Trend, Cycle and Expectation Formation*

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Abstract. We characterize how forecasters form expectations when they cannot perfectly distinguish between trends and cycles. This model is motivated by a set of findings from the Survey of Professional Forecasters, which reveal various patterns in forecasting behaviors across different forecast horizons. These facts are inconsistent with the common assumption in the expectation formation literature that trends are stable or observable. Our framework can be applied to account for changes in forecasting behavior following the introduction of explicit inflation targeting in 2012. We also extend the model to incorporate behavioral biases so as to address empirical puzzles documented in the literature.

Keywords. expectation formation, trend, cycle

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1 Introduction

There is a growing interest in understanding how forecasters form expectations and make forecasts on macroeconomic variables. The literature on expectation formation primarily focuses on how forecasters make predictions for macroeconomic variables following a stationary process, with fewer studies investigating how deal with trends and cycles in their expectation formation process.

In this paper, we introduce a simple framework to characterize how forecasters update their beliefs, form expectations and make forecasts when macroeconomic variables consist of trends and forecasters cannot perfectly distinguish between trends and cycles. This theoretical exploration is empirically relevant. We provide a set of facts concerning forecasting behaviors across forecast horizons to inform the process of expectation formation and show that they are not consistent with the conjecture that forecasters believe trends are stable or observable.

We begin by constructing proxies for forecasters' beliefs regarding trend and cyclical components by utilizing data from the Survey of Professional Forecasters (SPF). To proxy beliefs about trends, we rely on their three-year-ahead forecasts for the macroeconomic variables (e.g., real GDP growth rate and the unemployment rate). To proxy forecasters' beliefs on short-term cyclical components, we use the deviation between the now-cast of the relevant macroeconomic variable and its three-year-ahead forecast.

Utilizing the proxies, we design a new empirical test in which we construct the "across-period changes" in trend beliefs and across-period changes in cyclical beliefs, and use their correlation to inform the process of belief updating. Specifically, we proxy changes in one's trend beliefs as the difference between their three-year ahead forecasts between quarter t and $t-1$. Similarly, we proxy changes in one's cyclical beliefs by examining the corresponding cross-period changes in the cyclical component we constructed. If the forecasted variable follows a stationary data generation process (e.g., an AR(1) process), the standard forecasting model would predict that the correlation is zero. That's because cross-period changes in cyclical beliefs reflect innovations in cyclical components and new information between quarter t and $t-1$. The changes in trend beliefs reflect measurement errors if forecasters believe the trend is stable.

We estimate the correlation between changes in trend beliefs and cyclical beliefs using forecast data for various macroeconomic variables. We find that most of the correlations are significantly negative, which contradicts the prediction of standard models with a stationary process.

Furthermore, we examine how the dispersion of forecasts varies over the forecast horizon. This pattern is also informative about how forecasters form expectations. Specifically, utilizing the SPF data, we observe that for most macroeconomic variables (after being transformed into growth rate), except for inflation, forecast dispersion increases as the forecast horizon extends from zero to four quarters ahead. To address any concerns that a four-quarter forecast horizon may not be sufficiently long, we conduct a number of additional robustness tests. For example, utilizing year-level expectation data for real GDP and the unemployment rate, we show that the year-level forecast dispersion also increases as the forecast horizon expands from one to three years (or even longer horizon), providing further corroborating evidence.

This finding is also inconsistent with models that assume a stationary process for the forecasted variables. In those models, the significance of dispersed private information regarding the current state diminishes for forecasting future states. As the impact of state innovation decreases over longer horizons, heterogeneity across forecasters diminishes as the forecast horizon expands. For instance, if the state follows an AR(1) process with a specific long-run mean, the forecasts by various forecasters eventually converge to that mean with a sufficiently long forecast horizon. Consequently, there will be little forecast dispersion among forecasters for the long-term forecast.

While this set of facts contradicts the predictions of the standard models that assume stable trends, can an augmented model, which allows for a non-stationary trend component, account for them? We show that such an augmented model still fails to produce the empirical patterns if the trend component is ex post observable. For the first pattern, the changes in trend beliefs reflect innovations in trend components that are independent of innovations in cyclical components. Therefore, the correlation between changes in trend beliefs and cyclical beliefs in this model is still zero. For the second pattern, the dispersion of forecasts caused by heterogeneity in trend beliefs across forecasters remains stable over the forecast horizon. Therefore, the model always predicts a decreasing dispersion of forecasts over the horizon, which is caused by heterogeneity in cyclical beliefs across forecasters.

Motivated by those findings, we propose an otherwise standard forecasting model that explicitly incorporates a non-stationary, unobservable trend component in the data generation process. Specifically, in this model, the state variable consists of a non-stationary random walk trend component and a cyclical component that follows the standard AR(1) process. The goal of forecasters is to minimize the squared error of their forecasts. The actual value of the state, which is the sum of these two components, is publicly announced and observed by forecasters at the end of each period.

The key assumption is that forecasters cannot directly observe the actual realizations of the trend and cyclical components. Instead, in each period, they receive two private noisy signals on the trend and cyclical components, respectively. This means that they are unable to differentiate the two components perfectly and have to make inferences about them.

In such a setting, forecasters will need to update their beliefs about the trend and cyclical components *twice* in each period. At the beginning of each period, forecasters receive private signals regarding the trend and cyclical components and then revise their beliefs on each component. Forecasters use this set of posterior beliefs to make forecasts that minimize the expected forecasting errors. At the end of each period, the actual state value is disclosed, which is informative about the trend and cyclical components as well. Therefore, forecasters will have to update their beliefs again, making revisions to their beliefs about the two components. That is the key difference from the situation where forecasters could differentiate trends and cycles perfectly. In that case, upon observing the actual state value, forecasters know the state perfectly, rendering their beliefs about the two components when they make forecasts useless.

In this model, forecasters would make forecasting errors on both the trend and cyclical components. In other words, they are to some extent confused about the two components. For example, one might believe that the trend component is stronger than it actually is, leading them to simultaneously believe that the cyclical component is weaker than it actually is. In the following, we show that the confusion regarding the trend and cyclical components helps account for the documented empirical patterns.

Specifically, in the presence of this confusion mechanism, a positive trend signal plays a dual role. First, it provides information about the trend, indicating a strong trend component in the current period. Consequently, forecasters revise their posterior beliefs regarding the trend component upwards, from the prior beliefs inherited from the previous period. Second, the positive trend signal is useful for updating beliefs on the cyclical component. Forecasters rationally interpret the positive trend signal as indicating three possibilities: a positive state innovation in the trend, a positive noise in the signal, as well as an underestimation of the trend component in the preceding period. Recognizing the likelihood of having underestimated the trend component previously, forecasters would conclude that they had likely overestimated the cyclical component previously. Consequently, they would revise their current beliefs regarding the cyclical component downward.[1]

In summary, the confusion between trend and cyclical components leads forecasters to rationally update their beliefs about these components in opposite directions. This mechanism gives rise to a negative correlation between changes in forecasters' trend beliefs and changes in their cyclical beliefs.

This mechanism can also account for the observed increase in forecast dispersion over horizons. In this model, for any forecast horizon, the dispersion of forecasts can be broken down into three parts: the dispersion caused by heterogeneous beliefs about the cyclical and trend components, as well as their covariance. Similar to the predictions of the standard model, the dispersion caused by heterogeneous beliefs about

[1] Likewise, upon receiving a positive signal about the cyclical component, forecasters would revise their posterior beliefs about the cyclical component upwards from the prior beliefs inherited from the previous period. Additionally, they would revise their belief about the trend component downwards, as they aim to rectify the forecasting errors in their prior beliefs about the cyclical component.

the cyclical component decreases over the forecast horizon, as the cyclical component becomes less influential for longer-term forecasts. Further, the dispersion caused by heterogeneous beliefs about the trend component is constant over the forecast horizon, as the trend component is equally important for all horizons. The third part, characterized by the negative covariance of beliefs regarding the two components, is a novel aspect of the model. It stems from forecasters' inability to perfectly distinguish between trends and cycles, and its significance diminishes over the forecast horizon as the cyclical component itself becomes less influential in forecasting. Therefore, the overall dispersion could either increase or decrease over horizon. Our model predicts that forecast dispersion increases as the forecast horizon extends, under the condition that the trend is neither too volatile nor stable.

Application. Our framework not only helps organize the documented empirical findings but also offers insights into various policy-relevant issues related to expectation formation. To illustrate this, we examine the impact of implementing inflation targeting policy on forecasting behaviors. In 2012, the United States introduced an explicit inflation target of 2 percent for the first time (Shapiro and Wilson (2019)). Through the lens of our framework, this policy can be interpreted as a shift in the data-generating process for inflation.

To uncover the corresponding change, we conduct structural estimation using the simulated method of moments (SMM) for both pre-2012 and post-2012 periods. We find that the only change is a substantial decrease in the persistence of the cyclical component during the post-2012 period, while the estimated values for all other parameters remain largely the same across both periods. This observed change is intuitive, suggesting that after the policy shift, the central bank would respond more to short-term deviations from the long-term target, thereby diminishing the persistence of the cyclical component.

Furthermore, we have documented a set of changes in the pattern of inflation expectations across the two periods. First, using the inflation expectation data of the pre-2012 subsample, we find a statistically significant negative correlation between changes in forecasters' trend beliefs and those of the cyclical beliefs. However, after 2012, this correlation becomes positive and statistically insignificant. Second, we observe a steeper decline in forecast dispersion over the horizon in the subsample post-2012 compared to the preceding period. We show that these empirical patterns align both qualitatively and quantitatively with the predictions of our model, contingent upon a decrease in the persistence of cyclical components.

Extension. Furthermore, we demonstrate that our framework can be extended to incorporate behavioral biases of forecasters that have been studied in the literature. Specifically, we explore a scenario where forecasters exhibit overconfidence in new information, leading them to believe that the variances of signal noise are smaller than their actual values. By investigating the interaction between overconfidence and the new confusion mechanism, we unveil qualitatively different model predictions compared to models lacking either of these features. We show that our model predictions are empirically relevant: if the overconfidence is moderate, the now-cast error in the current period will be positively correlated with the now-cast error in the previous period, which is consistent with empirical findings from the SPF data.

Discussion. We also consider an alternative scenario where confusion between trends and cycles arises because forecasters misinterpret signals. In this model, although forecasters observe the trend and cyclical components at the end of a period, they must still infer these components in the following period before making forecasts based on trend and cycle signals. Some forecasters may mistakenly interpret a trend signal as a cyclical one, and vice versa. We demonstrate that such misinterpretation can lead to a negative correlation between the trend and cyclical beliefs among forecasters who interpret the signals correctly and those who do not. On the one hand, this model could also predict an increasing forecast dispersion over forecast horizons when the fraction of forecasters who misinterpret signals is neither too large or too small. On the other hand, this model always predicts a non-negative correlation between changes in trend beliefs and cyclical beliefs, which contradicts the finding in the data.

Literature Review. Our work complements recent studies that utilize survey data to investigate expectation formation. Research in the paradigm of noisy information has revealed that forecasters tend to under-react to new information at the aggregate level (Coibion and Gorodnichenko 2015), while exhibiting overreactions at the individual level (Bordalo et al. 2020; Broer and Kohlhas 2022). New contributions to this literature further expand its scope. For instance, Kohlhas and Walther (2021) explore why individual forecast errors are negatively correlated with current realizations, while Rozsypal and Schlafmann (2023) examine how forecaster characteristics influence individual forecasts errors.

A common feature of these studies is that they assume the data-generating process for the state is stationary, often an AR(1) process. Our work examines a scenario in which the data generation process of the forecasted state incorporates a non-stationary trend component which is not observable.[2] Even in its simplest form, this framework yields several predictions that align with a set of empirical facts concerning how forecast behaviors vary over the forecast horizon.

Our work is related to Afrouzi et al. (2023). In their lab experiment, they show that forecasting behaviors could vary over the forecast horizon, e.g., overreaction is stronger for longer forecast horizons. We document how forecasting behaviors vary

[2]Early studies such as Nelson and Plosser (1982) and Harvey (1985) have demonstrated the presence of a non-stationary trend component in GDP growth. Similar findings have also been observed in studies analyzing inflation data, such as Cogley and Sargent (2005) and Cogley and Sbordone (2008).

over the forecast horizon in the survey data and find that they can be informative about how forecasters update beliefs and form expectations.

A number of studies have documented that forecast dispersion tend to be larger in the long run.[3] Lahiri and Sheng (2008) and Patton and Timmermann (2010) assume that forecasters possess a diverse set of prior beliefs. As the forecast horizon extends, forecasters assign less weight to new information and instead rely more on their prior beliefs. Our model differs in that the confusion mechanism is rational rather than behavioral. Andrade et al. (2016) consider a case where forecasters can only occasionally observe the state value (i.e., the sticky information assumption) and the current trend shock has a more pronounced effect on the future state compared to its impact on the current state. Our model features noisy information and the trend component holds equal importance across all horizons. In addition, our model predicts that the changes in trend beliefs and changes in cyclical beliefs can be negatively correlated.

Finally, the mechanism of confusion in our model is informational rather than behavioral, distinguishing our approach from theoretical explorations that incorporate behaviorial biases, such as diagnostic expectations (Bordalo et al. 2018, Bianchi et al. 2021), overconfidence (Broer and Kohlhas 2022), ambiguity aversion (Chen et al. 2024, Huo et al. 2023), cognitive discounting (Gabaix 2020), level-K thinking (García-Schmidt and Woodford 2019, Farhi and Werning 2019), narrow thinking (Lian 2021), adaptive learning (Adam et al. 2012, Kuang and Mitra 2016), autocorrelation averaging (Wang 2021) and loss aversion (Elliott and Timmermann 2008, Capistrán and Timmermann 2009). However, we demonstrate that this framework can be extended to accommodate behavioral biases and the interaction between rational confusion and behavioral biases is useful for addressing empirical puzzles in the literature.

2 Evidence

In this section, using the Survey of Professional Forecasters (SPF) of the U.S., we present two empirical findings. Firstly, we document a negative correlation between changes in forecasters' trend beliefs and changes in their cyclical beliefs. Secondly, we show that forecast dispersion among forecasters tends to increase as the forecast horizon extends for most macroeconomic variables.

2.1 Survey of Professional Forecasters Data

The Survey of Professional Forecasters (SPF) of the U.S. is a source of predictions made by professional forecasters regarding a broad range of macroeconomic variables. The data is collected quarterly and goes back to 1968Q4. The Fed of Philadelphia surveys

[3] Farmer et al. (2024) provides a Bayesian learning model and stress that they focus on uncertainty about the data generation process as a key information friction, instead of noisy information. Their model could address a set of anomalies at the consensus forecast, leaving out discussion of forecast dispersion. Our work, based on the paradigm of noisy information, examines the heterogeneity in forecasting behaviors at the individual level over the forecast horizon.

approximately 35 professional forecasters each quarter, assigning a unique ID number to each forecaster to track their forecast history.

For each variable, a forecaster provides six predictions, including one back-cast toward the previous period, a now-cast (forecast for the current quarter), and forecasts for the subsequent four quarters. In addition, they are asked to provide The annual projection of this variable for the current year, and the next year. Since 1991Q4, the survey has included an extra question regarding the Consumer Price Index (CPI) for a ten-year forecast. Since 1992Q1, the first quarter survey has included an additional question about the GDP for a ten-year forecast, while since 1996Q3, the third quarter survey has incorporated an additional question regarding the natural unemployment rate. Starting from 2009, SPF has expanded to encompass year-level forecasts of the unemployment rate and real GDP for two- and three-year periods.

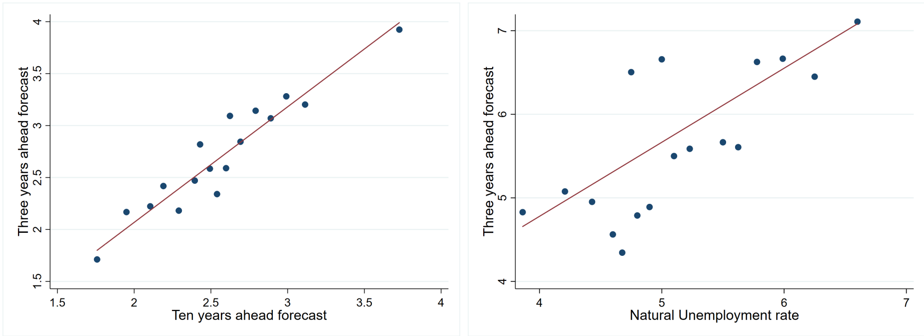
The survey is conducted before the end of each quarter, following the Bureau of Economic Analysis' (BEA) advance report of the national income and product accounts (NIPA) release. The BEA reports macroeconomic variables (e.g., GDP estimates) for the preceding quarter. At the beginning of the questionnaire, forecasters will be provided with the BEA reported value of the macro variable for the previous quarter. Therefore, when giving their predictions for current and future quarters, forecasters have access to information about the values of forecasted variables up to the last quarter.

2.2 Trend Beliefs and Cyclical Beliefs

To explore the influence of trends and cycles in the expectation formation process over the horizon, we start our investigation by constructing proxies for forecasters' beliefs regarding trend and cyclical components. As discussed earlier, since 2009, the Survey of Professional Forecasters (SPF) has asked forecasters each quarter to report their long term forecasts for the unemployment rate and real GDP, precisely three years ahead. Therefore, we employ forecaster i's three-year ahead forecast at quarter t, denoted as $F_{i,t}y_{t+3Y}$, to represent her belief regarding the trend component. Furthermore, we utilize the deviation of forecaster i's now-cast at quarter t, denoted as $F_{i,t}y_t$, from the three-year ahead forecast to proxy her belief of the cyclical component. Specifically, forecaster i's belief of the cyclical component is constructed as follows:

$$Cyc_{i,t} = F_{i,t}y_t - F_{i,t}y_{t+3Y}.$$

However, there might be a concern that the three-year forecast horizon may not be long enough to capture the long-term trend. To examine this conjecture, we utilize two forecast series with even longer horizons: forecasts of real GDP ten years ahead (available every first quarter since 1992Q1) and forecasts of the natural unemployment rate (available every third quarter since 1996Q3). We compare the predictions



(a) Real GDP (b) Unemployment

Figure 1. *Three years ahead predictions and predictions of longer horizons. Note: The sample period is from 2009 to 2019, given the data availability. Predictions of the natural unemployment rate are only available in the third quarter survey, and predictions of the ten-year-ahead real GDP are only available in the first quarter survey. Figure 1(a) depicts the real GDP prediction for three years ahead and ten years ahead. The correlation between the forecast on a three-year horizon and a longer horizon, illustrated in the upper two figures, is 0.903 (Real GDP) and 0.886 (Unemployment).*

for the three-year horizon with those for the longer horizon in the corresponding quarters, specifically real GDP in every first quarter since 1992 and unemployment in every third quarter since 1996. Figure 1 presents bin-scatter plots the relationship between forecasts made for a three-year horizon with those for extended forecast horizons. Specifically, Figure 1(a) illustrates the correlation between the forecasts of real GDP three years ahead and those for a ten-year period, while Figure 1(b) presents the correlation between forecasts of the unemployment rate for a three-year horizon and forecasts of the natural unemployment rate. We find a high correlation between forecasts for the 3-year horizon and forecasts of longer horizons, reaching 0.903 for real GDP and 0.886 for the unemployment rate, respectively. This indicates that the 3-year horizon is likely adequate for proxying trend.

In addition, we provide further evidence in Appendix A.3 showing that the three-year-ahead forecasts are highly correlated with trend estimates derived from actual data using the HP filter. Trend estimates obtained using the HP filter do not reflect the heterogeneous trend beliefs of forecasters, but the high correlation between these estimated trends and the trend beliefs we construct is reassuring.

2.3 Changes in Trend Beliefs and Changes in Cyclical Beliefs

In this section, we construct a novel empirical test using changes in forecaster's trend beliefs and cyclical beliefs across periods. The correlation of these changes informs their roles in the expectation formation process. Specifically, if the forecasted variable follows an AR(1) process or a general stationary data generation process, the standard forecasting model predicts that the correlation is zero. This is because cross-period changes in cyclical beliefs reflect new information regarding the cyclical components

Table 1. Changes in Trend Beliefs and Changes in Cyclical Beliefs

	Dependent Variable: Trend belief changes			
	Baseline		Time FE	
	Unemployment	Real GDP	Unemployment	Real GDP
	(1)	(2)	(3)	(4)
Cyclical belief changes	-0.948***	-0.252***	-0.930***	-0.776***
	(0.063)	(0.052)	(0.021)	(0.052)
Time FE	NO	NO	YES	YES
Obs.	794	783	788	779
R-sq	0.753	0.273	0.928	0.783

Note: This table shows the coefficients estimated from Equation 1. The sample period using the three-year ahead forecast spans from 2009Q1 to 2019Q4. Columns (1) and (2) present the baseline estimation result. Columns (3) and (4) present the estimation result with year-quarter fixed effect. The estimation result indicates a negative correlation between the belief changes in the two components.

between quarters t and $t-1$, while updates in trend beliefs reflect new information regarding the trend components between quarters t and $t-1$. This prediction is characterized in section 4.1.

