Métodos computacionales

Alejandra Preciado* and Luisa Sanchez**
Universidad de los Andes, Bogotá, Colombia.
(Dated: 31 de agosto de 2022)

I. 5 TEÓRICO

A. 5a

Show that the $D^4 f$ operator is given by:

Sabemos que:

$$Df = \frac{f(x_i + 1) - f(x_j)}{h}$$

$$D^2 f = D[D[f]] = \frac{f(x_i + 1) - f(x_j)}{h}$$

$$\frac{\frac{f(x_j+2)-f(x_j)}{h} - \frac{f(x_j)-f(x_j-1)}{h}}{h}$$

$$\frac{f(x_j + 2) - 2f(x_j) + f(x_j - 1)}{h^2}$$

$$D^3 f = D[D^2 f]$$

$$= \frac{\frac{f(x_j+2)-2f(x_j+1)+2f(x_j-1)-f(x_j-2)}{2h^2}}{h^2}$$

$$= \frac{f(x_j + 2) - 2f(x_j + 1) + 2f(x_j - 1) - f(x_j - 2)}{2h^3}$$

$$D^4 f = D[D^3 f]$$

$$= \frac{f'(x_j + 2) - 2f'(x_j + 1) + 2f'(x_j - 1) - f'(x_j - 2)}{2h^3}$$

$$= \frac{f'(x_j + 3) - 3f'(x_j + 2) + 3f'(x_j + 1) - f'(x_j)}{h^3}$$

$$= \frac{f'(x_j + 4) - 4f'(x_j + 3) + 6f'(x_j + 2) - 4f'(x_j + 1) + f(x_j)}{h^4}$$

Corremos el intervalo dos posiciones:

$$= \frac{f'(x_j + 2) = 4f'(x_j + 1) + 6f'(x_j) - 4f'(x_j - 1) + f(x_j - 2)}{h^4}$$

* Correo institucional: d.preciador@uniandes.edu.co
** Correo institucional: lm.sanchezr1@uniandes.edu.co

B. 5b

For this operator, what is the order ($O(h^k)$) of the approximation?

Usamos el proceso matricial:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & -1 & -2 \\ 4 & 1 & 0 & 1 & 4 \\ 8 & 1 & 0 & -1 & -8 \\ 16 & 1 & 0 & 1 & 16 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 6 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 8 \\ 8 \\ 32 \end{bmatrix}$$

El proceso matricial se realiza dos veces por lo tanto ese será el orden Oh^2