Declining Business Dynamism

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2 Model

This model is a simple extension of Hopenhayn's (1992) [citationinsertlabel] with essential inherited characteristics and critical market structure developments. In detail, this model takes on the settings of representative households, firms' linear production functions, and exit & enter dynamics with necessary modifications. Meanwhile, introducing monopolistic and duopolistic markets leads to endogenous markups of firms and diversifies the decisions of entrants, launching a trade-off between innovations and imitations.

2.1 Households

Representative households consume goods and supply labor to get utility:

$$U(C,L) = \sum_{t=0}^{\infty} \beta^t [logC_t - aL_t] \tag{1}$$

Here, C_t is the aggregate consumption of households, L_t is the labor supply, and β is the discount factor. The log form means a unit intertemporal elasticity of consumption. The disutility of labor supply is linear with marginal disutility, a.

Though supplying labor brings about disutility, the households rely on wage income to support consumption. Moreover, as the owners of firms, households gain profits in each period. The budget constraint is as follows:

$$P_t \cdot C_t = \Pi_t + \int_{j=0}^{N} \sum_{i}^{M} p_{ijt} c_{ijt} dj = \Pi_t + w_t L_t$$

Here, P_t is the aggregate price index. Combined with aggregate consumption, C_t, it generates total spending. Equivalently, it is the sum of spending on each local product, c_j. Equivalent to the total spending are the total profits Π_t, and the wage income generated by the labor supply, L_t, and the wage rate, w_t.

In equilibrium, the entry of firms draws the total profits to zero, leading to the following budget constraint:

$$1 \times C_t = \int_{j=0}^{N} \sum_{i}^{M} p_{ijt} c_{ijt} dj = w_t L_t \tag{2}$$

Here, I normalize the aggregate price index as 1 without loss of generality.

2.2 Firms

Firms locate in different local markets and produce distinctive goods. The good of each local market substitutes with each other by a constant elasticity of substitution θ and sums up as follows:

$$Y_t = \left[\int_{0}^{N} y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{3}$$

Here, N is the endogenous number of local markets. Two forces decide its magnitude, horizontal innovations and disappear of local markets. The CES aggregator reveals the relationship between the aggregated global good Y_t and products in the local market j in period t, y_{jt}.

Within each local market, products substitute with each other by the elasticity of substitution η. Thus, the product indicator in each local market is as follows:

$$y_{jt} = \left[\sum_{i=1}^{M_j} y_{ijt}^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}}$$

y_{ijt} is the production of firm i in local market j during the period t. η is the within-market ealsticity of substitution. Generally, goods in the same local market, either in a geographical sense or in a class sense, have higher substitution. So we have $\eta > \theta$ and focus on the case that η is ∞ at first for simplicity. In such a case, the aggregation of local goods follows a linear form:

$$y_{jt} = \sum_{i=1}^{M_j} y_{ijt} \tag{4}$$

Here, M_j ($M_j = 1$ or 2) is the number of firms in each local market. To decide the optimal quantity y_{ijt}, firm i with productivity z_{ijt} has a Cournot-quantity competition with firm j. It hires l_{ijt} labor and applies a linear production technology:

$$y_{ijt} \equiv f(z_{ijt}, l_{ijt}) = z_{ijt} l_{ijt} \tag{5}$$

The productivity of each firm tracks an idiosyncratic random path under the same random process, which is the only disturbance on the model, and its log value follows an AR(1) process:

$$\begin{aligned} logz_t = \rho logz_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2) \\ z_0 \sim H(\cdot) \end{aligned} \tag{6}$$

Here, $\rho \in (0, 1)$ represents the inertia of productivity. A higher ρ means a higher probability of keeping itself at a similar level. ϵ follows a white noise. Any entrant's productivity z_0 comes from a distribution with cumulative productivity density $H(\cdot)$.

In each period t, firm i sells goods at the price of p_{ijt} and pays workers at the wage of w_t, The remainings are the profits, a component of households' total income:

$$\pi_{ijt} = p_{ijt}(y_{ijt}, \mathbf{y}_{-ijt}, Y_t) y_{ijt} - w_t l_{ijt} \tag{7}$$

p_{ijt} is a function of firm's products, y_{ijt}, the competitors' products, $\mathbf{y}_{-ijt}$, and the aggregate products, Y_t.

2.3 Exit and Enter

At the beginning of each period, the economy witnesses δ proportion of local markets disappears. Equivalently, $N \cdot (1-\delta)$ kinds of products survive. This process abstract varies phenomenons in reality based on the illustration of local markets. If local markets represent industries, then the process above means the death of some industries or even the offshoring (if the spendings on such goods are relatively tiny). If local markets imply the destruction by creations or horizontal innovations, then the exit mimics the products' iterations. This exogenous exit is the only way to exit since this model has no running costs.

After the exit process, new firms appear, draw their initial productivity, and pay the sunk entry costs. This sunk entry cost with no running cost indicates that firms always choose to enter. These entrants pay the entry costs depending on the types they want to be, an innovator or an imitator. If the entrant chooses to innovate, it pays C_M amount of aggregate goods, installs a new industry, and becomes a monopolist. Alternatively, it can choose to imitate, pays a C_D amount of aggregate goods, enters a randomly picked monopolistic local market, and begins a Cournot competition with the incumbent. No entrants can enter a duopolistic local market because of the market capacity issues by assumption.

If $V_1(z, Y)$ and $V_2(z, z_-, Y)$ are values of firms in monopolistic and duopolistic markets with productivity z, aggregate products Y, and competitor's productivity z_-. Then entrants prefer innovation if and only if $C_M - C_D < \mathbb{E}V_1(z, Y) - \mathbb{E}\mathcal{V}_2(z, z_-, Y)$.