To investigate, we estimate the following equation:

$$\underbrace{F_{i,t}y_{t+3Y} - F_{i,t-1}y_{t-1+3Y}}_{change\ in\ trend\ belief} = \alpha + \beta_2 \underbrace{(Cyc_{i,t} - Cyc_{i,t-1})}_{change\ in\ cyclical\ belief} + \epsilon_{i,t}. \tag{1}$$

The left-hand side of Equation (1) approximates changes in trend beliefs by utilizing the difference between three-year ahead forecasts across periods. The right-hand side represents the difference in cyclical beliefs across periods. The coefficient β_2 captures the correlation between changes in trend beliefs and changes in cyclical beliefs.

Columns (1) and (2) of Table 1 presents the estimation results of Equation (1), using unemployment and real GDP forecast data. Columns (3) and (4) of Table 1 present the estimation results, after controlling quarter fixed effects. The estimated coefficients indicate a significant, negative correlation between changes in trend beliefs and changes in cyclical beliefs. In other words, when a forecaster adjusts their belief upwards regarding cyclical components from that of the last period, they tend to adjust their beliefs for trend components downwards from that of the last period, and vice versa.

We conducted a robustness check by using one-year-ahead forecasts as a proxy for trend beliefs, in place of the three-year-ahead forecasts used to proxy trend beliefs. The advantage of this approach is that it allows us to explore a broader range of macroeconomic variables, given the data availability. The disadvantage is that we anticipate that this measure of trend beliefs may be contaminated by cyclical information, as a one-year horizon may not be sufficiently long to capture long-term trends cleanly. If

Table 2. Correlation between Changes in Trend Beliefs and Cyclical Beliefs

	Baseline		Time FE		
	β	SD	β	SD	Obs
Forecast Variable	(1)	(2)	(3)	(4)	
Nominal GDP	-0.702***	0.010	-0.774***	0.038	3,296
Real GDP	-0.468***	0.044	-0.806***	0.081	3,359
GDP price index inflation	-0.752***	0.017	-0.757***	0.037	3,274
Real consumption	-0.539***	0.020	-0.709***	0.045	3,234
Industrial production	-0.226	0.156	-0.899***	0.086	3,047
Real nonresidential investment	-0.078***	0.022	-0.709***	0.033	3,140
Real residential investment	-0.109***	0.020	-0.527***	0.055	3,145
Real federal government consumption	-0.188	0.170	-0.694***	0.153	2,980
Real state and local government consumption	-0.354***	0.057	-0.811***	0.043	2,918
Housing start	-0.111***	0.025	-0.332***	0.037	3,235
Unemployment	-0.869***	0.085	-0.910***	0.021	3,757
Inflation rate (CPI)	-0.051*	0.030	-0.151***	0.050	3,696
Three-month Treasury rate	-0.755***	0.038	-0.871***	0.034	3791
Ten-year Treasury rate	-0.794***	0.024	-0.889***	0.018	3102

Note: This table shows the coefficients from estimating Equation 1 using one-year ahead forecast as proxy of the trend belief. Column (1) presents the baseline estimation result. Column (3) presents the estimation result with year-quarter fixed effect.

this is the case, the correlation under consideration should be positive, as both changes are driven by the same set of new information.

Table 2 presents the results for a broad range of macroeconomic variables with and without quarter fixed effects. In this robustness check, we continue to observe a negative relationship between changes in trend beliefs and cyclical beliefs for most macroeconomic variables.

In summary, these empirical findings challenge the commonly used assumption of stationary forecasted macroeconomic variables. It's worth noting that even if the data generation process allows non-stationary trends, changes in trend beliefs and cyclical beliefs should still be uncorrelated, provided that innovations in trends and cycles are independent over time.

2.4 Forecast Dispersion over Forecast Horizon

In this section, we explore whether the dispersion in forecasts across forecasters varies as the forecast horizon extends. This analysis is informative in understanding the role

__Table 3.__ Forecast dispersion over forecast horizon

	Dependent Variable: Forecast Dispersion				
	Variance of forecasts		50 percentile difference		
	β_1	SE	β_1	SE	Obs
Forecast Variable	(1)	(2)	(3)	(4)	
Nominal GDP	0.337***	0.026	0.204***	0.008	1,025
Real GDP	0.242***	0.022	0.162***	0.007	1,025
GDP price index inflation	0.118***	0.008	0.119***	0.004	1,025
Real consumption	0.125***	0.013	0.127***	0.006	770
Industrial production	0.860***	0.062	0.320***	0.014	1,025
Real nonresidential investment	1.647***	0.127	0.497***	0.018	770
Real residential investment	6.021***	0.547	0.932***	0.039	770
Real federal government consumption	1.284***	0.102	0.393***	0.019	770
Real state and local government consumption	0.317***	0.028	0.210***	0.009	770
Housing start	0.004***	0.000	0.020***	0.001	1,024
Unemployment	0.034***	0.002	0.081***	0.003	1,014
CPI	-0.066***	0.021	-0.073***	0.012	770
Three-month Treasury rate	0.053***	0.002	0.106***	0.005	560
Ten-year Treasury rate	0.045***	0.001	0.094***	0.003	560

Note: This table shows the coefficients from estimating Equation 2. The sample period is from 1968Q4 to 2019Q4. In column (1), we directly use the forecast variance. In column (3), we use the difference between the 25% percentile and 50% percentile. All the standard error is clustered by year-quarter.

of beliefs concerning trends and cycles.[4] We estimate the following equation:

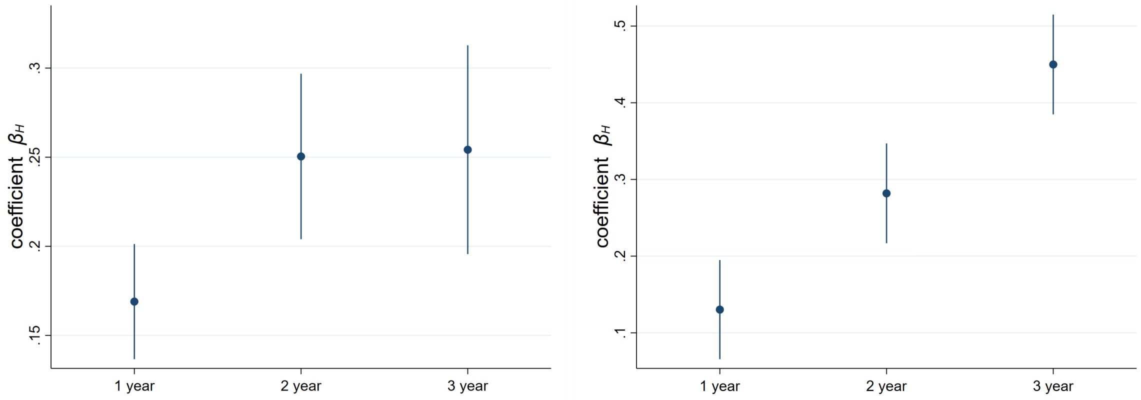
$$\text{Forecast dispersion}_{th} = \alpha + \beta_1 h + \epsilon_t, \tag{2}$$

where Forecast dispersion$_{th}$ represents the cross-forecaster dispersion in forecasts $F_{i,t}y_{t+h}$ provided by forecaster i at period t for h quarters ahead and the forecast horizon is defined as $h = 0, 1, 2, 3, 4$. The standard error is clustered at the year-quarter level.

We use forecast data for different macroeconomic variables to estimate Equation (2). We consider two measures of forecast dispersion: the variance of forecasts and the difference between the 75th percentile and the 25th percentile. The estimated coefficient β_1 is of particular interest and is presented in Table 3.

Column (1) of Table 3 presents the results using forecast variance as the measure of forecast dispersion. The coefficient for the forecast horizon h is positive ($\beta_2 > 0$) and statistically significant for most variables, indicating that forecasts among forecasters become more dispersed as the forecast horizon increases. The only exception is CPI (inflation). We will revisit the analysis of inflation expectations in section 5.1. In column (3), we repeat our estimations using the difference between the 75th and

[4]In the literature, several studies have investigated this particular pattern. Lahiri and Sheng (2008) use the *Consensus Forecasts* data and show that the forecast dispersion of real GDP growth is larger in a longer forecast horizon for all the G7 countries. Patton and Timmermann (2010) utilize the same data and find that both the forecast dispersion regarding the U.S. GDP growth and inflation is higher at longer horizons. Andrade et al. (2016) study the data from Blue Chip Survey and find a steady increase in the dispersion of Federal Fund rate forecasts as the forecast horizon extends.



(a) Real GDP rate (b) Unemployment rate

Figure 2. *Variance of the year level prediction. Note: The figure shows the estimation result from Equation (3). The left figure shows the estimation result for Real GDP, and the right figure shows the result using the unemployment rate. The sample period is from 2009Q1 to 2019Q4. In both cases, β_H is greater than zero and increases as H increases, which indicates a larger dispersion as the forecast horizon expands.*

25th percentiles as the measure of forecast dispersion. The results are rather similar. To confirm that the pattern is robust to the inclusion of time fixed effect, we report the estimation results with time fixed effect in appendix A1.

A potential concern is that forecasting four quarters ahead is indicative of a medium-run forecast, and the findings in Table 3 may not be informative about the pattern of long-run forecasts. We employ a number of additional tests to address this concern. First, we focus on a subset of variables with yearly forecast data spanning an extended horizon. Beginning from 2009Q1, the U.S. SPF incorporates forecasts for real GDP and the unemployment rate one year, two years, and three years into the future. To investigate whether the observed pattern persists, we employ this dataset and estimate the following specification:

$$Var(F_{i,t}y_{t+H}) = \alpha_2 + \sum_{H=1}^{3} \beta_H \, \mathrm{horizon}_H + \epsilon_t, \tag{3}$$

where $\mathrm{horizon}_H$ is a dummy variable for horizon H, taking the value 1 if the forecast horizon is $H = 1$ year, 2 years, or 3 years ahead; and 0 otherwise. The coefficient β_H captures the difference in forecast dispersion between forecasts H years ahead and current year predictions ($H = 0$).

Figure 2(a) on the right illustrates the estimation result using real GDP data, while Figure 2(b) presents the counterpart for the unemployment rate. In both cases, the coefficients β_H are positive and greater as the forecast horizon becomes larger. This is consistent with our previous finding that the dispersion of forecasts increases as the forecast horizon expands.

In addition, we provide an additional test that can be informative about the forecast dispersion across different horizons is to examine the forecast dispersion of U.S. government treasury securities with different maturities, specifically three months (Treasury bills) and ten years (Treasury bonds). The price of both securities is affected by the same information set. The key distinction between them lies in their respective maturities. In other words, the price of Treasury bonds is contingent upon beliefs about the state of the U.S. economy over a longer time horizon. When forecasters exhibit greater disagreement in their predictions for the long-term U.S. economy, it is expected that the forecast dispersion for Treasury bonds will be larger compared to Treasury bills. Appendix A.4 provides evidence confirming this conjecture.

3 Forecasting Model with Trend-cycle Confusion

3.1 Setup

Utility function. In this model, there exists a continuum of forecasters, indexed by $i \in [0,1]$, who make forecasts about a stochastic state variable y_t. The objective of the forecasters is to minimize forecasting errors. We consider a standard quadratic utility function, which is given by:

$$U(F_{i,t}y_{t+h}) = -(F_{i,t}y_{t+h} - y_{t+h})^2, \tag{4}$$

where y_{t+h} is the actual value of the state in period $t+h$ and $F_{i,t}y_{t+h}$ denotes the forecast made by forecaster i at period t for the state h periods in the future.

Data generation process. We assume that the state variable y_t is composed of two components: a trend component, μ_t, representing long-term trend, and a cyclical component, x_t, capturing short-term fluctuations. In particular, the trend follows a random walk process, while the cycle is modeled as an AR(1) process. Specifically, the data generation process for the state can be described as follows:

$$y_t = \mu_t + x_t,$$

$$\mu_t = \mu_{t-1} + \gamma_t^{\mu},$$

$$x_t = \rho x_{t-1} + \gamma_t^{x},$$

where ρ is the persistence for the AR(1) process and γ_t^{μ} and γ_t^{x} are the innovations of the trend and cyclical components, both of which are normally distributed with zero mean and variances of σ_μ^2 and σ_x^2, respectively, i.e.,$\gamma_t^{\mu} \sim N(0, \sigma_\mu^2)$ and $\gamma_t^{x} \sim N(0, \sigma_x^2)$. We use $\boldsymbol{\theta_t} = (\mu_t, x_t)'$ to denote the state components in period t. Consistent with the previous literature, we assume that the data generating process (DGP) is common knowledge for all forecasters.

In each period, forecasters receive private noisy signals for each component, that is, $\boldsymbol{s_{i,t}} = (s^{\mu}_{i,t}, s^{x}_{i,t})'$, where

$$s^{\mu}_{i,t} = \mu_t + \epsilon_{i,t}; \quad \text{and} \quad s^{x}_{i,t} = x_t + e_{i,t}. \tag{5}$$

We assume that the error terms of the signals are independent and normally distributed. The variance-covariance matrix of i's private signals is given by:

$$\boldsymbol{\Sigma_s} = \begin{pmatrix} \sigma^2_\epsilon & 0 \\ 0 & \sigma^2_e \end{pmatrix}.$$

At the end of each period t, we allow forecasters to observe the actual state variable y_t but not the trend and cyclical components. Therefore, upon the announcement of the actual state value, forecasters revise their beliefs regarding the trend and cyclical components. The updated beliefs about the two components become the prior beliefs for the next period.

Throughout the paper, we use $\boldsymbol{\theta}^i_{1,t}$ to represent forecaster i's posterior belief after forecaster i receive signals about the trend and cyclical components in period t (i.e., the first update). Similarly, $\boldsymbol{\theta}^i_{2,t}$ represents forecaster i's posterior belief after they observe the actual realization of the state in period t (i.e., the second update). The subscript 2 stands for the second time updating in period t.

Timeline. We summarize the timeline of our setting in Figure 3:

- At the beginning of period t, forecaster i is endowed with the prior belief $\boldsymbol{\theta}^i_{2,t-1}$, which is the posterior of the second updating from the period $t-1$.
- Forecaster i observes the private signal $\boldsymbol{s_{i,t}}$ and then update her belief accordingly (the first updating).
- Given the updated beliefs $\boldsymbol{\theta}^i_{1,t}$, forecasters choose their optimal forecasts of the current and future period $F_{i,t}y_{t+h}$.
- At the end of period t, y_t is revealed.
- Forecasters revise their beliefs again, forming beliefs $\boldsymbol{\theta}^i_{2,t}$ (the second updating).

3.2 Equilibrium Characterization

In this section, we turn to the characterization of forecasters' optimal forecasts. We start our analysis by considering the posterior belief obtained from the second update

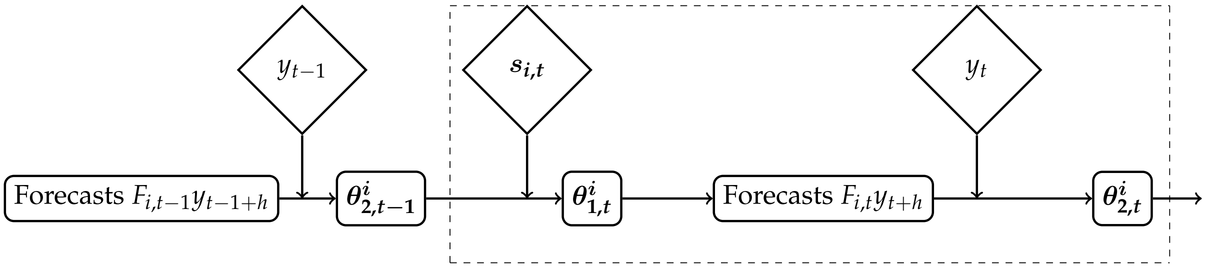


Figure 3. *Timeline. In each period t, forecaster i will update his belief twice. Firstly, based on the observed private signals, forecaster i adjusts his beliefs and provides forecasts for the current and future periods,i.e., $F_{i,t}y_{t+h}$. Secondly, forecaster i revises his belief regarding the trend and cycle upon observing the actual realization of the state variable. The diamond box is on behalf of the exogenous information. The squared box stands for the forecaster's belief.*

in period $t-1$, which is the prior belief of forecaster i at the beginning of period t:

$$\boldsymbol{\theta^i_{2,t-1}} = (\mu^i_{2,t-1}, \rho x^i_{2,t-1})',$$

where $\mu^i_{2,t-1}$ and $x^i_{2,t-1}$ are forecaster i's beliefs about trend and cyclical components at the end of period $t-1$, respectively. This set of beliefs $\mu^i_{2,t-1}$ and $x^i_{2,t-1}$ can always be written in the form:

$$\mu^i_{2,t-1} \equiv \mu_{t-1} + z_{i,t-1} \quad \textit{and} \quad x^i_{2,t-1} \equiv x_{t-1} - z_{i,t-1}, \tag{6}$$

$$s.t. \;\; \mu^i_{2,t-1} + x^i_{2,t-1} = y_{t-1},$$

where $z_{i,t-1}$ captures the error in forecaster i's beliefs regarding the trend and cyclical components at the end of period $t-1$. Throughout the remainder of the paper, we denote $z_{i,t-1}$ as the *separation error*. Given the restriction that actual y_{t-1} is released and observed at the end of $t-1$, the two components $\mu^i_{2,t-1}$ and $x^i_{2,t-1}$ must sum up to y_{t-1}. In other words, this condition imposes a restriction such that the error terms in the two components are of the same magnitude but opposite in sign.

Denote the variance of $z_{i,t-1}$ as $\sigma^2_{z,t-1}$, then the variance-covariance matrix of $\boldsymbol{\theta^i_{2,t-1}}$ follows:

$$\boldsymbol{\Sigma}_{\boldsymbol{\theta^i_{2,t-1}}} = \begin{pmatrix} \sigma^2_{z,t-1} + \sigma^2_\mu & -\rho\sigma^2_{z,t-1} \\ -\rho\sigma^2_{z,t-1} & \rho^2\sigma^2_{z,t-1} + \sigma^2_x \end{pmatrix}.$$

The covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\theta^i_{2,t-1}}}$ indicates that forecasters subjectively perceive a negative correlation between the trend and cyclical components. Intuitively, if a forecaster believes that the trend is stronger than it actually is, she will tend to believe that the cyclical component is weaker than it actually is, and vice versa. Note that when forecasters can perfectly distinguish between the trend and cyclical components, the covariance will be zero.

Lemma 1. Suppose $z_{i,t-1}$, the separation error in period $t-1$, is normally distributed.

Then $z_{i,t}$ must also be normally distributed, and there exists a steady state σ_z^2 for the variance $\sigma_{z,t}^2$.

The proof and subsequent proofs are collected in Appendix B. Firstly, if the separation error follows a normal distribution in one particular period, it will continue to be normally distributed indefinitely, given that both the state innovations and signals are also normally distributed. Secondly, the variance $\sigma_{z,t}^2$ always converges to a steady-state value, σ_z^2, which represents the extent of confusion in distinguishing between the trend and cyclical components. Throughout the paper, we assume that the separation error z_i is normally distributed and in the steady state.

