MATH 115 Fall 2018: Assignment 7 **Due at 8:30am on Friday, November 30, 2018**

It is important that you read the assignment submission instructions and suggestions available on LEARN.

1. (**7 marks**) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.
   (a) Find the characteristic polynomial of A. Fully factor your answer.
   (b) Find the eigenvalues of A.
   (c) For each eigenvalue of A, find a basis for the corresponding eigenspace.
   (d) Is A diagonalizable? Explain why or why not.

2. (**7 marks**) Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 3 \end{bmatrix}$.
   (a) Find the characteristic polynomial of A. Fully factor your answer.
   (b) Find the eigenvalues of A.
   (c) For each eigenvalue of A, find a basis for the corresponding eigenspace.
   (d) Is A diagonalizable? Explain why or why not.

3. (**8 marks**) Let $A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$.
   (a) Diagonalize A, that is, find an invertible matrix P and a diagonal matrix D so that $A = PDP^{-1}$.
   (b) Find A^k, where k is a positive integer.

4. (**6 marks**) Let A be an $n \times n$ matrix with characteristic polynomial $c_A(\lambda)$.
   (a) Prove that $c_{A^T}(\lambda) = c_A(\lambda)$.
   (b) Prove that if $r \in \mathbb{R}$ and $r \neq 0$, then $c_{rA}(\lambda) = r^n\, c_A(\frac{\lambda}{r})$.

5. (**7 marks**) For an integer $k \geq 0$, let h_k denote the number of hawks in a given region in year k, and let m_k denote the number of mice in that same region in year k. Assume the populations are related as follows:

$$\begin{aligned} h_{k+1} &= \tfrac{1}{2}h_k + \tfrac{1}{100}m_k \\ m_{k+1} &= -\tfrac{50}{4}h_k + \tfrac{5}{4}m_k \end{aligned}$$

   If there are initially 50 hawks and 1600 mice ($h_0 = 50$ and $m_0 = 1600$), determine the long term populations of the hawks and mice.

6. (**6 marks**) Let

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}$$

   (a) Determine the volume of the parallelepiped determined by $\vec{v}_1, \vec{v}_2, \vec{v}_3$.
   (b) Show that $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ a basis for $\mathbb{R}^3$.
   (c) Apply the Gram-Schmidt Algorithm to $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ to obtain an *orthonormal* basis $\mathcal{C}$ of $\mathbb{R}^3$.
   (d) If $\vec{v} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$, find $[\vec{v}]_{\mathcal{C}}$.

Total: 41 marks.