Punto 3:

$$m = \frac{a+b}{2}$$

$$f(x) \approx p_2(x) = \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)}f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}f(b), \quad \forall x \in [a,b]. \tag{1.86}$$

$x_m = m$ por facilidad

De modo que la integral tiene un valor aproximado:

$$\int_a^b f(x)dx \cong \int_a^b p_2(x)dx = \frac{h}{3}(f(a) + 4f(x_m) + f(b)). \tag{1.87}$$

$$\int_a^b \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)}f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}f(b)\, dx$$

$$\int_a^b \frac{(x-b)(x-m)}{(a-b)(a-m)}f(a)\,dx + \int_a^b \frac{(x-a)(x-b)}{(m-a)(m-b)}f(m)\,dx + \int_a^b \frac{(x-a)(x-m)}{(b-a)(b-m)}f(b)\,dx$$

$$\frac{f(a)}{(a-b)(a-m)}\int_a^b (x-b)(x-m)\,dx + \frac{f(m)}{(m-a)(m-b)}\int_a^b (x-a)(x-b)\,dx + \frac{f(b)}{(b-a)(b-m)}\int_a^b (x-a)(x-m)\,dx$$

$$\int_a^b x^2 - mx - bx + bm\, dx$$

$$\left[\frac{x^3}{3} - \frac{mx^2+bx^2}{2} + bmx\right]_a^b$$

entonces

$$\frac{f(a)}{(a-b)(a-m)}\left[\frac{x^3}{3} - \frac{mx^2+bx^2}{2} + bmx\right]_a^b + \frac{f(m)}{(m-a)(m-b)}\left[\frac{x^3}{3} - \frac{bx^2+ax^2}{2} + abx\right]_a^b$$

$$+ \frac{f(b)}{(b-a)(b-m)}\left[\frac{x^3}{3} - \frac{mx^2+ax^2}{2} + amx\right]_a^b$$

$$\frac{f(a)}{(a-b)(a-m)}\left[\frac{b^3}{3} - \frac{mb^2+b^3}{2} + b^2m - \frac{a^3}{3} + \frac{ma^2+ba^2}{2} - bam\right]$$

$$= \quad '' \quad \left[\frac{-b^3+3b^2m-2a^3+3a^2m+3a^2b}{6} - bam\right]$$

$$\frac{f(a)}{(a-b)(a-\frac{a+b}{2})}\left[\frac{-3ab^2+b^3+3a^3+3a^2b-4a}{12}\right]$$

$$\frac{f(a)_a}{\frac{a^2-2ab+b^2}{2}} \text{ " } = \text{ " } \rightarrow \frac{2f(a)}{(a-b)^2}\left[\frac{-3ab^2+b^3+3a^3+3a^2b-4a}{12}\right]$$

$$\frac{f(m)}{(m-a)(m-b)}\left[\frac{b^3}{3}-\frac{b^3+ab^2}{2}+ab^2-\frac{a^3}{3}+\frac{ba^2+a^3}{2}+a^2b\right]$$

$$\frac{f(m)}{(m-a)(m-b)}\left[\frac{-b^3+3ab^2+a^3-3a^2b}{6}\right]$$

$$\frac{f(m)_4}{-\frac{a^2-2ab+b^2}{4}} \text{ " } = \text{ " } \rightarrow \frac{-4f(m)}{(a-b)^2}\left[\frac{-(b-a)^3}{6}\right]$$

$$\frac{f(b)}{(b-a)(b-m)}\left[\frac{b^3}{3}-\frac{b^2m+ab^2}{2}+abm-\frac{a^3}{3}-\frac{ma^2+a^3}{2}+a^2m\right]$$

$$\text{" } = \text{ "}\left[\frac{2b^3-3b^2m-3ab^2}{6}-\frac{a^3+3a^2m}{6}+abm\right]$$

$$\left[\frac{2b^3-3b^2m-3ab^2-a^3-3a^2m}{6}+abm\right]$$

$$\frac{f(b)_4}{\frac{b^2-2ab+a^2}{2}}\left[\frac{b^3-3ab^2-5a^3+3a^2b}{12}\right] \Rightarrow$$