Lemma 2. In period t, after acquiring the private signals $\boldsymbol{s}_{i,t}$, forecaster i updates her beliefs on the trend and cyclical components and form her beliefs $\boldsymbol{\theta}_{1,t}^i$, which is joint-normally distributed. The expectations of these beliefs are given by:

$$\boldsymbol{\theta}_{1,t}^i = \boldsymbol{\theta}_{2,t-1}^i + \boldsymbol{\kappa} \times (\boldsymbol{s}_{i,t} - \boldsymbol{\theta}_{2,t-1}^i), \tag{7}$$

where $\boldsymbol{\kappa}$ is the Kalman gain and $(\boldsymbol{s}_{i,t} - \boldsymbol{\theta}_{2,t-1}^i)$ is the surprise from signals:

$$\boldsymbol{\kappa} = \begin{pmatrix} \frac{V+\sigma_e^2(\sigma_z^2+\sigma_\mu^2)}{\Omega} & -\frac{\rho\sigma_\epsilon^2\sigma_z^2}{\Omega} \\ -\frac{\rho\sigma_e^2\sigma_z^2}{\Omega} & \frac{V+\sigma_\epsilon^2(\sigma_x^2+\rho^2\sigma_z^2)}{\Omega} \end{pmatrix} \quad \text{and} \quad \boldsymbol{s}_{i,t} - \boldsymbol{\theta}_{2,t-1}^i = \begin{pmatrix} s_{i,t}^\mu - \mu_{2,t-1}^i \\ s_{i,t}^x - \rho x_{2,t-1}^i \end{pmatrix}.$$

The variance-covariance matrix of $\boldsymbol{\theta}_{1,t}^i$ is given by:

$$(\boldsymbol{\Sigma}_s^{-1} + \boldsymbol{\Sigma}_{\boldsymbol{\theta}_{2,t-1}^i}^{\mathbf{-1}})^{-1} = \begin{pmatrix} \mathrm{Var}^T & \widetilde{COV} \\ \widetilde{COV} & \mathrm{Var}^C \end{pmatrix} = \begin{pmatrix} \frac{\sigma_\epsilon^2[\Omega-\sigma_\epsilon^2(\sigma_x^2+\sigma_e^2+\rho^2\sigma_z^2)]}{\Omega} & -\frac{\rho\sigma_e^2\sigma_\epsilon^2\sigma_z^2}{\Omega} \\ -\frac{\rho\sigma_e^2\sigma_\epsilon^2\sigma_z^2}{\Omega} & \frac{\sigma_e^2[\Omega-\sigma_e^2(\sigma_\epsilon^2+\sigma_\mu^2+\sigma_z^2)]}{\Omega} \end{pmatrix}, \tag{8}$$

where Ω and V are positive constants:

$$\Omega = (\sigma_z^2+\sigma_\mu^2+\sigma_\epsilon^2)(\sigma_x^2+\sigma_e^2+\rho^2\sigma_z^2) - \rho^2\sigma_z^4 \quad \text{and} \quad V = (\sigma_z^2+\sigma_\mu^2)(\sigma_x^2+\rho^2\sigma_z^2) - \rho^2\sigma_z^4.$$

The Kalman gain matrix $\boldsymbol{\kappa}$ has two parts. The elements on the main diagonal resemble those in the standard belief updating. That is, forecasters use signals about the trend (cycle) to update their beliefs on the trend (cycle).

When there is no confusion (i.e., σ_z^2 goes to zero), the model reduces to the standard Bayesian case. In this scenario, the Kalman gain for the trend component reduces to $\sigma_\mu^2/(\sigma_\mu^2+\sigma_\epsilon^2)$, and for the cyclical component, it reduces to $\sigma_x^2/(\sigma_x^2+\sigma_e^2)$. When there is confusion (i.e., $\sigma_z^2 > 0$), the Kalman gain becomes larger than the Bayesian case without confusion. In other words, the confusion mechanism leads to less precise prior beliefs, and forecasters rely more on the signals, which provide new information.

A similar argument holds true for the Kalman gain for the cyclical component.

Crucially, the non-zero elements on the sub-diagonal of the Kalman gain matrix indicate that forecasters incorporate the signal on the trend (cycle) component when updating their beliefs about the cyclical (trend) component. Consider a scenario where the private signal indicates that the cyclical component is stronger than the forecaster's prior belief. This situation could arise from three possibilities: Firstly, it might reflect a substantial innovation in the cyclical component itself. Secondly, it could be due to positive noise in the signal. Thirdly, it might suggest that the actual value of the cyclical component in the previous period was larger than what the forecaster believed. As forecasters cannot know the true value of each component with certainty, they will adjust their prior beliefs by increasing their estimate of the cyclical component from the last period and correspondingly decreasing their estimates of the trend component for both the last and current periods.

The variance-covariance matrix in Equation (8) warrants further discussion. Firstly, the elements on the main diagonal correspond to the perceived variance of the trend and cyclical components, which are influenced by the confusion mechanism. These variances are larger compared to the case where there is no confusion (i.e., the components can be perfectly observed). We denote them as Var^C and Var^T, respectively.

Secondly, the elements on the sub-diagonal components are non-zero and negative. That is, forecasters cannot perfectly distinguish between the trend and cycle, which gives rise to a negative covariance between the beliefs of these two components. Intuitively, when there are strong positive signals about the cyclical component, forecasters will simultaneously revise the cyclical component upward and the trend component downward. We denote this covariance as $\widetilde{\text{COV}}$.

Finally, we turn to the stage of making forecasts. Forecaster i makes a series of forecasts about the state in h periods ahead. Under a quadratic utility function, her optimal prediction is the expected value of the state variable.

Lemma 3. The optimal forecast of forecaster i over horizon h is determined by their beliefs of trend and cyclical components, i.e.,

$$F_{i,t}y_{t+h} = E_{i,t}[\mu_t + \rho^h x_t] = \mu^i_{1,t} + \rho^h x^i_{1,t}.$$

This lemma says that the trend and cyclical beliefs play different roles over forecast horizons: the trend belief consistently influences predictions across all horizons, while the influence of the cyclical belief diminishes as the forecast horizon extends.

4 Forecasts over Horizon: Main Results

4.1 A Special Case with Observable Trends and Cycles

Before presenting our model predictions regarding forecasting behaviors over the forecast horizon, we first examine a special case where trends remain stochastic, but forecasters can observe the actual trend component along with the state value at the end of each period. This means forecasters can perfectly distinguish between the trend and cyclical components. In essence, the key information friction in our model is absent in this special case, while all other assumptions remain unchanged. Consequently, the separation error becomes zero (i.e., $z_{i,t} = 0$) and the variance of the separation error also reduces to zero (i.e., $\sigma_z^2 = 0$). Contrasting this special case and our benchmark model helps illustrate the importance of the information friction arising from trends and cycles not being separable.

When the forecasters can perfectly separate the two components, both the Kalman gain matrix in Equation (7) and the variance-covariance matrix in Equation (8) become standard:

$$\boldsymbol{\kappa} = \begin{pmatrix} \frac{\sigma_\mu^2}{\sigma_\mu^2+\sigma_\epsilon^2} & 0 \\ 0 & \frac{\sigma_x^2}{\sigma_x^2+\sigma_e^2} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \text{Var}_s^T & \widetilde{\text{COV}}_s \\ \widetilde{\text{COV}}_s & \text{Var}_s^C \end{pmatrix} = \begin{pmatrix} \frac{\sigma_\epsilon^2\sigma_\mu^2}{\sigma_\epsilon^2+\sigma_\mu^2} & 0 \\ 0 & \frac{\sigma_e^2\sigma_x^2}{\sigma_x^2+\sigma_e^2} \end{pmatrix}.$$

In this scenario, the sub-diagonal elements of the Kalman gain matrix are zero, indicating that forecasters do not use information from the trend or cyclical component to update their beliefs about the other. That is, they treat these components as independent, resulting in zero covariance (i.e., $\widetilde{COV}_s = 0$).

In the following, we investigate whether this model could help address the two empirical patterns documented in section 2. We first examine the relationship between changes in trend beliefs and cyclical beliefs. In the empirical section, we proxy trend belief using the three-year (i.e., $h = 3Y$) ahead forecast. The changes in trend beliefs and cyclical beliefs can be written as follows:

$$F_{i,t}y_{t+3Y} - F_{i,t-1}y_{t-1+3Y} = (E_{i,t}[\mu_t] - E_{i,t-1}[\mu_{t-1}]) + \rho^{3Y}(E_{i,t}[x_t] - E_{i,t-1}[x_{t-1}]),$$

$$Cyc_{i,t} - Cyc_{i,t-1} = (1-\rho^{3Y})(E_{i,t}[x_t] - E_{i,t-1}[x_{t-1}]).$$

Therefore, the model predicts a non-negative correlation between changes in trend and cyclical beliefs:

$$cov(F_{i,t}y_{t+3Y} - F_{i,t-1}y_{t-1+3Y}, Cyc_{i,t} - Cyc_{i,t-1}) = \rho^{3Y}(1-\rho^{3Y})Var(E_{i,t}[x_t] - E_{i,t-1}[x_{t-1}]) \geq 0.$$

It holds because the belief updating of trend and cyclical components is independent (i.e., $\widetilde{COV}_s = 0$) and the covariance between the changes in trend beliefs and cyclical beliefs is zero, i.e., $cov(E_{i,t}[\mu_t] - E_{i,t-1}[\mu_{t-1}], E_{i,t}[x_t] - E_{i,t-1}[x_{t-1}])$.

Furthermore, in this special case, the forecast variance across forecasters can be decomposed into two components:

$$Var(F_{i,t}y_{t+h}) = \rho^{2h}Var(E_{i,t}[x_t]) + Var(E_{i,t}[\mu_t]) = \rho^{2h}\mathrm{Var}_s^C + \mathrm{Var}_s^T. \tag{9}$$

As the forecast horizon increases, the dispersion across forecasters caused by their noisy information on the cyclical component becomes less significant, i.e., ρ^{2h} decreases in h quickly. However, the dispersion caused by their noisy information on the trend component remains stable over the horizon. As a result, the total dispersion decreases monotonically over the forecast horizon.

In summary, in this special case where trends and cycles are separable, the model fails to generate either of the two empirical patterns documented. In fact, its predictions are exactly opposite to the observed patterns in the data. We further extend this special case by allowing the data generation process to be a general ARMA model instead of an AR(1). However, this does not alter the model predictions. Further discussion of this result is provided in Appendix B. Moving forward, we will elaborate on the scenario where the two components are not perfectly separable, and show that the model predictions can be reversed.

4.2 Covariance of Beliefs and Confusion

The key difference between our benchmark model and the previously discussed special case is that forecasters cannot perfectly observe trends and cycles. As a result, their beliefs about these two components are correlated, even when they are, in fact, independent. In this section, we will first analyze the covariance between forecasters' beliefs regarding trends and cycles after they have observed their private signals. Then, we will turn to the covariance after they have observed the actual value of the state. We refer to the former as "covariance of beliefs" and the latter as "confusion."

The covariance of beliefs is captured by $\widetilde{\mathrm{COV}}$ in Equation (8). It is expected that it depends on volatility of each component and the persistence of the cyclical components. Lemma 4 provides the corresponding characterization.

Lemma 4. (i) There exists a threshold $\widetilde{\sigma}_\mu^2$ for the variance of the trend innovation, such that the magnitude of the covariance between the trend and cyclical beliefs increases with σ_μ^2 when $\sigma_\mu^2 \in (0, \widetilde{\sigma}_\mu^2]$ and decreases with σ_μ^2 when $\sigma_\mu^2 \in (\widetilde{\sigma}_\mu^2, +\infty)$. (ii) The magnitude of the covariance increases with the persistence of the cyclical component (ρ).

To understand part (i), recall the covariance is characterized by $\widetilde{\mathrm{COV}} = -\rho\sigma_\epsilon^2\sigma_e^2\sigma_z^2/\Omega$. As the variance of trend innovations (σ_μ^2) increases, two effects emerge. Firstly, Lemma

1 has shown that forecaster i's confusion, represented by σ_z^2, increases. Secondly, forecaster i's uncertainty about the state, represented by Ω, also increases. When the variance of trend innovations remains relatively small, the increase in confusion (σ_z^2) dominates. Conversely, when it is relatively large, the increase in overall variance (Ω) dominates. Consider the following two polar cases. When the trend is stable (i.e., $\sigma_\mu^2 = 0$), there is no confusion (i.e., $\sigma_z^2 = 0$). Therefore, the covariance is zero. When the innovation is very large (i.e., $\sigma_\mu^2 \to \infty$), forecaster i's uncertainty about the state is also very large (i.e., $\Omega \to \infty$), the confusion mechanism is less relevant, and the covariance converges to zero too.

To understand part (ii), we first examine an extreme scenario where the persistence of the cyclical component approaches zero ($\rho = 0$). In this instance, the cyclical component becomes independent over time. Consequently, signals regarding the cyclical components offer information solely about the cyclical components, which are uninformative for the trend components. As a result, the covariance of beliefs regarding the two components is rendered to be zero. As the persistence of the cyclical component increases, signals regarding the cyclical components become more valuable for revising trend beliefs, giving rise to a larger covariance in magnitude.

Next, we turn to the analysis of confusion. After the forecasters have observed the actual value of the current state (y_t), they revise their beliefs again. This set of posterior beliefs becomes the prior beliefs for the next period. The forecasting error present in this set of posterior beliefs is the separation error ($z_{i,t}$). Lemma 5 characterizes its construction.

Lemma 5. Upon observing the actual state value y_t, the separation error $z_{i,t}$ present in the posterior beliefs is given by:

$$z_{i,t} = \frac{(\mathrm{Var}^T + \widetilde{\mathrm{COV}})(x_t - x_{1,t}^i) - (\mathrm{Var}^C + \widetilde{\mathrm{COV}})(\mu_t - \mu_{1,t}^i)}{(\mathrm{Var}^T + \widetilde{\mathrm{COV}}) + (\mathrm{Var}^C + \widetilde{\mathrm{COV}})}. \tag{10}$$

The variance of the separation error (σ_z^2) is bounded:

$$0 \leq \sigma_z^2 \leq \min\{\mathrm{Var}^C, \mathrm{Var}^T\}. \tag{11}$$

Furthermore, σ_z^2 increases as σ_μ^2, σ_x^2, σ_e^2, and σ_ϵ^2 increase, and converges to zero if any of these parameters goes to zero.

Recall that Var^T and Var^C represent the variances of forecasters' posterior beliefs regarding the trend and cyclical components, respectively, while $\widetilde{\mathrm{COV}}$ denotes the corresponding covariance between the two components, as shown in Equation (8).

Lemma 5 states that the separation error after forecasters observe the actual state, is a weighted combination of the error terms in forecasters' beliefs regarding the trend and cyclical components *before* they observe the actual state. If they over-predict the trend component (i.e., $\mu_t - \mu^i_{1,t} < 0$), then $z_{i,t}$ tends to be positive. Conversely, if they over-predict the cyclical component (i.e., $x_t - x^i_{1,t} < 0$), then $z_{i,t}$ tends to be negative.[5]

Note that after observing the actual state value, the covariance between beliefs regarding the trend and cyclical components is represented as $-\sigma_z^2$. The extent of confusion, denoted by σ_z^2, is influenced by two primary factors: the quality of signals and the volatility of the state variables. First, forecasters receive private signals about each component in every period, which help them differentiate between the two. Consequently, more accurate signals decrease the level of confusion. Second, when the state innovations in the trend or cyclical component are more volatile, it becomes more difficult to identify each component, resulting in a higher level of confusion. Intuitively, the confusion is upper bounded by the uncertainty in either the trend or cyclical components

4.3 Correlation between changes of trend beliefs and cyclical beliefs

In this section, we investigate the model prediction of the relationship between changes in trend beliefs and cyclical beliefs. We show that, with the confusion mechanism, our model can produce either a positive or negative correlation. Proposition 1 presents relevant necessary and sufficient conditions.

We begin our analysis by decomposing both the right-hand side (RHS) and left-hand side (LHS) of Equation (1). The changes in belief regarding the forecast for h periods ahead (represented by $F_{i,t}y_{t+h} - F_{i,t-1}y_{t-1+h}$) consist of changes in one's beliefs about both the cyclical and trend components:

$$F_{i,t}y_{t+h} - F_{i,t-1}y_{t-1+h} = \mu^i_{1,t} - \mu^i_{1,t-1} + \rho^h(x^i_{1,t} - x^i_{1,t-1}).$$

When horizon h is sufficiently long, the term $F_{i,t}y_{t+h} - F_{i,t-1}y_{t-1+h}$ captures the change in trend beliefs, as ρ^h becomes very small. The changes in the short-run beliefs consist only the belief changes regarding the cyclical component:

$$Cyc_{i,t} - Cyc_{i,t-1} = (1-\rho^h)(x^i_{1,t} - x^i_{1,t-1}).$$

Therefore, the covariance between the changes regarding the trend beliefs and

[5]Consider a special case nested in Equation (10). When the trend is stable (i.e., $\sigma_\mu^2 = 0$), forecasters can predict the trend component perfectly. Therefore, the error term in their beliefs regarding the trend component is zero. In this scenario, both the variance of the belief regarding the trend component (Var^T) and the covariance ($\widetilde{\mathrm{COV}}$) would also be zero. As a result, the separation error in this case would be zero.

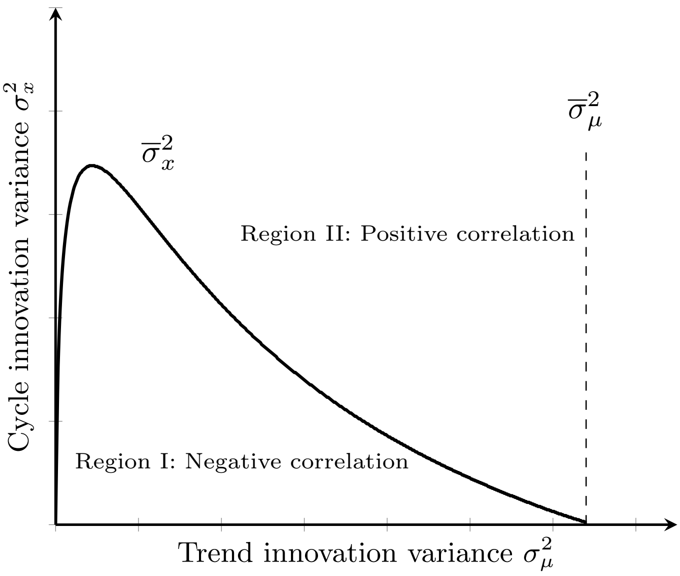


Figure 4. *The state innovation and the correlation of changing in trend and changing in cycle. For a pair of state innovation σ_μ^2, σ_x^2, the model predicts a negative correlation between the updating in the short-run and the updating in the long-run if it lies inside the line(Region I), and a positive correlation if it lies in Region II. For this particular illustration, we have chosen $h = 4$, $\sigma_\epsilon^2 = 3$, and $\sigma_e^2 = 2$.*

cyclical beliefs can be decomposed into two parts:

$$cov(F_{i,t}y_{t+h} - F_{i,t-1}y_{t-1+h}, Cyc_{i,t} - Cyc_{i,t-1}) \tag{12}$$
$$= (1-\rho^h)\left\{ \underbrace{cov(\mu^i_{1,t} - \mu^i_{1,t-1}, x^i_{1,t} - x^i_{1,t-1})}_{\text{covariance between changes in trend and cycle } (-)} + \rho^h \underbrace{var(x^i_{1,t} - x^i_{1,t-1})}_{\text{variance of changes in cycle } (+)} \right\}.$$

The first term within the bracket of Equation (12) denotes the covariance between the change in forecaster i's belief regarding the trend component and the change in belief regarding the cyclical component. This term is always negative because it reduces to the subjective covariance between beliefs regarding trends and cycles ($\widetilde{\text{COV}}$), which says any new information that increases the forecaster's belief regarding the trend component will simultaneously decrease their belief in the cyclical component, and vice versa. The second term represents the variance of forecaster i's belief changes regarding the cyclical component, which is always positive. Proposition 1 presents the necessary and sufficient conditions for the sum of the two terms to be negative.

Proposition 1. *There exists a threshold $\overline{\sigma}_\mu^2$ for the variance of the trend component innovation, such that:*
(i) for any $\sigma_\mu^2 \in [\overline{\sigma}_\mu^2, +\infty)$, changes in the trend beliefs and changes in cyclical beliefs are positively correlated.

(ii) for any $\sigma_\mu^2 \in (0, \overline{\sigma}_\mu^2)$, there exists a threshold $\overline{\sigma}_x^2$ such that changes in trend beliefs and changes in cyclical beliefs are negatively correlated if and only if $\sigma_x^2 < \overline{\sigma}_x^2$; and they are positively correlated, otherwise.

Figure 4 illustrates how the sign of this correlation changes as the variance of the trend and cyclical innovation varies. For a given pair of signal precisions (σ_ϵ^2 and σ_e^2), the model predicts a negative correlation when the trend component is moderately stable, and the cyclical component is not excessively volatile (i.e., within the region enclosed by the solid line in Figure 4).

Intuitively, changes in trend beliefs and cyclical beliefs exhibit a negative correlation when the covariance between beliefs about the two components is dominant. As shown in Lemma 4, this scenario only occurs when the trend component is neither too stable nor too volatile. In addition, as the variance of the cyclical innovation (σ_x^2) increases, the variance of belief changes concerning the cyclical component (represented by the second term of Equation (12)) also increases. However, if the cyclical component is too volatile, the confusion mechanism becomes less relevant. We show the existence of a threshold $\overline{\sigma}_x^2$ for this volatility, such that changes in trend beliefs and cyclical beliefs exhibit a negative correlation when σ_x^2 is lower than this threshold.

4.4 Forecast dispersion

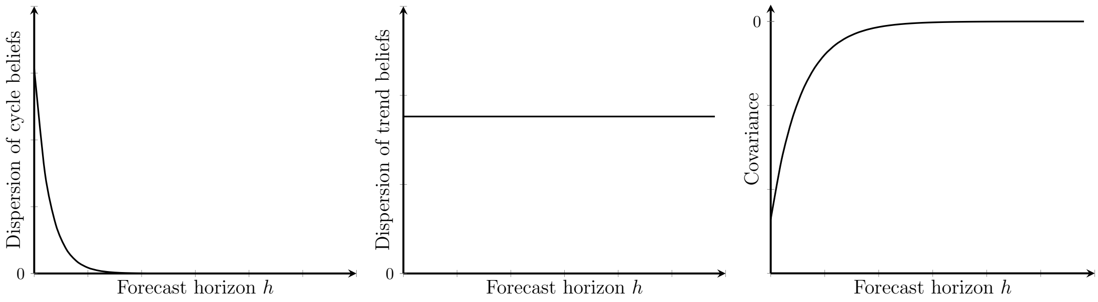
We proceed to examine the prediction of our model regarding the relationship between the forecast dispersion and the forecast horizon. In our model, the forecast dispersion can either increase or decrease as the forecast horizon becomes longer. Proposition 2 characterizes the necessary and sufficient conditions for the forecast dispersion to increase or decrease over the forecast horizon.

