ITERATIVE REGULARIZED IMAGE RESTORATION USING LOCAL CONSTRAINTS

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ABSTRACT

In this paper, we propose a spatially adaptive image restoration algorithm, using local statistics. The local variance, mean and maximum value are utilized to constraint the solution space. These parameters are computed at each iteration step using partially restored image. A parameter defined by the user determines the degree of local smoothness imposed on the solution. The resulting iterative algorithm exhibits increased convergence speed when compared with the nonadaptive algorithm. In addition, a smooth solution with a controlled degree of smoothness is obtained. Experimental results demonstrate the capability of the proposed algorithm.

1. INTRODUCTION

When an image is formed or recorded by an imaging system, the image may be degraded due to uniform motion, defocusing, long-term atmosphere turbulence, or any combination of them. The blurred image may be more seriously degraded by the additive noise which comes from the image formation process, transmission medium, recording process, or any combination of them.

A typical degradation model is of the form [2, 6]

$$y = Dx + n, \tag{1}$$

where the vectors y, x, n are of size $MN \times 1$, and represent the lexicographically ordered observed image, original image, and the additive noise, respectively, of size $M \times N$. D is the degradation matrix of size MN x MN which may represent a spatially invariant or spatially varying point spread function.

Least-squares regularization has been used for obtaining solutions to Eq. (1). According to the regularization approach, the following functional is minimized with respect to x [2, 7]

$$M(x) = ||y - Dx||^2 + \alpha ||Cx||^2, \tag{2}$$

where α, the regularization parameter, controls the trade-off between fidelity to the data and smoothness, and C represents typically a high pass operator.

The prior knowledge used in the formulation represented by Eq. (2) is that the original image is smooth. Since such knowledge constraints the solution space, meaningless solutions can be avoided. However, this is a global requirement and therefore not very effective in terms of local smoothness. The solution of Eq. (2) represents a spatially invariant filter.

In this paper, an adaptive image restoration algorithm using local smoothing constraints is proposed. We follow the formulation represented by Eq. (2) and propose to bring knowledge about the local properties of the original image into the restoration process, so that prior knowledge and the spatial adaptivity are incorporated on the solution. The basic idea is to constrain locally the range of values the restored image can take, leading to increased convergence speed of the iterative algorithm and signal to noise ratio (SNR) improvement. The proposed algorithm differs from all other spatially adaptive restoration algorithms proposed in the literature (for a review see [6, 3]).

This paper is organized as follows. In section 2, the iterative regularized image restoration is reviewed. The proposed spatially adaptive image restoration algorithm is described in section 3. Experimental results are presented in section 4, and finally conclusions are reached in section 5.

2. BACKGROUND

The steepest descent iteration with a constant step size (equal to 1) applied to (2) results in

$$x_{k+1} = x_k + [D^T y - (D^T D + \alpha C^T C)x_k] = T x_k. \quad (3)$$

There exist various ways for determining the regularization parameter α [4]. According to [5], the regularization parameter is determined by partially restored image at each iteration step,

$$\alpha(x_k) = \frac{||y - Dx||^2}{\theta - ||Cx||^2}, \quad (4)$$

where $\theta \geq 2||y||^2$.

Constraints can be imposed on the partially restored image and can be incorporated into equation (3). That is, iteration (3) takes the form

$$\begin{aligned} \hat{x}_k &= P x_k, \\ x_{k+1} &= T\hat{x}_k = TPx_k, \end{aligned} \quad (5)$$

where P denotes a projection operator (or concatenation of operator) of a signal onto a set of signals with desirable properties.

3. ADAPTIVE IMAGE RESTORATION ALGORITHM USING LOCAL CONSTRAINT

In this section, we describe a way to choose a set onto which the partially restored image in Eq. (3) is projected. In order to define local smoothing constraint, it is necessary to determine the parameters which describe the local properties of an image. In our work, we use the local variance for local spatial activity and local maximum intensity value. For the image $x_k(i,j)$, the local mean $m_{x_k}(i,j)$ and the local variance $\sigma^2_{x_k}(i,j)$ at coordinate (i,j) are defined by