3 Solutions

The framework above provides the following solutions.

3.1 Households' Optimal Decisions

Households optimize their utilities under budget constraints. The first-order conditions with respect to consumption and labor supply are:

$$C_t = \frac{1}{a} w_t \quad L_t^S = \frac{1}{a} \tag{8}$$

Since the wage is the only resource of income to support consumption in equilibrium, a higher wage leads to higher consumption. Meanwhile, the labor supply chosen by households causes a balance between utility earned from consumption and disutility brought about by labor.

3.2 Firms' Production and Pricing Decisions

The demands for the local markets' products follow the results of the Dixit-Stiglitz framework:

$$1 \equiv P_t = \left(\int_0^1 p_{jt}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \tag{9}$$

$$y_{jt} = (p_{jt})^{-\theta} Y_t \tag{10}$$

Here, p_{jt} is the price setting in the j^{th} local market during period t. The price index P_t is the $L_{1-\theta}$-norm of all local markets' prices. Higher elasticity of substitution θ leads to a lower price index P_t, meaning a cheaper cost to increase a marginal utility unit. y_{jt} and Y_t are products of local market j and the aggregate products. Higher price setting and elasticity of substitution both decrease the demand through the competition of other goods.

Among each local market, firms choose the quantities of products to maximize their profits. Monopolists face the following profits:

$$\pi_{ijt} = p_{ijt} y_{ijt} - w_t l_{ijt} = \left(y_{ijt}^{-\frac{1}{\theta}} Y_t^{\frac{1}{\theta}} \right) \cdot y_{ijt} - w_t \cdot \frac{y_{ijt}}{z_{ijt}} \tag{11}$$

Equations 5 and 9 are necessary to make the second equality stand. By solving the first-order conditions, monopolists set the quantity as follows:

$$y_{ijt} = \left[\frac{\theta}{\theta - 1} \left(\frac{w_t}{z_{ijt}} \right) \right]^{-\theta} Y_t \tag{12}$$

Equivalently, the price is:

$$p_{jt} = \frac{\theta}{\theta - 1} \frac{w_t}{z_{ijt}} \tag{13}$$

The price has two components, the marginal cost $\frac{w_t}{z_{ijt}}$ and the markup $\frac{\theta}{\theta-1}$. Lower elasticity of substitution causes a higher markup and profit, a standard result of CES framework. Consequently, the monopoly profits are:

$$\pi_1(z_{ijt}, Y_t) = \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\right)^{1-\theta}\left(\frac{w_t}{z_{ijt}}\right)^{1-\theta} Y_t \tag{14}$$

Compared to monopolists, duopolists face a more complex situation since their strategies influent each other. Specifically, their profits and corresponding quantity decisions include their competitors'. Take the derivative with respect to quantity y_{ijt} on Equation 7, duopolists have the following first-order condition:

$$\frac{\partial \pi_{ijt}}{\partial y_{ijt}} = Y_t^{\frac{1}{\theta}}\left\{(y_{ijt} + y_{-ijt})^{-\frac{1}{\theta}} - \frac{1}{\theta}(y_{ijt} + y_{-ijt})^{-\frac{1}{\theta}-1} y_{ijt}\right\} - \frac{w_t}{z_{ijt}} = 0$$

Their competitors face a similar first-order condtion:

$$\frac{\partial \pi_{-ijt}}{\partial y_{-ijt}} = Y_t^{\frac{1}{\theta}}\left\{(y_{ijt} + y_{-ijt})^{-\frac{1}{\theta}} - \frac{1}{\theta}(y_{ijt} + y_{-ijt})^{-\frac{1}{\theta}-1} y_{-ijt}\right\} - \frac{w_t}{z_{-ijt}} = 0$$

After some calculation (details shown in the Appendix), these firms choose their optimal quantities:

$$y_{ijt} = \frac{(1-\theta)z_{-ijt} + \theta z_{ijt}}{z_{ijt} + z_{-ijt}} y_{jt}$$

Since perfect substitution between goods in the same local market leads to uniform pricing, $p_{ijt} = p_{jt} = p_{-ijt}$. The market share becomes the ratio of products:

$$s_{ijt} = \frac{y_{ijt}}{y_{ijt} + y_{-ijt}} = \frac{y_{ijt}}{y_{jt}}$$

Combined with the formula of the optimal quantites, the market share in form of both firms' productivities is as follows:

$$s_{ijt}(z_{ijt}, z_{-ijt}) = \frac{(1-\theta)z_{-ijt} + \theta z_{ijt}}{z_{ijt} + z_{-ijt}} \tag{15}$$

Given the optimal quantities, the price of the local market is as follows:

$$p_{ijt} = p_{-ijt} = p_{jt} = \frac{\theta}{2\theta - 1} w_t \left(\frac{1}{z_{ijt}} + \frac{1}{z_{-ijt}}\right) = \frac{\theta}{\theta - s_{ijt}} \frac{w_t}{z_{ijt}} \tag{16}$$

The above pricing and quantity choices only happen when both firms' productivities are close. When the competitor's productivity is higher than the threshold, $\frac{\theta}{\theta-1}z_{ijt}$, the firm should wisely be inactive, produce nothing, and earn zero profit. Oppositely, if the competitor's productivity is lower than the threshold given by $\frac{\theta-1}{\theta}z_{ijt}$, the firm will corner the market, behave like a monopolist, and earn a monopoly profit. To sum up, the optimal productions are as follows:

$$y_{ijt} = \begin{cases} y_{jt} & z_{-ijt} < \frac{\theta-1}{\theta}z_{ijt} \\ s_{ijt}y_{jt} = \frac{(1-\theta)z_{-ijt}+\theta z_{ijt}}{z_{ijt}+z_{-ijt}}y_{jt} & \frac{\theta-1}{\theta}z_{ijt} \leq z_{-ijt} \leq \frac{\theta}{\theta-1}z_{ijt} \\ 0 & z_{-ijt} > \frac{\theta}{\theta-1}z_{ijt} \end{cases} \tag{17}$$

