ECE 486 Lab Pair-up Problem

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- Here we describe a pairing-up problem and try to compute how many pairs there are such that the names of lab partners of each pair would appear *consecutively* in the roster in alphabetic order.

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- A **consecutive names** problem in lab pairing up. First the problem is stated as what I have been *observing* for ten semesters.

  Problem: In a class of 16 students, each student has to work with *one* other person to form a pair. In the final pairing-up, there are *always* at least one pair whose names in alphabetic order appear consecutively in the roll call list.

  Remark 1: It might be that the class size is a factor. Suppose the class size is 2. There is no choice for the two students in the class, and the claim of the problem above is *trivially* true.

  Remark 2: However the claim in the problem above is not always true, for example, consider the case where four students with last name initials A, B, C, D. In the final pairing-up, student A is the partner of student C and student B has no choice but to be paired up with student D. This situation will make the claim false since in the roster, names of partners students A and C (or B and D) appear alternately in alphabetic order. As in this case, of class size of 4, the probability of the claim to be true is $\frac{2}{3}$, making this situation a rather probable case. As the class size increases, the probability of the claim to be true maybe also increases. That is why the claim of problem seems plausible in the first place.

  Remark 3: Since if we follow Remark 2 above, the original problem cannot be proved true all the time, we shall modify the problem as follows,

  Problem*: In a class of given size of $2N$ ($N = 1, 2, \ldots$) students, each student has to work with *one* other person to form a pair. In the final pairing-up, what is the *probability* of the situation where there are at least one pair whose names in alphabetic order appear consecutively in the roll call list.

Remark 4: All we have to do with the modified problem* is to find out all the cases when there is *not* a single pair whose names appear consecutively in alphabetic order in the roster.

Remark 5: Follow Remark 4, we tested different class sizes. When $N = 4$, there is one pair out of three where the pairs' names appear non-consecutively in the roster. When $N = 6$, there are five out of 15; $N = 8$, 36 out of 105; and $N = 10$, 329 out of 945. Further cases are documented as below in the Table 1.

Table 1: Tested cases with different class sizes N

Class size N	Total combinations	Non-consecutive cases	Probability
4	3	1	0.3333333333
6	15	5	0.3333333333
8	105	36	0.3428571429
10	945	329	0.3481481481
12	10395	3655	0.3516113516
14	135135	47844	0.3540459540
16	2017025	721315	0.3576133166

Remark 6: As seen in Table 1, when the class size increases, the possibility of having no pairs such that their names appear consecutively in the roster also *increases slowly*. What is the limit of this probability when $N \to \infty$?
Remark 7: There is a closed form formula for calculation of the second column of the table above.

$$\text{Total combinations } (N) = \frac{\binom{N}{2}\binom{N-2}{2}\cdots\binom{4}{2}\binom{2}{2}}{\left(\frac{N}{2}\right)!}$$

- From the above discussion, given a class size $N > 0$ (also even), can we compute the number of non-consecutive cases in the third column of the table above with a closed form formula associated with N? If not possible, is it possible to find the limit of the probability in the fourth column as $N \to \infty$?

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