It is important that you read the assignment submission instructions and suggestions available on LEARN.

1. (**6 marks**)

   (a) Give conditions on $s, t \in \mathbb{R}$ such that the matrix $A = \begin{bmatrix} s & t \\ st & 1 \end{bmatrix}$ is symmetric.

   (b) A square matrix C is called *skew-symmetric* if $C^T = -C$. Give an example of a *non-zero* 2×2 skew-symmetric matrix.

   (c) Show that if B is *any* square matrix, then the matrix $B^T - B$ is skew-symmetric.

2. (**4 marks**) Solve for the matrix A if

$$4A - 9\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} = \left(2A^T - 5\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}\right)^T.$$

3. (**7 marks**) Let A be an $m \times n$ matrix, $\vec{x} \in \mathbb{R}^n$ and $\vec{b} \in \mathbb{R}^m$ with $\vec{b} \neq \vec{0}$. The equation $A\vec{x} = \vec{b}$ represents a non-homogeneous system of m equations in n variables. The system $A\vec{x} = \vec{0}$ is the corresponding homogeneous system (the resulting system of equations we get by replacing $\vec{b}$ with $\vec{0} \in \mathbb{R}^m$). Let $\vec{y} \in \mathbb{R}^n$ satisfy the non-homogeneous system and $\vec{z} \in \mathbb{R}^n$ satisfy the corresponding homogeneous system.

   (a) Show that the vector $\vec{y} + t\vec{z}$ satisfies the non-homogeneous system for any scalar t.

   (b) Find all scalars s so that the vector $s\vec{y} + \vec{z}$ satisfies the non-homogeneous system.

   (c) Find all scalars s so that the vector $s\vec{y} + \vec{z}$ satisfies the corresponding homogenous system.

4. (**7 marks**) A University of Waterloo Engineering student exists in three states: eating, sleeping and studying. A student remains in a given state for one hour.

   - If a student is eating during one hour, then they will not eat during the next hour, but are equally likely to sleep or study during the next hour.
   - If a student is sleeping during one hour, then during the next hour they will eat with probability 1/4, they will sleep with probability 1/2 and they will study with probability 1/4.
   - If a student is studying during one hour, then they will always sleep during the next hour

   (a) Write down a stochastic matrix P that describes this Markov Chain (assume the states are eating, sleeping, studying in that order).

   (b) If the student is initially sleeping, what is the probability that they are studying 2 hours later?

   (c) Show that the matrix P computed in part (a) is regular.

   (d) Find the steady state vector of this Markov Chain.

5. (**4 marks**)

(a) Compute A^T and A^H given that

$$A = \begin{bmatrix} 1+j & 2-j & 3-2j \\ 6j & 45 & 1-4j \\ 1-j & 3-5j & 2+j \end{bmatrix}$$

(b) Let B be an $n \times n$ matrix with complex entries. Prove that $B^T = B^H$ if and only if the entries of B are real.

6. (**5 marks**) Let $A = \begin{bmatrix} 2 & 5 & 1 \\ 1 & 2 & 0 \\ 4 & 5 & -2 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 10 \\ 4 \\ 9 \end{bmatrix}$ and consider the equation $A\vec{x} = \vec{b}$.

(a) Write out the system of equations represented by the equation $A\vec{x} = \vec{b}$.

(b) Use the inversion algorithm to find A^{-1}. Verify that your answer is correct by showing that $A^{-1}A = I$.

(c) Use A^{-1} to find the solution to the system $A\vec{x} = \vec{b}$

(d) Using your answer from part (c), express $\vec{b}$ as a linear combination of the columns of A.

Total: 33 marks.