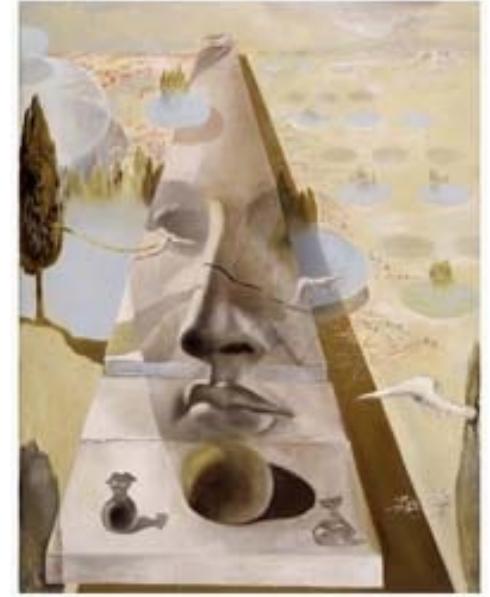


# Lecture 3

## Camera Models 2 & Camera Calibration



Professor Silvio Savarese

Stanford Vision and Learning Lab

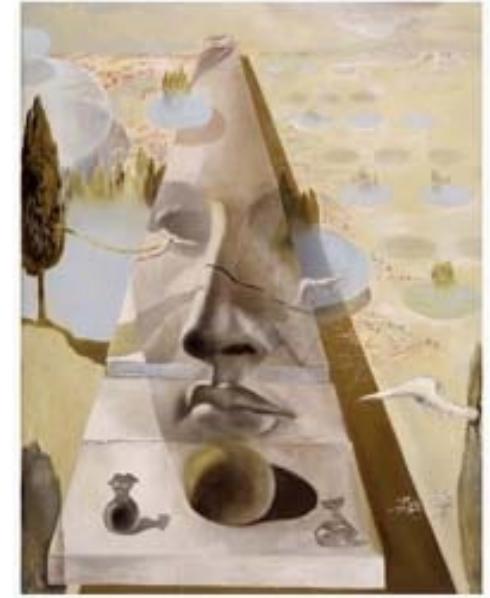
# Lecture 3

## Camera Models 2 & Camera Calibration

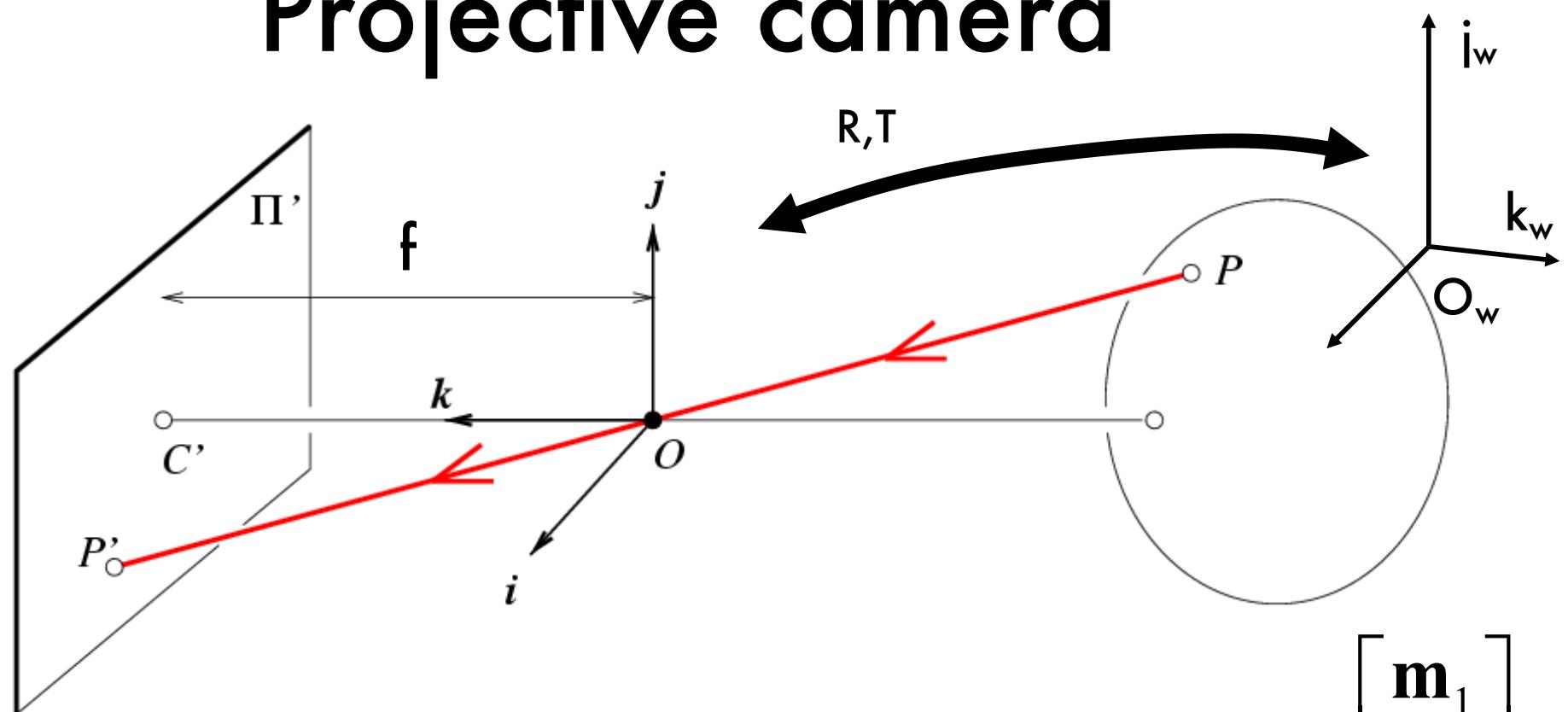
- Recap of camera models
- Camera calibration problem
- Camera calibration with radial distortion
- Example

Reading:      **[FP]** Chapter 1 “Geometric Camera Calibration”  
**[HZ]** Chapter 7 “Computation of Camera Matrix P”

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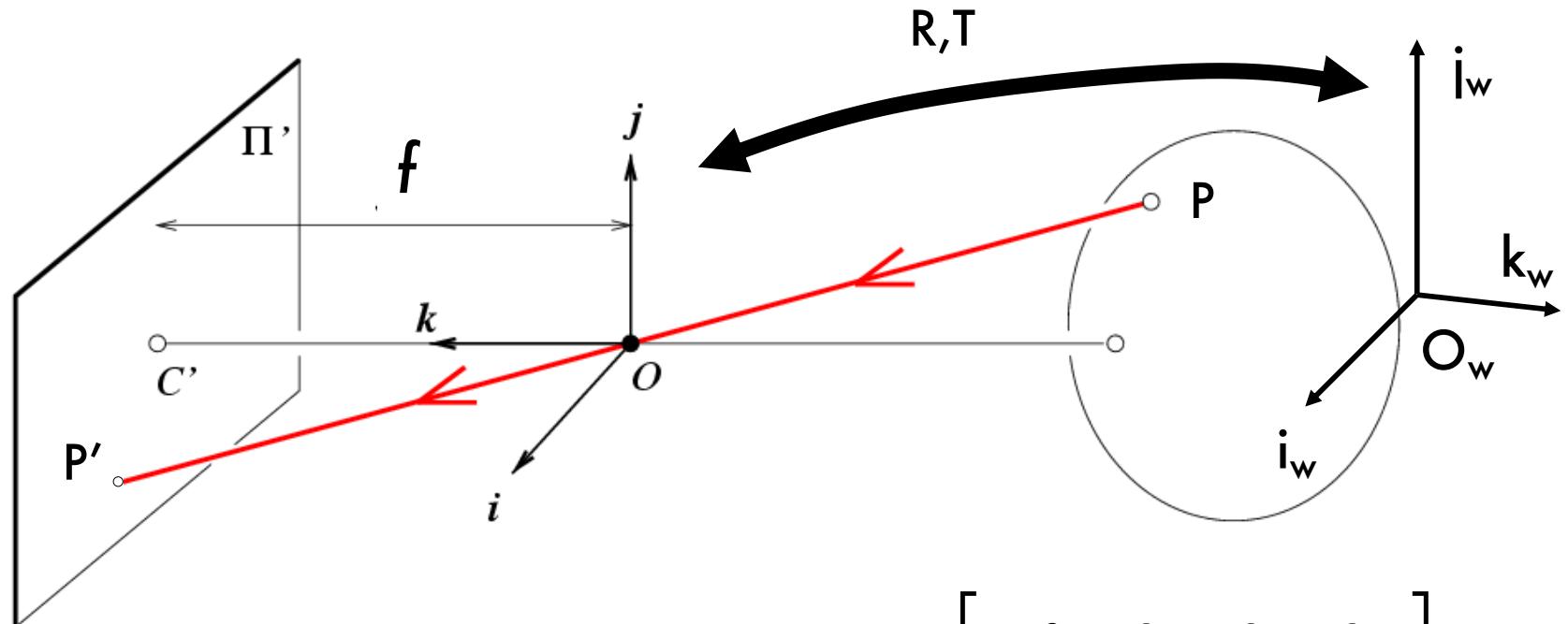


# Projective camera



$$\begin{aligned}
 P'^{3 \times 1} &= M P_w = K_{3 \times 3} \begin{bmatrix} R & T \end{bmatrix}_{3 \times 4} P_w^{4 \times 1} & M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} \\
 &= \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} P_w = \begin{bmatrix} \mathbf{m}_1 P_w \\ \mathbf{m}_2 P_w \\ \mathbf{m}_3 P_w \end{bmatrix} & \xrightarrow{\text{Eq. } E} P_E^E = \left( \frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w} \right) \quad [\text{Eq. 1}]
 \end{aligned}$$

# Exercise!



$$M = K \begin{bmatrix} R & T \end{bmatrix} = K \begin{bmatrix} I & 0 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\rightarrow P'_E = \left( \frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w} \right) = \left( f \frac{x_w}{z_w}, f \frac{y_w}{z_w} \right)$$

$$P_w = \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

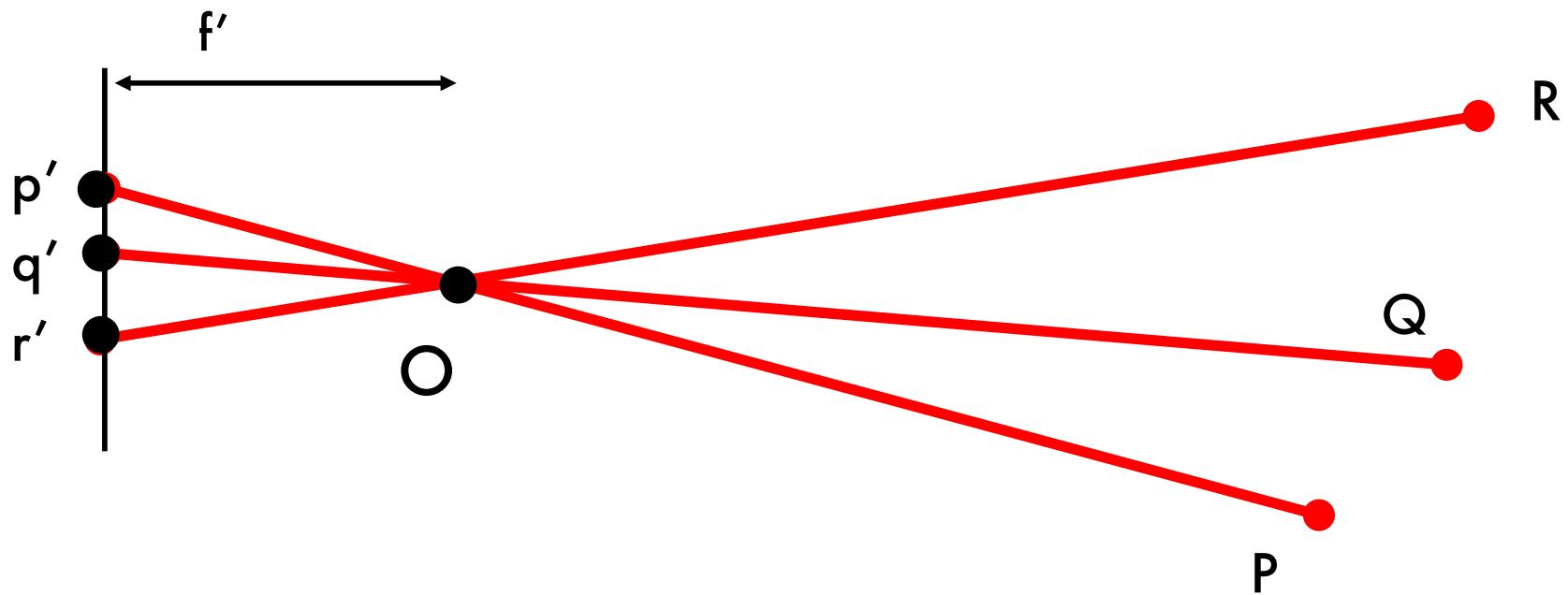
# Canonical Projective Transformation

$$P' = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_M \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad P' = M P$$

