Available at
www.ComputerScienceWeb.com
POWERED BY SCIENCE @ DIRECT®

Signal Processing 83 (2003) 1001–1012

SIGNAL PROCESSING

www.elsevier.com/locate/sigpro

Wavelet-domain iterative center weighted median filter for image denoising

S.M. Mahbubur Rahman, Md. Kamrul Hasan*

Department of Electrical and Electronic Engineering, Bangladesh University of Engineering and Technology, Dhaka 1000, Bangladesh

Received 6 May 2002; received in revised form 8 December 2002

Abstract

A new median filter termed as the iterative center weighted median filter (ICWMF) in the wavelet coefficient domain is proposed for image denoising. Exploiting both inner- and inter-scale dependencies of the image wavelet coefficients, an improved estimation of the variance field is obtained using the proposed filter. This filter iteratively smoothes the noisy wavelet coefficients' variances preserving the edge information contained in the large magnitude wavelet coefficients. The variance field estimated using the ICWMF is then used in a minimum mean-square error estimator to denoise the noisy image wavelet coefficients. Simulation results show that higher peak-signal-to-noise ratio can be obtained as compared to other recent image denoising methods.

Keywords: Image denoising; Wavelet transform; Statistical modeling; ICWMF

1. Introduction

Denoising images corrupted by additive white Gaussian noise (AWGN) is a classical problem to the signal-processing community. The corruption of images by noise is common during its acquisition or transmission. The aim of denoising is to remove the noise while keeping the signal features as much as possible. Traditional algorithms perform image denoising in the pixel domain [7,8,13]. However, in recent years, wavelet transform-based image denoising algorithms show a remarkable success [1–4,6,10,11,14]. For the underlying problem, wavelet shrinkage method in [6] can provide an asymptotically minimax-optimal solution [14]. But for many image-processing applications, e.g. medical imagery, astronomical imagery, remote-sensing imagery, minimum mean-square error (MMSE) metric is more preferable [1–4,10,11,14]. Image denoising methods based on the overcomplete expansion of the wavelet transform show significant improvement in MSE than in the case of critically sampled wavelet transform [2,4,14]. In this paper our objective, however, is to obtain an improved image denoising performance over recently reported methods like [1,10,11], where critically sampled wavelet transform is used.

Assuming the transform coefficients within subbands are independent identically distributed (i.i.d.) random variables with generalized Gaussian (GG) distribution, the optimal MMSE estimator (Wiener filter) performs better than wavelet coefficient threshold methods [14]. However, such assumption may

* Corresponding author. Fax: +880-2-8613046.
E-mail addresses: khasan@eee.buet.ac.bd, kamrul_hasanbd@yahoo.com (Md. Kamrul Hasan).

0165-1684/03/$ - see front matter © 2003 Elsevier Science B.V. All rights reserved.
doi:10.1016/S0165-1684(02)00502-9

not be always realistic, considering the statistics of wavelet coefficients of natural images. The wavelet coefficients within subbands are, in fact, locally stationary and have dependency both in scale and across scales [1,9–11,14]. Considering such dependency with the neighbors, wavelet coefficients within each subband is modeled as independent Gaussian random variables with zero-mean and slowly varying variance [1,9–11] rather than GG distribution [2,3]. Estimation of the variance field using the Bayes-rule with a single exponential prior, inferred from the approximate histogram of the true image coefficients in each scale, as in [11], is a crude one especially in the finest scale for two reasons. First, the assumed prior neglects the high occurrence of very low-magnitude coefficients in the histogram. Second, the local statistics of the signal or noise is not the same as that of the whole subband. Thus, the estimation of the variance field in [11] motivated by the histogram analysis of the wavelet coefficients is unrealistic. The image denoising algorithm proposed in [1] adopts another approach considering both inner- and inter-scale dependency of the wavelet coefficients. In this method, if the parent value of an initial estimate of a coefficient variance is less than a predefined threshold, then it is set to zero. The denoising performance achieved by this method is close to that of reported in [11].

This paper proposes a new smoothing process in the wavelet domain using an iterative center weighted median filter (ICWMF) for better estimation of the variance field incorporating the local characteristics of the wavelet coefficients. For smoothing the variance field within the scale, the ICWMF does not assume any kind of particular distribution as in [1–3,10,11,14]. An exponentially decaying inter-scale model for image wavelet coefficients is also suggested for the adaptation of the ICWMF across scales. The key idea of the smoothing process is to suppress more noise from a preliminary estimate of the variance field without losing image features that may degrade the quality of the denoised image. The preliminary estimation is performed assuming that the wavelet coefficients are conditionally i.i.d. as in [9–11]. For re-estimation of the variance field, first, we classify the overall scales as the noise-dominant scales and the signal-dominant scales. The ICWMF is operated only on the noise-dominant scales. Due to the presence of a trace of signal component in these scales, the proposed ICWMF is designed as a special type of median filter that intelligently identifies the signal-dominant regions from the noise-dominated ones, and adapts itself in a predefined way during the smoothing process. Since high-energy wavelet coefficients correspond to the signal features of sharp variation like edges and textures, and low-energy corresponds to the smooth regions, during smoothing process we compare the center value within the mask to a prescribed threshold level to identify the signal- or noise-dominant regions in a scale. Because the adaptation of the proposed median filter depends on the magnitude of the center value within the mask, it is termed as the ICWMF. Finally, the variance field estimated from the ICWMF is used in the MMSE estimator for denoising wavelet coefficients.

2. Proposed denoising method using the ICWMF

2.1. Overview of statistical models

Many researchers have exploited the statistics of wavelet coefficients for wavelet coding and denoising of natural images. Considering the spatial clusters, coefficients within each band can be globally modeled as independent Gaussian random variables with zero-mean and slowly varying variance field [1,9–11]. The slow variation of the variance field can roughly be incorporated by a stochastic prior as in [11]. To obtain better accuracy, by an additional smoothing, we consider the absolute value of the wavelet coefficients as a measure of the signal feature instead of assuming any global distribution throughout the scale concerned. Also the magnitude of the wavelet coefficients of a typical image are strongly correlated across scales. Embedded zero tree wavelet (EZW) coder by Shapiro [12] and hidden Markov tree (HMT) model by Crouse et al. [5] exploited the dependency between a parent coefficient and its children. Here we instead take a relation that represents the average signal level distribution across scales. A typical image usually consists of smooth regions separated by a finite number of discontinuities. This results in a $1/f$-type spectral behavior, which leads to the near exponential decay of the wavelet coefficients across scales [1]. The standard deviation of the coefficients in each band represents the average signal level in that scale. Then it is expected