Correspondly, the profits are:

$$\pi_2(z_{ijt}, z_{-ijt}, Y_t) = \begin{cases} \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\right)^{1-\theta}\left(\frac{w_t}{z_{ijt}}\right)^{1-\theta}Y_t = \pi_1(z_{ijt}, Y_t) & z_{-ijt} < \frac{\theta-1}{\theta}z_{ijt} \\ \frac{s_{ijt}^2}{\theta}\left(\frac{\theta}{\theta-s_{ijt}}\right)^{1-\theta}\left(\frac{w_t}{z_{ijt}}\right)^{1-\theta}Y_t & \frac{\theta-1}{\theta}z_{ijt} \leq z_{-ijt} \leq \frac{\theta}{\theta-1}z_{ijt} \\ 0 & z_{-ijt} > \frac{\theta}{\theta-1}z_{ijt} \end{cases} \tag{18}$$

4 Equilibrium

With the solutions above, equilibrium analysis begins with discussing the compositions of markets. Based on the compositions, the law of motion of productivity distributions and value functions have their final forms. In the end, a summary lists all conditions necessary to define an equilibrium.

4.1 Compositions of Markets

In equilibrium, N local markets and λ proportion of duopolistic markets exist. The values of N and λ keep constant, but the compositions of markets changes. These changes decide the coefficients in value functions and steady-state productivity distributions.

During the exit process, $(1-\lambda)\delta \cdot N$ monopolistic and $\lambda\delta \cdot N$ duopolistic markets disappear, leaving $(1-\lambda)(1-\delta) \cdot N$ monopolistic and $\lambda(1-\delta) \cdot N$ duopolistic markets surviving. The productivities of firms in these surviving markets experience the transition defined by the log-AR(1) process above before entrants appear. $\delta \cdot N$ entrants choose to innovate and form monopoly markets. Meanwhile, $\lambda\delta \cdot N$ entrants choose to imitate and compete with monopolists, forming new duopoly markets.

At the beginning of the next period, there are still N local markets with $\lambda \cdot N$ duopoly markets. Among those duopoly markets, $1-\delta$ proportion is those old ones while δ are new ones combined from monopolists and entrants. Among those monopoly markets, the $\frac{\delta}{1-\lambda}$ ratio is upstarts, and $1-\delta-\frac{\lambda\delta}{1-\lambda}$ is old nobles.

4.2 Steady-state Productivity Distributions

With the compositions of marekts, the productivity distribution of monopoly markets is as follows:

$$\begin{aligned} g_M(z') =& \frac{\delta}{1-\lambda} g(z') + \left[1-\delta-\frac{\lambda\delta}{1-\lambda}\right] \langle \Gamma(z'|z), g_M(z) \rangle \\ =& \frac{\delta}{1-\lambda} h(z') + \left[1-\delta-\frac{\lambda\delta}{1-\lambda}\right] \int_z \Gamma(z'|z) g_M(z) dz \end{aligned} \tag{19}$$

The productivity density (PDF) of monopoly markets $g_M(z')$ is a weighted average of two PDFs. The former is the PDF of entrants' productivity, $h(z')$. The late is the PDF of monopolists after the transition of the corresponding Markov process represented by the inner product between the transition matrix (Mrkov Kernel), $\Gamma(z'|z)$, and the PDF yesterday, $g_M(z)$.

Similarly, the joint PDF of duopoly markets' productivity is as follows:

$$\begin{aligned}
g_D(z') \cdot g_D(z'_-) =& (1-\delta)\langle\Gamma(z'|z), g_D(z)\rangle \cdot \langle\Gamma(z'_-|z_-), g_D(z_-)\rangle + \\
& \frac{\delta}{2}[\langle\Gamma(z'|z), g_M(z)\rangle \cdot h(z'_-)] + \\
& \frac{\delta}{2}[h(z') \cdot \langle\Gamma(z'_-|z_-), g_M(z_-)\rangle] \\
=& (1-\delta)\left(\int_z \Gamma(z'|z), g_D(z)dz\right) \cdot \left(\int_{z_-} \Gamma(z'_-|z_-) g_D(z_-)dz_-\right) + \\
& \frac{\delta}{2}\left[\left(\int_z \Gamma(z'|z), g_M(z)dz\right) \cdot h(z'_-)\right] + \\
& \frac{\delta}{2}\left[h(z') \cdot \left(\int_{z_-} \Gamma(z'_-|z_-), g_M(z_-)dz_-\right)\right]
\end{aligned} \tag{20}$$

Here, $g_D(z') \cdot g_D(z'_-$ represents the joint PDF with the first firm's productivity as z' and its competitor' as z'_-. This joint PDF has three compositions. First, both firms were duopolists in the last period and their productivity z', and z'_- come from the transition. Here, $g_D(\cdot)$ was the marginal PDF yesterday and $\langle\Gamma(\cdot'|\cdot), g_D(\cdot)\rangle$ is the one today. Second, the first firm is a monopolist and the second firm is an entrant. So the first firm's productivity follows the productivity density in monopoly markets after the transition $\langle\Gamma(z'|z), g_M(z)\rangle$, and the second firm's comes from the productivity density of entrants, $h(z')$. The third case is the opposite of the second case.

4.3 Value Functions

In equilibrium, firms evaluate their values as follows.