$$\mathbb{R}^4 \xrightarrow{H} \mathbb{R}^3$$

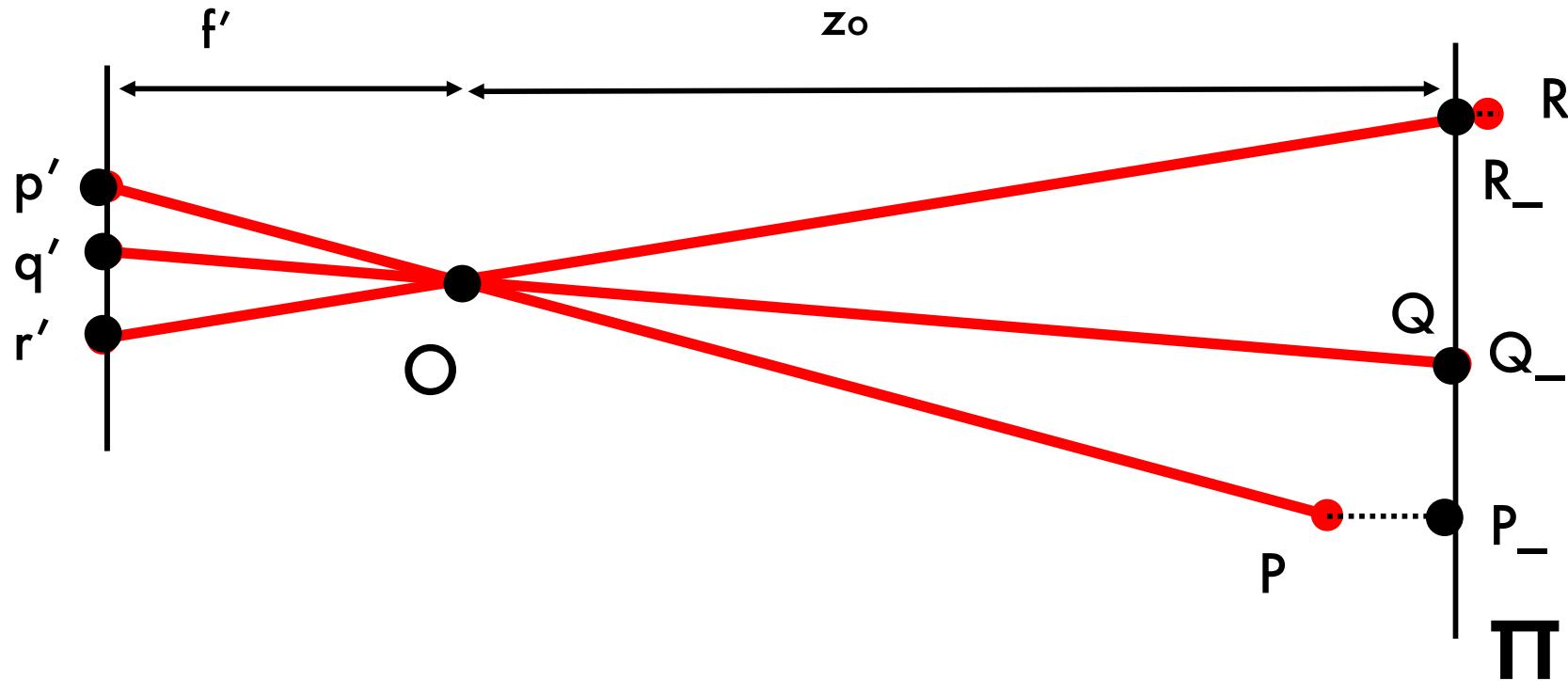
$$P_i' = \begin{bmatrix} \frac{x}{z} \\ \frac{y}{z} \end{bmatrix}$$

# Projective camera

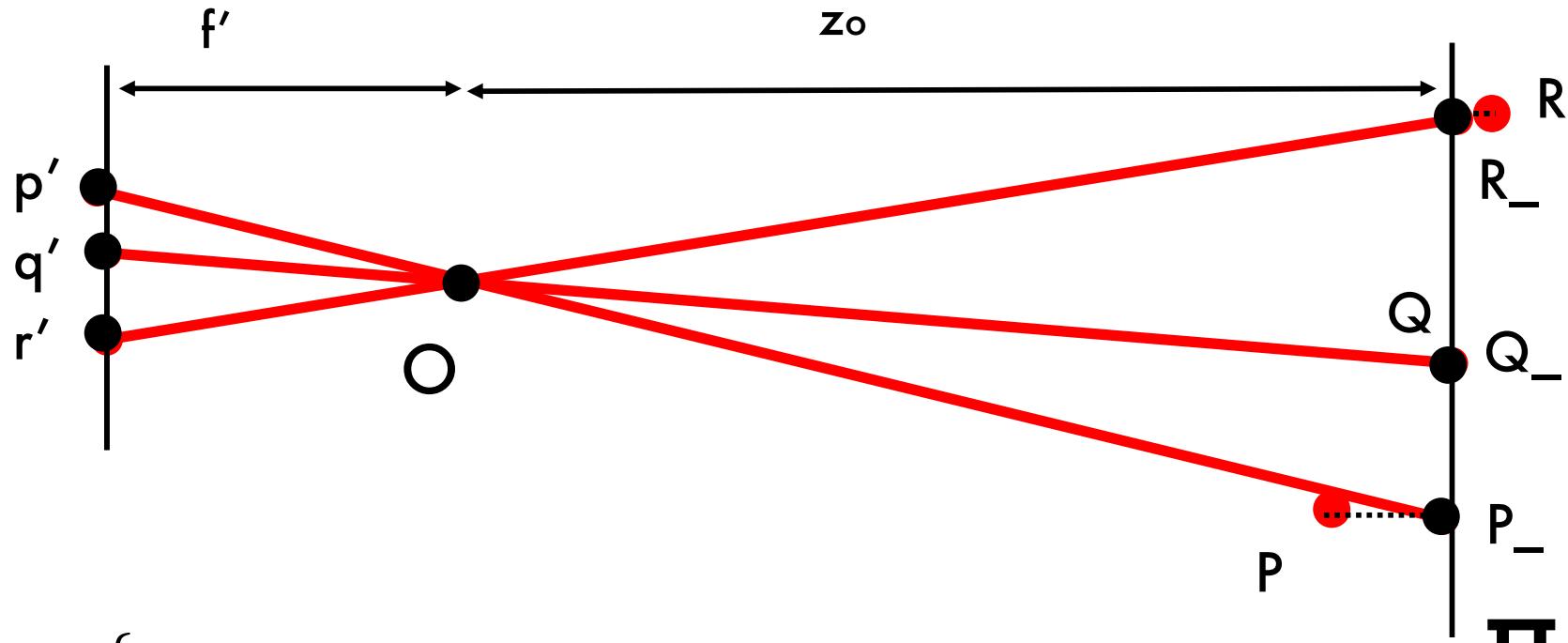


# Weak perspective projection

When the relative scene depth is small compared to its distance from the camera



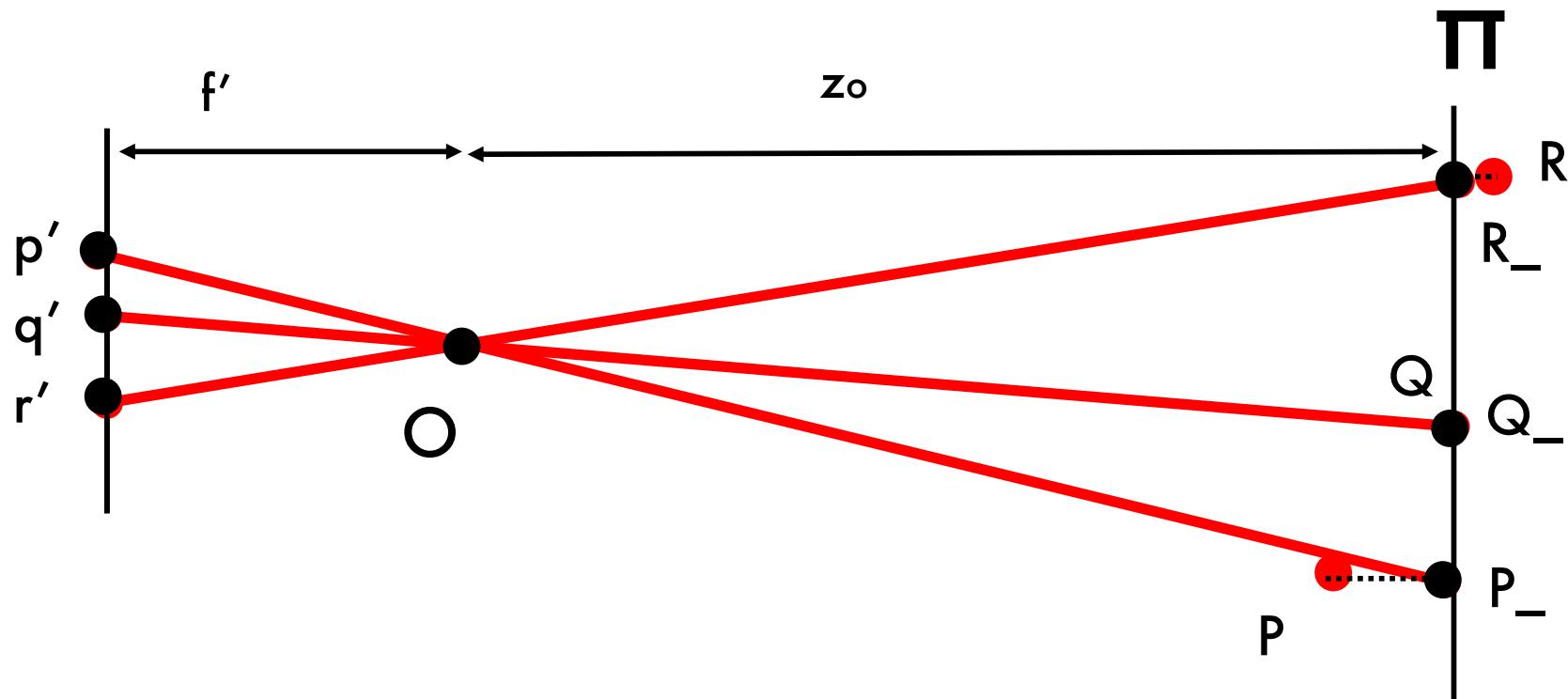
# Weak perspective projection



$$\left\{ \begin{array}{l} x' = \frac{f'}{z} x \\ y' = \frac{f'}{z} y \end{array} \right. \rightarrow \left\{ \begin{array}{l} x' = \frac{f'}{z_0} x \\ y' = \frac{f'}{z_0} y \end{array} \right.$$

Magnification  $m$

# Weak perspective projection



Projective (perspective)

Weak perspective

$$M = K \begin{bmatrix} R & T \end{bmatrix} = \begin{bmatrix} A & b \\ v & 1 \end{bmatrix} \rightarrow M = \begin{bmatrix} A & b \\ 0 & 1 \end{bmatrix}$$

$$P' = M P_w = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} P_w = \begin{bmatrix} \mathbf{m}_1 P_w \\ \mathbf{m}_2 P_w \\ \mathbf{m}_3 P_w \end{bmatrix}$$

$$M = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{v} & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

$$\overset{E}{\rightarrow} \left( \frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w} \right)$$

Perspective

---

$$P' = M P_w = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} P_w = \begin{bmatrix} \mathbf{m}_1 P_w \\ \mathbf{m}_2 P_w \\ 1 \end{bmatrix}$$

