\frac{2}{Q_v} [(\frac{\partial D}{\partial d(r,s)})h]^t (D^{-1})^t h \qquad (B2)$$

In computing the partial derivatives of D we need to incorporate the enforced linear relations described in Section III. After substitution of these relations and some mathematical manipulations we have

$$[\frac{\partial D}{\partial d(r,s)} h]^t [D^{-1}]^t h = \{ \sum_{p,q\in E_D(r,s)} h_{pq}$$

$$- \frac{|E_D(r,s)|}{|C_D|} \sum_{k,\ell\in C_D} h_{k\ell}\}^t (D^{-1})^t h, \qquad (B3)$$

where h_{pq} denotes the image h shifted circularly over the vector (p,q).

To evaluate the second term in (B1) we make use of theorem 2. However, due to the enforced relations between the blur coefficients we need to reconsider the partial derivative of the DFT of $d(m,n)$ ($c(m,n)$) with respect to $d(r,s)$ (cf. Eq. (A5)). It can be shown that

$$\frac{\partial c(m,n)}{\partial d(r,s)} = \sum_{p,q\in E_D(r,s)} \exp(\frac{-2\pi j}{N^2}(pm+qn))$$

$$- \frac{|E_D(r,s)|}{|C_D|} \sum_{k,\ell\in C_D} \exp(\frac{-2\pi j}{N^2}(km+\ell n)) \qquad (B4).$$

Combining first Eq. (B4) and (A4), and next combining the result with (B3) yields equation (17a).

References

[1] Andrews, H.C. and B.R. Hunt, Digital Image Restoration, Prentice Hall Inc., Englewood Cliffs, NJ, 1977.

[2] Tekalp, A.M., H. Kaufman and J.W. Woods, "Identification of Image and Blur Parameters for the Restoration of Noncausal Blurs", IEEE Trans. on ASSP, vol. 34, no. 4, Aug. 1986, pp. 963-972.

[3] Biemond, J., F.G. van der Putten and J.W. Woods, "Identification and Restoration of Images with Symmetric Noncausal Blurs", accepted for publication, IEEE Trans. on Cas., 1987.

[4] Kashyap, R.L., "Maximum Likelihood Identification of Stochastic Linear Systems", IEEE Trans. on AC, vol. 15, no. 1, Feb. 1970.

[5] Åström, K.J., "Maximum Likelihood and Prediction Error Methods", Automatica, vol. 16, pp. 551-574, 1980.

[6] Lagendijk, R.L., D.L. Angwin, H. Kaufman and J. Biemond, "Recursive and Iterative Methods for Image Identification and Restoration", submitted to the European Conference on Signal Processing Eusipco'88, September 1987.