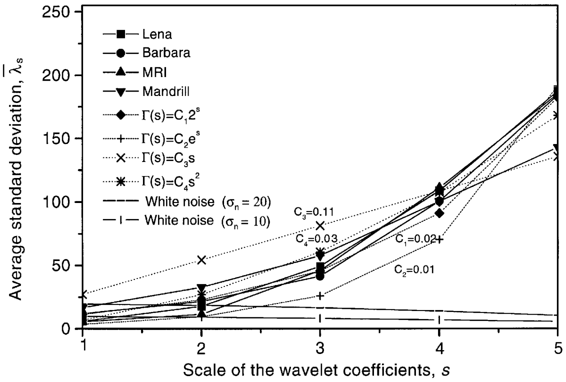

Fig. 1. Average signal level distribution across scales for different images ((——) distribution of $\bar{\lambda}_s$; (– – –) $\Gamma(s)=C\beta^s$ distribution; (· · ·) $\Gamma(s)=Cs^p$ distribution).

that standard deviations of the wavelet coefficients for a typical image have near exponential distribution across the scales. Under this circumstance, if $\bar{\lambda}_s$ is the average of three standard deviations of the wavelet coefficients of the three high subbands HH_s, HL_s, LH_s, for a given scale s, then $\bar{\lambda}_s$ should have a near exponential distribution across scales. The plot of $\bar{\lambda}_s$ across scales for different images is shown in Fig. 1. A normalized function $\Gamma(s)=C\beta^s, \beta>1$ or $\Gamma(s)=Cs^p$ may be used to match such distribution, with C being a suitable constant, β denotes a base parameter, and p is an integer exponent. Since the dilation parameter in case of dyadic expansion of the wavelet transform is 2^{-s}, $\Gamma(s)=C2^s$ can be a good choice to represent the distribution of $\bar{\lambda}_s$. Most interestingly, for a number of images it is found that $\Gamma(s)=C\beta^s$ with $\beta=2$ shows the best match in the least-squares sense.

2.2. Denoising algorithm

In this work, we assume that image pixels are corrupted by AWGN with known variance σ_n^2. If σ_n^2 is unknown, it may be estimated by applying the median absolute deviation (MAD) method [6] in the highest subband of the wavelet coefficients. Let $F_s(k)$ represent the wavelet coefficients of the true image for a given scale s. We use k as a brief notation to represent any two-dimensional indices (k_1,k_2) of the image wavelet coefficients. The corresponding wavelet coefficients of the noisy image are given by $G_s(k)=F_s(k)+N_s(k)$,

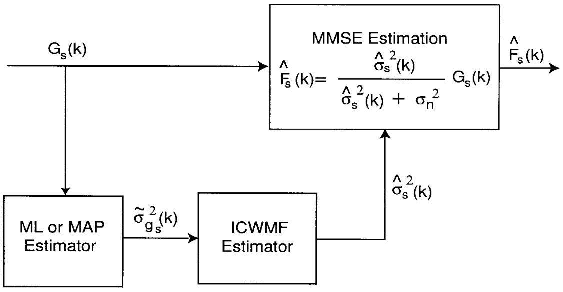

Fig. 2. Block diagram of the denoising algorithm.

where $N_s(k)$ are the wavelet coefficients of the additive noise.

The proposed denoising algorithm operates in three steps. First, we perform a preliminary estimation of the variance field $\tilde{\sigma}_{gs}^2(k)$ for the noisy coefficients, using maximum likelihood (ML) estimator proposed in [10] or maximum a posteriori (MAP) estimator proposed in [11]. In the second step, more accurate noisy coefficient variances $\hat{\sigma}_{gs}^2(k)$ are re-calculated by our proposed ICWMF. Finally, the denoised wavelet coefficients $\hat{F}_s(k)$ are estimated by the MMSE estimator given by

$$\hat{F}_s(k)=\frac{\hat{\sigma_s}^2(k)}{\hat{\sigma}_{gs}^2(k)}G_s(k)=\frac{\hat{\sigma}_s^2(k)}{\hat{\sigma}_s^2(k)+\sigma_n^2}G_s(k). \qquad (1)$$

Here $\hat{\sigma_s}^2(k)$ are the estimated variances of the noise-free coefficients using the proposed method. The block diagram of the proposed denoising algorithm is shown in Fig. 2.

2.3. Estimation of the underlying variance field

The performance of the MMSE-type estimator depends highly on the quality of the estimation of $\hat{\sigma}_{gs}^2(k)$ or simply $\hat{\sigma}_s^2(k)$. As mentioned in Section 2.2, preliminary estimation of the variance field is a prerequisite to the proposed smoothing process. For this estimation, assuming the correlation between the variance of the neighboring coefficients is high in the given data field $G_s(k)$, one can set $\tilde{\sigma}_{gs}^2(k)\approx\tilde{\sigma}_{gs}^2(j)$ for all $j\in\mathcal{N}(k)$. Here $\mathcal{N}(k)$ is a square-shaped local neighborhood centered at $G_s(k)$. Variance of the noisy coefficients, $\tilde{\sigma}_{gs}^2(k)$, can be estimated using the ML

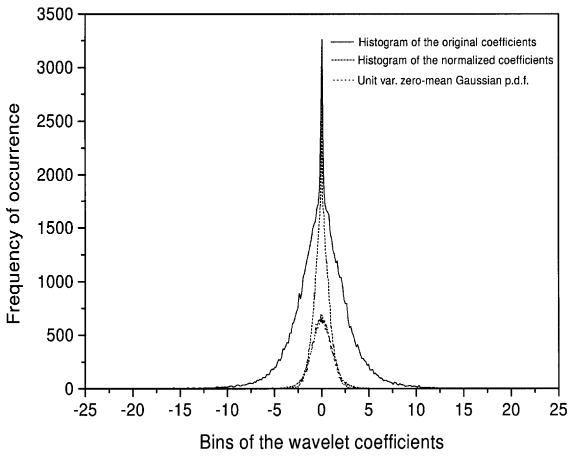

Fig. 3. Histogram of the original high band HH_1 wavelet coefficients of the image *Lena*.

estimator [10] as

$$\tilde{\sigma}^2_{gs}(k) = \frac{1}{M} \sum_{j \in \mathcal{N}(k)} G_s^2(j). \tag{2}$$

Here, it is assumed that noisy coefficients $G_s(j)$ have a Gaussian distribution with zero-mean and variance $\tilde{\sigma}^2_{gs} = \tilde{\sigma}^2_s + \sigma^2_n$, and M is the number of coefficients in $\mathcal{N}(k)$.