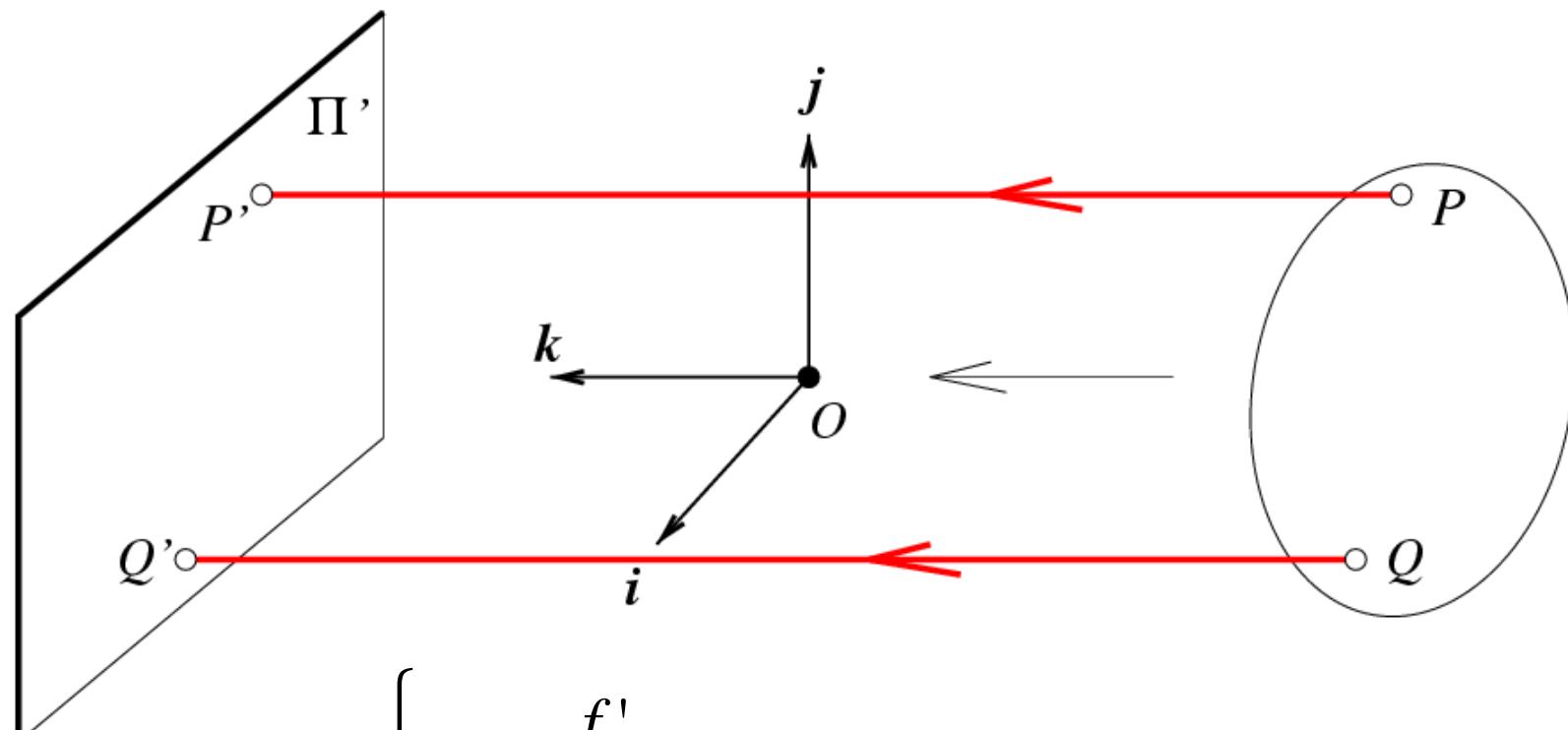
$$M = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

↑      ↑  
magnification

Weak perspective

# Orthographic (affine) projection

Distance from center of projection to image plane is infinite

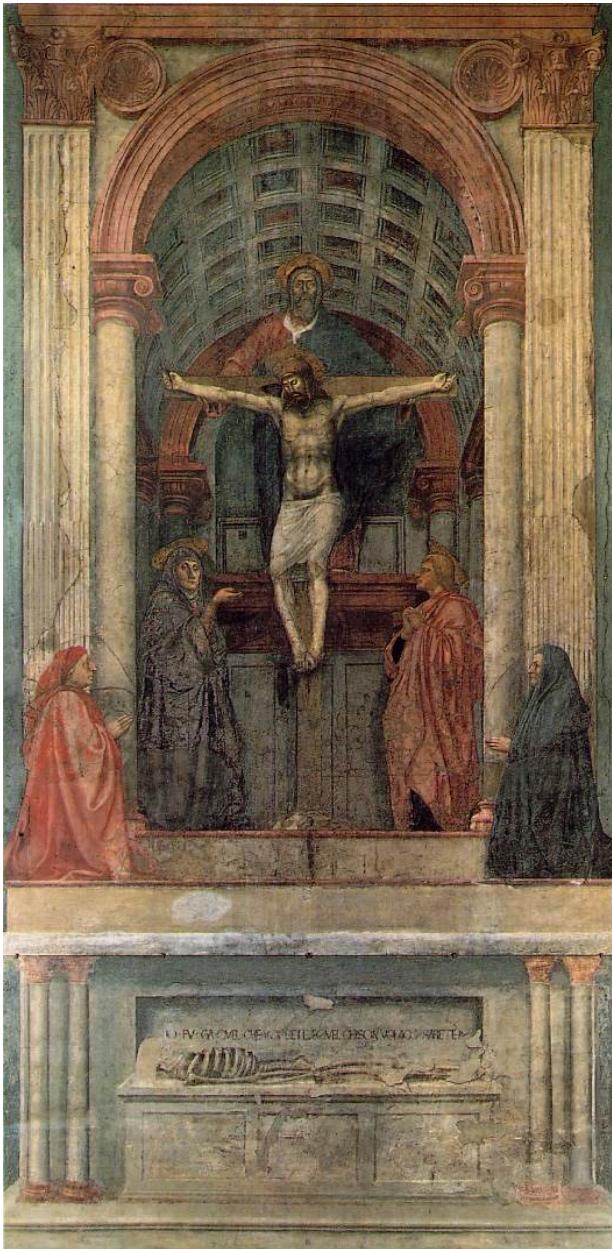


$$\begin{cases} x' = \frac{f'}{z} x \\ y' = \frac{f'}{z} y \end{cases} \rightarrow \begin{cases} x' = x \\ y' = y \end{cases}$$

# Pros and Cons of These Models

- Weak perspective results in much simpler math.
  - Accurate when object is small and distant.
  - Most useful for recognition.
- Pinhole perspective is much more accurate for modeling the 3D-to-2D mapping.
  - Used in structure from motion or SLAM.

# One-point perspective



Masaccio, *Trinity*,  
Santa Maria  
Novella, Florence,  
1425-28

il Canaletto The Piazzetta, Venice,



# Weak perspective projection



*The Kangxi Emperor's Southern Inspection Tour (1691-1698)* By Wang Hui

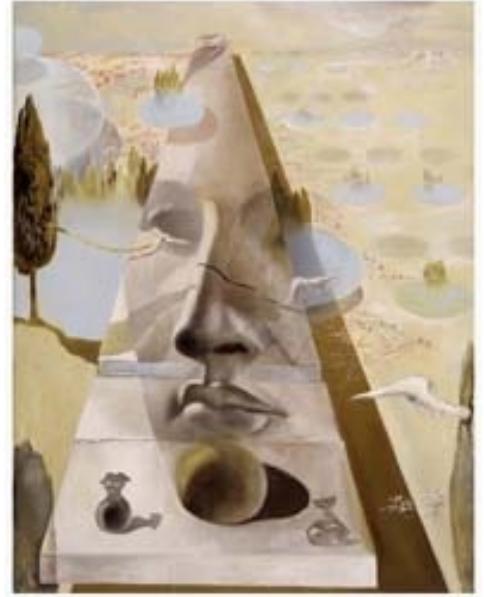
# Weak perspective projection



*The Kangxi Emperor's Southern Inspection Tour (1691-1698)* By Wang Hui

# Lecture 3

## Camera Calibration



- Recap of camera models
- Camera calibration problem
- Camera calibration with radial distortion
- Example

Reading:      **[FP]** Chapter 1 “Geometric Camera Calibration”  
**[HZ]** Chapter 7 “Computation of Camera Matrix P”

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# Projective camera

$$P' = M P_w = \boxed{K} \begin{bmatrix} R & T \end{bmatrix} P_w$$

Internal parameters      External parameters

$$\mathcal{M} = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{pmatrix}_{3 \times 4}$$

$$K = \begin{bmatrix} \alpha & -\alpha \cot \theta & u_o \\ 0 & \frac{\beta}{\sin \theta} & v_o \\ 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \quad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

# Goal of calibration

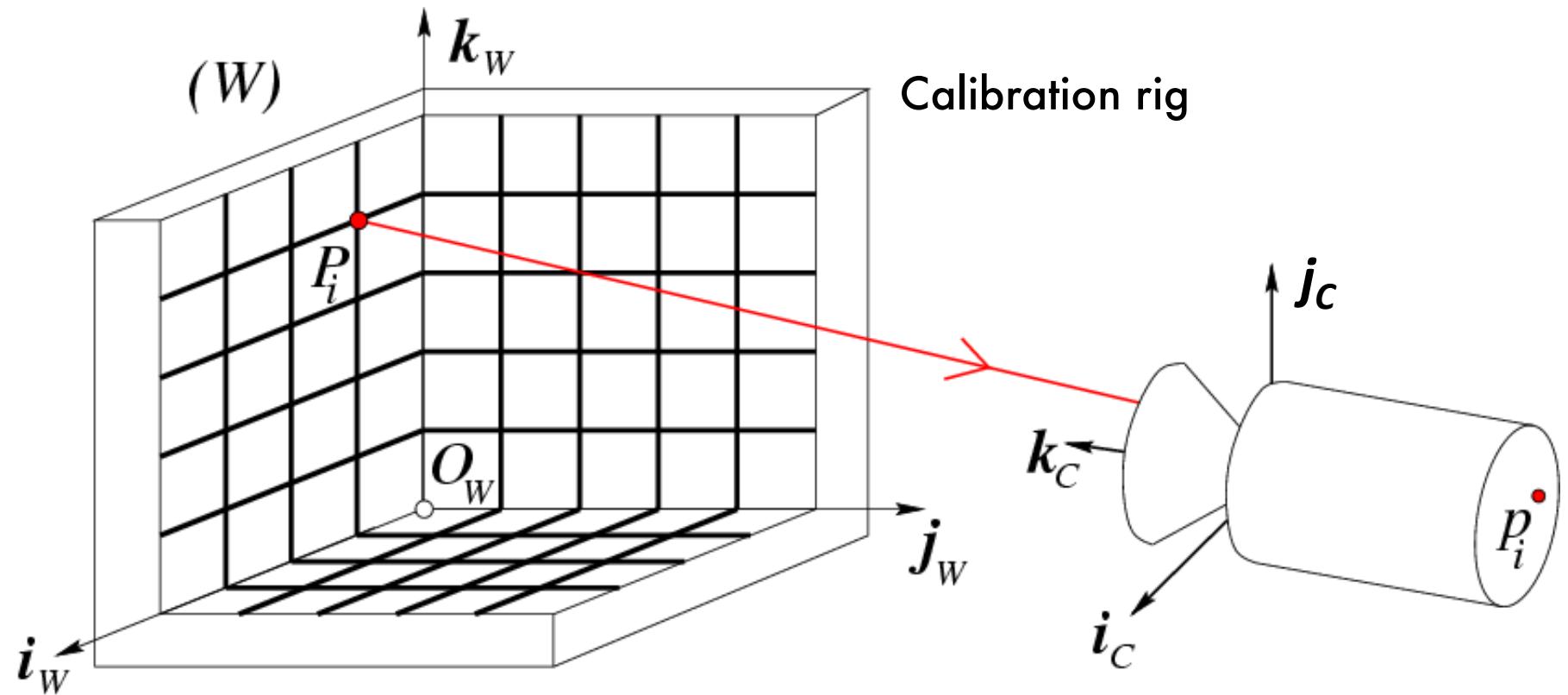
$$P' = M P_w = \begin{bmatrix} K & [R \quad T] \end{bmatrix} P_w$$

Internal parameters                      External parameters

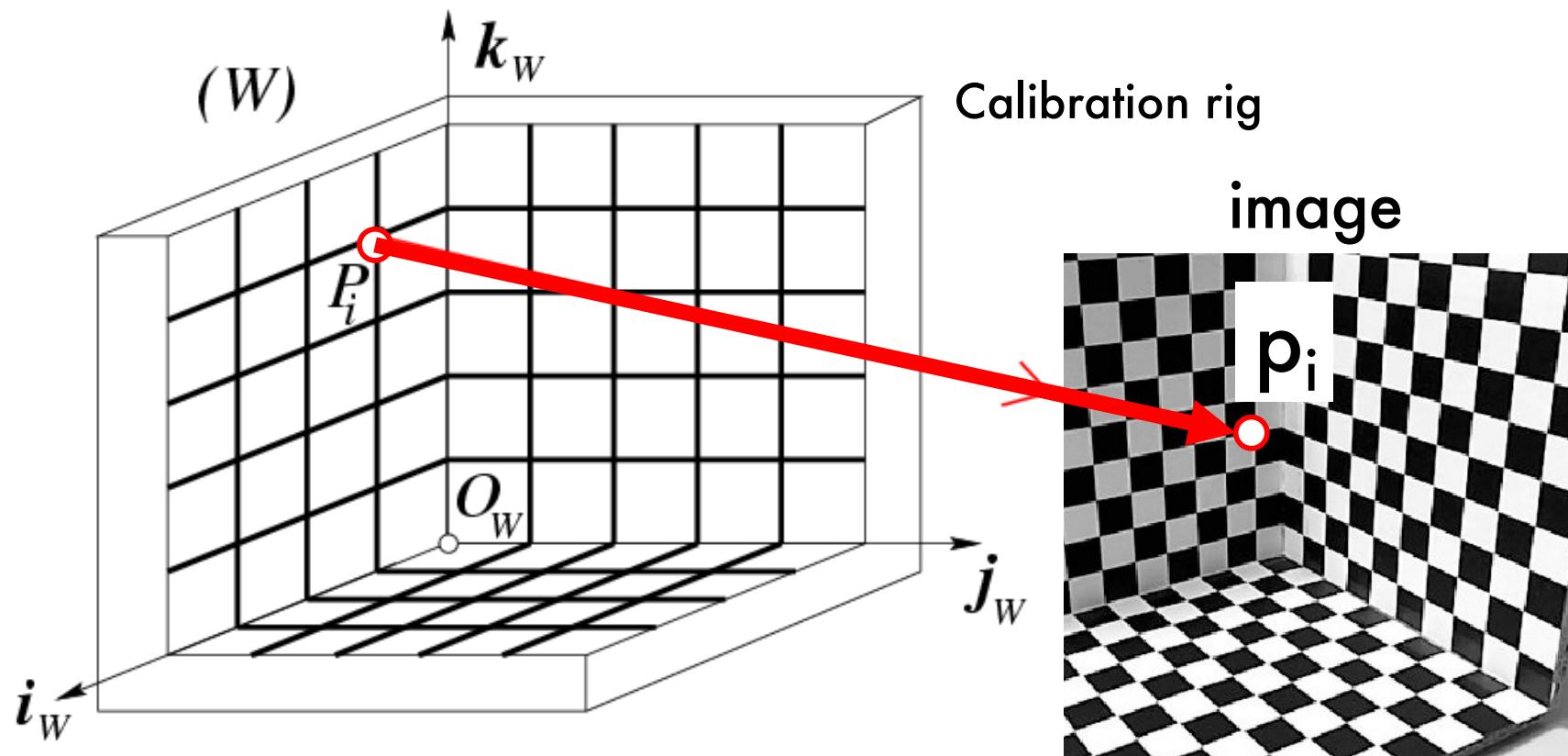
Estimate intrinsic and extrinsic parameters from 1 or multiple images