$$\frac{2f(b)}{(b-a)^2}\left[\frac{b^3-3ab^2-5a^3+3a^2b}{12}\right]$$

todo junto:

$$\frac{\cancel{2} f(a)}{(a-b)^2}\left[\frac{-3ab^2+b^3+3a^3+3a^2b-4a}{\cancel{12}}\right] + \frac{\cancel{4}4f(m)}{(a-b)^2}\left[\frac{4(b-\cancel{a)^3}}{6}\right]$$

$$+\frac{\cancel{2} f(b)}{(b-a)^2}\left[\frac{b^3-3ab^2-5a^3+3a^2b}{\cancel{12}}\right]$$

$$= \frac{f(a)}{\cancel{(a-b)}^2}\left[\frac{\overbrace{-3ab^2+b^3+3a^3+3a^2b-4a}^{\text{Poli } \cancel{(a-b)}^3 \text{ NK } 6}}{6}\right] + \underset{x_m}{4f(m)}\left[\frac{\overbrace{(b-a)}^{h}}{3}\right] +$$

$$\frac{f(b)}{(\cancel{b-a})^2}\left[\frac{\overbrace{b^3-3ab^2-5a^3+3a^2b}^{\hookrightarrow 4(\cancel{b-a})^3}}{6}\right]$$

entonces, con $h=\frac{a-b}{2}$, se obtiene

$$= \frac{h}{3}\left[f(a)+4f(x_m)+f(b)\right]$$ ☺

$$\int_a^b p_2(x)dx$$

Punto 4:

Verificar →

$$E=\int_a^b \epsilon(x)dx = \int_a^b \frac{f'''(\xi)}{4!}(x-a)(x-b)(x-(a+b)/2)dx = 0, \qquad a \le \xi \le b. \quad (1.89)$$

$$\int_a^b \frac{f'''(\xi)}{4!}(x-a)(x-b)\left(x-\left(\frac{a+b}{2}\right)\right)dx$$

$$\frac{f^{(3)}(\xi)}{4!}\int_a^b (x-a)(x-b)\left(\frac{2x-a-b}{2}\right)dx$$

$$" = " \int_a^b (x^2-bx-ax+ab)\left(\frac{2x-a-b}{2}\right)dx$$

$$" = " \int_a^b \frac{2x^3 + (-3ax^2) - 3bx^2 + 4abx + b^2x + a^2x - a^2b - ab^2}{2} \, dx$$

$$\frac{f^{(3)}(\xi)}{2(4!)} \int_a^b 2x^3 - 3ax^2 - 3bx^2 + 4abx + b^2x + a^2x - a^2b - ab^2 \, dx$$

$$\left[\frac{2x^4}{4} - \frac{3ax^3}{3} - \frac{3bx^3}{3} + \frac{4abx^2}{2} + \frac{b^2x^2}{2} + \frac{a^2x^2}{2} - a^2bx - ab^2x \right]_a^b$$

$$\left[\frac{1}{2}x^4 - ax^3 - bx^3 + 2abx^2 + \frac{b^2x^2}{2} + \frac{a^2x^2}{2} - a^2bx - ab^2x \right]_a^b$$

$$\left[\frac{1}{2}(b^4) - ab^3 - b^4 + 2ab^3 + b^4/2 + \frac{a^2b^2}{2} - a^2b^2 - ab^3 \right] -$$

$$\left[\frac{1}{2}a^4 - a^4 - a^3b + 2a^3b + \frac{b^2a^2}{2} + \frac{a^4}{2} - a^3b - a^2b^2 \right]$$

$$\cancel{b^4} - \cancel{b^4} + \cancel{ab^3} + \cancel{\frac{a^2b^2}{2}} - \cancel{a^2b^2} - \cancel{ab^3} - \cancel{a^4} + \cancel{a^4} + \cancel{a^3b} - \cancel{a^3b} - \cancel{\frac{b^2a^2}{2}} + \cancel{a^2b^2}$$

$$= 0$$

entonces

$$= \frac{f^{(3)}(\xi)}{2(4!)}(0) \rightarrow 0 = E = \int_a^b \epsilon(x)dx \;"$$