Facing the exit, a monopolist exits with probability δ and has no value. Meanwhile, it survives with probability $1-\delta$ and experiences the transition of productivity. The probability that this monopolist has productivity z' is $(1-\delta)\Gamma(z'|z)$. After the entry process, this monopolist stays monopoly with probability $\left[(1-\delta) - \frac{\lambda\delta}{1-\lambda}\right]$, having the corresponding value $V_1(z', Y')$ with probability $\left[(1-\delta) - \frac{\lambda\delta}{1-\lambda}\right]\Gamma(z'|z)$. Alternatively, it becomes a duopolist with probability $\left[\frac{\lambda\delta}{1-\lambda}\right]$, combined with an entrant with productivity z' with probability $h(z')$(i.e., the PDF of CDF $H(z')$), and has the corresponding value $\mathcal{V}_2(z', z'_-, Y')$ with probability $\left[\frac{\lambda\delta}{1-\lambda}\right]\Gamma(z'|z)h(z'_-)$. To sum up, the motion of monopolists' values is as follows:

$$V_1(z) = \pi_1(z,Y) + \beta\left\{\delta\cdot 0 + \left[(1-\delta) - \frac{\lambda\delta}{1-\lambda}\right]\int_{z'=0}^{\infty}\Gamma(z'|z)V_1(z')dz' + \left[\frac{\lambda\delta}{1-\lambda}\right]\int_{z'=0}^{\infty}\Gamma(z'|z)\mathcal{F}\left(\int_{z'_-=0}^{\infty}h(z'_-)\mathcal{V}_2(z',z'_-)dz'_-\right)\right\}$$

Here, $\mathcal{F}(h,V)$ is a mapping gives out the weighted average of value function V based on the probability density h. In the case of duopoly markets, a transition matrix Γ functions similarly as h here.

Since duopolists only experience exit and transition, their value functions follow a more straightforward form:

$$\mathcal{V}_2(z,z_-) = \pi_2(z',z'_-) + \beta\left\{\delta\cdot 0 + (1-\delta)\int_{z'=0}^{\infty}\Gamma(z'|z)\mathcal{F}\left(\int_{z'_-=0}^{\infty}\Gamma(z'_-|z_-)\mathcal{V}_2(z',z'_-)dz'_-\right)\right\}$$

However, the piecewise profit function complicates the form. Let us set $V_{2M}(z,z_-,Y)$, $V_{2D}(z,z_-,Y)$, and $V_{20}(z,z_-,Y)$ as the corresponding value functions of a firm with productivity z in cornering the market, duopoly competition, and inactivity when facing aggregate products Y and a competitor with productivity z_-. Their specific forms with some notation abuse are as follows:

$$V_{2M}(z,z_-) = \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\right)^{1-\theta}\left(\frac{w}{z}\right)^{1-\theta}Y + \beta(1-\delta)\int_{z'=0}^{\infty}\Gamma(z'|z)\left[\int_{z'_-=0}^{\underline{z'}}\Gamma(z'_-|z_-)V_{2M}(z',z'_-)dz'_- + \int_{z'_-=\underline{z'}}^{\overline{z'}}\Gamma(z'_-|z_-)V_{2D}(z',z'_-)dz'_- + \int_{z'_-=\overline{z'}}^{\infty}\Gamma(z'_-|z_-)V_{20}(z',z'_-)dz'_-\right] \quad (21)$$

$$V_{2D}(z,z_-) = \frac{s^2}{\theta}\left(\frac{\theta}{\theta-s}\right)^{1-\theta}\left(\frac{w}{z}\right)^{1-\theta} Y +$$

$$\beta(1-\delta)\int_{z'=0}^{\infty}\Gamma(z'|z)\left[\int_{z'_-=0}^{\underline{z'}}\Gamma(z'_-|z_-)V_{2M}(z',z'_-)dz'_- + \int_{z'_-=\underline{z'}}^{\overline{z'}}\Gamma(z'_-|z_-)V_{2D}(z',z'_-)dz'_- + \int_{z'_-=\overline{z'}}^{\infty}\Gamma(z'_-|z_-)V_{20}(z',z'_-)dz'_-\right] \quad (22)$$

$$V_{20}(z,z_-) = \beta(1-\delta)\int_{z'=0}^{\infty}\Gamma(z'|z)\left[\int_{z'_-=0}^{\underline{z'}}\Gamma(z'_-|z_-)V_{2M}(z',z'_-)dz'_- + \int_{z'_-=\underline{z'}}^{\overline{z'}}\Gamma(z'_-|z_-)V_{2D}(z',z'_-)dz'_- + \int_{z'_-=\overline{z'}}^{\infty}\Gamma(z'_-|z_-)V_{20}(z',z'_-)dz'_-\right] \quad (23)$$

Here, $\overline{z'}$ and $\underline{z'}$ are abbrevates of $\frac{\theta}{\theta-1}z'$ and $\frac{\theta-1}{\theta}z'$

Similarly, the final form of monopolists' value functions is as follows:

$$V_1(z) = \pi_1(z,Y) + \beta\left\{\left[(1-\delta) - \frac{\lambda\delta}{1-\lambda}\right]\int_{z'=0}^{\infty}\Gamma(z'|z)V_1(z')dz' + \left[\frac{\lambda\delta}{1-\lambda}\right]\int_{z'=0}^{\infty}\Gamma(z'|z)\left[\int_{z'_-=0}^{\underline{z'}}h(z'_-)V_{2M}(z',z'_-)dz'_- + \int_{z'_-=\underline{z'}}^{\overline{z'}}h(z'_-)V_{2D}(z',z'_-)dz'_- + \int_{z'_-=\overline{z'}}^{\infty}h(z'_-)V_{20}(z',z'_-)dz'_-\right]\right\} \quad (24)$$

4.4 Summary of Equilibrium Conditions

In equilibrium, the following conditions meet.