Change notation:  
 $P = P_w$   
 $p = P'$

# Calibration Problem

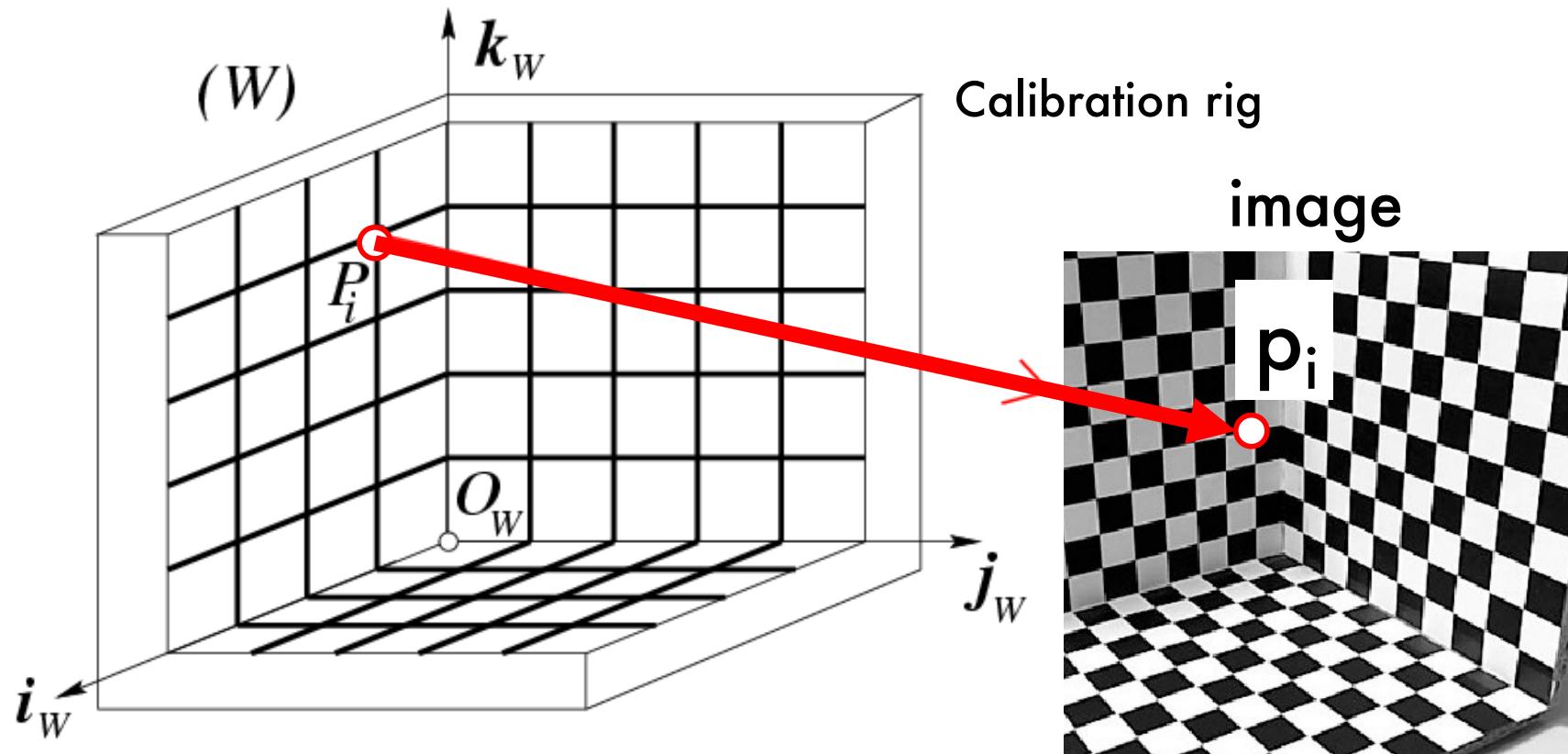


# Calibration Problem



- $P_1 \dots P_n$  with **known** positions in  $[O_w, i_w, j_w, k_w]$
  - $p_1, \dots p_n$  **known** positions in the image
- Goal:** compute intrinsic and extrinsic parameters