$$m_{x_k}(i,j) = K \sum_{m=i-U}^{i+U} \sum_{n=j-V}^{j+V} x_k(m,n), \quad (6)$$

$$\sigma^2_{x_k}(i,j) = K \sum_{m=i-U}^{i+U} \sum_{n=j-V}^{j+V} [x_k(m,n) - m_{x_k}(i,j)]^2, \quad (7)$$

where $K^{-1} = (2U+1)(2V+1)$ is the extent of the analysis window which is symmetric about the point (i,j). The local maximum value, $x_{k,max}(i,j)$, is simply defined as

$$x_{k,max}(i,j) = \max_{(m,n)\in S_{i,j}} x_k(m,n), \quad (8)$$

where $S_{i,j}$ represents the support region for determining the local maximum value at (i,j). In the experiments, it is the same with analysis window used for local mean and variance computation.

From Eqs. (7) and (8), the projection operator P to the set expressing local smoothness is defined as

$$P(x_k(i,j)) = \begin{cases} m_{x_k}(i,j) - L \cdot B(i,j) \\ \quad \text{if } x_k(i,j) < m_{x_k}(i,j) - L \cdot B(i,j), \\ m_{x_k}(i,j) + L \cdot B(i,j) \\ \quad \text{if } x_k(i,j) > m_{x_k}(i,j) + L \cdot B(i,j), \\ x_k(i,j) \text{ otherwise}, \end{cases} \quad (9)$$

where L is a threshold to be determined and $B(i,j)$ is equal to $\frac{x^2_{k,max}(i,j)}{\sigma^2_{x_k}(i,j)}$. Smaller $B(i,j)$ represents tighter bound for flat regions, resulting in an oversmoothed image with most of the noise removed (the blur does not change much the flat regions of th image), while larger $B(i,j)$ (looser bound) for high activity. This is in agreement with the noise masking property in areas of high spatial activity of the human visual system [1].

4. EXPERIMENTAL RESULTS

n our experiments, we used the 256×256 pixels lena image. The original image is degraded by 7x7 uniform motion blur and by 10 dB Gaussian noise. The degraded image is shown in Fig. 1 We tested the proposed algorithm for various signal to noise ratios (SNR) and images. For evaluating the performance of the algorithm, the improvement in SNR (dB) was utilized. It is defined at the kth iteration step by

$$\Delta_{SNR} = 10 \log_{10} \frac{||y - x||^2}{||x_k - x||^2}. \quad (10)$$

The criterion

$$\frac{||x_{k+1} - x_k||^2}{||x_k||^2} \leq 10^{-5} \quad (11)$$

was used for terminating the iteration.

For $L = 0.001$, the proposed algorithm converges after 9 iterations ($\Delta_{SNR} = 3.05$), while the nonadaptive algorithm after 74 iterations ($\Delta_{SNR} = -4.57$). Figs. 2 and 3 show the restored images by iterations (3) and (9) with the use of (4). When tighter bounds (smaller L) are used, the convergence becomes faster. However, the tighter bounds result in oversmoothed images. On the basis of our experiments, $0.01 \leq L \leq 0.0001$ is a good range with respect to convergence speed and performance. Figures 4 and 5 show the pixels outside and inside the region defined by the bounds in Eq. (9). Black pixels denote the location where the intensity values are below the lower threshold, white pixels the location where the intensity values are above the upper threshold, and gray pixels the location where the intensity values are in between the two bounds. Clearly by comparison Figs. 4 and 5 considerably more pixels do not satisfy the constraint at the beginning of the iteration than at convergence. Figs. 6 and 7 respectively shows comparison of the convergence rates and the mean squared error between the nonadaptive and adaptive algorithms.

5. CONCLUSIONS

In this paper, we propose an adaptive iterative regularized image restoration algorithm using local smoothing constraints. Each pixel in an image is projected onto local smoothing set which is determined by the local mean, variance, and maximum intensity value of the partially restored image. These parameters are utilized in defining the convex set. We are currently investigating the use of the local smoothness constraint in the blind deconvolution problem.