Households maximize their utility (Equation 1) by optimally allocating their consumption and providing fair labor under budget constraints (Equation 2). Firms choose their labor demand and the supply of goods to maximize their profits (Equations 12, 13, 16, and 17).

The aggregate labor demand equals the aggregate labor supply, clearing the labor market, and the supply equals the demand for the goods, clearing the goods market.

$$L_t^S = \int_{j=0}^{N} \sum_{i=1}^{M_j} \frac{y_{ijt}}{z_{ijt}} dj, \quad C_t = Y_t \quad \forall t \tag{25}$$

Meanwhile, the productivity distributions in monopoly and duopoly markets are stationary.

$$G_M(z') = G_M(z), \quad G_D(z') = G_D(z) \tag{26}$$

The Number of the local markets N and the proportion of the duopoly markets λ are steady.

At last, the zero-profit lead (expected) values to be equivalent to entry costs, in both monopoly and duopoly markets.

$$\begin{aligned} C_M = \mathbb{E}V_1(z) = \int_z V_1(z)h(z)dz \\ C_D = \mathbb{E}\left\{C_D|z\right\} =& \mathbb{E}_z\mathbb{E}_{z'}\mathcal{V}_2(z, z_-, Y) \\ =& \int_z \Big\{ \int_0^{\underline{z_-}} g_D(z_-)V_{20}(z, z_-, Y)dz_- + \\ & \int_{\underline{z_-}}^{\overline{z_-}} g_D(z_-)V_{2D}(z, z_-, Y)dz_- + \\ & \int_{\overline{z_-}}^{\infty} g_D(z_-)V_{2M}(z, z_-, Y)dz_- \Big\} h(z)dz \end{aligned} \tag{27}$$

Since the value of imitation is a weighted average of V_{2M}, V_{2D}, and V_{20}, entrants need to consider the productivity distribution of the duopoly markets.

5 Numerical Solution

The steady-state PDF and CDF of productivity distribution of the monopoly markets given the values of λ are as follows:

The steady-state PDF and CDF of marginal productivity distribution of the duopoly markets given the values of λ are as follows:

Algorithm 1 Numerical Solution of this Model

Input: parameters $\delta, a, H(z), \Gamma(z', z)$,

Output: Value Functions $V_1, \mathcal{V}_2$, steady-state distribution $G_M(z), G_D(z)$, Number of local markets N, proportion of duopoly markets λ, the equilibrium wage level w.

1: Initialize the parameters

2: Guess the value of wage w and the proportion of duopoly markets λ

3: Given the guess of w and λ, by iterating the value functions (Equations 24, 21, 22, and 23), get $V_1|\lambda, w$, $\mathcal{V}_2|\lambda, w$

4: Given the value of λ, by solving the discretization form of steady-state productivity distributions equation (Equation 26), get the steady-state productivity distributions $G_M(z)$ and $G_D(z)$

5: Adjust the guess of w and λ until entry conditions (Equation 27) meet.

6: Use the labor market clearing condition (Equation 25) to pin down the value N

Figure 1: PDF and CDF of Productivity Distributions in Monopoly Markets

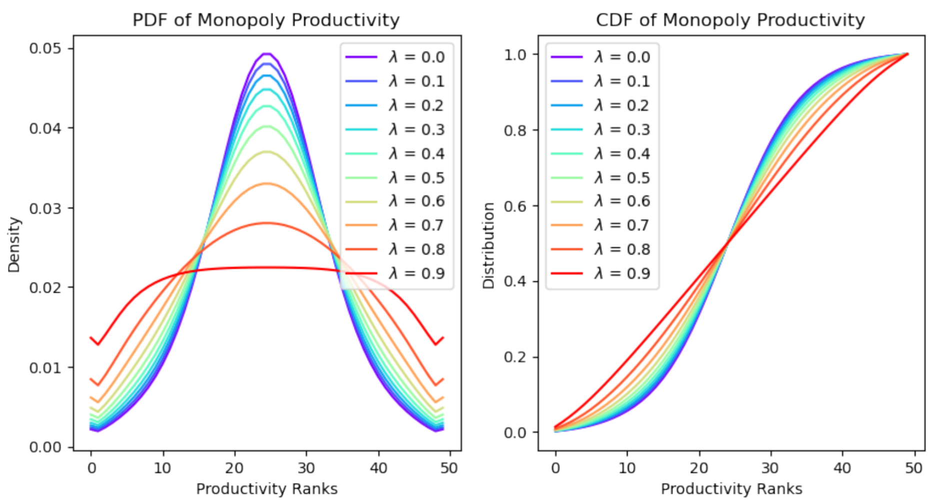
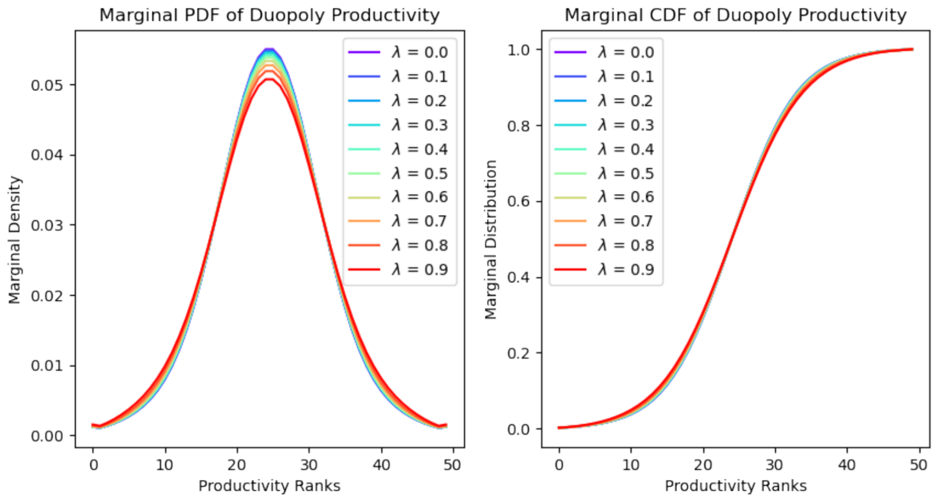