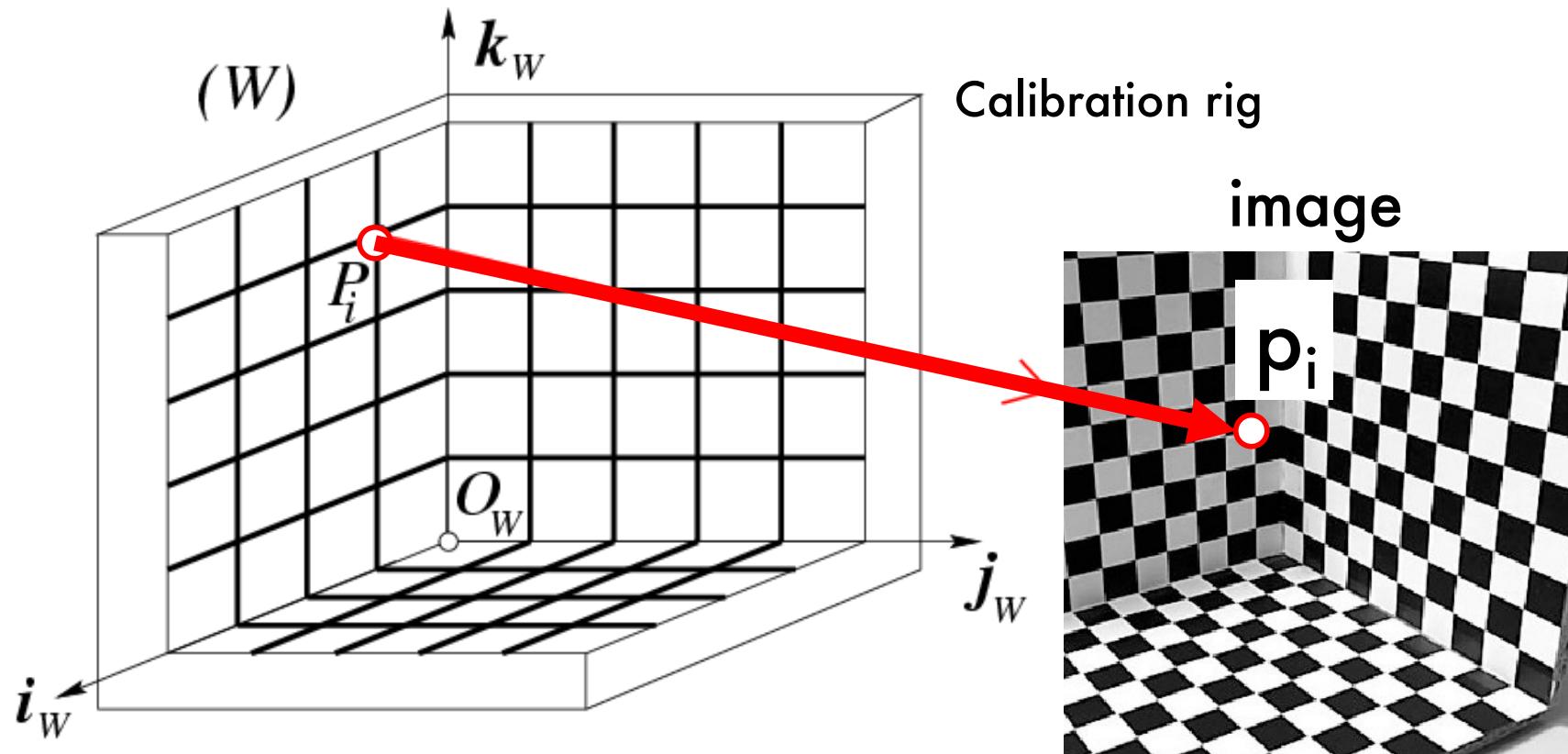
# Calibration Problem



## How many correspondences do we need?

- $M$  has 11 unknowns • We need 11 equations

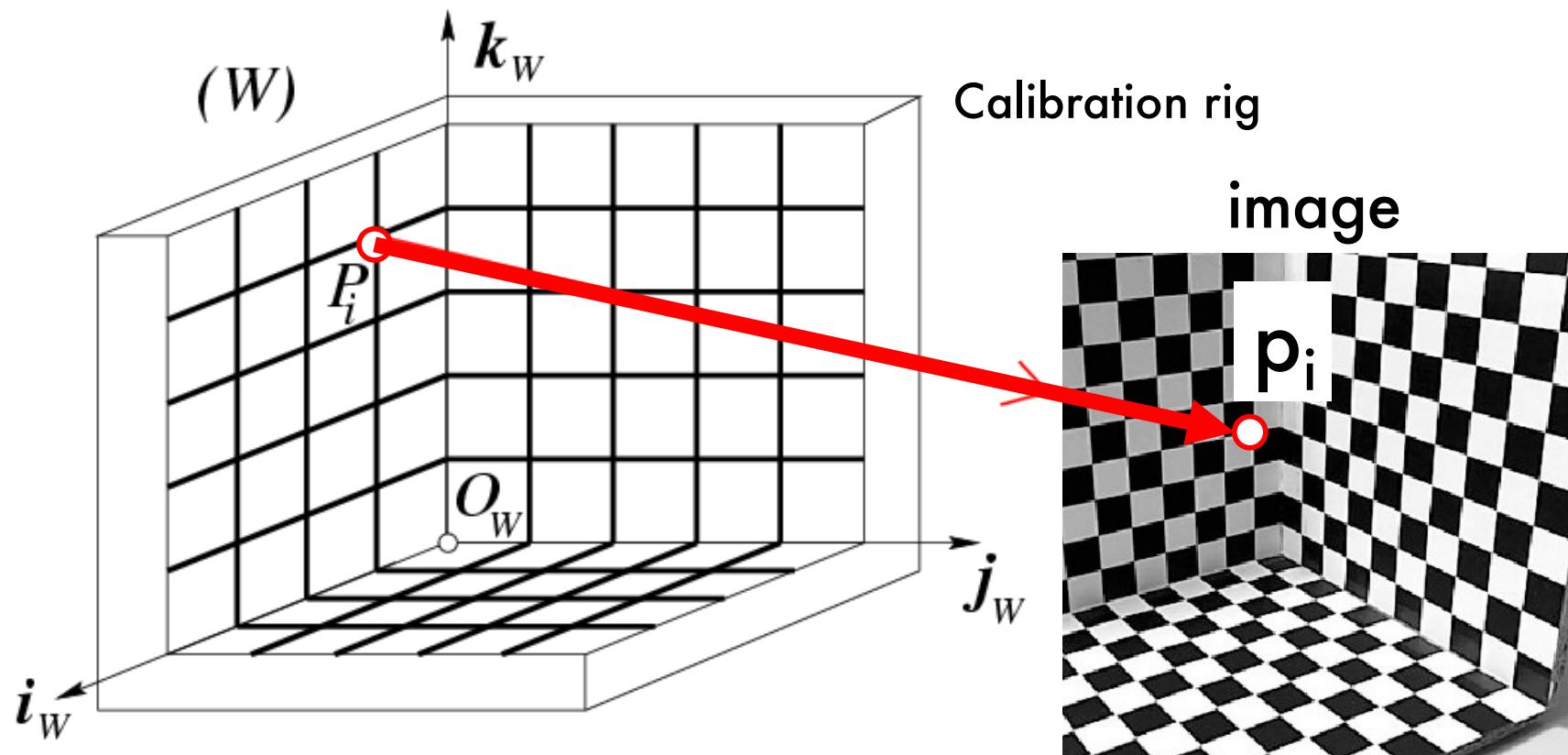
# Calibration Problem



## How many correspondences do we need?

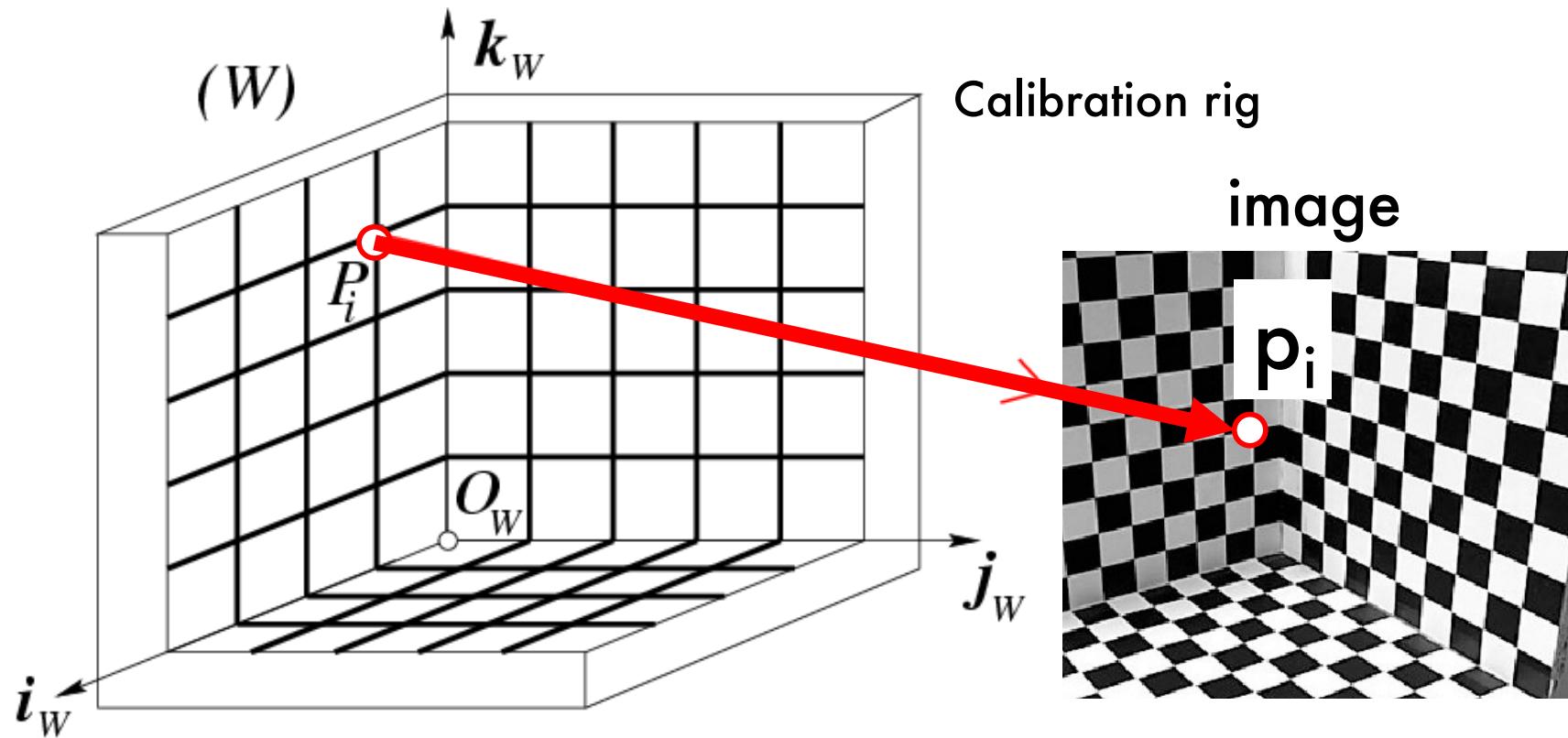
- $M$  has 11 unknowns • We need 11 equations • 6 correspondences would do it

# Calibration Problem



In practice, using more than 6 correspondences enables more robust results

# Calibration Problem



$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{m}_1}{\mathbf{m}_3} P_i \\ \frac{\mathbf{m}_2}{\mathbf{m}_3} P_i \end{bmatrix} = M P_i$$

[Eq. 1]

$$M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

in pixels

# Calibration Problem

[Eq. 1]

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{m_1 P_i}{m_3 P_i} \\ \frac{m_2 P_i}{m_3 P_i} \end{bmatrix}$$

$$u_i = \frac{m_1 P_i}{m_3 P_i} \rightarrow u_i(m_3 P_i) = m_1 P_i \rightarrow u_i(m_3 P_i) - m_1 P_i = 0$$

$$v_i = \frac{m_2 P_i}{m_3 P_i} \rightarrow v_i(m_3 P_i) = m_2 P_i \rightarrow v_i(m_3 P_i) - m_2 P_i = 0$$

[Eqs. 2]

# Calibration Problem

$$\left\{ \begin{array}{l} u_1(\mathbf{m}_3 P_1) - \mathbf{m}_1 P_1 = 0 \\ v_1(\mathbf{m}_3 P_1) - \mathbf{m}_2 P_1 = 0 \\ \vdots \\ u_i(\mathbf{m}_3 P_i) - \mathbf{m}_1 P_i = 0 \quad [\text{Eqs. 3}] \\ v_i(\mathbf{m}_3 P_i) - \mathbf{m}_2 P_i = 0 \\ \vdots \\ u_n(\mathbf{m}_3 P_n) - \mathbf{m}_1 P_n = 0 \\ v_n(\mathbf{m}_3 P_n) - \mathbf{m}_2 P_n = 0 \end{array} \right.$$

# Block Matrix Multiplication

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

What is  $AB$  ?

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

# Calibration Problem

$$\left\{ \begin{array}{l} -u_1(\mathbf{m}_3 P_1) + \mathbf{m}_1 P_1 = 0 \\ -v_1(\mathbf{m}_3 P_1) + \mathbf{m}_2 P_1 = 0 \\ \vdots \\ -u_n(\mathbf{m}_3 P_n) + \mathbf{m}_1 P_n = 0 \\ -v_n(\mathbf{m}_3 P_n) + \mathbf{m}_2 P_n = 0 \end{array} \right. \quad [\text{Eqs. 3}]$$



known      unknown  
 $\mathbf{P}$   $\mathbf{m}$  = 0      [Eq. 4]

Homogenous  
linear system

$$\mathbf{P} \stackrel{\text{def}}{=} \left( \begin{array}{ccc} \mathbf{P}_1^T & \mathbf{0}^T & -u_1 \boxed{\mathbf{P}_1^T}^{1 \times 4} \\ \mathbf{0}^T & \mathbf{P}_1^T & -v_1 \mathbf{P}_1^T \\ \vdots & & \\ \mathbf{P}_n^T & \mathbf{0}^T & -u_n \mathbf{P}_n^T \\ \mathbf{0}^T & \mathbf{P}_n^T & -v_n \mathbf{P}_n^T \end{array} \right)_{2n \times 12}$$

$$\mathbf{m} \stackrel{\text{def}}{=} \left( \begin{array}{c} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{array} \right)_{12 \times 1}$$

# Homogeneous $M \times N$ Linear Systems

$M = \text{number of equations} = 2n$

$N = \text{number of unknown} = 11$

$$\begin{matrix} & N \\ P & \times \\ M & \end{matrix} \quad m = 0$$

The diagram shows a rectangular system of linear equations. On the left, there is a large rectangle divided into four quadrants by dashed lines. The top-left quadrant contains the letter 'P', the bottom-left quadrant contains the letter 'M', and the top-right quadrant contains the letter 'N'. To the right of this large rectangle is a smaller square containing the letter 'm'. An equals sign follows the smaller square, and to its right is another small square containing the letter '0'.

Rectangular system ( $M > N$ )

- 0 is always a solution
- To find non-zero solution

Minimize  $|P m|^2$

under the constraint  $|m|^2 = 1$

# Calibration Problem

$$\mathbf{P} \mathbf{m} = 0$$

- How do we solve this homogenous linear system?
- Via SVD decomposition!

# Calibration Problem

$$\boxed{\mathbf{P} \mathbf{m} = 0}$$

SVD decomposition of  $\mathbf{P}$

$$\boxed{\mathbf{U}_{2n \times 12} \ \mathbf{D}_{12 \times 12} \ \mathbf{V}^T_{12 \times 12}}$$

Last column of  $\mathbf{V}$  gives  $\mathbf{m}$

Why? See pag 592 of HZ

$$\mathbf{m} \stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{pmatrix}$$

$M$

# Extracting camera parameters

$$M = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{pmatrix} \rho$$

# Extracting camera parameters

See [FP],  
Sec. 1.3.1

$$\frac{M}{\rho} = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T \\ \mathbf{r}_3^T \end{pmatrix} \begin{pmatrix} \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ t_z \end{pmatrix}$$

$A$                              $\mathbf{b}$

Box 1

$$A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\rho = \frac{\pm 1}{|\mathbf{a}_3|} \quad u_o = \rho^2 (\mathbf{a}_1 \cdot \mathbf{a}_3) \\ v_o = \rho^2 (\mathbf{a}_2 \cdot \mathbf{a}_3)$$

$$\cos \theta = \frac{(\mathbf{a}_1 \times \mathbf{a}_3) \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_1 \times \mathbf{a}_3| \cdot |\mathbf{a}_2 \times \mathbf{a}_3|}$$

# Extracting camera parameters

See [FP],  
Sec. 1.3.1

$$\frac{M}{\rho} = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T \\ \mathbf{r}_3^T \end{pmatrix} \begin{pmatrix} \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ t_z \end{pmatrix}$$

**A**                                   **b**

Box 1

$$A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\alpha = \rho^2 |\mathbf{a}_1 \times \mathbf{a}_3| \sin \theta$$

$$\beta = \rho^2 |\mathbf{a}_2 \times \mathbf{a}_3| \sin \theta$$

# Extracting camera parameters

See [FP],  
Sec. 1.3.1

$$\frac{M}{\rho} = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T \\ \mathbf{r}_3^T \end{pmatrix} = \mathbf{K} [\mathbf{R} \quad \mathbf{T}]$$

$\mathbf{A}$                                      $\mathbf{b}$

Box 1

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

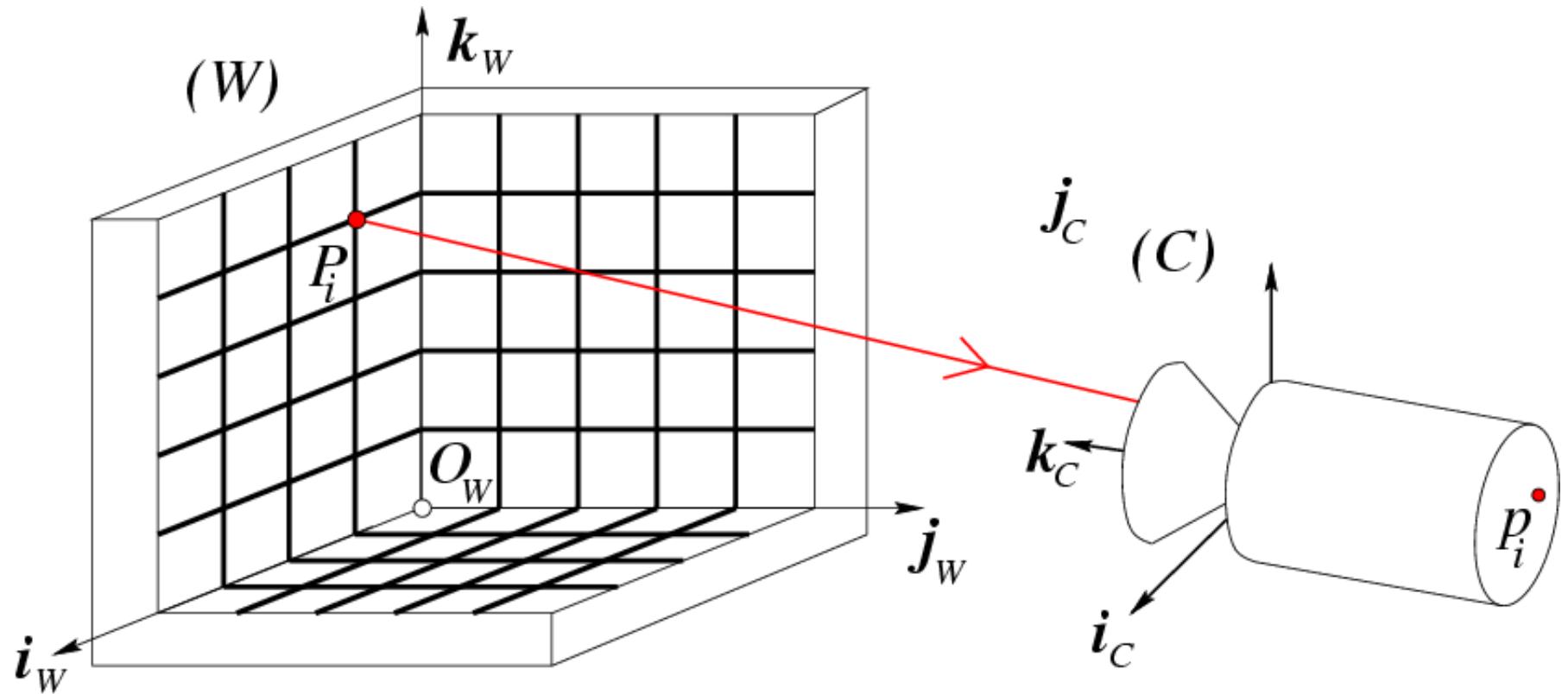
Estimated values

**Extrinsic**

$$\mathbf{r}_1 = \frac{(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_2 \times \mathbf{a}_3|} \quad \mathbf{r}_3 = \frac{\pm \mathbf{a}_3}{|\mathbf{a}_3|}$$