If the average power level of the signal coefficients in each subband, with a prior marginal distribution $f_{\tilde{\sigma}_s}(\tilde{\sigma}^2_s) = (1/\lambda_s)\exp(-\tilde{\sigma}^2_s/\lambda_s)$, is λ_s, then $\tilde{\sigma}^2_{gs}(k)$ can be estimated using the MAP estimator [11] as

$$\tilde{\sigma}^2_{gs}(k) = \frac{M\lambda_s}{4}\left[-1 + \sqrt{1 + \frac{8}{\lambda_s M^2}\sum_{j \in \mathcal{N}(k)} G_s^2(j)}\right]. \tag{3}$$

It is expected that the MAP estimator calculates the coefficients' variances more accurately than the ML estimator. But there is still an opt to improve the estimation accuracy, since the prior marginal distribution $f_{\tilde{\sigma}_s}$ of the MAP estimator in [11] does not match very closely with the actual distribution. Here the image priori function $f_{\tilde{\sigma}_s}$ is defined motivating from the normalized histogram of the true image wavelet coefficients, which approximately resembles the zero-mean, unit-variance, Gaussian probability density function (p.d.f.). Fig. 3 shows the histogram of the true image wavelet coefficients of high band HH_1 for the image *Lena*. It is clear from this histogram that the MAP estimator using a single exponential prior as in [11] will provide a good accuracy in estimating coefficient variances having high magnitude, since the reported prior function closely match the true distribution near the tail end. On the other hand, the exponential prior cannot incorporate satisfactorily the very high occurrence of the low-magnitude coefficients in the distribution represented by the sharp peak in the histogram. Then it can be concluded that the low-energy coefficient variances are overestimated in magnitude in case of the MAP estimator reported in [11]. In this paper, for renovation of the preliminary estimated variance field, we introduce a new smoothing process that is adapted to both inter- and inner-scale exploiting the statistics of wavelet coefficients. The adaptation is such that the smoothing process reduce the noise bias remaining in the preliminary estimated variance, which is most likely to be in the low-magnitude coefficients. It is evident from Fig. 1 that approximate wavelet coefficients of practical images mostly represent the signal coefficients. We can neglect the smoothing of the noisy coefficients in the scales where signal coefficients dominate, by defining a threshold in terms of λ_s. A subband having λ_s higher than the threshold value is assumed to have mostly signal coefficients with insignificant noise coefficients and vice versa. In the proposed method, we apply the modified median filter termed as the ICWMF to the noise-dominated high bands. If the preliminary estimate of the local variances using an existing estimator, e.g. ML or MAP, is $\tilde{\sigma}^2_{gs}(k)$, then the re-estimated variance $\hat{\sigma}^2_{gs}(k)$ after passing through the proposed filter is given by

$$\hat{\sigma}^2_{gs}(k) = \begin{cases} \tilde{\sigma}^2_{gs}(k) & \text{if } \lambda_s \geqslant \lambda_{\text{th}}, \\ \text{med}^i_{\text{cw}}(\tilde{\sigma}^2_{gs}(j)) & \text{if } \lambda_s < \lambda_{\text{th}}, \end{cases} \tag{4}$$

for all $j \in \mathcal{N}(k)$, where med^i_{cw} represents the center weighted median value of the given set of data for the ith iteration and λ_{th} denotes a threshold value for discriminating signal-dominant scales from the noise-dominant ones. The parameter λ_s can be calculated from the standard deviation of the preliminary estimate of the signal coefficients' variances $\tilde{\sigma}^2_s(k)$ in each scale [11]. To define the discriminating threshold λ_{th} we refer to the distribution of standard deviations of the true and noisy image coefficients across scales as shown in Fig. 4. It can be seen that the difference between the plots of noise-free and noisy λ_s is

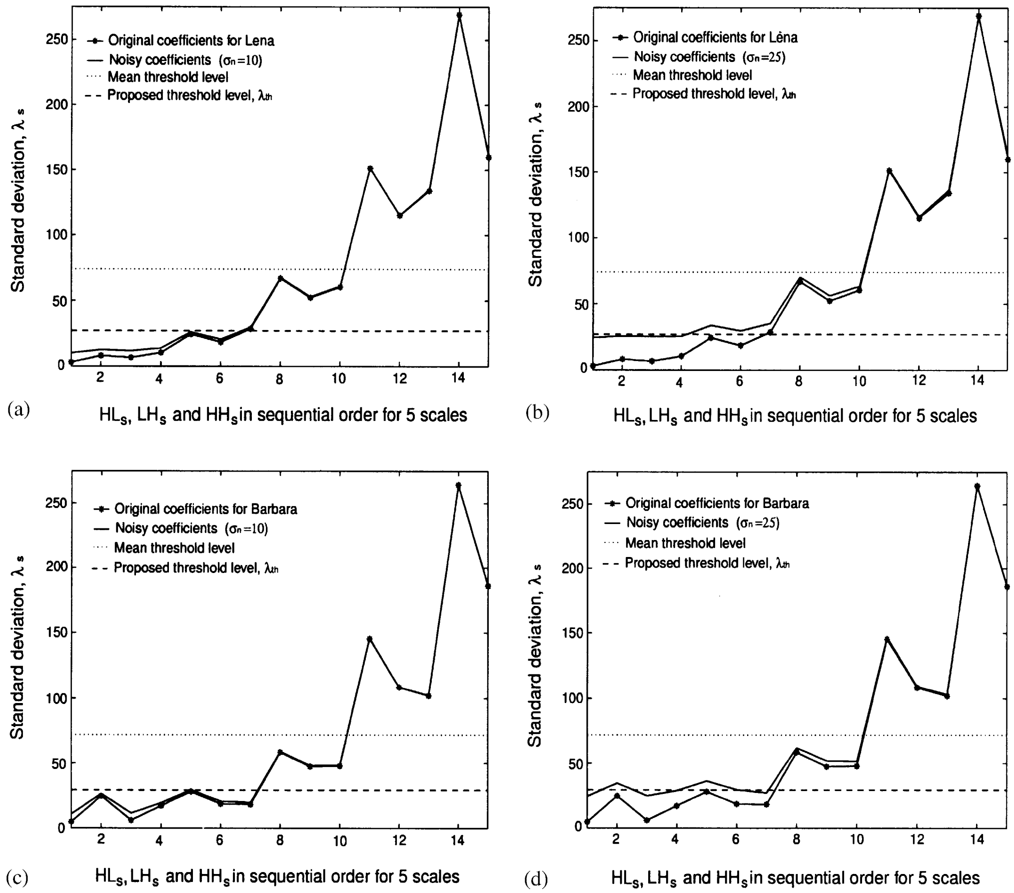

Fig. 4. Distribution of λ_s for original and noisy images across scales (scales represented by asterisks below the proposed threshold level λ_{th} will undergo smoothing process).