Figure 2: PDF and CDF of Marginal Productivity Distributions in Duopoly Markets



And the steady-state joint CDF of productivity distribution of the duopoly markets given the values of $\lambda = 0.2$ is as follows:

The value function of monopolists given $w = 0.7$ and $\lambda = 0.5$ is as follows:

The value functions of duopolists given $w = 0.7$ and $\lambda = 0.5$ is as follows:

Figure 3: Joint CDF of Productivity in Duopoly Markets

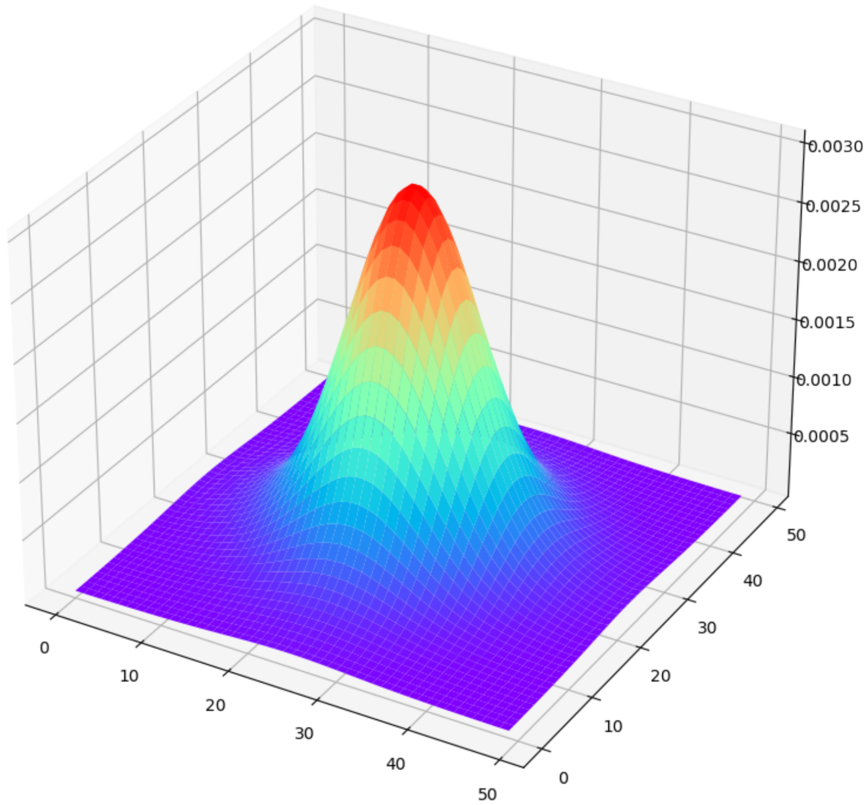


Figure 4: Value Functions of Monopolists

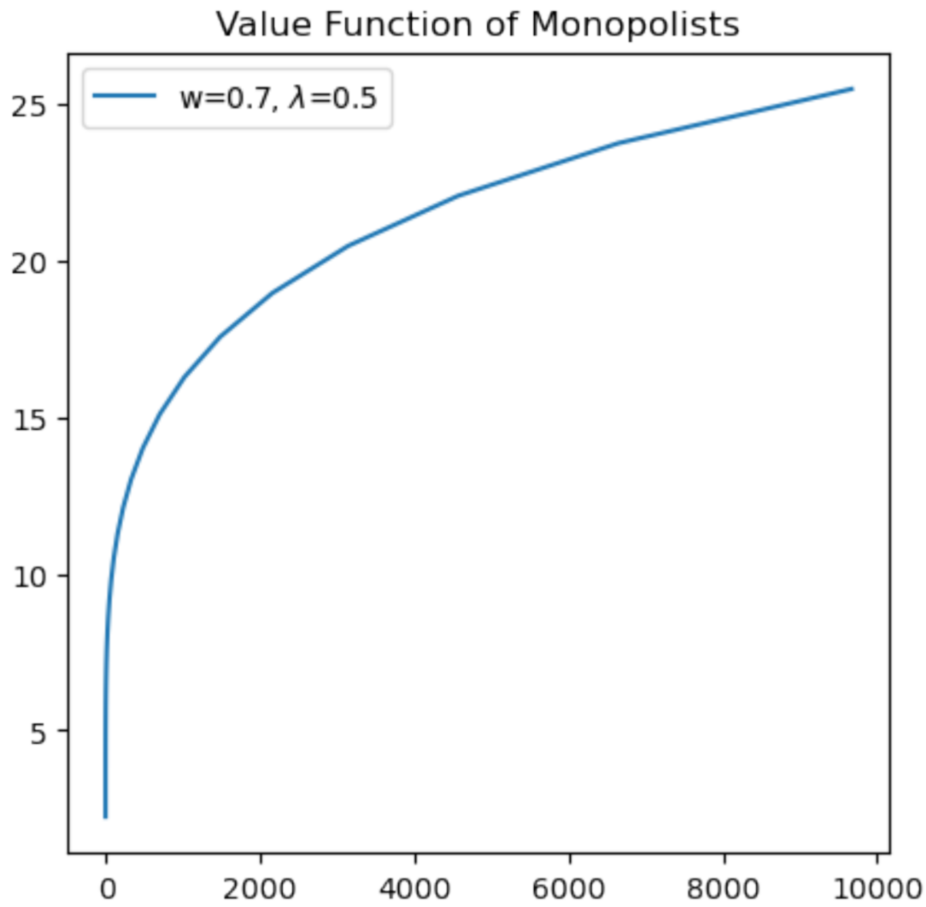
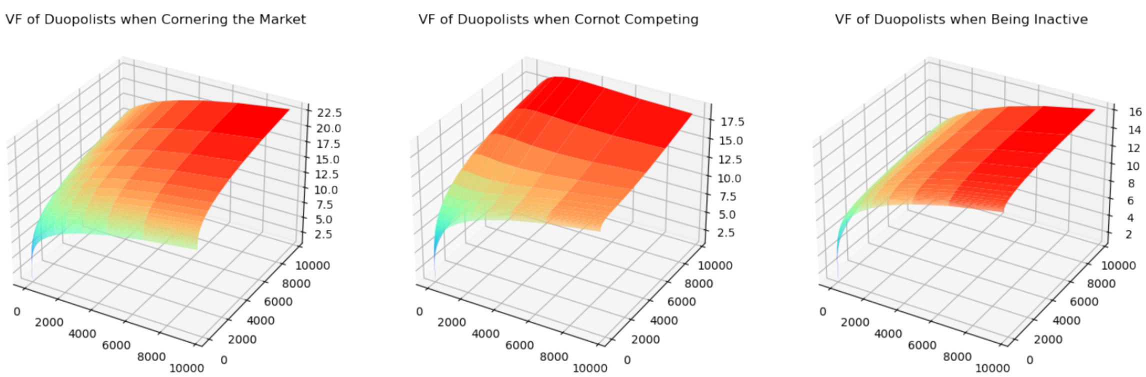