$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1 \quad \mathbf{T} = \rho \mathbf{K}^{-1} \mathbf{b}$$

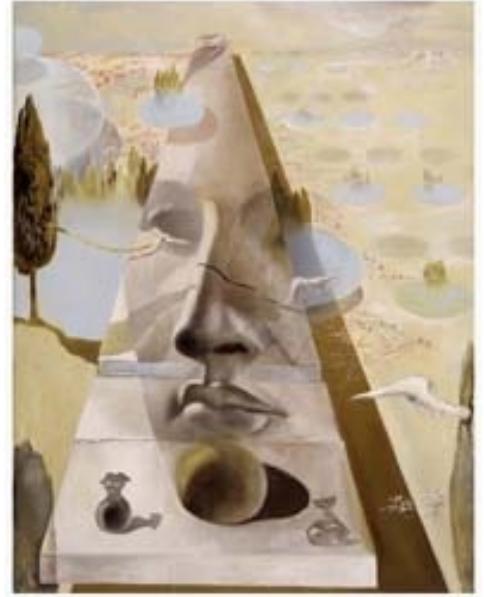
# Degenerate cases



- $P_i$ 's cannot lie on the same plane!
- Points cannot lie on the intersection curve of two quadric surfaces [FP] section 1.3

# Lecture 3

## Camera Calibration



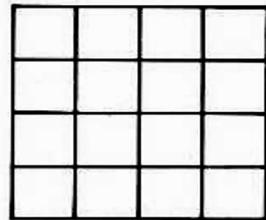
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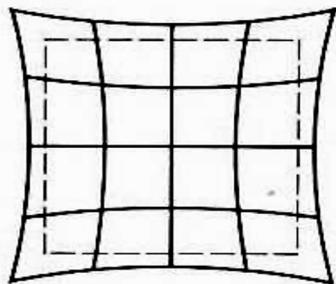
Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

# Radial Distortion

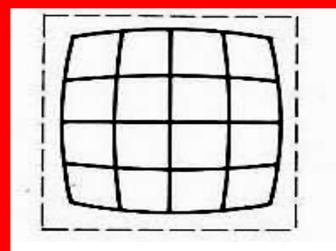
- Image magnification (in)decreases with distance from the optical axis
- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens



No distortion



Pin cushion

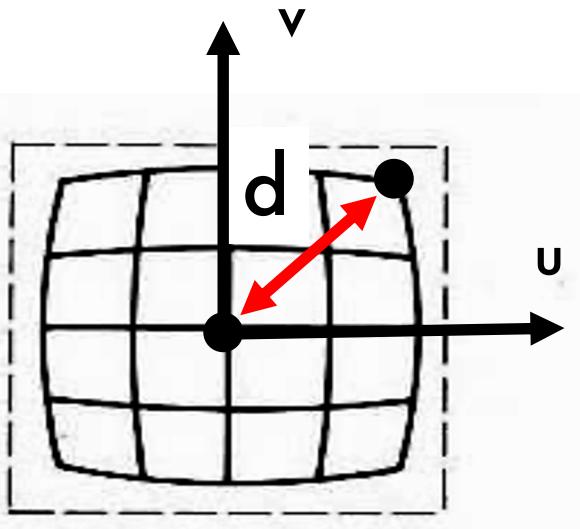


Barrel



# Radial Distortion

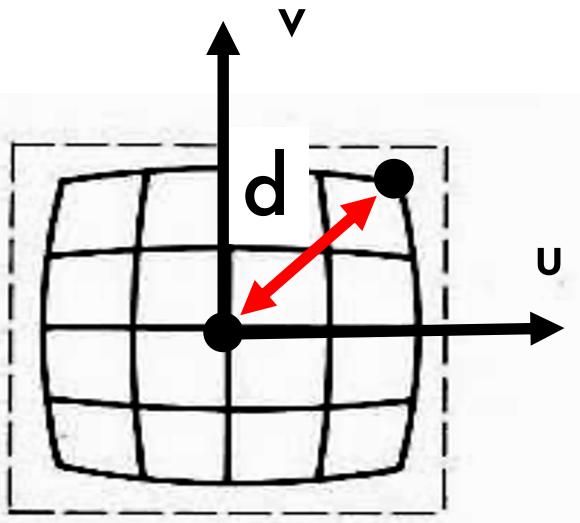
Image magnification decreases with distance from the optical center



$$S_{\lambda} \quad \begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i$$

# Radial Distortion

Image magnification decreases with distance from the optical center



$$S_{\lambda} \quad \begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i$$

$$\lambda = 1 \pm \sum_{p=1}^3 \kappa_p d^{2p}$$

[Eq. 5]  Polynomial function

Distortion coefficient

$$d^2 = a u^2 + b v^2 + c u v$$

To model radial behavior

[Eq. 6]

# Radial Distortion

$$\boxed{\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix}} M P_i \rightarrow$$

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i$$

$$Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Q

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{q_1 P_i}{q_3 P_i} \\ \frac{q_2 P_i}{q_3 P_i} \end{bmatrix}$$

Is this a linear system of equations?

$$\left\{ \begin{array}{l} u_i q_3 P_i = q_1 P_i \\ v_i q_3 P_i = q_2 P_i \end{array} \right.$$

No!

[Eqs.7]

# General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{q_1 P_i}{q_3 P_i} \\ \frac{q_3 P_i}{q_3 P_i} \\ \frac{q_2 P_i}{q_3 P_i} \\ \frac{q_3 P_i}{q_3 P_i} \end{bmatrix} \xrightarrow{\text{measurements}} X = f(Q) \quad [\text{Eq .8}]$$

parameters

$i=1\dots n \quad f( )$  is the nonlinear mapping

-Newton Method

-Levenberg-Marquardt Algorithm

- Iterative, starts from initial solution
- May be slow if initial solution far from real solution
- Estimated solution may be function of the initial solution (because of local minima)
- Newton requires the computation of J, H
- Levenberg-Marquardt doesn't require the computation of H

# General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{q_1 P_i}{q_3 P_i} \\ \frac{q_3 P_i}{q_3 P_i} \\ \frac{q_2 P_i}{q_3 P_i} \\ \frac{q_3 P_i}{q_3 P_i} \end{bmatrix} \xrightarrow{\text{measurements}} X = f(Q) \quad [\text{Eq .8}]$$

$i=1\dots n$        $f( )$  is the nonlinear mapping

parameters

## A possible algorithm

1. Solve linear part of the system to find approximated solution
2. Use this solution as initial condition for the full system
3. Solve full system using Newton or L.M.

# General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{q_1 P_i}{q_3 P_i} \\ \frac{q_2 P_i}{q_3 P_i} \end{bmatrix} \xrightarrow{\text{measurements}} X = f(Q) \quad [\text{Eq .8}]$$

$i=1\dots n$        $f( )$  is the nonlinear mapping

parameters

Typical assumptions:

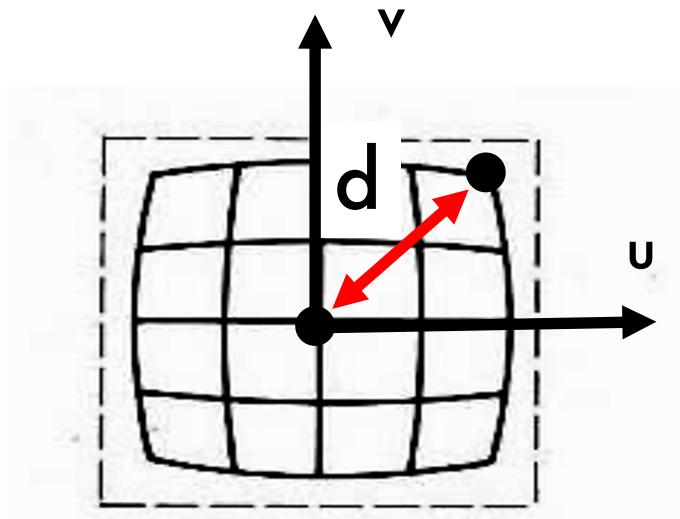
- zero-skew, square pixel
- $u_o, v_o$  = known center of the image

# Radial Distortion

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{q_1 P_i}{q_3 P_i} \\ \frac{q_2 P_i}{q_3 P_i} \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \frac{m_1 P_i}{m_3 P_i} \\ \frac{m_2 P_i}{m_3 P_i} \end{bmatrix}$$

Can we estimate  $m_1$  and  $m_2$  and ignore the radial distortion?

Hint:



$$\frac{u_i}{v_i} = \text{slope}$$

# Radial Distortion

Tsai [87]

Estimating  $\mathbf{m}_1$  and  $\mathbf{m}_2$ ...

$$\mathbf{p}_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \frac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \frac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix} \rightarrow \frac{u_i}{v_i} = \frac{\frac{(\mathbf{m}_1 P_i)}{(\mathbf{m}_3 P_i)}}{\frac{(\mathbf{m}_2 P_i)}{(\mathbf{m}_3 P_i)}} = \frac{\mathbf{m}_1 P_i}{\mathbf{m}_2 P_i}$$

[Eq .9]

[Eq .10]

$$\begin{cases} v_1(\mathbf{m}_1 P_1) - u_1(\mathbf{m}_2 P_1) = 0 \\ v_i(\mathbf{m}_1 P_i) - u_i(\mathbf{m}_2 P_i) = 0 \\ \vdots \\ v_n(\mathbf{m}_1 P_n) - u_n(\mathbf{m}_2 P_n) = 0 \end{cases}$$

[Eq .11]

$$L \mathbf{n} = 0$$



Get  $\mathbf{m}_1$  and  
 $\mathbf{m}_2$  by SVD

$$\mathbf{L} \stackrel{\text{def}}{=} \begin{pmatrix} v_1 \mathbf{P}_1^T & -u_1 \mathbf{P}_1^T \\ v_2 \mathbf{P}_2^T & -u_2 \mathbf{P}_2^T \\ \vdots & \vdots \\ v_n \mathbf{P}_n^T & -u_n \mathbf{P}_n^T \end{pmatrix}$$

$$\mathbf{n} = \begin{bmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \end{bmatrix}$$

# Radial Distortion

Once that  $\mathbf{m}_1$  and  $\mathbf{m}_2$  are estimated...

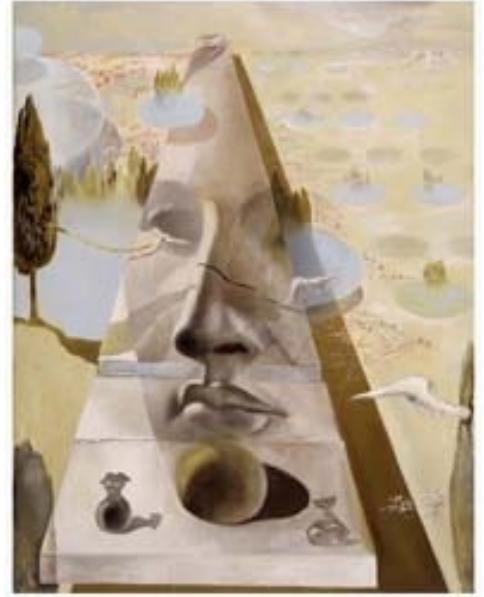
$$\mathbf{p}_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \frac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \frac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

$\mathbf{m}_3$  is non linear function of  $\mathbf{m}_1$ ,  $\mathbf{m}_2$ ,  $\lambda$

There are some degenerate configurations for which  $\mathbf{m}_1$  and  $\mathbf{m}_2$  cannot be computed

# Lecture 3

## Camera Calibration



- Recap of projective cameras
- Camera calibration problem
- Camera calibration with radial distortion
- Example