significant only in the lower scales. This also verifies that higher scales should be set aside during smoothing. Under this circumstance a demarcation line has to be drawn to decide which scales should undergo smoothing process. A very simple demarcation line between signal and noise-dominant scales may be the arithmetic mean of λ_s. But in this case as shown in Fig. 4 some scales would undergo smoothing though not desired. Doing this is expected to contribute in increasing distortion rather than removing noise. To remain in the safe side an weighted average of λ_s may be considered that will produce a threshold level below the aforementioned threshold value. Since, the higher the scale value s, the higher the signal component on average, intuitively we give lower weights to the higher scales and higher weights to the lower scales. One choice of weighting function can be the inverse of the proposed near exponential prior $\lambda_s = C2^s$, with C being a suitable constant. Then the threshold parameter λ_{th} can be estimated as

$$\lambda_{\mathrm{th}} = \frac{\sum_s \lambda_s 2^{-s}}{\sum_s 2^{-s}}. \tag{5}$$

It is to be noted that λ_{th} thus estimated may not be the best choice, but it gives a good parametric value to isolate the scales with sufficient noise margin.

In our proposed denoising algorithm only the noise-dominated scales will undergo smoothing. It is obvious that in these scales there exist noise along with the signal component. Therefore, our objective is to remove noise from these bands with less possible distortion of the signal components. To achieve

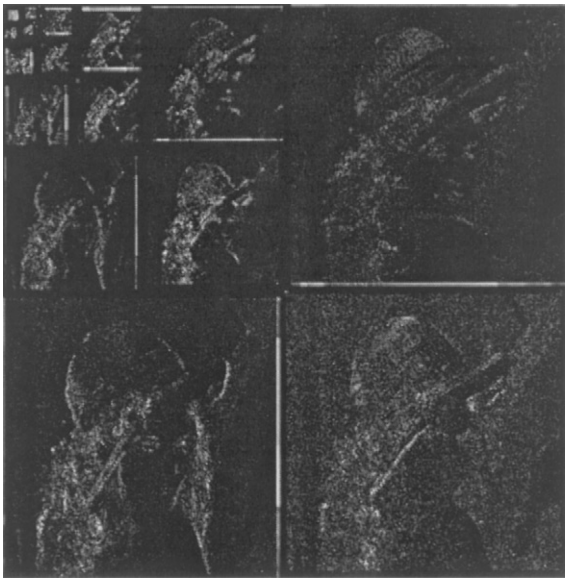

Fig. 5. Five-level wavelet decomposition of the image *Lena*. White pixel means the large magnitude coefficients, and black means the small magnitude.

this goal, the proposed ICWMF first identify the signal or noise-dominant regions within the scale and performs smoothing in a prescribed way. Within a scale to distinguish signal-dominant regions from the noise-dominant ones, we choose power level of the center coefficient of the mask as a determining factor. Magnitude of a wavelet coefficient and therefore its energy is a distinctive indicator of the image feature like edge, texture. This can be verified from Fig. 5, which shows the wavelet decomposition of the original *Lena* image. For a noisy image, it is expected that the magnitude of the wavelet coefficients will be increased on average due to the addition of noise. This reveals that the low-energy coefficients corresponding to the smooth regions in an image are dominated by the noise. We expect that in this region smoothing of the preliminary estimated variance field by a straight median filter can produce better estimate of its noise-free value. On the contrary, high-energy coefficients, which correspond to the regions of sharp transition in an image, are dominated by the signal. Using a straight median filter in this region would have adverse affect on the signal coefficients due to over smoothing. If we preserve the center value as it is in this region, it ensures the accuracy of the preliminary estimate. But our aim is to improve the performance in this region if possible. Let us take a hypothetical situation when center value of the variance field is less than the other values within mask. If we add a second term to the center value which is a weighted median of the differences of center to other values within the mask concerned, it is expected that center value will be boosted up and thereby enhance the edge information. In the other cases, the second term will simply smooth the gradient of the preliminary estimates and, hence, increase the convergence of smoothing process. To distinguish whether the variance is dominated by noise or by signal, we prescribe a scale-independent threshold level which is a function of noise variance. Finally, our proposed adaptive median filter $\mathrm{med}^i_{\mathrm{cw}}$ can be defined as

$$\mathrm{med}^i_{\mathrm{cw}}(\tilde{\sigma}^2_{gs}(j)) = \begin{cases} \mathrm{med}(\tilde{\sigma}^2_{gs}(j)) & \text{if } \tilde{\sigma}^2_{gs}(k) \leqslant \gamma\sigma^2_n \\ \tilde{\sigma}^2_{gs}(k) + \mathrm{med_w}(\tilde{\sigma}^2_{gs}(j) & \\ \quad -\tilde{\sigma}^2_{gs}(k)) & \text{if } \tilde{\sigma}^2_{gs}(k) > \gamma\sigma^2_n. \end{cases} \tag{6}$$

The parameter γ is a tuning factor that controls weight of the center value and the median value according to the mask size. The larger the mask size, the greater the chance that median value can affect the center value, and hence the smaller the value of γ. Since a straight median operator on the second term of the second condition in Eq. (6) represents nothing but the first condition, it necessitates that the second term be weighted. Let us take a test case to show how our proposed median smoothing is very likely to give better estimate of the coefficient variances. Fig. 6 represents the results of different kinds of iterative median smoothing to denoise the first level detail wavelet coefficient variances of an arbitrary square pulse having a width of 32 samples and a magnitude of +255, which is corrupted by AWGN with noise variance 25. A simple median smoothing just ignores the high-magnitude signal variances as shown in Fig. 6(a). For a signal with high peak-signal-to-noise ratio (PSNR), we can assume that the larger the peak value of the variance field, the greater is its chance to be signal dominant. Based on this fact the center value within the mask can be kept unaltered during smoothing, if the coefficient variance is greater than the proposed threshold