To illustrate the mechanism, we decompose the dispersion of forecasts across forecasters for any horizon into three components: the variance arising from heterogeneous beliefs about the trend component, the cyclical component, and their covariance. To be specific, the forecast variance is given by:

$$E[(F_{i,t}y_{t+h} - E[F_{i,t}y_{t+h}])^2] = \rho^{2h}\text{Var}^C\phi^C + \text{Var}^T\phi^T + 2\rho^h\widetilde{\text{COV}}\phi^{COV}, \tag{13}$$

where $0 < \phi^C < 1$, $0 < \phi^T < 1$ and $0 < \phi^{COV} < 1$ are positive scalars, which are contained in the proof of Proposition 2.

Figure 5 depicts the changes in the magnitude of each part as the forecast horizon extends. Figure 5(a) demonstrates that as the forecast horizon extends, the variance caused by heterogeneous beliefs about the cyclical component diminishes. This reduction in variance is due to the decreased influence of the cyclical component in longer-term forecasts. Figure 5(b) shows that the forecast variance caused by the heterogeneous beliefs regarding the trend component remains constant across all forecast



(a) Cycle beliefs (b) Trend beliefs (c) Covariance term

Figure 5. *Dispersion changes as the horizon extends. Note: This figure shows how the magnitude of each part of the variance changes as the forecast horizon extends. For this specific illustration, we use* $\sigma_e^2 = 2$, $\sigma_\epsilon^2 = 3$, $\sigma_x^2 = 2$, $\sigma_\mu^2 = 3$ *and* $\rho = 0.7$.

horizons. It is intuitive because the influence of the trend component is the same across the forecast horizon. How these two components vary over different horizons resembles the behavior observed in the standard model (see section 4.1).

Figure 5(c) depicts the magnitude of the covariance term, which decreases as the forecast horizon extends. That is also caused by the diminished importance of the cyclical component over horizons. This feature, though intuitive, is crucial for understanding our model results. On the one hand, the negative covariance term reduces overall forecast dispersion across forecasters for any horizon. On the other hand, as the forecast horizon extends, the influence of the covariance term diminishes, leading to an increase in observed forecast dispersion.

Whether the forecast dispersion increases or decreases in a longer forecast horizon is determined by the relative strength between the two forces: the diminishing force that originates from the cyclical variance and the increasing force that stems from the covariance term.

Proposition 2. *The dispersion of forecasts across forecasters is strictly increasing in the forecast horizon h, if and only if:*

$$h > \underline{h} = \frac{1}{\ln \rho} \ln \frac{-\widetilde{COV}}{Var^C} \frac{\phi^{COV}}{\phi^C} W; \tag{14}$$

where $W < 1$ *is a positive scalar given by* $E[(z_{i,t} - E[z_{i,t}])^2]/\sigma_z^2$ *and* $\ln \rho < 0$.

Proposition 2 states that the forecast dispersion is increasing in the forecast horizon when h is large enough. To understand, we note that the forecast horizon h has a greater impact on the variance of cyclical beliefs than on the covariance between trend and cyclical beliefs. That is, the former converges more rapidly as the forecast horizon extends than the latter, which is evident from Equation (13). Therefore, when the forecast horizon is sufficiently long, the increasing influence of the covariance becomes dominant, leading to an increasing dispersion.

Interestingly, when the threshold is negative ($\underline{h} \leq 0$), forecast dispersion always increases over the forecast horizon. This scenario occurs when the variance of the trend innovation is neither too small nor too large. As shown in Lemma 4, in such cases, the impact of the covariance between trend and cyclical beliefs (i.e., $\widetilde{COV}$) is greatest. In the appendix, we offer a full characterization of how $\underline{h}$ varies across parameters.

In the special case characterized in section 4.1, forecasters can distinguish between the trend and cyclical components perfectly, rendering the covariance term always equal to zero. As a result, the threshold value in the right-hand side of Equation (14) goes towards infinity ($\underline{h} \to \infty$), implies that the forecast dispersion always decreases over the forecast horizon.

5 Application, Extension and Discussion

In the preceding sections, we have presented a model of expectation formation in which forecasters cannot perfectly separate cyclical component from the trend component. We show that this model is useful to account for empirical regularities documented using SPF data. In this section, we will explore how this framework can be utilized to analyze policy-relevant issues (section 5.1) and how it can be extended to accommodate behavioral bias studied in the existing literature (section 5.2). We will also discuss an alternative approach to modeling confusion and its implications (section 5.3).

5.1 Inflation targeting and forecasts

In this section, we examine the effects of a significant policy change in the United States in 2012 - the introduction of explicit inflation targeting. This new approach to monetary policy implementation began with an announcement on January 25th by Ben Bernanke, the Chairman of the U.S. Federal Reserve, who set a specific inflation target of 2%. Prior to this policy change, the United States did not have an explicit inflation target, relying instead on regularly announced desired target ranges for inflation. Through the lens of our model, the implication of this policy for forecasters is that the underlying data generation process for inflation could undergo changes which would necessitate changes in forecasting behaviors.

To assess the impact of explicit inflation targeting, we start by dividing the whole sample into two sub-samples: the period before 2012 and the period after. Table 4 presents a statistical summary for these two sub-samples. Column (1) displays the average inflation rate (i.e., CPI) in the two sub-samples, revealing that the average inflation rate drops from 3.2 to 1.5 following the policy implementation. Columns (2) and (4) present the mean forecasts for each subsample, showing that after 2012, the means of both the now-cast and the ten-year ahead forecasts decline, coming closer

Table 4. Summary Statistics

	Summary Statistics				
	Data	Now-cast		Ten-year forecast	
	Mean	Mean	Variance	Mean	Variance
Sub-sample	(1)	(2)	(3)	(4)	(5)
Pre-2012	3.215	2.956	0.993	2.75	0.262
Post-2012	1.578	1.755	0.737	2.26	0.136

Note: This table provides a summary of statistics for the now-cast, the ten-year ahead forecast, and the corresponding real data. Column (1) presents the mean of the actual inflation rate in the two sub-samples. Columns (2) and (3) report the mean and average forecast variance across quarters for the pre-2012 subsample. Columns (4) and (5) report the mean and average forecast variance for the post-2012 subsample.

to 2%. Additionally, we compute the variance of both the now-cast and the ten-year ahead forecasts for each quarter and report the average variances for each period in columns (3) and (5), showing that both variances decrease in the post-2012 period.

To quantify the underlying changes caused by the policy implementation, we structurally estimate this model using moments obtained from both the pre- and post-2012 samples. We then assess the estimated changes in the data generation process and examine how they quantitatively impact the empirical patterns of forecasts. While all the details of the estimation are relegated to Appendix A.5, we provide a summary of the estimation procedures below.

Specifically, our model can be fully specified by two sets of parameters. First, there are three parameters related to the data generating process, that is $\{\rho, \sigma_\mu^2, \sigma_x^2\}$. Second, there are two parameters that capture the precision of the signals, that is $\{\sigma_\epsilon^2, \sigma_e^2\}$. To structurally estimate the value of these parameters $\Theta = \{\rho, \sigma_\mu^2, \sigma_x^2, \sigma_\epsilon^2, \sigma_e^2\}$, we follow Chen et al. (2024) in computing Laplace-type estimators (LTE) with an Markov Chain Monte Carlo approach.

As our goal is to identify the changes in the underlying parameters, we estimate this set of parameters for each subsample period. We compute the variances of forecasts for different horizons, specifically for $h = 0, 1, 2, 3, 4$, using the subsamples before and after 2012. These variances will be treated as the target moments in our estimation and denoted as $\hat{m}$. Furthermore, we construct the model counterpart of $\hat{m}$ and define the distance between the two as follows:

$$\Lambda(\Theta) = [m(\Theta) - \hat{m}]' \hat{W} [m(\Theta) - \hat{m}], \tag{15}$$

where $\hat{W}$ is the weighting matrix, where the diagonal elements represent the precision of the moments $\hat{m}$. We solve for the parameter values (Θ) to minimize the constructed distance, that is, finding the set of parameter values that best matches the forecast

Table 5. *Estimated Model Parameters*

	Parameter Estimation					
	Pre-2012			Post-2012		
	Mean	90%HPDI	95% HPDI	Mean	90%HPDI	95% HPDI
σ_μ^2	1.19	(0.83,1.68)	(0.75,1.69)	1.18	(0.81,1.67)	(0.74,1.68)
σ_x^2	1.64	(1.32,1.85)	(1.37,2.02)	1.64	(1.32,1.84)	(1.37,2.01)
σ_ϵ^2	1.27	(0.94,1.49)	(0.89,1.57)	1.26	(0.93,1.49)	(0.88,1.56)
σ_e^2	1.15	(0.85,1.42)	(0.84,1.45)	1.15	(0.85,1.42)	(0.84,1.45)
ρ	0.92	(0.85,0.99)	(0.85,0.99)	0.64	(0.54,0.76)	(0.54,0.79)

Note: This table presents the estimated parameter values for the pre-2012 and post-2012 periods. We provide the mean values, as well as the 90% and 95% Highest Posterior Density Intervals (HPDI).

variance at each forecast horizon.

The estimated parameters for each subsample are reported in Table 5 together with the 90% and 95% high posterior density interval (HPDI). A comparison of the two sets of estimated parameters reveals that there are minimal changes in the innovations in trends and cycles (i.e., σ_μ^2 and σ_x^2) and the precision of signals on trends and cycles (i.e., σ_ϵ^2 and σ_e^2) following the policy change in 2012. This indicates that this set of parameters remain relatively stable before and after the policy change.

The only noteworthy change is the significant, sizable decrease in the persistence of the cyclical component. Before the policy change, the estimated persistence of the cyclical component was 0.92, aligning with previous literature. For instance, Carvalho et al. (2023) estimated a value of $\rho = 0.87$, while Bianchi et al. (2021) estimated a persistence of $\rho = 0.70$. After the policy change, the estimated persistence dropped to 0.64, indicating that short-term fluctuations have become less persistent. The observed change in the estimated persistence of the cyclical component is intuitive. Following the policy change, the central bank would respond more aggressively to short-term deviations from the long-term target. Consequently, the persistence of the cyclical component would decrease.

Next, we will examine whether this estimated model can replicate the set of facts documented in section 2 regarding inflation forecasts before and after 2012. Towards this end, we first reproduce the empirical patterns of inflation expectations by re-estimating Equations (1) and (2) using the two sub-samples.

The results are presented in Table 6. In Panel A, we present the results of estimating Equation (1) in columns (1) and (2) for the period before and after 2012, respectively. In the pre-2012 sub-sample, we observe a significant negative correlation between changes in trend beliefs and changes in cyclical beliefs. In the post-2012 sub-sample, we find this correlation becomes positive and insignificant. In Panel B, we

Table 6. *Estimation result: effect of inflation targeting*

	Data		Estimated	
	Pre-2012	Post-2012	Pre-2012	Post-2012
	(1)	(2)	(3)	(4)
Panel A. Dependent Variable: Trend belief changes.				
Cyclical belief changes	-0.019***	0.009	-0.033**	0.008
	(0.006)	(0.009)	(0.014)	(0.021)
Obs	2,047	1,236	2178	1271
R-sq.	0.012	0.004	0.002	0.001
Panel B. Dependent Variable: Forecast Dispersion.				
Forecast horizon h	-0.058***	-0.094***	-0.043***	-0.070***
	(0.022)	(0.017)	(0.005)	(0.007)
Obs.	610	160	610	160
R-sq.	0.512	0.315	0.194	0.492

Note: This table presents the coefficients obtained Equation 1 and Equation 2. The estimation results are reported in columns (1) and (2) using the SPF data before and after 2012, respectively. Columns (3) and (4) report the estimation result using the simulated data with estimated parameters before and after 2012.

display the estimation results of Equation (2) in columns (1) and (2) for the pre-2012 and post-2012 periods, respectively. Comparing the two periods, we observe that after the implementation of explicit inflation targeting, the forecast dispersion exhibits a more pronounced decline over the forecast horizon, evidenced by an increase in the magnitude of the slope from -0.058 to -0.094, both being statistically significant.

This set of changes observed after the implementation of inflation targeting is in line with the predictions of our model. According to Lemma 4 (i.e., part (ii)), when the cyclical component becomes less persistent (i.e., ρ decreases), the negative covariance between forecasters' beliefs about trends and cycles diminishes. In other words, as the cyclical components become less persistent, distinguishing between trends and cycles becomes less important. Consequently, the empirical patterns should more closely align with those predicted by the standard model, where the confusion mechanism is absent. Specifically, our model predicts that following the policy change, changes in trend belief and changes in cyclical beliefs become more likely to be positively correlated, and forecast dispersion would decrease at a faster rate over the forecast horizon.

While our model can generate predictions that qualitatively align with the empirical evidence, we now aim to evaluate the extent to which this estimated model can quantitatively explain the observed changes in the data following the policy change. Specifically, we simulate the model with the two sets of estimated parameters and re-estimate the Equation (1) and Equation (2) with simulated data for each sub-period.

Columns (3) and (4) of Table 6 present the regression results from simulation data, using parameter values estimated for the periods before and after 2012, respectively. In panel A, we show the estimation result of Equation (1) in column (3), which is untargeted. The simulated data exhibits a significant negative correlation between changes in trend beliefs and cyclical beliefs (-0.033). In panel B, the estimation result of Equation (2) using the simulated data is significantly negative (-0.043) and close to the actual coefficient (-0.058) in terms of magnitude.

Column (4) of Table 6 displays the estimation results using simulation data for the period after 2012. In Panel A, the estimated coefficient is not statistically significant (0.008), which is in close proximity to the actual coefficient in the data (0.009). In Panel B, the estimated coefficient after 2012 is significantly negative (-0.070), and its magnitude is greater than that before 2012 (-0.094).

In summary, despite its simplicity and the limited number of parameters, our model can effectively capture the shift in forecasting patterns following the policy change and quantitatively resembles those changes observed in the actual data.

5.2 Rational Confusion and Behavioral Bias

In the previous sections, we show how the confusion mechanism plays a role by assuming that forecasters are rational and use the Bayesian rule to update their beliefs. In this section, we demonstrate that our framework can be extended to incorporate behavioral biases studied in the literature on expectation formation. We emphasize that the new confusion mechanism we introduce can interact with these biases and provide insights into various issues in the literature. Specifically, we showcase this by introducing the feature of overconfidence, where forecasters subjectively believe that the variances of the signal noise are smaller than their actual values (e.g., Daniel et al. 1998, Kohlhas et al. 2019). We demonstrate how this extension of the benchmark model could explain why now-cast errors might persist across periods, which is a well-known puzzle in the literature on expectation formation.

Following Ma et al. (2020), we examine the correlation between the now-cast errors across periods, using the SPF data. To be specific, we estimate the following equation:

$$\underbrace{y_t - F_{i,t}y_t}_{FE_{i,t}} = \alpha + \beta(\underbrace{y_{t-1} - F_{i,t-1}y_{t-1}}_{FE_{i,t-1}}) + \epsilon_{i,t}, \tag{16}$$

Table (7) displays the estimation results. In column (1), we observe that the estimated coefficient is significantly positive for the majority of macro variables. This suggests that the forecast error exhibits persistence over time: a larger (lower) now-cast error in the previous period is associated with a larger (lower) now-cast error in the current period.

The set of estimation results has two important implications. Firstly, in our benchmark model without behavioral bias, the estimated coefficients should be zero. It is straightforward that the now-cast error in the last period is already known to the forecasters when they provide their forecast in the current period. Therefore, the now-cast

Table 7. Forecast error persistence

	Dependent Variable: Now-cast error		
	β	SE	Obs
Forecast Variable	(1)	(2)	
Nominal GDP	0.154***	0.048	5,872
Real GDP	0.194***	0.057	5,907
GDP price index inflation	0.147**	0.064	5,803
Real consumption	-0.109*	0.064	4,122
Industrial production	0.303***	0.084	5,497
Real nonresidential investment	0.070	0.073	4,046
Real residential investment	0.120**	0.059	4,038
Real federal government consumption	-0.066	0.081	3,880
Real state and local government consumption	0.040	0.062	3,800
Housing start	0.261***	0.060	5,599
Unemployment	0.192***	0.056	5,489
Inflation rate (CPI)	0.044	0.065	4,188

Note: This table shows the coefficients from Equation (16). The sample period is from 1968Q4 to 2019Q4. All the standard error is clustered by individual and year-quarter.

error across periods should be independent when the forecasters are fully rational. The significant estimated coefficients arising from this estimation indicate that forecasters deviate from the rational benchmark, highlighting the necessity of incorporating behavioral bias in our model.

Secondly, in a model where the confusion mechanism is absent and forecasters are overconfident, the now-cast errors across periods should still be zero. This is because the now-cast error in each period consists only of a weighted average of the state innovation and the signal noise. Overconfidence distorts the weights assigned to each component. However, both the innovations and signal noises are independent across periods; therefore, the correlation between now-cast errors across periods remains zero.

In the following, we investigate how the interplay between the two mechanisms – confusion and overconfidence – could account for this the documented empirical pattern. Specifically, to incorporate overconfidence, we consider a scenario where forecasters perceive the signal variances of the trend and cyclical components as $m_1\sigma_\epsilon^2$ and $m_2\sigma_e^2$ respectively. When $m_1 < 1$ ($m_2 < 1$), it indicates that forecasters subjectively believe the trend (cyclical) signal is more precise than it actually is.

Proposition 3. *(i) When forecasters are overconfident in the trend signal ($m_1 < 1$), the now-cast errors across periods are positively correlated if and only if*

$$\frac{\rho\sigma_e^2\sigma_\mu^2}{\sigma_\epsilon^2[\sigma_x^2 + (1-\rho)\sigma_e^2]} = \underline{m}_1 < m_1 < 1, \tag{17}$$

and negatively correlated otherwise. (ii) When forecasters are overconfident in the cyclical *signal ($m_2 < 1$), the now-cast errors across periods are positively correlated if and only if*

$$1 < \frac{1}{m_2} < \frac{1}{\underline{m}_2} = \frac{\sigma_e^2[\rho\sigma_\mu^2 - (1-\rho)\sigma_\epsilon^2]}{\sigma_\epsilon^2\sigma_x^2} \tag{18}$$

and negatively correlated otherwise.

To explicate the proposition, we observe that the now-cast error of period t consists of three parts: the state innovations of period t, the noise of the new signals, and the separation error inherited from the previous period ($z_{i,t-1}$). Since the state innovations and the noise in the private signals are independent across periods, the component of the now-cast error generated by the current state innovation and signal noise must be independent of the now-cast error from the previous period ($FE_{i,t-1}$). In other words, the correlation between the now-cast errors in the last period ($FE_{i,t-1}$) and the current period ($FE_{i,t}$) must be driven by their correlations with the separation error from the last period ($z_{i,t-1}$).

We first analyze the correlation between the separation error from the last period ($z_{i,t-1}$) and the now-cast error in the last period ($FE_{i,t-1}$). As shown in Lemma 5, the separation error from the last period is a weighted combination of the error terms of the beliefs regarding the trend and cyclical components. When forecasters are overconfident in the trend signal (i.e., $m_1 < 1$), the perceived variance of the trend component is smaller than it actually is. Consequently, in the separation error $z_{i,t-1}$, the error term in the trend component is assigned an excessive weight compared to the Bayesian scenario without overconfidence. That force drives the correlation between the separating error ($z_{i,t-1}$) and the now-cast error for the previous period ($FE_{i,t-1}$) to be negative. Specifically, we show the covariance is given by:

$$cov(z_{i,t-1}, FE_{i,t-1}) = -(1-m_1)\sigma_\epsilon^2\sigma_e^2\phi_O^T\frac{V_1}{\Omega_1} < 0,$$

where ϕ_O^T is a positive scalar, and V_1 and Ω_1 are counterparts of V and Ω, respectively. Detailed expressions for all of these variables are provided in Appendix 3.

For example, suppose there exists a positive strong trend signal in period $t-1$. Forecasters would overreact to it, resulting in an overestimation of the trend component. Consequently, the now-cast $FE_{i,t-1}$ is smaller compared to the Bayesian case without overconfidence and by construction it implies a larger separation error $z_{i,t-1}$. That gives rise to a negative correlation between $FE_{i,t-1}$ and $z_{i,t-1}$.