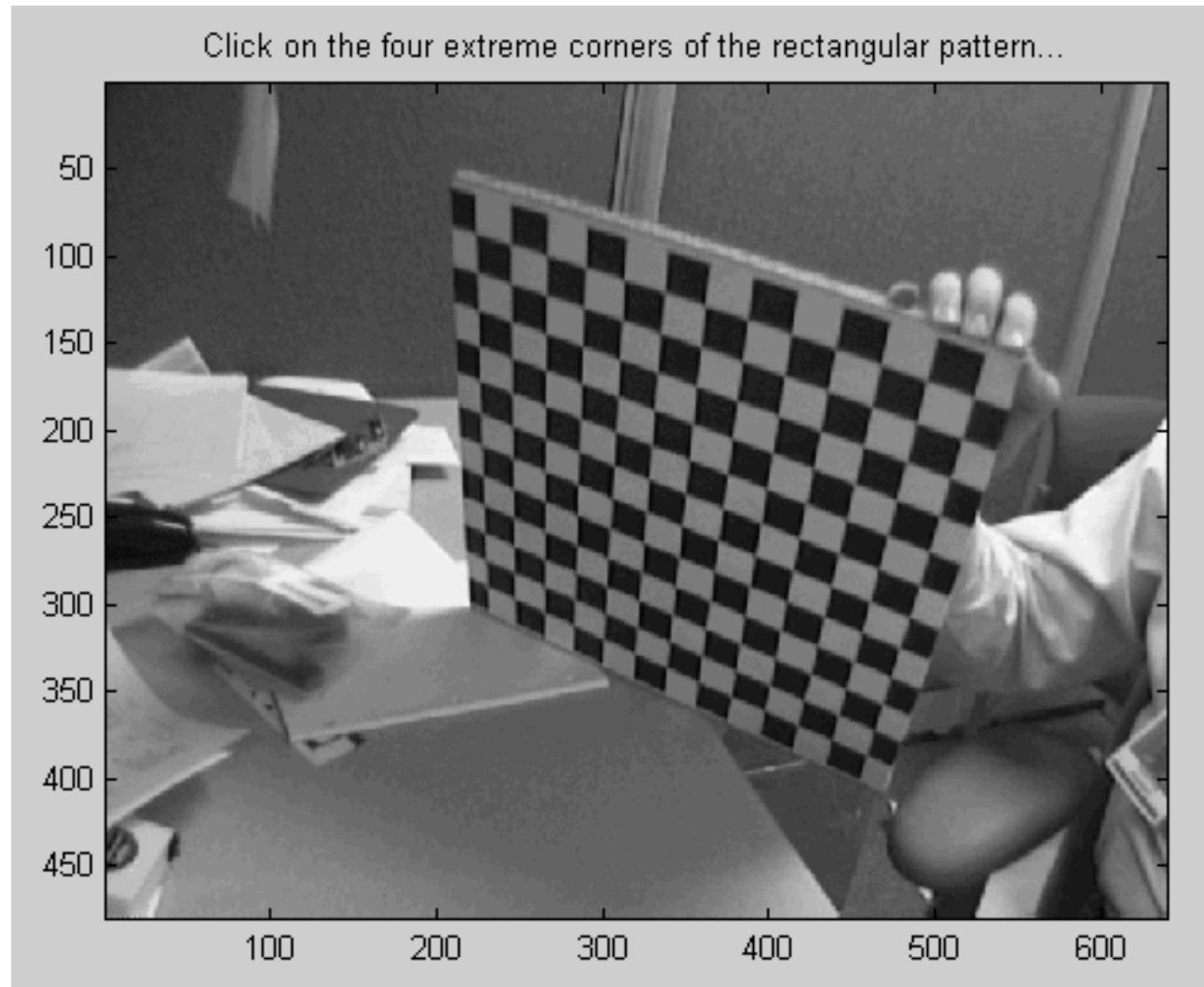
Reading:      **[FP]** Chapter 1 “Geometric Camera Calibration”  
**[HZ]** Chapter 7 “Computation of Camera Matrix P”

Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

# Calibration Procedure

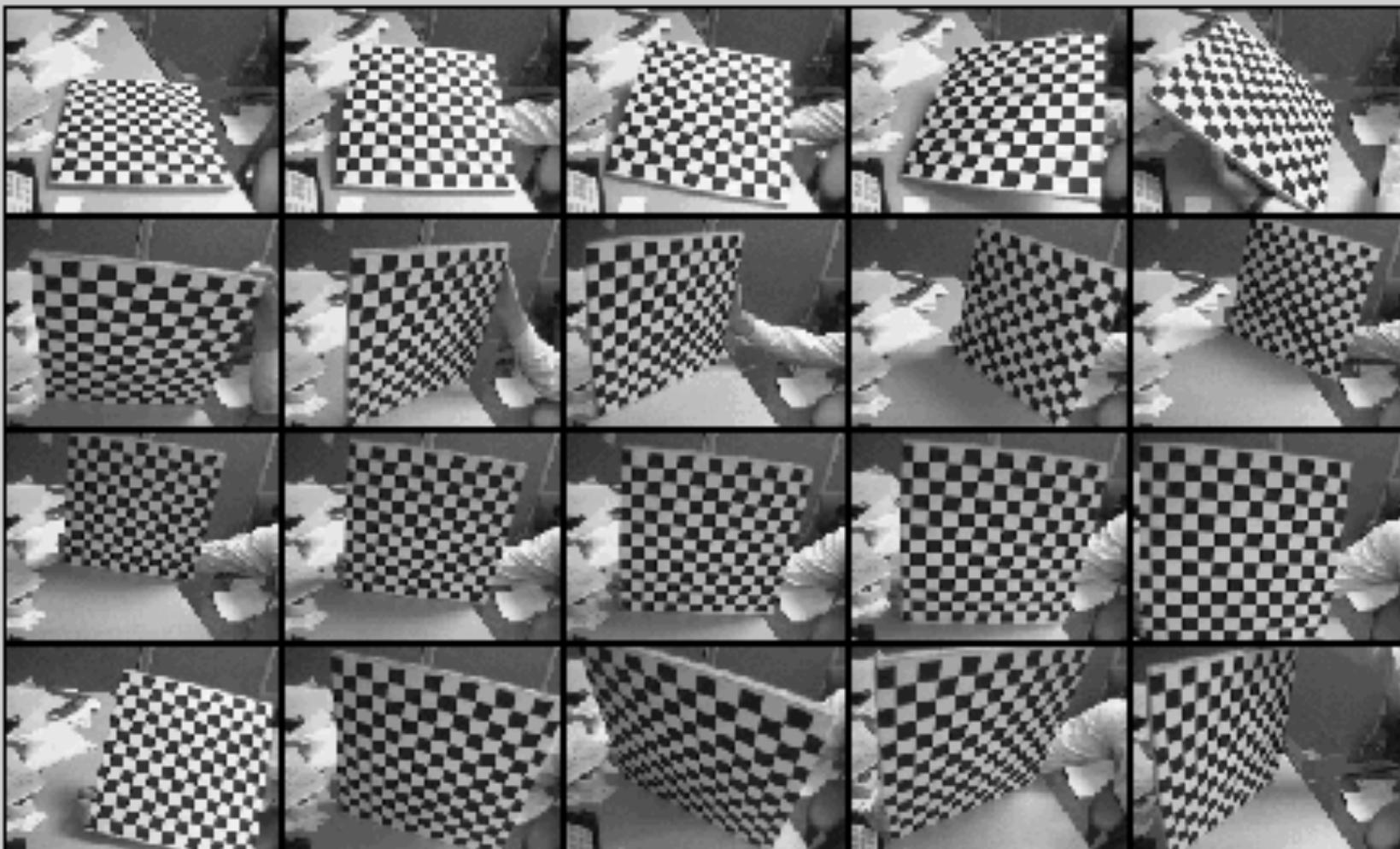
*Camera Calibration Toolbox for OpenCV*  
J. Bouguet – [1998-2000]

[http://www.vision.caltech.edu/bouguetj/calib\\_doc/index.html#examples](http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#examples)



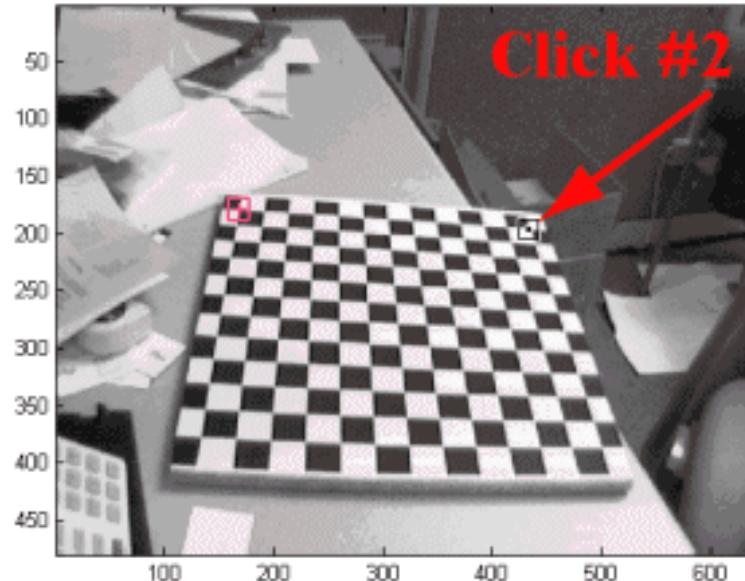
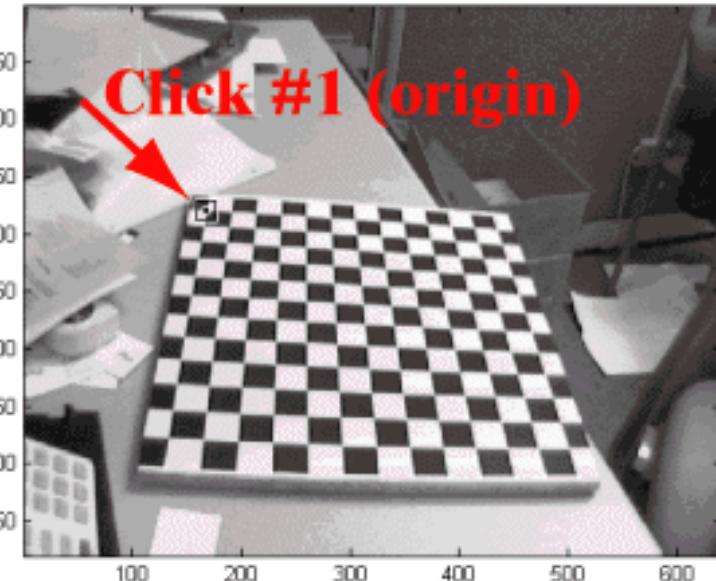
# Calibration Procedure

Calibration images

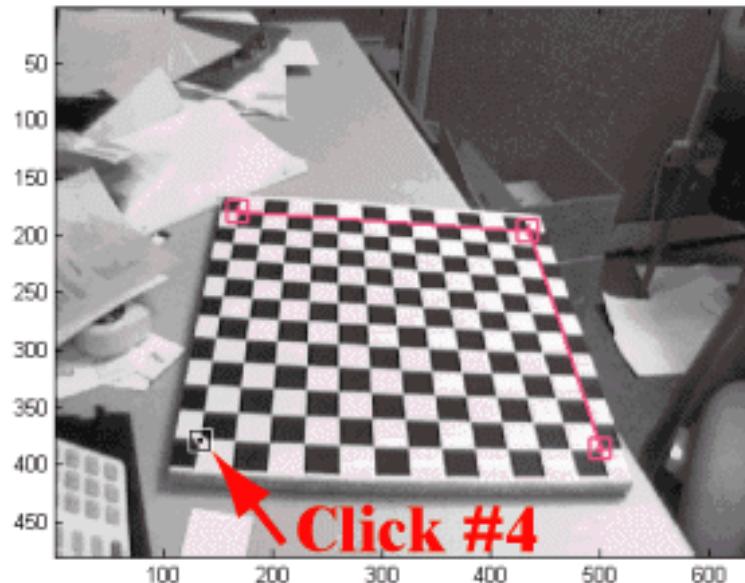
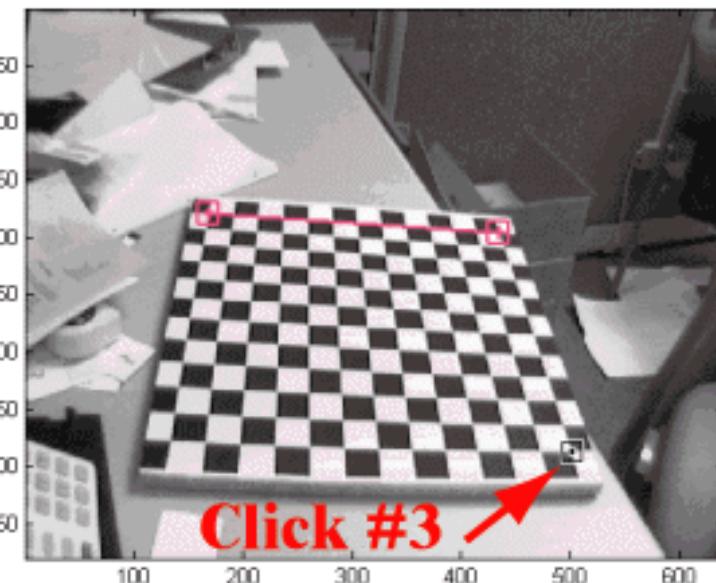