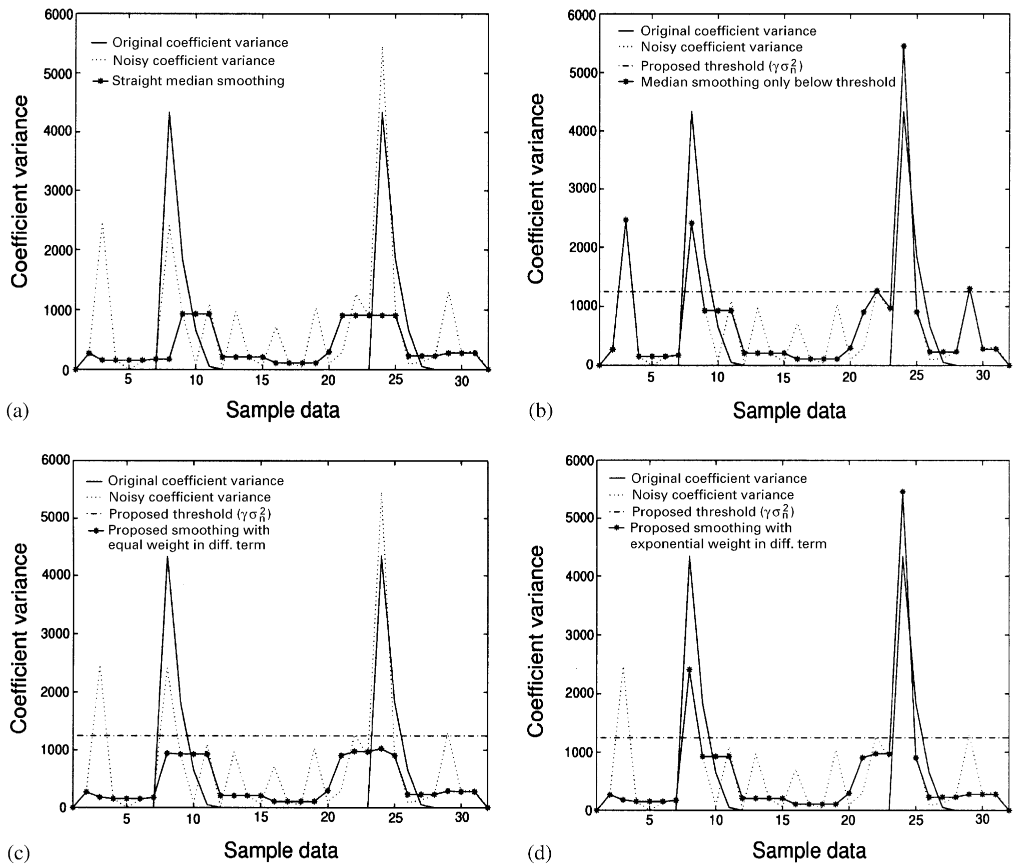

Fig. 6. Comparing the performances of the simple median and weighted median smoothing (values represented by asterisks denote the estimated variances after 40 iterations with smoothing window size 5).

$\gamma\sigma_n^2$. Result of such kind of smoothing is shown in Fig. 6(b). As can be seen, in this case some of the coefficient variances due to noise remain unaltered along with the signal-dominant coefficient variances, which is undesirable. Figs. 6(c) and (d) show results using a special type of smoothing where variances having magnitude larger than the threshold $\gamma\sigma_n^2$ are modified by adding the weighted median value of the difference terms derived from the local neighborhood. We choose the weight space B such that

$$|B|_{\mathrm{w}} = \sum_{j \in \mathcal{N}(k)} w(j) = 1, \tag{7}$$

to ensure that the weighting to the additive term is less than unity. A very simple idea to define the weighting function $w(j)$ is to take equal weight for all the difference terms within the mask. That means the additive quantity for modification will be the median value of the difference terms, each multiplied by $1/M$. But this will simply lead to the destiny of the overall median operation after few iterations, which can be verified from Fig. 6(c). A remedy can be a selection of a more appropriate weighting function. It is to be chosen in such a way that it gives less weight to the signal-dominant coefficient variances than to the noise-dominant ones present in the difference terms. To meet such a requirement an exponential weight can be a good choice because the signal coefficient distribution within a scale can be assumed to be exponential as shown in Fig. 3. The result shown in Fig. 6(d) is obtained using such an exponential weighting function defined as $w(j) = \exp(-\sigma_s^2(j)/\lambda_s)$. As can be seen, this type of median smoothing can only preserve the two peaks of the signal coefficient variance field, while other noisy variances are sufficiently smoothed. Based on these observations, we choose the weighting function $w(j)$ to be a function of negative exponential of the preliminary estimated variance field $\tilde{\sigma}_s^2(j)$ in our proposed image denoising algorithm. The parameter λ_s represents the expected value of $\tilde{\sigma}_s^2(k)$ in a given scale s. Multiplying the exponent term in the weighting exponential by $1/\lambda_s$ will slightly increase the smoothing rate but it is only optional. Then the weighting elements B can be defined as

$$B(j) = \frac{\exp(-\tilde{\sigma}_s^2(j)/\lambda_s)}{\sum_{j\in\mathcal{N}(k)} \exp(-\tilde{\sigma}_s^2(j)/\lambda_s)} \tag{8}$$

for all $j \in \mathcal{N}(k)$. Finally, the weighted median operator $\mathrm{med}_{\mathrm{w}}$ can be obtained as

$$\mathrm{med}_{\mathrm{w}}(\tilde{\sigma}_{gs}^2(j)) = \mathrm{med}(\tilde{\sigma}_{gs}^2(j)B(j)) \tag{9}$$

for all $j \in \mathcal{N}(k)$. Since our proposed median smoothing is an iterative process, a terminating criterion is required using the convergence characteristics. Convergence properties of iterative median and special type of weighted median filters are available in the recent literature [7,13]. Our smoothing filter ICWMF consists of two parts—one is the simple median operator and the other is the proposed weighted median filter. The first part is assured to converge in a single pass [13]. In the second part, the center value of the window has given the highest weight. Additive term in this part to modify the center value is also derived from a special median operation as described previously. For a symmetric center weighted median filter if $w(k) \geqslant \sum_{j\in\mathcal{N}(k), j\neq k} w(j)$, then it is assured to converge after a finite number of passes [13]. Since in our case $|B|_{\mathrm{w}}$ is unity, we can expect that our proposed median filter will also converge. To show that our proposed ICWMF possesses good convergence characteristics, we select an index that measures the extent of randomness of a given data set. If in successive iterations this index remains almost unchanged, then it may be assumed that the algorithm has converged and no further iteration is required. Log-entropy is known to be a good index to measure the degree of randomness of a given data set. If it is used as the cost function, then its value is expected to decrease with the progress of iteration; because in every iteration a certain amount of smoothing is performed on the data set corrupted by random noise. This can be justified using the results of our test case as shown in Fig. 7. As can be seen, after few iterations (e.g., 20) there is almost no change in the cost function. Thus the difference in the successive values of the cost function can be used as a terminating criterion. Finally, the cost function in the estimated noisy coefficient variance field is given by