Then, we turn to analyze how this separation error ($z_{i,t-1}$) affects the now-cast error in the current period ($FE_{i,t}$). The correlation of these two depends on how forecasters revise their beliefs regarding the trend and cyclical components when signals are observed. When forecasters are overconfident in the trend signal, the covariance be-

Table 8. Overconfidence, Confusion and Forecast Error Persistence

Overconfident	$Cov(FE_{i,t-1}, z_{i,t-1})$	$Cov(z_{i,t-1}, FE_{i,t})$	$Cov(FE_{i,t-1}, FE_{i,t})$
Trend signal	Negative	Negative, iif. $\underline{m}_1 < m_1$	Positive, iif. $\underline{m}_1 < m_1 < 1$
Cyclical signal	Positive	Positive, iif. $\underline{m}_2 < m_2$	Positive, iif. $\underline{m}_2 < m_2 < 1$

Note: This table summarizes the implications of overconfidence and confusion mechanisms on the relationship between the now-cast error in the previous period, the now-cast error in the current period, and the separation error.

tween the separation error and the now-cast error for the current period can be written as follows:

$$cov(z_{i,t-1}, FE_{i,t}) = \frac{\sigma_z^2}{\Omega_1}[\underbrace{-m_1\sigma_\epsilon^2(\sigma_x^2+\sigma_e^2)}_{trend\ prior\ effect} + \underbrace{\rho\sigma_e^2(\sigma_\mu^2+m_1\sigma_\epsilon^2)}_{cyclical\ prior\ effect}], \tag{19}$$

Recall that the separation error inherited from period $t-1$ is present in the prior belief for period t, which exert opposite effects on prior beliefs regarding trend and cyclical components (see Equation (6)). Specifically, if the separation error $z_{i,t-1}$ is positive, compared to the case without a separation error, the trend prior leads to a larger now-cast and a lower $FE_{i,t}$; whereas the cyclical prior leads to a smaller now-cast and a larger $FE_{i,t}$. That is why the effect of the trend prior is negative and the effect of the cyclical prior is positive in Equation (19).

When forecasters are overconfident about the trend signal, they tend to place greater reliance on the trend signal to infer the trend component in period t and rely less on the prior belief inherited from period $t-1$. That is why the effect of the trend prior is discounted with $m_1 < 1$ in Equation (19).

As the extent of overconfidence in the trend signal increases, the effect of the trend prior is more likely to be dominated by the effect of the cyclical prior. In other words, when m_1 becomes smaller, the covariance between the separation error ($z_{i,t-1}$) and the current now-cast error ($FE_{i,t}$) is more likely to be positive. Consider a polar case where m_1 goes to zero, the correlation is strictly positive.

Part (i) of Proposition 3 states that when both the confusion and overconfidence mechanisms are present, the now-cast errors can be positively correlated over time, on condition that the extent of overconfidence in the trend signal is moderate. Table 8 summarizes the correlations between the now-cast error in the previous period, the now-cast error in the current period, and the separation error.

The inequality in Equation (17) characterizes the condition under which the effect of the trend prior dominates the effect of the cyclical prior. Consider the case where the cyclical is very volatile, that is, σ_x^2 is large enough and as a result $\underline{m}_1$ converges to zero. Forecasters would place very limited reliance on the prior belief regarding the cyclical component and rely heavily on new information about the cycle. Therefore, the effect of the trend prior always dominates, which drives a negative correlation between the separation error and the now-cast error of the current period. As a result, the covariance between the now-cast errors across periods is positive for any $m_1 < 1$.

The analysis of the case where forecasters are overconfident about the cyclical signal is analogous. We relegate the relevant discussion in Appendix C.

5.3 Misinterpretation of Signals

In our benchmark model, forecasters can be confused about the trend and cyclical components because they are not directly observable. However, confusion can be modeled alternatively: although forecasters can observe the components at the end of each period, they may misinterpret the signals before making forecasts, mistaking a trend signal for a cyclical one or vice versa. Interestingly, under certain conditions, this model of misinterpretation predicts an increase in forecast dispersion as the forecast horizon extends. But it consistently predicts a non-negative correlation between changes in trend beliefs and cyclical beliefs, which contrasts with the negative correlation documented in the data.

The misinterpretation model differs from our benchmark model in two ways. First, we assume that forecasters can observe not only the state value (i.e., y_{t-1}) at the end of each period but also the trend and cyclical components perfectly (i.e., μ_{t-1} and x_{t-1}). Consequently, there is no confusion about these components at the end of each period. In the following period t, they still observe signals about trends and cycles. Second, we introduce the possibility of forecasters misinterpreting the signals before they make forecasts. Specifically, there is a probability τ that a forecaster may interpret the trend signal as a cyclical one and, at the same time, treat the cyclical signal as a trend signal.

In this model, each forecaster updates their beliefs and forms expectations using the Bayesian rule, even though there is a possibility of misinterpreting signals. That is,

$$\boldsymbol{\theta}_{1,t}^{i} = \boldsymbol{\theta}_{2,t-1} + \boldsymbol{\kappa} \times (\boldsymbol{s}_{i,t} - \boldsymbol{\theta}_{2,t-1}), \tag{20}$$

where $\boldsymbol{\theta}_{2,t-1}$ is $(\mu_{t-1}, x_{t-1})'$ for all forecasters, as the actual value of the components from the previous period is perfectly observed and the Kalman gain matrix $\boldsymbol{\kappa}$ is standard:

$$\boldsymbol{\kappa} = \begin{pmatrix} \frac{\sigma_\mu^2}{\sigma_\epsilon^2 + \sigma_\mu^2} & 0 \\ 0 & \frac{\sigma_x^2}{\sigma_e^2 + \sigma_x^2} \end{pmatrix}.$$

For those who correctly interpret the signals, $\boldsymbol{s}_{i,t}$ is represented as $(s_{i,t}^{\mu}, s_{i,t}^{x})'$. And the variance-covariance matrix of their beliefs is given by:

$$\begin{pmatrix} \text{Var}_c^T & \widetilde{\text{COV}}_c \\ \widetilde{\text{COV}}_c & \text{Var}_c^C \end{pmatrix} = \begin{pmatrix} \frac{\sigma_\epsilon^2 \sigma_\mu^2}{\sigma_\epsilon^2 + \sigma_\mu^2} & 0 \\ 0 & \frac{\sigma_e^2 \sigma_x^2}{\sigma_x^2 + \sigma_e^2} \end{pmatrix}.$$

For those who wrongly interpret the signals, $\boldsymbol{s}_{i,t}$ is represented as $(s_{i,t}^x, s_{i,t}^\mu)'$, the corresponding variance-covariance matrix of their beliefs is:

$$\begin{pmatrix} \text{Var}_w^T & \widetilde{\text{COV}}_w \\ \widetilde{\text{COV}}_w & \text{Var}_w^C \end{pmatrix} = \begin{pmatrix} \frac{\sigma_\mu^2(\sigma_\epsilon^4 + \sigma_\mu^2 \sigma_e^2)}{(\sigma_\epsilon^2 + \sigma_\mu^2)^2} & 0 \\ 0 & \frac{\sigma_x^2(\sigma_e^4 + \sigma_x^2 \sigma_\epsilon^2)}{(\sigma_x^2 + \sigma_e^2)^2} \end{pmatrix}.$$

Next, we examine whether the misinterpretation model could account for the two documented empirical facts. Proposition 4 summarizes our the result regarding the dispersion of forecasts over horizon.

Proposition 4. *If the individual forecaster may misinterpret the signals with a probability τ, the dispersion of forecasts across forecasters is increasing in the forecast horizon h, if and only if:*

$$h > \underline{h}_m = \frac{1}{\ln \rho} \ln\left[\tau(1-\tau)\right] \frac{-\widetilde{COV}_m}{2[\tau \phi_w^C Var_w^C + (1-\tau)\phi_c^C Var_c^C]}, \tag{21}$$

where $0 < \phi_w^C < 1$, $0 < \phi_c^C < 1$, $\widetilde{COV}_m = -\frac{(\sigma_\mu^2 + \sigma_x^2)\sigma_x^2 \sigma_\mu^2}{(\sigma_\epsilon^2 + \sigma_\mu^2)(\sigma_e^2 + \sigma_x^2)}$ *and* $\ln \rho < 0$.

Proposition 4 states that, similar to the benchmark model, the forecast variance increases as the forecast horizon extends when h is larger than a threshold $\underline{h}_m$. Interestingly, when everyone correctly interprets the signals (i.e., $\tau = 0$), or everyone misinterprets the signals (i.e., $\tau = 1$), the threshold $\underline{h}_m$ approaches infinity. This implies that the forecast variance decreases monotonically over the horizon. Furthermore, the threshold $\underline{h}_m$ could be negative if the value of τ falls within the intermediate range. This implies that the forecast variance increases monotonically over the horizon.

To understand this result, we examine the forecast variance, which can be decomposed as the variance of the cyclical belief across forecasters, the variance of the trend belief across forecasters, and their covariance across forecasters:

$$\begin{aligned} Var(F_{i,t} y_{t+h}) = {} & \rho^{2h}[\tau \phi_w^C \text{Var}_w^C + (1-\tau)\phi_c^C \text{Var}_c^C] + [\tau \phi_w^T \text{Var}_w^T + (1-\tau)\phi_c^T \text{Var}_c^T] \\ & + \rho^h (1-\tau)\tau \widetilde{\text{COV}}_m. \end{aligned} \tag{22}$$

Since forecasters can observe the two components at the end of each period, the variance of the cyclical belief and the trend belief (i.e., the first two terms) is caused by the information heterogeneity prior to making forecasts.

The covariance across forecasters (i.e., the third term) arises because forecasters can be divided into two groups in this model: those who misinterpret the signals and those who correctly use them. To illustrate, consider there is a positive strong trend signal. This signal would increase the trend beliefs of those who correctly interpret it and the cyclical beliefs of those who misinterpret it as a cyclical signal. As a result, due to the presence of individuals who misinterpret the signal, the trend belief of the entire population, on average, is lower than it should be, while the cyclical belief is higher than it should be. This creates a negative covariance between beliefs about the trend and cyclical components, even though the beliefs of individuals regarding these components are independent.

As the forecast horizon extends, similar to the benchmark model, the variance of the cyclical component decreases, and the covariance term increases. If the increase in the covariance term is more pronounced, the forecast variance would increase as the forecast horizon extends.

While this misinterpretation model could generate an increasing forecast dispersion over the horizon under some conditions, can it also generate the negative correlation between changes in trend beliefs and changes in cyclical beliefs? It is important to note that the Kalman gain matrix in Equation (20) indicates that individuals' belief updating for the trend and cyclical components is independent. As a result, one's subjective beliefs regarding these components are also independent. Thus, we have $cov(E[\mu_{i,t}] - E[\mu_{i,t-1}], E[x_{i,t}] - E[x_{i,t-1}]) = 0$. Therefore, the covariance of the changes in trend beliefs and cyclical beliefs becomes:

$$cov(F_{i,t}y_{t+3Y} - F_{i,t-1}y_{t-1+3Y}, Cyc_{i,t} - Cyc_{i,t-1}) = (1-\rho^{3Y})\rho^{3Y} var(E[x_{i,t}] - E[x_{i,t-1}]) \geq 0.$$

In other word, the misinterpretation model predicts a non-negative correlation between the changes in trend beliefs and cyclical beliefs, which is inconsistent with the fact documented in section 2.3.

6 Conclusion

Our paper introduces a framework in which forecasters cannot perfectly separate the trend and cyclical components in the state variable. Qualitatively, we demonstrate that this crucial feature of our model can help to account for a set of observed empirical patterns. Quantitatively, we apply this model to study the impact of explicit inflation targeting policy in 2012 on forecasting behaviors. This framework can be extended to incorporate behavioral biases and address additional empirical puzzles in the literature on expectation formation. We also explore an alternative setting in which forecasters confuse trend and cycle signals, which turns out to be inconsistent with the observed empirical patterns.

In continuation of the current work presented in this paper, there are two promising avenues for further research. First, our model is flexible enough to incorporate various behavioral biases studied in the literature, and investigating their interaction with the confusion mechanism will provide valuable insights into the expectation formation process. Second, this framework has potential applications beyond forecasting models. For instance, it could be applied to understand the behavior of investors who cannot separate trend and cyclical components in the earnings of firms. As a result, their diverse beliefs may influence their choices in the financial market. We defer the exploration of these research questions to future developments of this framework.

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Appendix

A Data and Robustness Tests

A.1 Variable Definition

The data used in this paper are from the Survey of Professional Forecasters (SPF). Following Bordalo et al. (2020), we convert macroeconomic variables to annual growth rates. For variables that are already presented as rates, we use the original data directly. The procedures are a replication of Bordalo et al. (2020).

Variables changed to the annual growth rate: nominal GDP (NGDP), real GDP (RGDP), GDP price index inflation (PGDP), real consumption (RCONSUM), Industrial production (INDPROD), real nonresidential investment (RNRESIN), real residential investment (RRESINV), real federal government consumption (RGF), real state and local government consumption (RGSL).

- Questions: The level of **Variable name** in the current quarter and the next 4 quarters.
- Forecast of h period ahead: $(\frac{F_{i,t}y_{t+h}}{y_{t+h-4}} - 1) \times 100$, where $F_{i,t}y_{t+h}$ is the original survey forecast from the forecaster i provided in period t regarding the state variable y in h period ahead. y_{t+h-4} is the real state value of period $t + h - 4$ already released.

Variables directly use the survey data: Unemployment (UNEMP), housing start (HOUSING), CPI, Three-month Treasury rate (Tbills), Ten-year Treasury rate (Tbonds).

- Questions: The level of **Variable name** in the current quarter and the next 4 quarters.
- Forecast of h period ahead: $F_{i,t}y_{t+h}$, where $F_{i,t}y_{t+h}$ is the original survey forecast from the forecaster i provided in period t regarding the state variable y in h period ahead.

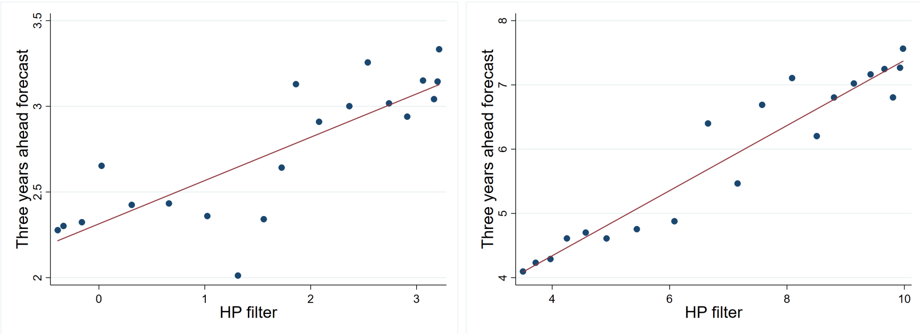
A.2 Robustness: Forecast dispersion over forecast horizon with time fixed effect

Table A1. *Forecast dispersion over forecast horizon with time FE*

	Dependent Variable: Forecast Dispersion					
	Variance of forecasts		50 percentile difference			
	β_1	SE	β_1	SE	Time FE	Obs
Forecast Variable	(1)	(2)	(3)	(4)		
Nominal GDP	0.337***	0.014	0.204***	0.005	Yes	1,025
Real GDP	0.242***	0.013	0.162***	0.004	Yes	1,025
GDP price index inflation	0.118***	0.005	0.119***	0.003	Yes	1,025
Real consumption	0.125***	0.008	0.127***	0.004	Yes	770
Industrial production	0.860***	0.034	0.320***	0.009	Yes	1,025
Real nonresidential investment	1.647***	0.068	0.497***	0.012	Yes	770
Real residential investment	6.021***	0.299	0.932***	0.026	Yes	770
Real federal government consumption	1.284***	0.065	0.393***	0.013	Yes	770
Real state and local government consumption	0.317***	0.016	0.210***	0.006	Yes	770
Housing start	0.004***	0.000	0.020***	0.001	Yes	1,024
Unemployment	0.034***	0.001	0.082***	0.002	Yes	1,014
Inflation rate (CPI)	-0.066***	0.013	-0.073***	0.008	Yes	770
Three-month Treasury rate	0.053***	0.002	0.106***	(0.003	Yes	560
Ten-year Treasury rate	0.045***	0.001	0.094***	0.002	Yes	560

Note: This table shows the coefficients from estimating Equation 2 with year-quarter fixed effect. The sample period is from 1968Q4 to 2019Q4. In column (1), we directly use the forecast variance. In column (3), we use the difference between the 25% percentile and 50% percentile.

A.3 Three years ahead forecast and trend estimates using HP filter



(a) Real GDP growth (b) Unemployment rate

Figure A1. *Bin-scatter plot for the three years ahead predictions and trend estimates using HP filter. Note: The sample period for the analysis spans from 2009 to 2019. The two figures presented compare the three-year-ahead forecasts for the real GDP growth rate and the unemployment rate with the estimated trends obtained using the HP filter. Following Hodrick and Prescott (1997), the smooth parameter is set to be* $\lambda = 1600$ *for the HP filter.*

A.4 Robustness: T-bills and T-bonds

To examine the idea presented in the main text, we estimate the following specification:

$$Var(F_{i,t}y_{t+h}) = \alpha + \beta_4 h + \beta_5 \text{T-bond} + \epsilon_t, \tag{A1}$$

where T-bond is a dummy, which takes the value of 1 for forecast dispersion corresponding to ten years treasury bonds forecast and takes the value of 0 for forecasts on Treasury bills. Therefore, β_5 reflect the difference in forecast dispersion between the security with a longer maturity (T-bonds) and the security with a shorter maturity (T-bills).

The results of our estimation are presented in Table A2. Columns (1) and (2) present significantly positive coefficients for both the forecast horizon and the T-bond dummy. This indicates that forecast dispersion increases as the forecast horizon extends for both variables, aligning with our earlier findings. Furthermore, the dispersion of forecasts for ten-year T-bonds is significantly larger compared to T-bills (its three-month counterpart). This observation suggests that forecasters demonstrate greater disagreement in their forecasts for the long-term U.S. economy.

Table A2. *Forecast dispersion of three-month and ten-year treasury bills*

	Dependent Variable: Forecast Dispersion	
	Variance of forecasts	50 percentile difference
	(1)	(2)
Forecast horizon h	0.049***	0.100***
	(0.002)	(0.003)
Ten-year Tbond	0.010*	0.062***
	(0.006)	(0.012)
Year FEs	YES	YES
Quarter FEs	YES	YES
Obs	1120	1120
Adj R-sq.	0.643	0.683

Note: This table shows the coefficients from Equation (A1). The sample period is from 1968Q4 to 2019Q4. Column (1) reports the result using forecast variance as the measure of dispersion. Column (2) shows the result using the difference between the 25% percentile and 50% percentile. In both columns, significant positive coefficients on the forecast horizon and the ten-year dummy implies that forecast dispersion increases as the forecast horizon extends for both variables, and the dispersion of ten-year treasury bills is higher compared to its three-month counterpart.

A.5 Estimation procedures

To estimate the set of parameters $\Theta = \{\rho, \sigma_\mu^2, \sigma_x^2, \sigma_e^2, \sigma_\epsilon^2\}$ before and after 2012, we begin by dividing the entire dataset into two subsets: one before 2012 and one after 2012. For each subset, we compute the average forecast variance for different forecast horizons ($h = 0, 1, 2, 3, 4$). These sets of forecast variances serve as the targets for estimation denoted as $\hat{m}$.

Next, we compute the precision of each estimation target. Specifically, for a given forecast horizon h, we calculate the standard error of the forecast variance across different quarters. We use the precisions of moments $\hat{m}$ as the weighting matrix, denoted as $\hat{W}$. Table A3 provides the summary statistic of the estimation moments.

Table A3. *Estimation Moments*

	Estimation Moments			
	Pre-2012		Post-2012	
	Target	SE	Target	SE
h=0	0.805	1.276	0.719	0.693
h=1	0.543	0.568	0.321	0.182
h=2	0.444	0.370	0.274	0.123
h=3	0.410	0.324	0.256	0.103
h=4	0.413	0.282	0.269	0.071

The distance is defined in Equation (15) as the weighted squared difference between the target moments $\hat{m}$ and the model prediction $m(\Theta)$, which represents the moments implied by the model for the given parameter set (Θ). Using MCMC with the Metropolis-Hastings algorithm, we choose the set of model parameters that minimize the distance $\Lambda(\Theta)$. The estimation of the parameter set before and after 2012 follows the exact same procedures, with different estimation targets derived from the respective subsets of the data.

B Proofs

Characterization of special case when the trend is observable in section 4.1.

Consider a special case where both the state and trend components are observable at the end of each period. Without loss of generality, we assume the cyclical component follows an AR(N) process:

$$x_t = \sum_{h=0}^{N} \rho^h L^h x_t + \gamma_t^x,$$

where L is the lag operator.