Figure 5: Value Functions of Duopolists



6 Appendix

6.1 Appendix A: Derivatives

6.1.1 Results of Nested-CES Framework

The demand structure of this paper follows the Nested-CES framework:

$$Y = \left(\int_0^1 y_j^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \tag{A.1}$$

$$y_j = \left(\sum_{i=1}^{n} y_{ij}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \tag{A.2}$$

The households maximize their consumption concerning their budget constraints. The Lagrange equation is as follows:

$$\mathcal{L} = Y + \delta(income - \int_0^1 p_j y_j dj) \tag{A.3}$$

Here, the δ is the Lagrange multiplier and equals the inverse of the price index because of its economic meaning. The first-order condition of this equation is as follows:

$$y_j = (\delta p_j)^{-\theta} Y = \left(\frac{p_j}{P}\right)^{-\theta} Y \tag{A.4}$$

Pluging the result into A.1, I get the formula of price index:

$$P = \left(\int_0^1 p_j^{1-\theta} dj \right)^{\frac{1}{1-\theta}} \tag{A.5}$$

Similar process provides the relations between y_{ij}, y_j, p_j and p_{ij}:

$$y_{ij} = \left(\frac{p_{ij}}{p_j}\right)^{-\eta} y_j \tag{A.6}$$

$$p_j = \left(\sum_{i=1}^{n} p_{ij}^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{A.7}$$

The following result is the combination of derivatives above:

$$y_{ij} = \left(\frac{p_{ij}}{p_j}\right)^{-\eta} \left(\frac{p_j}{P}\right)^{-\theta} Y \tag{A.8}$$

6.1.2 Results of Cournot Equilibrium When both Firms Run

As mentioned, the first-order conditions are:

$$\frac{\partial \pi_{ij}}{\partial y_{ij}} = Y^{\frac{1}{\theta}} \left\{ (y_{ij} + y_{-ij})^{-\frac{1}{\theta}} - \frac{1}{\theta}(y_{ij} + y_{-ij})^{-\frac{1}{\theta}-1} y_{ij} \right\} - \frac{w}{z_{ij}} = 0$$
$$\frac{\partial \pi_{-ij}}{\partial y_{-ij}} = Y^{\frac{1}{\theta}} \left\{ (y_{ij} + y_{-ij})^{-\frac{1}{\theta}} - \frac{1}{\theta}(y_{ij} + y_{-ij})^{-\frac{1}{\theta}-1} y_{-ij} \right\} - \frac{w}{z_{-ij}} = 0$$

First, I use the wage rate w and productivity z and z_- to represent the optimal price. By adding up the two first-order conditions, pluging in the equation $y_j = y_{ij} + y_{-ij}$, and considering the relationship between p_j and y_j. I have:

$$\begin{aligned} 2y_j^{-\frac{1}{\theta}} - \frac{1}{\theta} y_j^{-\frac{1}{\theta}} &= \left(\frac{w}{z_{ij}} + \frac{w}{z_{-ij}} \right) Y^{-\frac{1}{\theta}} \\ \frac{2\theta - 1}{\theta} &= \left(\frac{w}{z_{ij}} + \frac{w}{z_{-ij}} \right) y_j^{\frac{1}{\theta}} Y^{-\frac{1}{\theta}} \\ \frac{2\theta - 1}{\theta} &= \left(\frac{w}{z_{ij}} + \frac{w}{z_{-ij}} \right) \frac{1}{p_j} \\ p_j &= \frac{\theta}{2\theta - 1} w \left(\frac{1}{z_{ij}} + \frac{1}{z_{-ij}} \right) \\ p_j &= \frac{\theta}{2\theta - 1} \frac{w}{z_{ij}} \left(\frac{z_{ij} + z_{-ij}}{z_{-ij}} \right) \end{aligned} \tag{A.9}$$