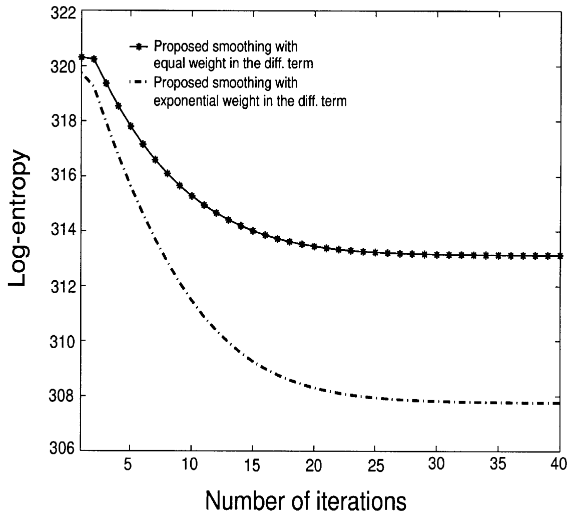

Fig. 7. Variation of 'log-entropy' with iteration for the proposed weighted median smoothing.

$$H^i(\tilde{\sigma}_{gs}^2) = \sum_{k=1}^{J_s} \ln[\tilde{\sigma}_{gs}^2(k)] \tag{10}$$

with J_s being the number of coefficients in a given subband s. If $|H^i - H^{i+1}|/|H^i|$ is less than a very small value ε, then the iteration is terminated. And the signal coefficients' variances can be estimated from $\hat{\sigma}_{gs}^2(k)$ as

$$\hat{\sigma}_s^2(k) = \max[0, \hat{\sigma}_{gs}^2(k) - \sigma_n^2]. \tag{11}$$

3. Simulation results

Simulations have been performed on a number of images, but only results for 512 × 512 gray-scale images *Lena* and *Mandrill* are presented here. The orthogonal wavelet Daubechies' of length-8 was used and decompositions were 5 levels deep. Centered square-shaped windows of sizes 3 × 3, 5 × 5 and 7 × 7 were applied to estimate each signal coefficient

Table 1
Results in PSNR (dB) for several denoising methods

Denoising algorithm	Noise standard deviation σ_n			
	10	15	20	25
Lena				
wiener2	32.99	30.43	28.53	26.95
SAWT [2]	—	32.36	31.04	30.04
Mihçak [10]	34.39	32.38	30.96	29.84
3 × 3 LAWML	33.79	31.41	29.63	28.22
3 × 3 ICWMML	34.48	32.40	30.92	29.78
3 × 3 LAWMAP	34.26	32.33	30.97	29.97
3 × 3 ICWMMAP	34.48	32.62	31.26	30.23
5 × 5 LAWML	34.22	32.05	30.51	29.29
5 × 5 ICWMML	34.55	32.55	31.20	30.18
5 × 5 LAWMAP	34.40	32.43	31.11	30.11
5 × 5 ICWMMAP	34.54	32.59	31.25	30.25
7 × 7 LAWML	34.26	32.17	30.73	29.59
7 × 7 ICWMML	34.44	32.41	31.08	30.01
7 × 7 LAWMAP	34.34	32.36	31.04	30.03
7 × 7 ICWMMAP	34.44	32.46	31.14	30.12
Mandrill				
wiener2	29.85	27.13	25.32	23.97
3 × 3 LAWML	30.08	27.47	25.74	24.46
3 × 3 ICWMML	30.29	27.76	26.14	24.96
3 × 3 LAWMAP	30.09	27.52	25.87	24.69
3 × 3 ICWMMAP	30.24	27.68	26.03	24.86
5 × 5 LAWML	30.24	27.69	26.05	24.85
5 × 5 ICWMML	30.32	27.82	26.24	25.09
5 × 5 LAWMAP	30.24	27.69	26.08	24.91
5 × 5 ICWMMAP	30.31	27.79	26.19	25.02
7 × 7 LAWML	30.26	27.73	26.13	24.96
7 × 7 ICWMML	30.30	27.79	26.21	25.06
7 × 7 LAWMAP	30.26	27.73	26.13	24.98
7 × 7 ICWMMAP	30.30	27.78	26.19	25.04

variance, $\hat{\sigma}_s^2(k)$. The parameter γ was set empirically. We have observed that the suitable value of γ lies in the range 1–4. The value ofthis parameter is found to be dependent on the mask size. In the simulation, we have used $\gamma = 3$ for the mask size 3 × 3, 2 for the mask size 5 × 5, 1.5 for the mask size 7 × 7.

A performance comparison of the proposed method in terms of PSNR with other reported methods, namely, *wiener2*, SAWT, Ref. [10], LAWML, and LAWMAP is shown in Table 1. Here *wiener2* is the MATLAB's 2-D noise removal filter available in the image-processing toolbox, SAWT refers to the spatially adaptive wavelet thresholding method [2], LAWML and LAWMAP, respectively, refer to locally adaptive window-based denoising using ML, and locally adaptive window-based denoising using MAP [11]. The entries in Table 1 corresponding to the SAWT method are based on the critically sampled wavelet transform as it is similar to our approach. Results using the proposed ICWMF are presented as two different methods, and they are categorized according to the two different preliminary estimates of the variance field. The method based on the approximate ML estimator to compute the variance field will be termed as the *iterative center weighted median-based denoising using ML* (ICWMML). And the other method, using the approximate MAP estimator to compute

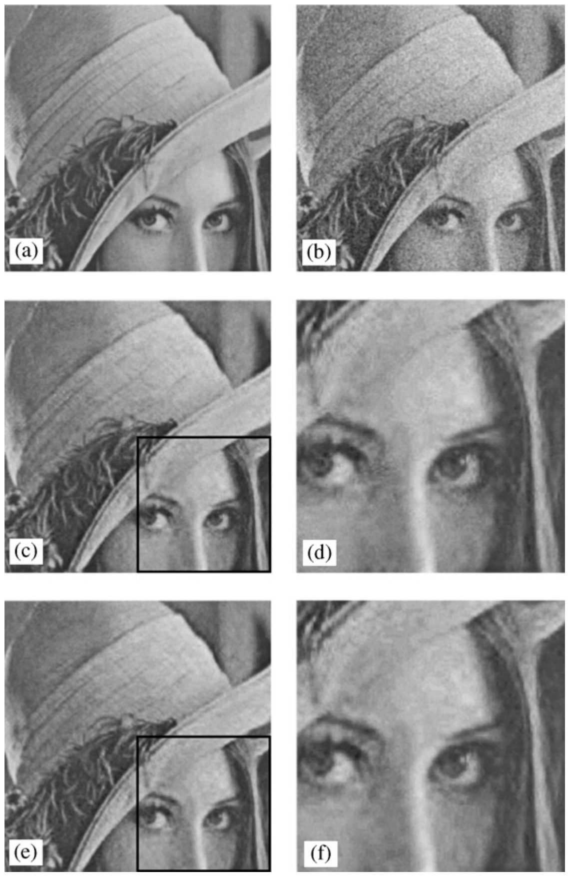

Fig. 8. Comparing the performances of the methods LAWML and ICWMML on the image *Lena* with $\sigma_n = 20$ and mask size 5 × 5: (a) original, (b) noisy observation, (c) denoising by the LAWML, (d) zoomed-in section of (c), (e) denoising by the ICWMML, and (f) zoomed-in section of (e).