The private signal of forecaster i is given by:

$$s_{i,t}^{\mu} = \mu_t + \epsilon_{i,t} \quad \text{and} \quad s_{i,t}^{x} = x_t + e_{i,t}.$$

Given the trend component is observable at the end of each period, one's prior belief before observing the signals is:

$$\boldsymbol{\theta}_{2,t-1}^{i} = \begin{pmatrix} \mu_{t-1} \\ \sum_{h=0}^{N} \rho^h L^h x_t \end{pmatrix}.$$

The posterior beliefs regarding the two components upon observing the signals is given by:

$$\boldsymbol{\theta}_{1,t}^{i} = \boldsymbol{\theta}_{2,t-1}^{i} + \boldsymbol{\kappa} \times (\boldsymbol{s}_{i,t} - \boldsymbol{\theta}_{2,t-1}^{i}),$$

where the Kalman gain matrix and the variance-covariance matrix is same as the ones in the main text:

$$\boldsymbol{\kappa} = \begin{pmatrix} \frac{\sigma_\mu^2}{\sigma_\mu^2+\sigma_\epsilon^2} & 0 \\ 0 & \frac{\sigma_x^2}{\sigma_x^2+\sigma_e^2} \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} \text{Var}_s^T & \widetilde{\text{COV}}_s \\ \widetilde{\text{COV}}_s & \text{Var}_s^C \end{pmatrix} = \begin{pmatrix} \frac{\sigma_\epsilon^2\sigma_\mu^2}{\sigma_\epsilon^2+\sigma_\mu^2} & 0 \\ 0 & \frac{\sigma_e^2\sigma_x^2}{\sigma_x^2+\sigma_e^2} \end{pmatrix}.$$

The forecast variance across forecasters is given by:

$$Var(F_{i,t}y_{y+h}) = \rho^{2h}\left(\frac{\sigma_x^2}{\sigma_x^2+\sigma_e^2}\right)^2\sigma_e^2 + \left(\frac{\sigma_\mu^2}{\sigma_\mu^2+\sigma_\epsilon^2}\right)^2\sigma_\epsilon^2.$$

It is evidence that the forecast variance across forecasters is decreasing, as the forecast horizon extends.

In addition, changes in trend beliefs and changes cyclical beliefs can be written as follows:

$$F_{i,t}y_{t+3Y} - F_{i,t-1}y_{t-1+3Y} = (\mu^i_{1,t} - E_{i,t-1}[\mu_{t-1}]) + \rho^{3Y}(E_{i,t}[\sum_{h=0}^{N}\rho^h L^h x_{t+3Y}] - E_{i,t-1}[\sum_{h=0}^{N}\rho^h L^h x_{t+3Y-1}]),$$

and

$$Cyc_{i,t} - Cyc_{i,t-1} = (1-\rho^{3Y})(E_{i,t}[\sum_{h=0}^{N}\rho^h L^h x_{t+3Y}] - E_{i,t-1}[\sum_{h=0}^{N}\rho^h L^h x_{t+3Y-1}]).$$

Following the same logic as the main text, the correlation between changes in the beliefs about the trend component and changes in beliefs about the cyclical component at any horizon should be non-negative. That is,

$$\begin{aligned}&cov(F_{i,t}y_{t+3Y} - F_{i,t-1}y_{t-1+3Y}, Cyc_{i,t} - Cyc_{i,t-1})\\ &= \rho^{3Y}(1-\rho^{3Y})Var(E_{i,t}[\sum_{h=0}^{N}\rho^h L^h x_{t+3Y}] - E_{i,t-1}[\sum_{h=0}^{N}\rho^h L^h x_{t+3Y-1}]) \geq 0.\end{aligned}$$

In this special case, where trends and cycles are observable at the end of each period, the model fails to replicate either of the two empirical patterns documented, even when we allow the data generation process for the cyclical component to follow an AR(N) process.

Proof of Lemma 1 and Lemma 2. We first show that if $z_{i,t-1}$ follows a normal distribution with a mean of zero and a variance denoted by $\sigma^2_{z,t-1}$, then $z_{i,t}$ will also be normally distributed. Furthermore, we show that the variance of $z_{i,t}$ converges to a unique steady state value of σ^2_z.

To begin, we note that with the prior belief and the signal structures given by Equation (5) and (6), the posterior belief of forecaster i after receiving signals is given by:

$$\begin{aligned}p(\boldsymbol{\theta}|\boldsymbol{s}_{i,t}) &\propto p(\boldsymbol{\theta}^i_{2,t-1})p(\boldsymbol{s}_{i,t}|\boldsymbol{\theta}^i_{2,t-1})\\ &\propto \exp\left\{-\frac{1}{2}[\boldsymbol{\theta}^T(\boldsymbol{\Sigma}_s^{-1} + \boldsymbol{\Sigma}^{-1}_{\boldsymbol{\theta}^i_{2,t-1}})\boldsymbol{\theta} - 2(\boldsymbol{\Sigma}_s^{-1} + \boldsymbol{\Sigma}^{-1}_{\boldsymbol{\theta}^i_{2,t-1}})^{-1}(\boldsymbol{\Sigma}_s^{-1} + \boldsymbol{\Sigma}^{-1}_{\boldsymbol{\theta}^i_{2,t-1}})(\boldsymbol{s}^T_{i,t}\boldsymbol{\Sigma}_s^{-1} + \boldsymbol{\theta}^{i,T}_{2,t-1}\boldsymbol{\Sigma}^{-1}_{\boldsymbol{\theta}^i_{2,t-1}})\boldsymbol{\theta}]\right\}\\ &\propto \exp[-\frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\theta}^i_{1,t})^T(\boldsymbol{\Sigma}_s^{-1} + \boldsymbol{\Sigma}^{-1}_{\boldsymbol{\theta}^i_{2,t-1}})(\boldsymbol{\theta} - \boldsymbol{\theta}^i_{1,t})],\end{aligned}$$

where

$$\boldsymbol{\theta}^i_{1,t} = (\boldsymbol{\Sigma}_s^{-1} + \boldsymbol{\Sigma}^{-1}_{\boldsymbol{\theta}^i_{2,t-1}})^{-1}(\boldsymbol{s}^T_{i,t}\boldsymbol{\Sigma}_s^{-1} + \boldsymbol{\theta}^{i,T}_{2,t-1}\boldsymbol{\Sigma}^{-1}_{\boldsymbol{\theta}^i_{2,t-1}})^T.$$

To be specific, $\boldsymbol{\theta}^i_{1,t} = (\mu^i_{1,t}, x^i_{1,t})'$ is given by:

$$\mu^i_{1,t} = \underbrace{\frac{\sigma^2_\epsilon(\rho^2\sigma^2_z + \sigma^2_x + \sigma^2_e)}{\Omega}}_{prior\ weight}\mu^i_{2,t-1} + \underbrace{\frac{V + \sigma^2_e(\sigma^2_z + \sigma^2_\mu)}{\Omega}}_{signal\ weight}s^\mu_{i,t} - \frac{\rho\sigma^2_\epsilon\sigma^2_z}{\Omega}\underbrace{(s^x_{i,t} - \rho x^i_{2,t-1})}_{surprise\ from\ cycle}, \quad \text{(A2)}$$

$$x^i_{1,t} = \underbrace{\frac{\sigma_e^2(\sigma_z^2+\sigma_\mu^2+\sigma_\epsilon^2)}{\Omega}}_{prior\ weight}\rho x^i_{2,t-1} + \underbrace{\frac{V+\sigma_\epsilon^2(\sigma_x^2+\rho^2\sigma_z^2)}{\Omega}}_{signal\ weight} s^x_{i,t} - \frac{\rho\sigma_e^2\sigma_z^2}{\Omega}\underbrace{(s^\mu_{i,t}-\mu^i_{2,t-1})}_{surprise\ from\ trend}. \tag{A3}$$

where Ω and V are constants:

$$\Omega = (\sigma_z^2+\sigma_\mu^2+\sigma_\epsilon^2)(\sigma_x^2+\sigma_e^2+\rho^2\sigma_z^2)-\rho^2\sigma_z^4,\ \ V=(\sigma_z^2+\sigma_\mu^2)(\sigma_x^2+\rho^2\sigma_z^2)-\rho^2\sigma_z^4.$$

Equations (A2) and (A3) can also be rewritten in the form of a Kalman filter, as shown in the main text:

$$\boldsymbol{\theta}^i_{1,t} = \boldsymbol{\theta}^i_{2,t-1} + \boldsymbol{\kappa}(\boldsymbol{s}_{i,t}-\boldsymbol{\theta}^i_{2,t-1}),$$

where $\boldsymbol{\kappa}$ is the Kalman gain:

$$\boldsymbol{\kappa} = \begin{pmatrix} \frac{V+\sigma_e^2(\sigma_z^2+\sigma_\mu^2)}{\Omega} & -\frac{\rho\sigma_\epsilon^2\sigma_z^2}{\Omega} \\ -\frac{\rho\sigma_e^2\sigma_z^2}{\Omega} & \frac{V+\sigma_\epsilon^2(\sigma_x^2+\rho^2\sigma_z^2)}{\Omega} \end{pmatrix}.$$

Before the value y_t revealed, forecaster i's beliefs about μ_t and x_t are given by a joint distribution:

$$f(\mu,x) \sim N(\boldsymbol{\theta}^i_{1,t}, (\boldsymbol{\Sigma}_{\boldsymbol{s}}^{-1}+\boldsymbol{\Sigma}^{-1}_{\boldsymbol{\theta}^i_{2,t-1}})^{-1}),$$

The observation of y_t imposes a constraint to the belief updating process. That is, the posterior beliefs regarding the trend and cyclical components after observing the actual state must sum up to y_t, i.e.,

$$\mu^i_{2,t} + x^i_{2,t} = y_t.$$

With this constraint, the joint distribution can be written as:

$$\begin{aligned} f(\mu_t, y_t-\mu_t) &\propto \exp\left\{-\frac{1}{2(1-r^2)}\left[\frac{(\mu-\mu^i_{1,t})^2}{\text{Var}^T} - \frac{2r(\mu-\mu^i_{1,t})(y_t-\mu-x^i_{1,t})}{\sqrt{\text{Var}^T\text{Var}^C}} + \frac{(y_t-\mu-x^i_{1,t})^2}{\text{Var}^C}\right]\right\} \\ &\propto \exp\{-\frac{1}{2(1-r^2)}[\frac{(\text{Var}^T+2r\sqrt{\text{Var}^T\text{Var}^C}+\text{Var}^C)\mu^2}{\text{Var}^T\text{Var}^C} \\ &\quad -2\mu\frac{\text{Var}^C\mu^i_{1,t}+r\sqrt{\text{Var}^T\text{Var}^C}(\mu^i_{1,t}+y_t x^i_{1,t})+\text{Var}^T(y_t-x^i_{1,t})}{\text{Var}^T\text{Var}^C}]\}. \end{aligned}$$

where Var^T, Var^C and r are given by:

$$\text{Var}^T = \frac{\sigma_\epsilon^2[\Omega-\sigma_\epsilon^2(\sigma_x^2+\sigma_e^2+\rho^2\sigma^2_{z,t-1})]}{\Omega} \quad \text{and} \quad \text{Var}^C = \frac{\sigma_e^2[\Omega-\sigma_e^2(\sigma_\epsilon^2+\sigma_\mu^2+\sigma_z^2)]}{\Omega};$$

and

$$r = \frac{cov(\mu, x)}{\sqrt{Var(\mu)Var(x)}} = -\frac{\rho\sigma_e^2\sigma_\epsilon^2\sigma_{z,t-1}^2}{\sqrt{\text{Var}^T\text{Var}^C\Omega}}.$$

Therefore, the updated belief of trend component follows $f(\mu) \sim N(\mu_{2,t}^i, \sigma_{z_t}^2)$,where

$$\mu_{2,t}^i = \frac{\text{Var}^C\mu_{1,t}^i + \text{Var}^T(y_t - x_{1,t}^i) + r\sqrt{\text{Var}^T\text{Var}^C}(\mu_{1,t}^i + y_t - x_{1,t}^i)}{\text{Var}^T + 2r\sqrt{\text{Var}^T\text{Var}^C} + \text{Var}^C}, \tag{A4}$$

and

$$\sigma_{z_t}^2 = \frac{(1-r^2)\text{Var}^T\text{Var}^C}{\text{Var}^T + 2r\sqrt{\text{Var}^T\text{Var}^C} + \text{Var}^C}. \tag{A5}$$

The steady state value σ_z^2 is a fixed point of the condition characterized by Equation (A5). Solving for the fixed point of Equation (A5) gives:

$$\sigma_z^2 = \frac{-\sigma_\mu^2[\Lambda + 2\rho(1-\rho)\sigma_e^2\sigma_\epsilon^2] + \sqrt{\sigma_\mu^2\Lambda[\sigma_\mu^2(\Lambda + 4\rho\sigma_e^2\sigma_\epsilon^2) + 4\sigma_e^2\sigma_\epsilon^2\sigma_x^2]}}{2[\Lambda + \rho^2\sigma_\mu^2(\sigma_e^2 + \sigma_\epsilon^2)]}, \tag{A6}$$

where $\Lambda = (1-\rho)^2\sigma_e^2\sigma_\epsilon^2 + \sigma_x^2(\sigma_e^2 + \sigma_\epsilon^2)$.

In the next step, we demonstrate that regardless of the initial variance of the separation error, denoted as $\sigma_{z_0}^2$, it always converges to a unique steady state value σ_z^2. We first simplify Equation (A5) to:

$$\sigma_{z,t}^2 = \frac{g_1(\sigma_{z,t-1}^2)}{g_2(\sigma_{z,t-1}^2)}, \tag{A7}$$

where

$$g_1(\sigma_{z,t-1}^2) = w_1\sigma_{z,t-1}^2 + \eta_1 \quad \text{and} \quad g_2(\sigma_{z,t-1}^2) = w_2\sigma_{z,t-1}^2 + \eta_2,$$

$$w_1 = \sigma_e^2\sigma_\epsilon^2(\rho^2\sigma_\mu^2 + \sigma_x^2);\ \eta_1 = \sigma_e^2\sigma_\epsilon^2\sigma_\mu^2\sigma_x^2;$$

$$w_2 = \rho^2(\sigma_e^2\sigma_\epsilon^2 + \sigma_e^2\sigma_\mu^2 + \sigma_\mu^2\sigma_\epsilon^2) + \sigma_e^2\sigma_\epsilon^2 + \sigma_e^2\sigma_x^2 + \sigma_\epsilon^2\sigma_x^2 - 2\rho\sigma_e^2\sigma_\epsilon^2;\ \eta_2 = \sigma_e^2\sigma_\epsilon^2(\sigma_\mu^2 + \sigma_x^2) + \sigma_\mu^2\sigma_x^2(\sigma_e^2 + \sigma_\epsilon^2).$$

Define the difference between $\sigma_{z,t+1}^2$ and $\sigma_{z,t}^2$ as:

$$D(\sigma_{z,t}^2) = \sigma_{z,t+1}^2 - \sigma_{z,t}^2 = \frac{g_1(\sigma_{z,t}^2)}{g_2(\sigma_{z,t}^2)} - \sigma_{z,t}^2.$$

To show the steady state is unique, it is sufficient to show that $D(\sigma_{z,t}^2)$ is monotonically decreasing. We first show that evaluated at $\sigma_{z,t}^2 = 0$, the derivative is negative.

$$\frac{\partial D(\sigma_{z,t}^2)}{\partial \sigma_{z,t}^2}\Big|_{\sigma_{z,t}^2=0} = \left[\frac{\sigma_e^2\sigma_\epsilon^2(\rho\sigma_\mu^2 + \sigma_x^2)}{\sigma_e^2\sigma_\epsilon^2(\rho\sigma_\mu^2 + \sigma_x^2) + \sigma_\mu^2\sigma_x^2(\sigma_e^2 + \sigma_\epsilon^2)}\right]^2 - 1 < 0.$$

Then we show that the first-order derivative of $D(\sigma^2_{z,t})$ is negative. The derivative is given by:

$$\frac{\partial D(\sigma^2_{z,t})}{\partial \sigma^2_{z,t}} = \frac{w_1\eta_2 - w_2\eta_1}{(w_2\sigma^2_{z,t} + \eta_2)^2} - 1 = \left[\frac{\sigma^2_e \sigma^2_\epsilon(\rho\sigma^2_\mu + \sigma^2_x)}{(w_2\sigma^2_{z,t} + \eta_2)}\right]^2 - 1. \tag{A8}$$

It is always decreasing, because we show that the second-order derivative is negative:

$$\frac{\partial^2 D(\sigma^2_{z,t})}{\partial(\sigma^2_{z,t})^2} = -2w_2 \frac{[\sigma^2_e \sigma^2_\epsilon(\rho\sigma^2_\mu + \sigma^2_x)]^2}{(w_2\sigma^2_{z,t} + \eta_2)^3} < 0.$$

Since $D(\sigma^2_{z,t})$ is monotonously decreasing and concave and the steady state exists, it is unique.

Proof of Lemma 3. Given the quadratic utility function, the forecaster's optimal forecasts are given by the following:

$$\begin{aligned} F_{i,t}y_{t+h} &= E_{i,t}[y_{t+h}] \\ &= E_{i,t}[\mu_t + \rho^h x_t] \\ &= \mu^i_{1,t} + \rho^h x^i_{1,t}. \end{aligned}$$

The first equality is derived from the first order condition of the standard quadratic utility function. With a quadratic utility function, forecasters would minimize the expected squared error, and the first-order condition is given by:

$$E_{i,t}[F_{i,t}y_{t+h} - y_{t+h}] = 0.$$

The second equality follows given the data generation process is known to forecasters. The third equality states that the expected value of the sum of μ_t and $\rho^h x_t$ is the sum of the expected values of the two components, a well known property using Fourier transform (Folland 2009).

Proof of Lemma 4 and Lemma 5. We provide proof for Lemma 4 and 5 jointly. From Equation (A4) in the proof of Lemma 1, we obtain:

$$\begin{aligned} z_{i,t-1} &= \mu^i_{2,t-1} - \mu_{t-1} \\ &= \frac{(\text{Var}^T + \widetilde{\text{COV}})(x_{t-1} - x^i_{1,t-1}) - (\text{Var}^C + \widetilde{\text{COV}})(\mu_{t-1} - \mu^i_{1,t-1})}{\text{Var}^T + \widetilde{\text{COV}} + \text{Var}^C + \widetilde{\text{COV}}}, \end{aligned}$$

which is the first part of Lemma 5.

For the second part of Lemma 5, we first show the steady state value of σ_z^2 increases in σ_μ^2. The proof is constructed using the convergence property of the model. Specifically, using Equation (A7), we can show that $\sigma_{z,t}^2$ increases in σ_μ^2:

$$\frac{\partial \sigma_{z,t}^2}{\partial \sigma_\mu^2} = \frac{\partial[g_1(\sigma_{z,t-1}^2)/g_2(\sigma_{z,t-1}^2)]}{\partial \sigma_\mu^2} = \frac{\sigma_e^4 \sigma_\epsilon^4 [\sigma_x^2 - \rho(1-\rho)\sigma_{z,t-1}^2]^2}{g_2(\sigma_{z,t-1}^2)^2} > 0.$$

Therefore, when the σ_μ^2 increases, the variance of the separation error of the current period $\sigma_{z,t}^2$ increases and monotonically converges to the new steady state, which is unique. That is because $D(\sigma_{z,t}^2)$ is decreasing and crossing zero from above and only once. As a result, the steady state value of the variance σ_z^2 always increases with an increase in σ_μ^2.

Furthermore, we can show that σ_z^2 is upper bounded. To see this, we note that $g_2(\sigma_{z,t-1}^2)^2$ increases with σ_μ^2 and the partial derivative $\partial\sigma_z^2/\partial\sigma_\mu^2$ decreases as σ_μ^2 increases. As σ_μ^2 approaches infinity, $\partial\sigma_z^2/\partial\sigma_\mu^2$ approaches zero. That is,

$$Z'_\mu \equiv \frac{\partial \sigma_z^2}{\partial \sigma_\mu^2} > 0 \quad \text{and} \quad Z''_\mu \equiv \frac{\partial^2 4\sigma_z^2}{(\partial \sigma_\mu^2)^2} < 0.$$

The comparative statics with respect to σ_x^2, σ_ϵ^2, and σ_e^2 are analogous.

It is worth noting that $z_{i,t}$ is obtained via Bayesian updating, using the prior belief $\mu_{1,t}^i$ and $y_t - x_{1,t}^i$ shown in Equation (A6). As the variance of the posterior belief is always smaller than the variance of both prior beliefs, we can obtain:

$$0 \le \sigma_z^2 \le \min\{\text{Var}^C, \text{Var}^T\}.$$

Similarly, considering the case that when the persistence of the cyclical component ρ changes:

$$\frac{\partial \sigma_{z,t}^2}{\partial \rho} = \frac{\sigma_{z,t-1}^2}{g_2(\sigma_{z,t-1}^2)^2} \left\{2\sigma_e^4 \sigma_\epsilon^4 [\rho\sigma_\mu^2 + \sigma_x^2][\sigma_\mu^2 + (1-\rho)\sigma_{z,t-1}^2]\right\} > 0.$$

Therefore, the steady state value of σ_z^2 is increasing in ρ. The logic underlying this statement is analogous.