6. REFERENCES

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Figure 1: Noisy blurred image (7x7 uniform blur, 10 dB Gaussian noise)

Figure 2: Restored image by nonadaptive approach; 74 iterations, $\Delta_{SNR} = -4.57$ dB

Figure 3: Restored image by proposed algorithm; 9 iterations, $\Delta_{SNR} = 3.05$ dB

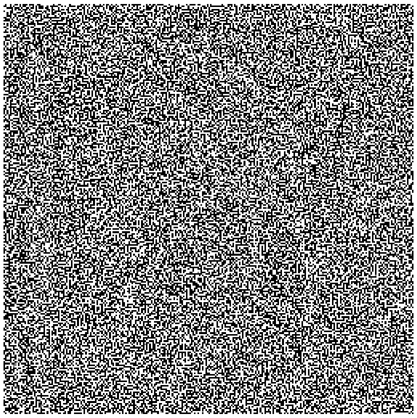

Figure 4: Black pixels : intensity below the lower bound ; white pixels : intensity above the upper bound ; gray pixels ; intensity between the bounds of Eq. (9) ; iteration 1

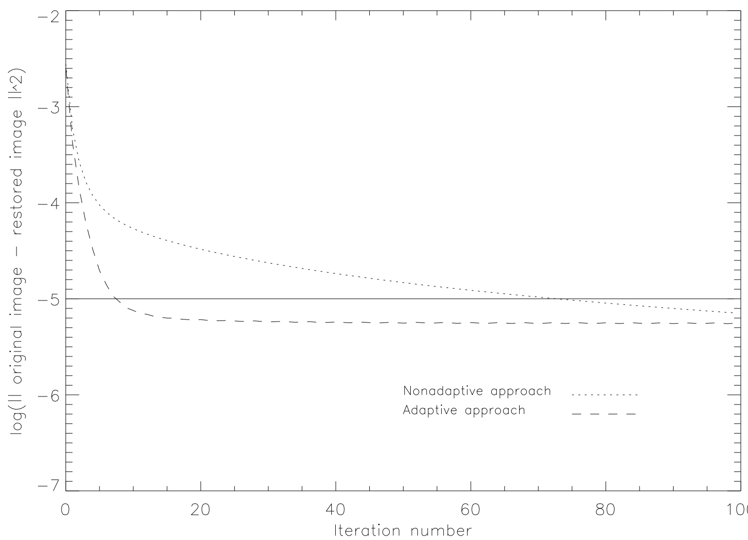

Figure 6: Comparison of convergence rates

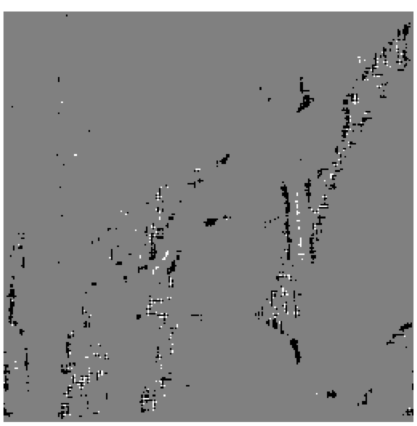

Figure 5: Black pixels : intensity below the lower bound ; white pixels : intensity above the upper bound ; gray pixels ; intensity between the bounds of Eq. (9) ; iteration 9

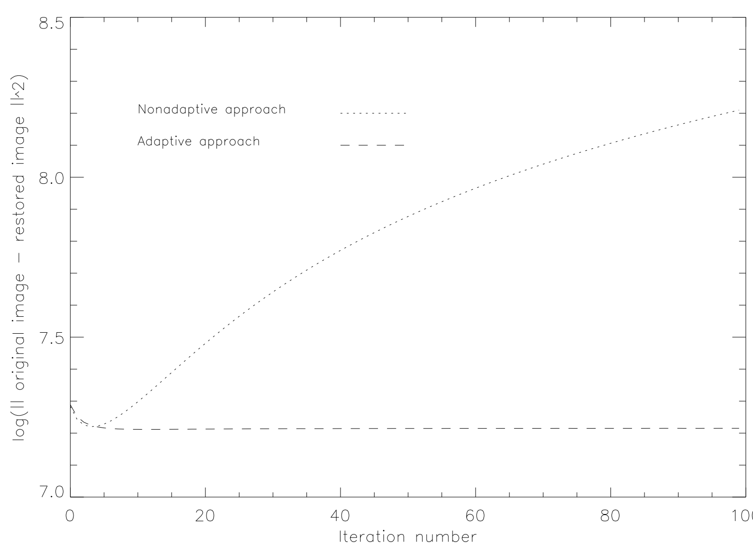

Figure 7: Mean squared error comparison