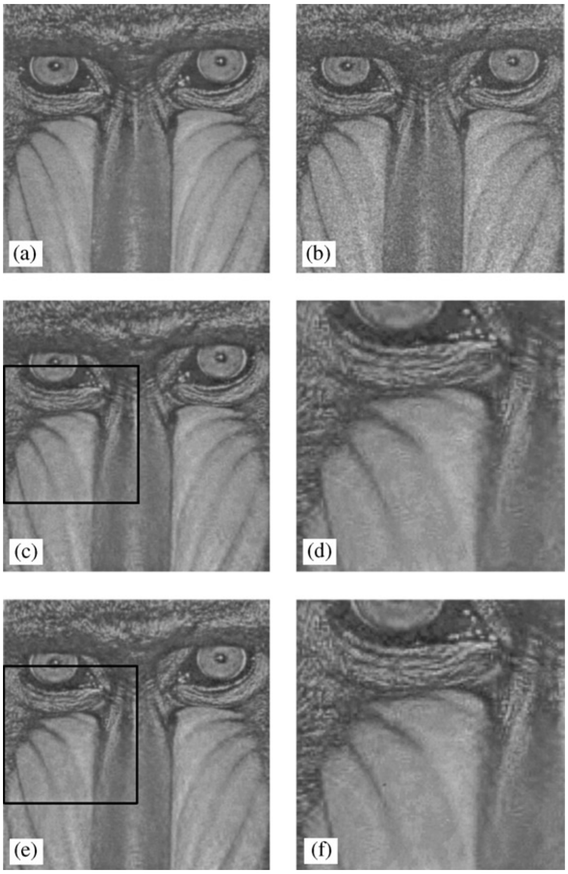

Fig. 9. Comparing the performances of the methods LAWMAP and ICWMML on the image *Mandrill* with $\sigma_n = 20$ and mask size 3×3: (a) original, (b) noisy observation, (c) denoising by the LAWMAP, (d) zoomed-in section of (c), (e) denoising by the ICWMML, and (f) zoomed-in section of (e).

the variance field will be termed as the *iterative center weighted median-based denoising using MAP* (ICWMMAP). Both the proposed methods are simulated for different terminating conditions by varying the value of ε as stated in the previous section. Simulations reveal that the arbitration of its value does not show any considerable degradation of the performance. The value of ε was set to $\sim 1.0\exp(-6)$ for the ICWMML method and $\sim 1.0\exp(-3)$ for the ICWMMAP method. It is evident from Table 1 that the proposed methods show better performance in terms of PSNR as compared to the other very often cited approaches in the literature for both the *Lena* and *Mandrill* images.

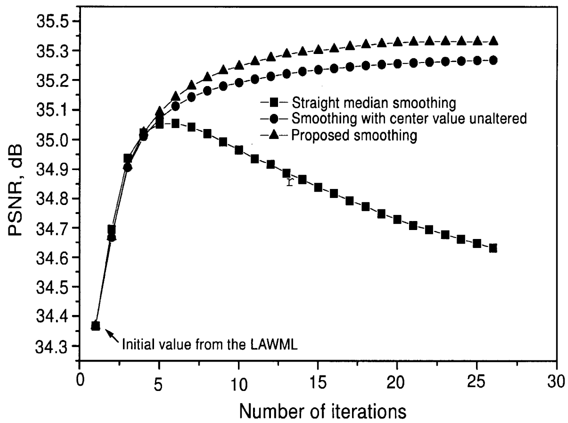

Fig. 10. Comparing the performances of straight median filter, ICWMML as in Eq. (6) except that the second term in its second condition is absent, and the proposed method ICWMML for HL_1 coefficients of the image *Lena* with $\sigma_n = 15$ and mask size 5×5.

Fig. 8 shows a comparison of the denoising performance of the LAWML method with the ICWMML method using a mask size 5×5 for the *Lena* image corrupted by noise level of $\sigma_n = 20$. A zoomed-in section (inside the square) of the image is displayed in order to view the details. Next, we compare the improvement of the proposed ICWMML method with the LAWMAP method for the image *Mandrill* with noise level $\sigma_n = 20$ using a mask size of 3×3. As shown in Fig. 9, the zoomed-in sections of the denoised image demonstrate noticeable improvement by the proposed one over the previous one.

Since the results in the coefficient domain may give more insight of the proposed technique, we simulate it also in the coefficient domain. Fig. 10 shows the performance comparison in terms of PSNR among three possible ways of smoothing by a median filter. Here the performances are simulated for denoising the coefficients in HL_1 of *Lena* image degraded by a noise level of $\sigma_n = 15$ using a mask size of 5×5. The performance curves in Fig. 10 reveal that the first one, a straight median filter degrades its performance in successive iterations after the first few iterations. The second method, which is a modified median filter as in Eq. (6) except that the second term in the second condition is absent, shows consistent improvement and better result than the straight median filter. The last method, which is the proposed ICWMML, shows