We proceed to show that $|\widetilde{\text{COV}}|$ first increases and then decreases in σ_μ^2. The first-order derivative is given by:

$$\frac{\partial|\widetilde{\text{COV}}|}{\partial \sigma_\mu^2} \propto Z'_\mu(\sigma_e^2 + \sigma_x^2)(\sigma_\epsilon^2 + \sigma_\mu^2) - \sigma_z^2(\rho^2\sigma_z^2 + \sigma_e^2 + \sigma_x^2).$$

We show that evaluated at $\sigma_\mu^2 = 0$,

$$\frac{\partial |\widetilde{\text{COV}}|}{\partial \sigma_\mu^2}|_{\sigma_\mu^2=0} \propto Z'_\mu (\sigma_e^2 + \sigma_x^2)\sigma_\epsilon^2 > 0.$$

That is because $\sigma_z^2 = 0$ when $\sigma_\mu^2 = 0$. The second-order derivative is given by:

$$\frac{\partial^2 |\widetilde{\text{COV}}|}{(\partial \sigma_\mu^2)^2} \propto Z''_\mu (\sigma_\mu^2 + \sigma_\epsilon^2)(\sigma_e^2 + \sigma_x^2) - 2\rho^2 \sigma_z^2 Z'_\mu < 0.$$

To see the inequality we note that $Z'_\mu > 0$, and $Z''_\mu < 0$. Therefore, there exists a unique $\widetilde{\sigma}_\mu^2 > 0$, such that $\partial |\widetilde{\text{COV}}| / \partial \sigma_\mu^2 = 0$. For any $\sigma_\mu^2 < \widetilde{\sigma}_\mu^2$, $|\widetilde{\text{COV}}|$ is increasing in σ_μ^2; and for any $\sigma_\mu^2 > \widetilde{\sigma}_\mu^2$, $|\widetilde{\text{COV}}|$ is decreasing in σ_μ^2. The property that $|\widetilde{\text{COV}}|$ increases and then decrease is implied.

Finally, it is straightforward that $|\widetilde{\text{COV}}|$ is always increasing in ρ, because

$$\frac{\partial |\widetilde{\text{COV}}|}{\partial \rho} = \frac{\sigma_e^2 \sigma_\epsilon^2}{\Omega^2} \left\{ (\sigma_x^2 + \sigma_e^2)[\sigma_z^4 + \rho Z'_\rho (\sigma_\mu^2 + \sigma_\epsilon^2)] + \sigma_z^2 (\sigma_\mu^2 + \sigma_\epsilon^2)[\sigma_x^2 + \sigma_e^2 - \rho^2 \sigma_z^2] \right\} > 0,$$

where $Z'_\rho \equiv \partial \sigma_z^2 / \partial \rho > 0$.

Proof of Proposition 1. The covariance between the changes in the constructed trend beliefs and the cyclical belief is given by:

$$\begin{aligned}
& cov(F_{i,t} y_{t+3Y} - F_{i,t-1} y_{t-1+3Y}, Cyc_{i,t} - Cyc_{i,t-1}) \\
&= (1 - \rho^{3Y}) \left[cov(\mu_{1,t}^i - \mu_{1,t-1}^i, x_{1,t}^i - x_{1,t-1}^i) + \rho^{3Y} var(x_{1,t}^i - x_{1,t-1}^i) \right] \\
&= (1 - \rho^h)(\widetilde{\text{COV}} + \rho^h \text{Var}^C) \\
&= \frac{(1 - \rho^h)\sigma_e^2}{\Omega} \left\{ \rho^h [\Omega - \sigma_e^2 (\sigma_\epsilon^2 + \sigma_\mu^2 + \sigma_z^2)] - \rho \sigma_\epsilon^2 \sigma_z^2 \right\} \\
& \propto \rho^h [\Omega - \sigma_e^2 (\sigma_\epsilon^2 + \sigma_\mu^2 + \sigma_z^2)] - \rho \sigma_\epsilon^2 \sigma_z^2.
\end{aligned}$$

Define $K \equiv \rho^h [\Omega - \sigma_e^2 (\sigma_\epsilon^2 + \sigma_\mu^2 + \sigma_z^2)] - \rho \sigma_\epsilon^2 \sigma_z^2$. Then the sign of the correlation between changes in trend beliefs and changes in cyclical beliefs depends on the sign of K.

To prove the properties in the proposition, we first show that for any given σ_μ^2, there is a threshold $\overline{\sigma}_x^2$ such that if and only if $\sigma_x^2 < \overline{\sigma}_x^2$, then $K < 0$; and otherwise, $K \geq 0$. To see this, we derive the first-order derivative of K with respect to σ_x^2:

$$\begin{aligned}\frac{\partial K}{\partial \sigma_x^2} &= \rho^h[\sigma_z^2 + \sigma_\mu^2 + \sigma_\epsilon^2 + \sigma_x^2 Z'_x + \rho^2 Z'_x(\sigma_\mu^2 + \sigma_\epsilon^2)] - \rho\sigma_\epsilon^2 Z'_x \\ &= Z'_x\left[\rho^h(\frac{\sigma_z^2 + \sigma_\mu^2 + \sigma_\epsilon^2}{Z'_x} + \sigma_x^2 + \rho^2\sigma_\mu^2 + \rho^2\sigma_\epsilon^2) - \rho\sigma_\epsilon^2\right].\end{aligned} \tag{A9}$$

According to Lemma 5, $Z'_x > 0$ and $Z''_x < 0$. Therefore, the sum of first two terms in Equation (A9), $(\sigma_z^2 + \sigma_\mu^2 + \sigma_\epsilon^2)/Z'_x + \sigma_x^2$, increases in σ_x^2.

If $\partial K/\partial\sigma_x^2 \geq 0$ when evaluated at $\sigma_x^2 = 0$, then it always holds $\partial K/\partial\sigma_x^2 \geq 0$. If $\partial K/\partial\sigma_x^2 < 0$ when evaluated at $\sigma_x^2 = 0$, $\partial K/\partial\sigma_x^2$ crosses zero only once from below. Note that $\partial K/\partial\sigma_x^2$ must be positive when σ_x^2 is sufficiently large.

Furthermore, we characterize how K changes in σ_x^2. When $\sigma_x^2 = 0$, $K = 0$. That is because $\sigma_z^2 = 0$. When $\sigma_x^2 > 0$, K is either always positive, or K initially decreases and then crosses zero from below. This property implies that for any given value of σ_μ^2, there exists a threshold $\overline{\sigma}_x^2 \geq 0$, such that $K|_{\sigma_x^2=\overline{\sigma}_x^2} = 0$, and for any $\sigma_x^2 < \overline{\sigma}_x^2$, $K < 0$.

Given this property, we start proving the first item in this proposition. Towards this end, we show the following claim.

Claim: When $\sigma_x^2 = 0$, there exists a threshold $\overline{\sigma}_\mu^2$ for σ_μ^2, such that when $\sigma_\mu^2 \geq \overline{\sigma}_\mu^2$, $\overline{\sigma}_x^2 = 0$; when $0 < \sigma_\mu^2 < \overline{\sigma}_\mu^2$, $\overline{\sigma}_x^2 > 0$; and when $\sigma_\mu^2 = 0$, $\overline{\sigma}_x^2 = 0$.

To prove this claim, we first evaluate $\partial K/\partial\sigma_x^2$ at $\sigma_x^2 = 0$:

$$\frac{\partial K}{\partial \sigma_x^2}|_{\sigma_x^2=0} = Z'_{x=0}\left[\rho^h(\frac{\sigma_\mu^2 + \sigma_\epsilon^2}{Z'_{x=0}} + \rho^2\sigma_\mu^2 + \rho^2\sigma_\epsilon^2) - \rho\sigma_\epsilon^2\right],$$

where $Z'_{x=0}$ is derivative of σ_z^2 evaluated at $\sigma_x^2 = 0$. It is given by:

$$Z'_{x=0} \equiv \frac{\partial \sigma_z^2}{\partial \sigma_x^2}|_{\sigma_x^2=0} = \begin{cases} \frac{2\rho(\sigma_\epsilon^2+\sigma_\epsilon^2)}{(1-\rho)(1+\rho)}\sigma_\mu^2 + \frac{2\sigma_\epsilon^2\sigma_\epsilon^2}{1+\rho}, & \text{if } \sigma_\mu^2 > 0. \\ 0, & \text{if } \sigma_\mu^2 = 0. \end{cases} \tag{A10}$$

There are only two cases. (i) When $\partial K/\partial\sigma_x^2|_{\sigma_x^2=0} \geq 0$, then K is always positive when $\sigma_x^2 > 0$ and $\overline{\sigma}_x^2 = 0$; and (ii) when $\partial K/\partial\sigma_x^2|_{\sigma_x^2=0} < 0$, K is negative and then crosses zero from below at $\sigma_x^2 = \overline{\sigma}_x^2 > 0$. Therefore, the necessary and sufficient condition for $\overline{\sigma}_x^2 > 0$ is given by $\partial K/\partial\sigma_x^2|_{\sigma_x^2=0} < 0$, which is equivalent to

$$\rho^h(\frac{\sigma_\mu^2 + \sigma_\epsilon^2}{Z'_{x=0}} + \rho^2\sigma_\mu^2 + \rho^2\sigma_\epsilon^2) - \rho\sigma_\epsilon^2 < 0$$

or using the expression of $Z'_{x=0}$ in Equation (A10),

$$\frac{2\rho^4(\sigma_e^2+\sigma_\epsilon^2)}{1-\rho^2}(\sigma_\mu^2)^2+\left[1+\frac{2\rho^2\sigma_e^2\sigma_\epsilon^2}{1+\rho}(1+\rho^2-\rho^{1-h})\right]\sigma_\mu^2-\left[\rho(\rho^{-h}-1)\frac{2\sigma_e^2\sigma_\epsilon^2}{1+\rho}+\rho^{1-h}\right]\sigma_\epsilon^2<0. \tag{A11}$$

The left-hand-side of Equation (A11) is quadric in σ_μ^2, therefore there are two roots. Note that The left-hand-side of Equation (A11) is decreasing and then increasing in σ_μ^2 and it is negative when $\sigma_\mu^2=0$. Therefore, there must exist a unique positive root $\overline{\sigma}_\mu^2>0$.

Therefore, when $\sigma_\mu^2\geq\overline{\sigma}_\mu^2$, $\overline{\sigma}_x^2=0$, which implies $K>0$ on condition that $\sigma_x^2>0$. The first item in this proposition is shown. When $0<\sigma_\mu^2<\overline{\sigma}_\mu^2$, $\overline{\sigma}_x^2>0$, which implies $K>0$ on condition that $\sigma_x^2>\overline{\sigma}_x^2$. The second item is shown.

Proof of Proposition 2. Given the optimal forecasts characterized by Lemma 3, the forecast variance across all forecasters is given by:

$$Var(F_{i,t}y_{t+h})=E[(\mu_{1,t}^i-\overline{E}[\mu_t])^2]+\rho^{2h}E[(x_{1,t}^i-\overline{E}[x_t])^2]+2\rho^hE[(\mu_{1,t}^i-\overline{E}[\mu_t])]E[(x_{1,t}^i-\overline{E}[x_t])].$$

$\overline{E}[\cdot]$ stands for the average forecast across all forecasters. To be specific:

$$E[(\mu_{1,t}^i-\overline{E}[\mu_t])^2]=\mathrm{Var}^T-\frac{\sigma_\epsilon^4(\rho^2\sigma_z^2+\sigma_x^2+\sigma_e^2)^2}{\Omega^2}-\frac{\rho^2\sigma_\epsilon^4\sigma_z^4}{\Omega^2}-\frac{\sigma_\epsilon^4(\sigma_e^2+\sigma_x^2)^2}{\Omega^2}=\mathrm{Var}^T\phi^T,$$

where ϕ^T is given by:

$$\begin{aligned}\phi^T&=1-\frac{\sigma_\epsilon^4}{\mathrm{Var}^T}\times\frac{\sigma_\mu^2(\rho^2\sigma_z^2+\sigma_x^2+\sigma_e^2)^2+\rho^2\sigma_x^2\sigma_z^4+(\sigma_e^2+\sigma_x^2)^2W\sigma_z^2}{\Omega^2}\\&=\frac{[V+\sigma_e^2(\sigma_\mu^2+\sigma_z^2)]^2+\rho^2\sigma_e^2\sigma_\epsilon^2\sigma_z^4+\sigma_\epsilon^2(\sigma_e^2+\sigma_x^2)^2W\sigma_z^2}{\Omega[\Omega-\sigma_\epsilon^2(\sigma_x^2+\sigma_e^2+\rho^2\sigma_z^2)]}<1.\end{aligned}$$

Note that $W=E[(z_{i,t-1}-\overline{E}[z_{i,t-1}])^2]/\sigma_z^2$ is a positive scalar in steady state and invariant in t. To obtain the numerator term $E[(z_{i,t}-\overline{E}[z_{i,t}])^2]$, we rewrite Equation (10) and express $z_{i,t}$ as the follows:

$$\begin{aligned}z_{i,t}=&\frac{\sigma_e^2\sigma_\epsilon^2}{\Omega(\mathrm{Var}^T+2\widetilde{\mathrm{COV}}+\mathrm{Var}^C)}\{-[\sigma_x^2+\rho(\rho-1)\sigma_z^2]\gamma_t^\mu+[\sigma_\mu^2+(1-\rho)\sigma_z^2]\gamma_t^x \\&+\sigma_e^2V\epsilon_{i,t}-\sigma_\epsilon^2Ve_{i,t}+(\rho\sigma_\mu^2+\sigma_x^2)z_{i,t-1}\}.\end{aligned} \tag{A12}$$

This allows us to obtain:

$$z_{i,t}-\overline{E}[z_{i,t}]=\frac{\sigma_e^2\sigma_\epsilon^2}{\Omega(\mathrm{Var}^T+2\widetilde{\mathrm{COV}}+\mathrm{Var}^C)}\left[\sigma_e^2 V\epsilon_{i,t}-\sigma_\epsilon^2 V e_{i,t}+(\rho\sigma_\mu^2+\sigma_x^2)(z_{i,t-1}-\overline{E}[z_{i,t-1}])\right].$$

and

$$E[(z_{i,t}-\overline{E}[z_{i,t}])^2]=\frac{(\sigma_e^2+\sigma_\epsilon^2)\sigma_z^2V^2}{(\sigma_e^2+\sigma_\epsilon^2)V^2+\sigma_e^2\sigma_\epsilon^2\{\sigma_\mu^2[\sigma_x^2+\rho\sigma_z^2(\rho-1)]^2+\sigma_x^2[\sigma_\mu^2+(1-\rho)\sigma_z^2]^2\}}.$$

Therefore, W is given by:

$$W=\frac{(\sigma_e^2+\sigma_\epsilon^2)V^2}{(\sigma_e^2+\sigma_\epsilon^2)V^2+\sigma_e^2\sigma_\epsilon^2\{\sigma_\mu^2[\sigma_x^2+\rho\sigma_z^2(\rho-1)]^2+\sigma_x^2[\sigma_\mu^2+(1-\rho)\sigma_z^2]^2\}}<1.$$

Similarly, $E[(x_{1,t}^i-\overline{E}[x_t])^2]$ and $E[(\mu_{1,t}^i-\overline{E}[\mu_t])]E[(x_{1,t}^i-\overline{E}[x_t])]$ can be written as:

$$E[(x_{1,t}^i-\overline{E}[x_t])^2]=\frac{[V+\sigma_\epsilon^2(\sigma_x^2+\rho^2\sigma_z^2)]^2+\rho^2\sigma_e^2\sigma_\epsilon^2\sigma_z^4+\rho^2\sigma_e^2(\sigma_\epsilon^2+\sigma_\mu^2)^2W\sigma_z^2}{\Omega[\Omega-\sigma_e^2(\sigma_\epsilon^2+\sigma_\mu^2+\sigma_z^2)]}\mathrm{Var}^C=\phi^C\mathrm{Var}^C,$$

and

$$\begin{aligned}&E[(\mu_{1,t}^i-\overline{E}[\mu_t])]E[(x_{1,t}^i-\overline{E}[x_t])]\\&=\frac{\sigma_\epsilon^2[V+\sigma_\epsilon^2(\sigma_x^2+\rho^2\sigma_z^2)]+\sigma_e^2[V+\sigma_e^2(\sigma_z^2+\sigma_\mu^2)]+\sigma_e^2\sigma_\epsilon^2(\sigma_x^2+\sigma_e^2)(\sigma_\epsilon^2+\sigma_\mu^2)W\sigma_z^2}{\Omega\sigma_e^2\sigma_\epsilon^2}\widetilde{\mathrm{COV}}\\&=\phi^{COV}\widetilde{\mathrm{COV}}.\end{aligned}$$

Therefore, the forecast variance of $F_{i,t}y_{t+h}$ across all forecasters can be written as:

$$Var(F_{i,t}y_{t+h})=E[(F_{i,t}y_{t+h}-\overline{E}[F_{i,t}y_{t+h}])^2]=\rho^{2h}\mathrm{Var}^C\phi^C+\mathrm{Var}^T\phi^T+2\rho^h\widetilde{\mathrm{COV}}\phi^{COV},$$

Take the derivative with respect to the forecast horizon h:

$$\frac{\partial Var(F_{i,t}y_{t+h})}{\partial h}=2\rho^h\ln\rho(\rho^h\mathrm{Var}^C\phi^C+\widetilde{\mathrm{COV}}\phi^{COV}).$$

The forecast variance is increasing in h if and only if $\partial Var(F_{i,t}y_{t+h})/\partial h>0$. That is,

$$h>\underline{h}=\frac{1}{\ln\rho}\ln\frac{-\widetilde{\mathrm{COV}}\phi^{COV}}{\mathrm{Var}^C\phi^C}.$$

Proof of proposition 3. Given the beliefs regarding the trend and cyclical components specified in Equations (A2) and (A3), the now-cast error in period t is given by:

$$\begin{aligned}
FE_{i,t} &= y_t - F_{i,t}y_t \\
&= \frac{\rho\sigma_e^2\sigma_z^2 + \sigma_\epsilon^2(\sigma_x^2 + \sigma_e^2 + \rho^2\sigma_z^2)}{\Omega}\gamma_t^\mu + \frac{\rho\sigma_\epsilon^2\sigma_z^2 + \sigma_e^2(\sigma_\mu^2 + \sigma_\epsilon^2 + \sigma_z^2)}{\Omega}\gamma_t^x \\
&- \frac{V + \sigma_e^2[(1-\rho)\sigma_z^2 + \sigma_\mu^2]}{\Omega}\epsilon_{i,t} - \frac{V + \sigma_\epsilon^2[\sigma_x^2 + (\rho^2 - \rho)\sigma_z^2]}{\Omega}e_{i,t} \\
&+ \frac{\rho\sigma_e^2(\sigma_\epsilon^2 + \sigma_\mu^2) - \sigma_\epsilon^2(\sigma_e^2 + \sigma_x^2)}{\Omega}z_{i,t-1}.
\end{aligned}$$

Since the state innovations and the signal noises (γ_t^μ, γ_t^x, $\epsilon_{i,t}$, $e_{i,t}$) are independent across periods, the correlation between the now-cast errors across periods is:

$$cov(FE_{i,t-1}, FE_{i,t}) = \frac{\rho\sigma_e^2(\sigma_\epsilon^2 + \sigma_\mu^2) - \sigma_\epsilon^2(\sigma_e^2 + \sigma_x^2)}{\Omega}cov(FE_{i,t-1}, z_{i,t-1}).$$

We first examine the correlation between the now-cast error at period $t-1$ (i.e., $FE_{i,t-1}$) and the separation error at the end of $t-1$ (i.e., $z_{i,t-1}$).

To begin with, we first demonstrate that in the rational case where $m_1 = m_2 = 1$, the covariance between the now-cast error in period $t-1$ and the separation error $z_{i,t-1}$ is zero. The now-cast error in period $t-1$ is given by:

$$\begin{aligned}
FE_{i,t-1} &= y_{t-1} - F_{i,t-1}y_{t-1} \\
&= (\mu_{t-1} - \mu_{1,t-1}^i) + (x_{t-1} - x_{1,t-1}^i).
\end{aligned}$$

The separation error $z_{i,t-1}$ is given by:

$$z_{i,t-1} = \frac{(\text{Var}^T + \widetilde{\text{COV}})(x_{t-1} - x_{1,t-1}^i) - (\text{Var}^C + \widetilde{\text{COV}})(\mu_{t-1} - \mu_{1,t-1}^i)}{\text{Var}^T + \text{Var}^C + 2\widetilde{\text{COV}}}.$$

Therefore, the covariance is:

$$cov(FE_{i,t-1}, z_{i,t-1}) = \frac{[(\text{Var}^T + \widetilde{\text{COV}})(\widetilde{\text{COV}} + \text{Var}^C) - (\text{Var}^C + \widetilde{\text{COV}})(\text{Var}^T + \widetilde{\text{COV}})]}{\text{Var}^T + \text{Var}^C + 2\widetilde{\text{COV}}} = 0.$$

Therefore, when $m_1 = m_2 = 1$, the now-cast error in the period $t-1$ is independent with the separation error $z_{i,t-1}$. Consequently, the now-cast errors across periods $t-1$ and t would also be zero.