Second, I put the optimal price back to the first F.O.C equation and get the value of the market share $s_{ij} = \frac{y_{ij}}{y_j}$:

$$\begin{aligned} \frac{1}{\theta} y_j^{-\frac{1}{\theta}-1} y_{ij} &= y_j^{-\frac{1}{\theta}} - \frac{w}{z_{ij}} Y^{-\frac{1}{\theta}} \\ \frac{y_{ij}}{y_j} &= \theta - \theta \frac{w}{z_{ij}} \frac{1}{p_j} \\ s_{ij} &= \theta - (2\theta - 1) \frac{z_{-ij}}{z_{ij} + z_{-ij}} \\ s_{ij} &= \frac{\theta z_{ij} + (1 - \theta) z_{-ij}}{z_{ij} + z_{-ij}} \end{aligned} \tag{A.10}$$

Last, I link the optimal price and the market share. I set $\kappa \equiv \frac{s_{ij}}{s_{-ij}}$. Then the optimal price becomes:

$$p_j = \frac{\theta}{2\theta - 1} \frac{w}{z_{ij}} (\kappa - 1) \tag{A.11}$$

Meanwhile, the market share becomes:

$$\begin{aligned} s_{ij} &= \frac{\theta\kappa + (1-\theta)}{1+\kappa} \\ \kappa &= \frac{\theta + s_{ij} - 1}{\theta - s_{ij}} \end{aligned} \tag{A.12}$$

Combining these two equations above, I get:

$$p_j = \frac{w}{z_{ij}} \frac{\theta}{\theta - s_{ij}} \tag{A.13}$$

6.1.3 Threshold Productivity

The threshold productivity ratio which corresponds to a zero market share is:

$$\kappa = \frac{\theta + 0 - 1}{\theta - 0} = \frac{\theta - 1}{\theta} \tag{A.14}$$

6.2 Appendix B: Numerical Solutions

6.2.1 Steady-State Productivity Distributions

The technology to solve the steady-state PDFs in this paper is first to transfer the Equations 19 and 20 into matrix forms and then to solve the discretized equations.

I list the transition of Equation 19 below:

$$\begin{aligned}
\mathbf{g_M} &= \frac{\delta}{1-\lambda}\mathbf{h} + \left[1-\delta-\frac{\lambda\delta}{1-\lambda}\right]\mathbf{\Gamma}^T \cdot \mathbf{g_M} \\
\left\{\mathbf{I} - \left[1-\delta-\frac{\lambda\delta}{1-\lambda}\right]\mathbf{\Gamma}^T\right\}\mathbf{g_M} &= \frac{\delta}{1-\lambda}\mathbf{h} \\
\mathbf{g_M} &= \left\{\mathbf{I} - \left[1-\delta-\frac{\lambda\delta}{1-\lambda}\right]\mathbf{\Gamma}^T\right\}^{-1}\frac{\delta}{1-\lambda}\mathbf{h} \qquad \text{(A.15)}
\end{aligned}$$

The bold fonts in this equations are K-dimensional vectors or K^2-dimemsional matrices corresponding to those variables in Equation 19. Since I apply column vectors as default in this paper, $\mathbf{\Gamma}^T \cdot \mathbf{g_M}$ represents the evolution of PDF.

Similarly, the transition of Equation 20 is:

$$\begin{aligned}
\mathbf{g_D} \cdot \mathbf{g_D}^T =& (1-\delta)\left(\mathbf{\Gamma}^T \cdot \mathbf{g_D}\right) \cdot \left(\mathbf{\Gamma}^T \cdot \mathbf{g_D}\right)^T + \\
& \frac{\delta}{2}[(\mathbf{\Gamma}^T \cdot \mathbf{g_M}) \cdot \mathbf{h}^T] + \frac{\delta}{2}[\mathbf{h} \cdot (\mathbf{\Gamma}^T \cdot \mathbf{g_M})^T] \\
\mathbf{g_D} \cdot \mathbf{g_D}^T =& (1-\delta)\mathbf{\Gamma}^T \cdot \mathbf{g_D} \cdot \mathbf{g_D}^T \cdot \mathbf{\Gamma} + \frac{\delta}{2}[\mathbf{\Gamma}^T \cdot \mathbf{g_M} \cdot \mathbf{h}^T] + \frac{\delta}{2}[\mathbf{h} \cdot \mathbf{g_M}^T \cdot \mathbf{\Gamma}] \\
\mathbf{g_J} \equiv \mathbf{g_D} \cdot \mathbf{g_D}^T =& (1-\delta)\mathbf{\Gamma}^T \cdot \mathbf{g_J} \cdot \mathbf{\Gamma} + \frac{\delta}{2}[\mathbf{\Gamma}^T \cdot \mathbf{g_M} \cdot \mathbf{h}^T] + \frac{\delta}{2}[\mathbf{h} \cdot \mathbf{g_M}^T \cdot \mathbf{\Gamma}] \\
vec(\mathbf{g_J}) =& (1-\delta)(\mathbf{\Gamma}^T \otimes \mathbf{\Gamma}^T)vec(\mathbf{g_J}) + \frac{\delta}{2}[(\mathbf{h} \otimes \mathbf{\Gamma}^T) + (\mathbf{\Gamma}^T \otimes \mathbf{h})]vec(\mathbf{g_M}) \\
vec(\mathbf{g_J}) =& \frac{\delta}{2}\left\{\mathbf{I} - (1-\delta)(\mathbf{\Gamma}^T \otimes \mathbf{\Gamma}^T)\right\}^{-1}[(\mathbf{h} \otimes \mathbf{\Gamma}^T) + (\mathbf{\Gamma}^T \otimes \mathbf{h})]vec(\mathbf{g_M})
\end{aligned}$$

$$\text{(A.16)}$$

Here, $\mathbf{g_D} \cdot \mathbf{g_D}^T$ is the outer product. $vec()$ is a vectorization operator and $\otimes$ means Kroneker product.