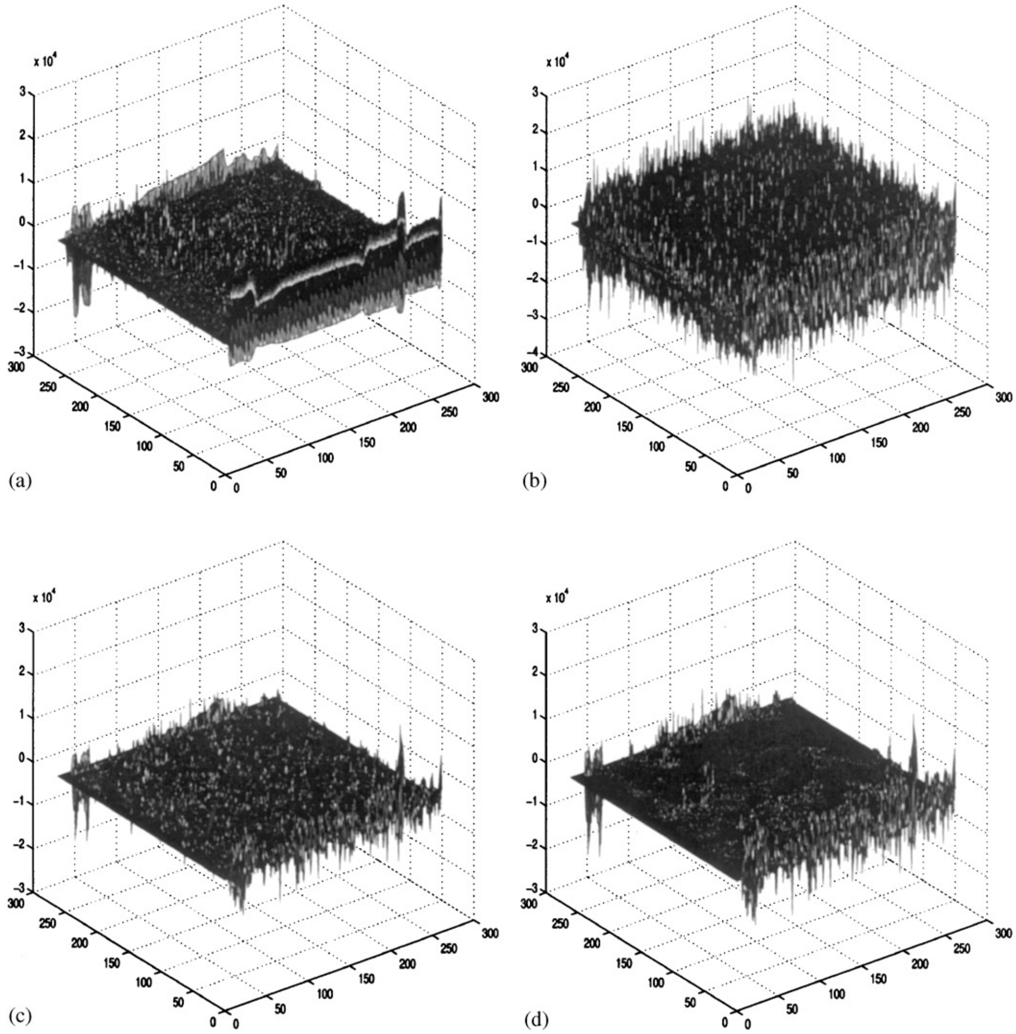

Fig. 11. Comparing the performances of LAWMAP method with ICWMML method for HL_1 coefficients of the image *Lena* with $\sigma_n = 15$ and mask size 5×5: (a) original, (b) noisy observation, (c) denoising by LAWMAP, and (d) denoising by ICWMML.

faster convergence and significant improvement than the other two in terms of PSNR. Here lies the justification of adding the second term in the second condition in Eq. (6). Also the PSNR improvement in these coefficients using the method ICWMML is ~ 0.90 dB when compared to the method LAWML and ~ 0.45 dB when compared to the method LAWMAP. The 3-D plot of these denoised wavelet coefficients using the ICWMML and LAWMAP is shown in Fig. 11 for further verification of the improved performance. As can be seen, smoothing of the coefficients is obtained preserving the detail information. Considering the results mentioned above, we can conclude that our proposed methods show superior performance over the usual denoising techniques both in image quality and MSE measure.

4. Conclusion

An effective wavelet domain iterative center weighted median filter (ICWMF) for image denoising has been proposed. The proposed filter operates on a preliminary estimate of the variance filed. For adaptation of the proposed filter, we have exploited both inner- and inter-scale statistics of the image wavelet coefficients. An exponentially decaying inter-scale model for image wavelet coefficients is used to classify the scales into the signal and noise-dominant ones. The ICWMF operates only on the noise-dominant scales. For smoothing process within these scales, the proposed filter exploits inner-scale statistics, where coefficient energy is chosen as an index for classification of signal and noise-dominant regions. The simulation results have shown substantial improvement in the denoising performance by the proposed methods over some of the recent methods both in visual quality and peak-signal-to-noise ratio (PSNR).

References

[1] Z. Cai, T.H. Cheng, C. Lu, K.R. Subramanium, Efficient wavelet-based image denoising algorithm, Electron. Lett. 37 (11) (2001) 683–685.

[2] S.G. Chang, B. Yu, M. Vettereli, Spatially adaptive wavelet thresholding with context modeling for image denoising, IEEE Trans. Image Process. 9 (9) (2000) 1522–1531.

[3] S.G. Chang, B. Yu, M. Vettereli, Adaptive wavelet thresholding for image denoising and compression, IEEE Trans. Image Process. 9 (9) (2000) 1532–1546.

[4] R.R. Coifman, D.L. Donoho, Translation-invariant de-noising, in: A. Antoniadis, G. Oppenheim (Eds.), Wavelets and Statistics, Springer, Berlin, Germany, 1995.

[5] M.S. Crouse, R.D. Nowak, R.G. Baraniuk, Wavelet-based statistical signal processing using hidden Markov models, IEEE Trans. Inform. Theory 45 (1999) 846–862.

[6] D.L. Donoho, Denoising by wavelet thresholding, IEEE Trans. Inform. Theory 41 (3) (1995) 613–627.

[7] F. Guichard, J.-M. Morel, Image iterative smoothing and PDE's, Lecture Notes, School of Mathematical Problems in Image Processing, The Abdus Salam International Centre For Theoretical Physics, September, 2000.

[8] S.S. Jiang, A.A. Sawchuk, Noise updating repeated Wiener filter and other adaptive noise smoothing filters using local image statistics, Appl. Opt. 25 (1986) 2326–2337.

[9] Juan Liu, P. Moulin, Information-theoritic analysis of interscale and intrascale dependencies between image wavelet coefficients, IEEE Trans. Image Process. 2001 (Preprint available at http://www.ifp.uiuc.edu//).

[10] M.K. Mihçak, I. Kozintsev, K. Ramchandran, Spatially adaptive statistical modeling of wavelet image coefficients and its application to denoising, Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing, Vol. 6, Phoenix, AZ, 1999, pp. 3253–3256.

[11] M.K. Mihçak, I. Kozintsev, K. Ramchandran, P. Moulin, Low-complexity image denoising based on statistical modeling of wavelet coefficients, IEEE Signal Process. Lett. 6 (12) (1999) 300–303.

[12] J.M. Shapiro, Embedded image coding using zerotrees of wavelet coefficients, IEEE Trans. Signal Process. 41 (1993) 3445–3462.

[13] L. Yin, R. Yang, M. Gabbouj, Weighted median filters: a tutorial, IEEE Trans. Circuits and Systems 43 (1996) 157–191.

[14] H. Zhang, A. Nosralinia, R.O. Wells Jr., Image denoising via wavelet-domain spatially adaptive FIR wiener filtering, Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing, Vol. 5, Istambul, Turkey, 2000, pp. 2179–2182.