When forecasters are overconfident in the trend signal, i.e., $m_1 < 1, m_2 = 1$. The posterior beliefs regarding the two components after observing the new signals can be written as:

$$\mu^T_{1,t,o} = \frac{m_1\sigma^2_\epsilon(\rho^2\sigma^2_{z,o}+\sigma^2_x+\sigma^2_e)}{\Omega_1}\mu^i_{2,t-1,o} + \frac{V_1+\sigma^2_e(\sigma^2_{z,o}+\sigma^2_\mu)}{\Omega_1}s^\mu_{i,t} - \frac{\rho m_1\sigma^2_\epsilon\sigma^2_{z,o}}{\Omega_1}(s^x_{i,t}-\rho x^i_{2,t-1,o}), \tag{A13}$$

$$x^T_{1,t,o} = \frac{\sigma^2_e(\sigma^2_{z,o}+\sigma^2_\mu+m_1\sigma^2_\epsilon)}{\Omega_1}\rho x^i_{2,t-1,o} + \frac{V_1+m_1\sigma^2_\epsilon(\sigma^2_x+\rho^2\sigma^2_{z,o})}{\Omega_1}s^x_{i,t} - \frac{\rho\sigma^2_e\sigma^2_{z,o}}{\Omega_1}(s^\mu_{i,t}-\mu^i_{2,t-1,o}), \tag{A14}$$

where Ω_1 and V_1 are constants:

$$\Omega_1 = (\sigma^2_{z,o}+\sigma^2_\mu+m_1\sigma^2_\epsilon)(\sigma^2_x+\sigma^2_e+\rho^2\sigma^2_{z,o}) - \rho^2\sigma^4_{z,o},\ \ V_1 = (\sigma^2_{z,o}+\sigma^2_\mu)(\sigma^2_x+\rho^2\sigma^2_{z,o}) - \rho^2\sigma^4_{z,o}.$$

The term $\sigma^2_{z,o}$ is the perceived variance of the separation error in the steady state in this case. The variance-covariance matrix regarding the beliefs of the trend and cyclical components is:

$$\begin{pmatrix} \mathrm{Var}^T_1 & \widetilde{\mathrm{COV}}_1 \\ \widetilde{\mathrm{COV}}_1 & \mathrm{Var}^C_1 \end{pmatrix} = \begin{pmatrix} \frac{m_1\sigma^2_\epsilon[\Omega_1 - m_1\sigma^2_\epsilon(\sigma^2_x+\sigma^2_e+\rho^2\sigma^2_{z,o})]}{\Omega_1} & -\frac{\rho\sigma^2_e m_1\sigma^2_\epsilon\sigma^2_{z,o}}{\Omega_1} \\ -\frac{\rho\sigma^2_e m_1\sigma^2_\epsilon\sigma^2_{z,o}}{\Omega_1} & \frac{\sigma^2_e[\Omega_1-\sigma^2_e(m_1\sigma^2_\epsilon+\sigma^2_\mu+\sigma^2_{z,o})]}{\Omega_1} \end{pmatrix}. \tag{A15}$$

Importantly, the perceived variances of both the trend and cyclical components, as well as their covariance (in magnitude), are lower:

$$\mathrm{Var}^T_a = \mathrm{Var}^T_1 + (1-m_1)\sigma^2_\epsilon\left(\frac{V_1+\sigma^2_e(\sigma^2_{z,o}+\sigma^2_\mu)}{\Omega_1}\right)^2 + \left(\frac{m_1\sigma^2_\epsilon(\sigma^2_x+\sigma^2_e)}{\Omega_1}\right)^2(\sigma^2_{z,a}-\sigma^2_{z,o}), \tag{A16}$$

$$\mathrm{Var}^C_a = \mathrm{Var}^C_1 + (1-m_1)\sigma^2_\epsilon\left(\frac{\rho\sigma^2_e\sigma^2_{z,o}}{\Omega_1}\right)^2 + \left(\frac{\rho\sigma^2_e(\sigma^2_\mu+m_1\sigma^2_\epsilon)}{\Omega_1}\right)^2(\sigma^2_{z,a}-\sigma^2_{z,o}), \tag{A17}$$

$$\begin{aligned}\widetilde{\mathrm{COV}}_a = \widetilde{\mathrm{COV}}_1 &- (1-m_1)\sigma^2_\epsilon\frac{\rho\sigma^2_e\sigma^2_{z,o}}{\Omega_o}\frac{V_1+\sigma^2_e(\sigma^2_{z,o}+\sigma^2_\mu)}{\Omega_1} \\ &- \frac{m_1\sigma^2_\epsilon(\sigma^2_x+\sigma^2_e)}{\Omega_1}\frac{\rho\sigma^2_e(\sigma^2_\mu+m_1\sigma^2_\epsilon)}{\Omega_1}(\sigma^2_{z,a}-\sigma^2_{z,o}).\end{aligned} \tag{A18}$$

The subscript a stands for the actual variance and covariance term when forecasters are overconfident in the trend signal. $\sigma^2_{z,a}$ is the actual variance of the separation error:

$$\sigma^2_{z,a} - \sigma^2_{z,o} = G^T(1-m_1)\sigma^2_\epsilon\left(\frac{\sigma^2_e V_1}{\Omega_1}\right)^2 > 0,$$

where G^T is given by:

$$G^T = \frac{\Omega_1^2}{[\Omega_1(\text{Var}_1^T + \text{Var}_1^C + 2\widetilde{\text{COV}}_1)]^2 - [m_1\sigma_e^2\sigma_\epsilon^2(\rho\sigma_\mu^2 + \sigma_x^2)]^2} > 0.$$

At the end of period $t-1$, when the actual state value y_{t-1} is observed by all the forecasters, they will revise their beliefs using the perceived variance-covariance matrix. The separation error $z_{i,t-1}$ in this case can be written as:

$$z_{i,t-1,o} = \frac{(\text{Var}_1^T + \widetilde{\text{COV}}_1)(x_{t-1} - x_{1,t-1,o}^T) - (\text{Var}_1^C + \widetilde{\text{COV}}_1)(\mu_{t-1} - \mu_{1,t-1,o}^T)}{\text{Var}_1^T + \text{Var}_1^C + 2\widetilde{\text{COV}}_1}.$$

The covariance between $FE_{i,t-1}$ and $z_{i,t-1,o}$ is given by:

$$\begin{aligned}
&cov(FE_{i,t-1}, z_{i,t-1,o}) \\
&= (\text{Var}_1^T + \widetilde{\text{COV}}_1)(\text{Var}_a^C + \widetilde{\text{COV}}_a) - (\text{Var}_1^C + \widetilde{\text{COV}}_1)(\text{Var}_a^T + \widetilde{\text{COV}}_a) \\
&= (\text{Var}_1^T + \widetilde{\text{COV}}_1)(\text{Var}_a^C - \text{Var}_1^C + \widetilde{\text{COV}}_a - \widetilde{\text{COV}}_1) \\
&- (\text{Var}_1^C + \widetilde{\text{COV}}_1)(\text{Var}_a^T - \text{Var}_1^T + \widetilde{\text{COV}}_a - \widetilde{\text{COV}}_1).
\end{aligned} \tag{A19}$$

The second equality in Equation (A19) holds because we subtract the following term to the right-hand-side:

$$(\text{Var}_1^T + \widetilde{\text{COV}}_1)(\text{Var}_1^C + \widetilde{\text{COV}}_1) - (\text{Var}_1^C + \widetilde{\text{COV}}_1)(\text{Var}_1^T + \widetilde{\text{COV}}_1) = 0. \tag{A20}$$

This term is zero because it is the perceived covariance $FE_{i,t-1}$ and $z_{i,t-1}$. Using Equations (A16) to (A19), we have:

$$cov(FE_{i,t-1}, z_{i,t-1,o}) = -(1-m_1)\sigma_\epsilon^2\phi_{over}^T\frac{\sigma_e^2 V_1}{\Omega_1}, \tag{A21}$$

where

$$\begin{aligned}
\phi_{over}^T = &\frac{1}{\Omega_1^3}[(1-\rho)\sigma_e^2(\sigma_\mu^2 + \sigma_{z,o}^2 + m_1\sigma_\epsilon^2) + V_1 + m_1\sigma_\epsilon^2\sigma_x^2][m_1\sigma_e^4\sigma_\epsilon^2 V_o(\rho\sigma_\mu^2 + \sigma_x^2)]G^T \\
&+ \frac{V_1 + \sigma_e^2[\sigma_\mu^2 + (1-\rho)\sigma_{z,o}^2]}{\Omega_1}[1 - \frac{G^T}{\Omega_1^2}m_1\sigma_e^4\sigma_\epsilon^2 V_1(\rho\sigma_\mu^2 + \sigma_x^2)] > 0.
\end{aligned}$$

Therefore, when forecasters exhibit overconfidence in the trend signal, there is always a negative correlation between the now-cast error $FE_{i,t-1}$ and the separation error $z_{i,t-1,o}$. Consequently, the covariance between the now-cast errors across periods can be expressed as:

$$cov(FE_{i,t-1}, FE_{i,t}) = \frac{\rho\sigma_e^2(m_1\sigma_\epsilon^2 + \sigma_\mu^2) - m_1\sigma_\epsilon^2(\sigma_e^2 + \sigma_x^2)}{\Omega_1} \underbrace{cov(FE_{i,t-1}, z_{i,t-1})}_{(-)}.$$

The condition for a positive correlation between the now-cast errors across periods then in given by:

$$\frac{\rho\sigma_e^2(m_1\sigma_\epsilon^2 + \sigma_\mu^2) - m_1\sigma_\epsilon^2(\sigma_e^2 + \sigma_x^2)}{\Omega_1} < 0,$$

which is equivalent to

$$\frac{\rho\sigma_\mu^2\sigma_e^2}{\sigma_\epsilon^2[(1-\rho)\sigma_e^2 + \sigma_x^2]} < m_1 < 1.$$

Following the same logic, when forecasters are overconfident in the cyclical signal, the variance-covariance matrix of regarding the beliefs of the trend and cyclical components is:

$$\begin{pmatrix} \text{Var}_2^T & \widetilde{\text{COV}}_2 \\ \widetilde{\text{COV}}_2 & \text{Var}_2^C \end{pmatrix} = \begin{pmatrix} \frac{\sigma_\epsilon^2[\Omega_2 - \sigma_\epsilon^2(\sigma_x^2 + m_2\sigma_e^2 + \rho^2\sigma_{z,o}^2)]}{\Omega_2} & -\frac{\rho m_2\sigma_e^2\sigma_\epsilon^2\sigma_{z,o}^2}{\Omega_2} \\ -\frac{\rho m_2\sigma_e^2\sigma_\epsilon^2\sigma_{z,o}^2}{\Omega_2} & \frac{m_2\sigma_e^2[\Omega_2 - m_2\sigma_e^2(\sigma_\epsilon^2 + \sigma_\mu^2 + \sigma_{z,o}^2)]}{\Omega_2} \end{pmatrix}. \tag{A22}$$

The covariance between the now-cast error at $t-1$ and the separation error $z_{i,t-1,o}$ is given by:

$$cov(FE_{i,t-1}, z_{i,t-1,o}) = (1-m_2)\sigma_e^2\phi_{over}^C\frac{\sigma_\epsilon^2 V_2}{\Omega_2}, \tag{A23}$$

where Ω_2 and V_2 are counterparts of Ω_1 and V_1:

$$\Omega_2 = (\sigma_{z,o}^2 + \sigma_\mu^2 + \sigma_\epsilon^2)(\sigma_x^2 + m_2\sigma_e^2 + \rho^2\sigma_{z,o}^2) - \rho^2\sigma_{z,o}^4, \;\; V_2 = (\sigma_{z,o}^2 + \sigma_\mu^2)(\sigma_x^2 + \rho^2\sigma_{z,o}^2) - \rho^2\sigma_{z,o}^4.$$

and

$$\begin{aligned}\phi_{over}^C = &\frac{\sigma_\epsilon^2[\sigma_x^2 - \rho(1-\rho)\sigma_{z,o}^2] + V_2}{\Omega_2}[1 - \rho G^C\frac{m_2\sigma_\epsilon^4\sigma_e^2 V_2(\sigma_\mu^2 + \sigma_x^2)}{\Omega_2^2}] \\ &+ \frac{1}{\Omega_2^3}[V_2 + m_1\sigma_e^2(\sigma_\mu^2 + (1-\rho)\sigma_z^2)][m_2\sigma_\epsilon^4\sigma_e^2 V_2(\sigma_\mu^2 + \sigma_x^2)]G^C > 0.\end{aligned}$$

G^C is the counterpart of G^T:

$$G^C = \frac{\Omega_2^2}{[\Omega_2(\text{Var}_2^T + \text{Var}_2^C + 2\widetilde{\text{COV}}_2)]^2 - [m_2\sigma_e^2\sigma_\epsilon^2(\rho\sigma_\mu^2 + \sigma_x^2)]^2} > 0.$$

Equation (A23) shows that when forecasters are overconfident in the cyclical signal, the correlation between the now-cast error at $t-1$ and the separation error $z_{i,t-1,o}$

would be positive. The covariance between the now-cast errors across periods is given by:

$$cov(FE_{i,t-1}, FE_{i,t}) = \frac{\rho m_2 \sigma_e^2(\sigma_\epsilon^2 + \sigma_\mu^2) - \sigma_\epsilon^2(m_2\sigma_e^2 + \sigma_x^2)}{\Omega_2} \underbrace{cov(FE_{i,t-1}, z_{i,t-1})}_{(+)}.$$

Thus, the now-cast errors are positively correlated across periods if and only if:

$$1 < \frac{1}{m_2} < \frac{1}{\underline{m}_2} = \frac{\sigma_e^2[\rho\sigma_\mu^2 - (1-\rho)\sigma_\epsilon^2]}{\sigma_\epsilon^2\sigma_x^2}.$$

Proof of proposition 4. For those who correctly use the signals, their expectations regarding the two components are:

$$E_{i,t}^c[\mu_t] = \frac{\sigma_\mu^2 s_{i,t}^\mu + \sigma_\epsilon^2 \mu_{t-1}}{\sigma_\mu^2 + \sigma_\epsilon^2} \quad \text{and} \quad E_{i,t}^c[x_t] = \frac{\sigma_x^2 s_{i,t}^x + \sigma_e^2 \rho x_{t-1}}{\sigma_x^2 + \sigma_e^2}.$$

The average beliefs regarding the two components of the group who correctly interpret the signals are:

$$E^C[\mu_t] = \mu_t - \frac{\sigma_\epsilon^2}{\sigma_\mu^2 + \sigma_\epsilon^2}\gamma_t^\mu, \quad \text{and} \quad E^C[x_t] = x_t - \frac{\sigma_e^2}{\sigma_x^2 + \sigma_e^2}\gamma_t^x.$$

For those who wrongly interpret the signals, their expectations regarding the two components are:

$$E_{i,t}^W[\mu_t] = \frac{\sigma_\mu^2 s_{i,t}^x + \sigma_\epsilon^2 \mu_{t-1}}{\sigma_\mu^2 + \sigma_\epsilon^2} \quad \text{and} \quad E_{i,t}^W[x_t] = \frac{\sigma_x^2 s_{i,t}^\mu + \sigma_e^2 \rho x_{t-1}}{\sigma_x^2 + \sigma_e^2}.$$

The average beliefs regarding the two components of the group who wrongly interpret the signals are:

$$E^W[\mu_t] = \frac{\sigma_\mu^2 x_t + \sigma_\epsilon^2 \mu_{t-1}}{\sigma_\mu^2 + \sigma_\epsilon^2}, \quad \text{and} \quad E^W[x_t] = \frac{\sigma_x^2 \mu_t + \sigma_e^2 \rho x_{t-1}}{\sigma_x^2 + \sigma_e^2}.$$

The forecast variance across all forecasters then is given by:

$$\begin{aligned} Var(F_{i,t}y_{t+h}) &= Var(\mu_{1,t}^i) + \rho^{2h} Var(x_{1,t}^i) + \rho^h E[(\mu_{1,t}^i - E[\mu_t])(x_{1,t}^i - E[x_t])] \\ &= \rho^{2h}[\tau\phi_w^C \mathrm{Var}_w^C + (1-\tau)\phi_c^C \mathrm{Var}_c^C] + [\tau\phi_w^T \mathrm{Var}_w^T + (1-\tau)\phi_c^T \mathrm{Var}_c^T] \\ &+ \rho^h (1-\tau)\tau\widetilde{\mathrm{COV}}_m, \end{aligned}$$

where ϕ_w^C, ϕ_c^C, ϕ_w^T, and ϕ_c^T are positive scalars between 0 and 1:

$$\phi_w^C = \frac{\sigma_x^2\sigma_\epsilon^2}{\sigma_e^4 + \sigma_x^2\sigma_\epsilon^2}; \qquad \phi_c^C = \frac{\sigma_x^2}{\sigma_e^2 + \sigma_x^2};$$
$$\phi_w^T = \frac{\sigma_\mu^2\sigma_e^2}{\sigma_\epsilon^4 + \sigma_\mu^2\sigma_e^2}; \qquad \phi_c^T = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\epsilon^2}.$$

The first-order derivative of the $Var(F_{i,t}y_{t+h})$ regarding the forecast horizon h is:

$$\frac{\partial Var(F_{i,t}y_{t+h})}{\partial h} = 2\rho^{2h}\ln\rho\left[\tau\phi_w^C\mathrm{Var}_w^C + (1-\tau)\phi_c^C\mathrm{Var}_c^C\right] + (1-\tau)\tau\rho^h\ln\rho\widetilde{\mathrm{COV}}_m.$$

The forecast variance is increasing in h, if and only if:

$$h > \underline{h}_m = \frac{1}{\ln\rho}\ln\left(-\tau(1-\tau)\right)\frac{\widetilde{\mathrm{COV}}_m}{2[\tau\phi_w^C\mathrm{Var}_w^C + (1-\tau)\phi_c^C\mathrm{Var}_c^C]}.$$

C Supplemental material: Overconfident in cyclical signal

When forecasters are overconfident in the cyclical signal, they perceive the variance of the cyclical component to be smaller than it actually is. Consequently, the error term in the cyclical belief is assigned an excessive weight compared to the Bayesian scenario. As a result, it drives the correlation between the separating error ($z_{i,t-1}$) and the now-cast error for the previous period ($FE_{i,t-1}$) to become positive. Specifically, we show that the covariance is given by:

$$cov(z_{i,t-1}, FE_{i,t-1}) = (1 - m_2)\sigma_e \phi_O^C \frac{\sigma_\epsilon^2 V_2}{\Omega_2} > 0,$$

where ϕ_O^C is a positive scalar shown in the proof of Appendix 4.

Similar to the case in the main text, when forecasters are overconfident in the cyclical signal, the covariance between the separation error and the now-cast error of the current period can also be decomposed into two parts:

$$cov(z_{i,t-1}, FE_{i,t}) = \frac{\sigma_z^2}{\Omega_2}[\underbrace{-\sigma_\epsilon^2(\sigma_x^2 + m_2\sigma_e^2)}_{trend\ prior\ effect} + \underbrace{\rho m_2 \sigma_e^2(\sigma_\mu^2 + \sigma_\epsilon^2)}_{cyclical\ prior\ effect}]. \tag{C24}$$

When forecasters are overconfident in the cyclical signal, they tend to place greater reliance on the cyclical signal and less on the prior belief inherited from the last period. Therefore, as the extent of overconfidence in the cyclical signal increases (i.e., m_2 becomes smaller), the effect of the cyclical prior is more likely to be dominated, and the covariance between the separation error ($z_{i,t-1}$) and the current now-cast error ($FE_{i,t}$) is more likely to be negative. Consider a polar case where m_2 goes to zero, the correlation is strictly negative.

Part (ii) of Proposition 3 states that when both the confusion and overconfidence mechanisms are present, the now-cast errors across periods can be positively correlated if the extent of overconfidence in the cyclical signal is moderate. The inequality in Equation (18) characterizes the condition under which the effect of the cyclical prior dominates the effect of the trend prior. Consider the case when the trend component is very volatile (i.e., σ_μ^2 is large enough). Forecasters would place limited reliance on the trend prior and rely heavily on the signal regarding the trend component. Therefore, the effect of the trend prior is always dominated, resulting in a positive correlation between the separation error and the now-cast error of the current period. Consequently, the covariance between the now-cast errors across periods is positive for any $m_2 < 1$.