# Calibration Procedure

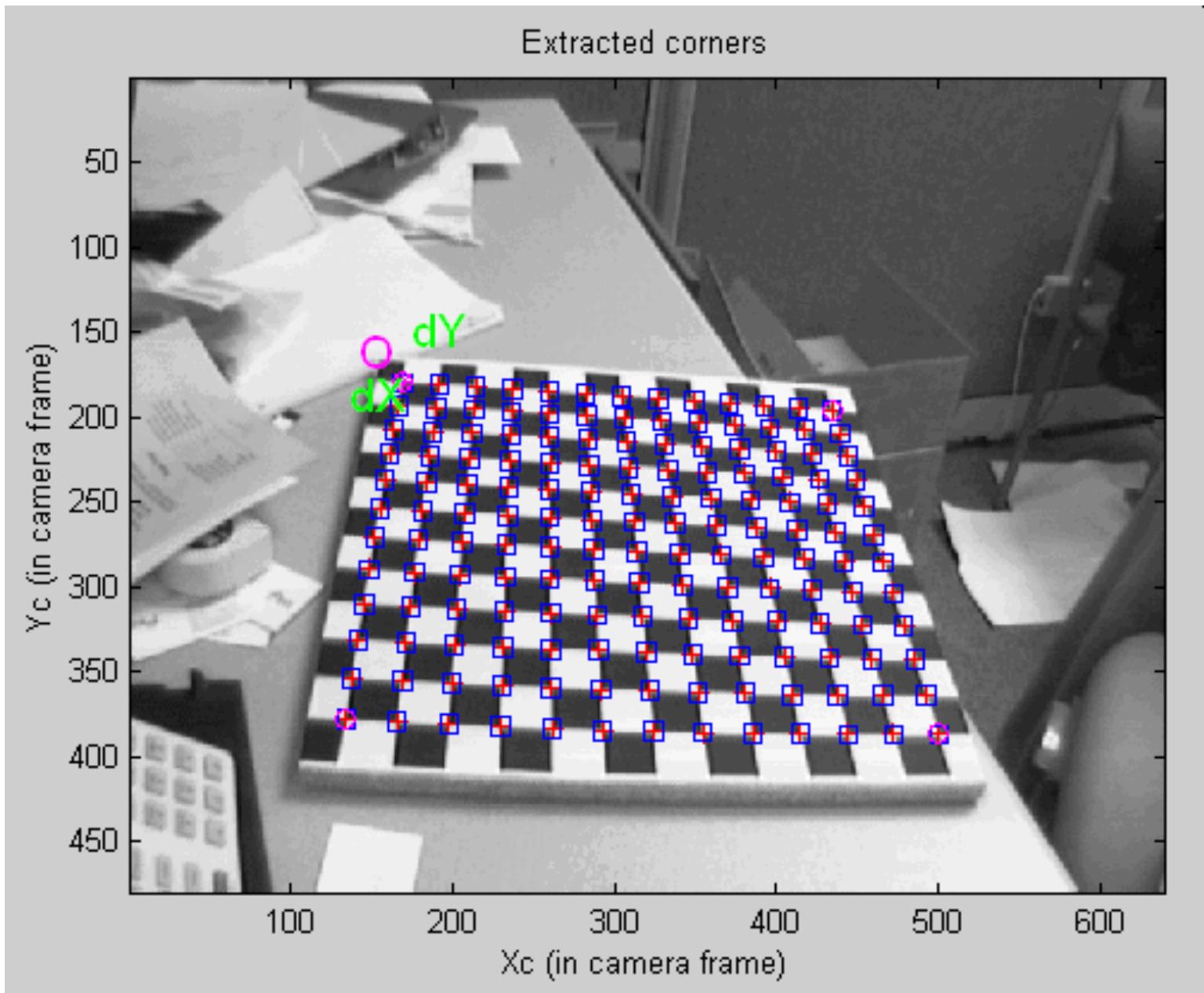
Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1 Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



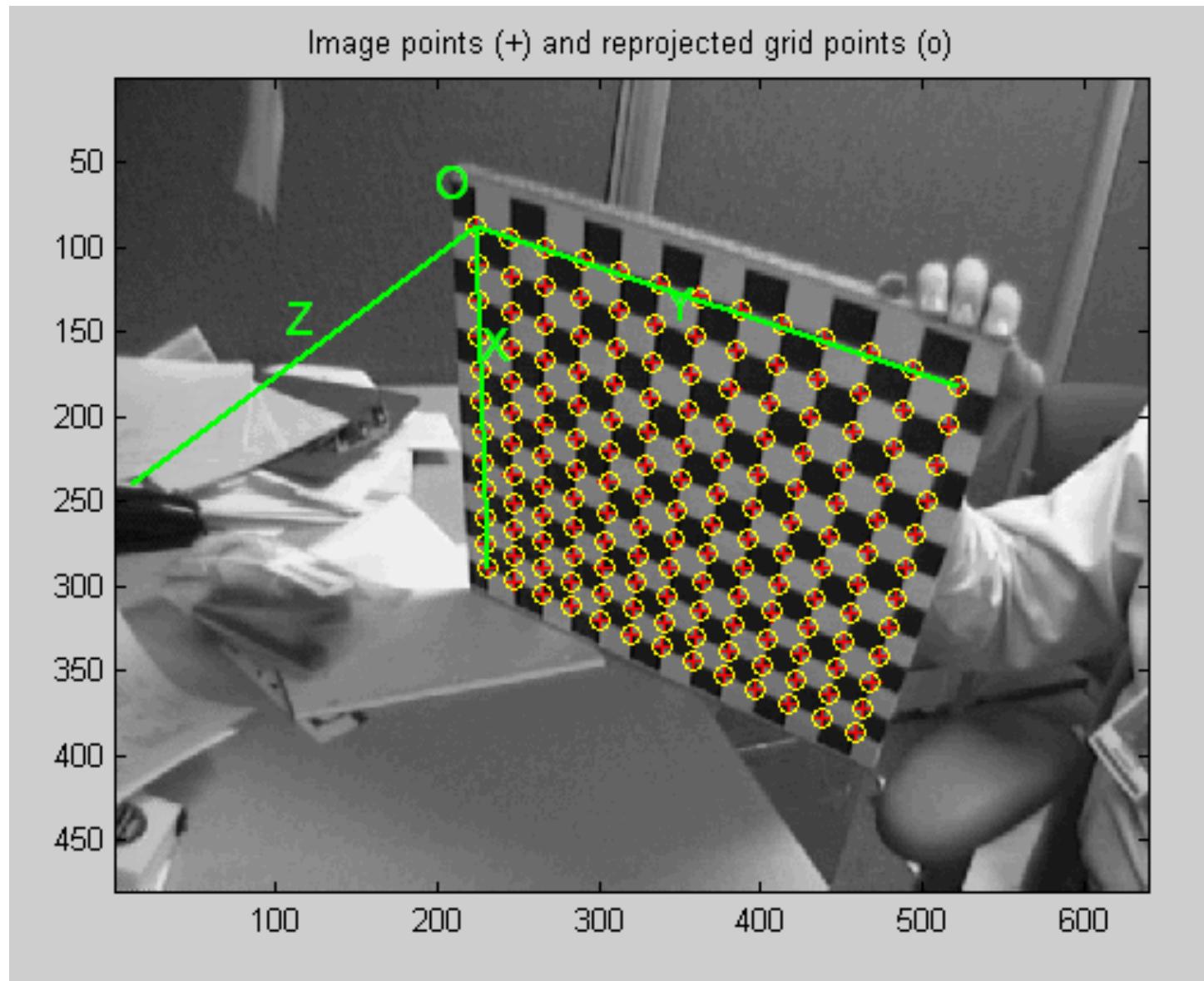
Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1 Click on the four extreme corners of the rectangular pattern (first corner = origin)... Image 1



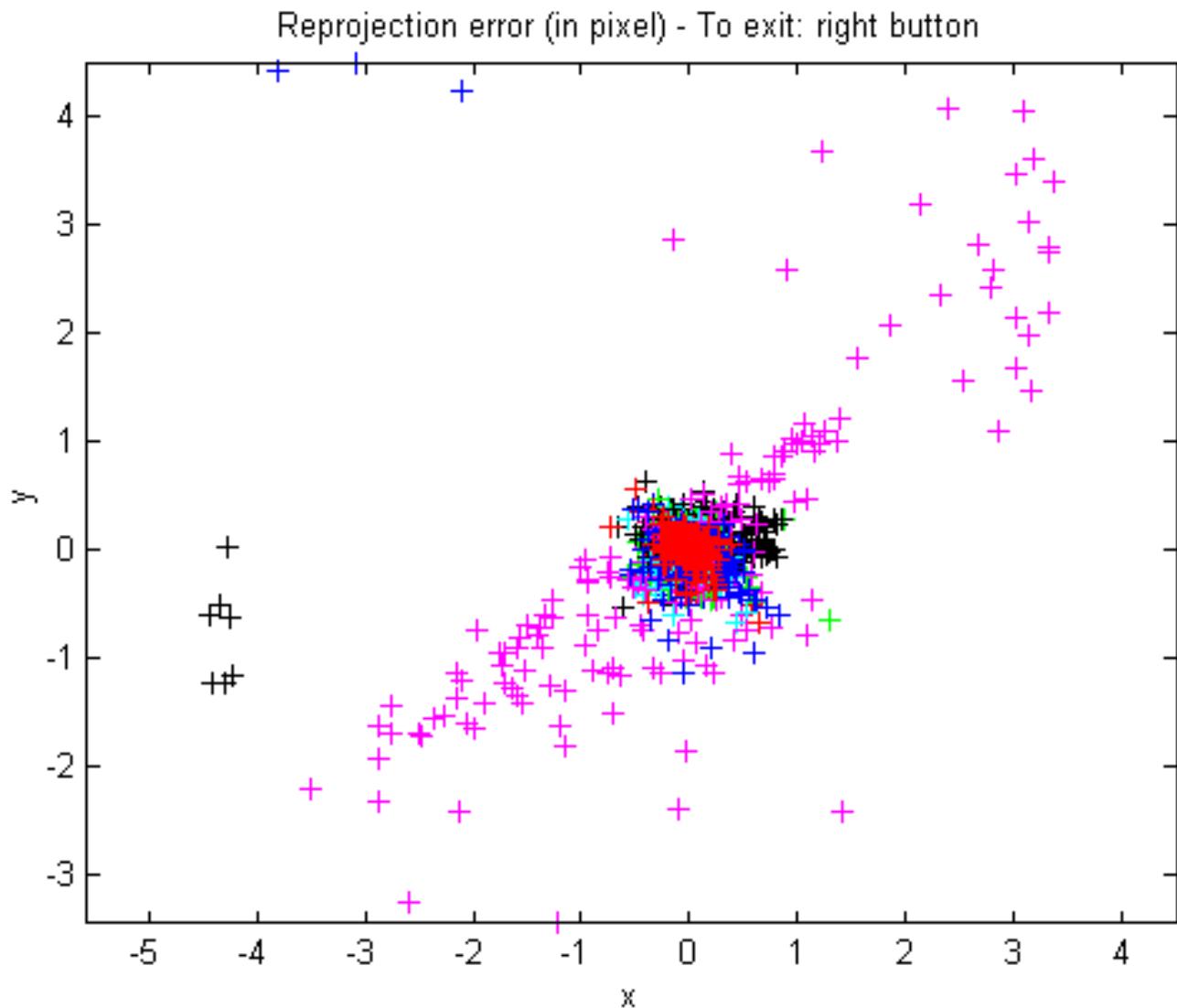
# Calibration Procedure



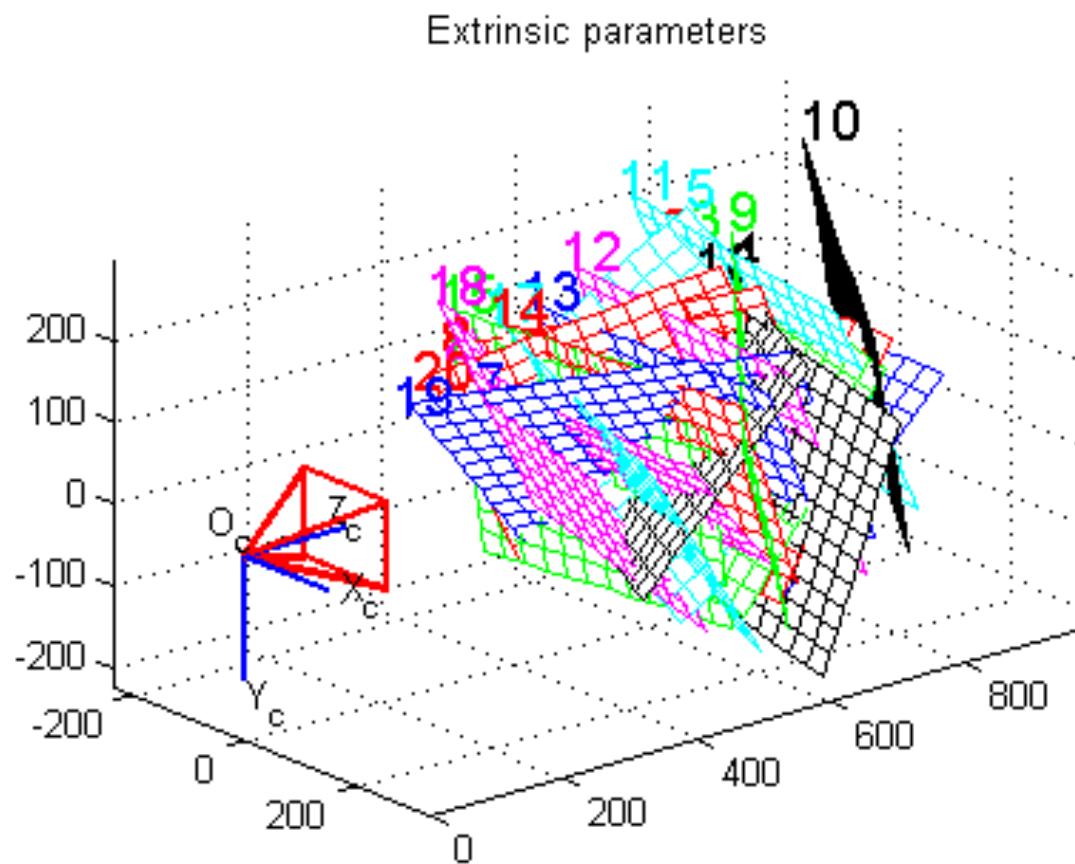
# Calibration Procedure



# Calibration Procedure

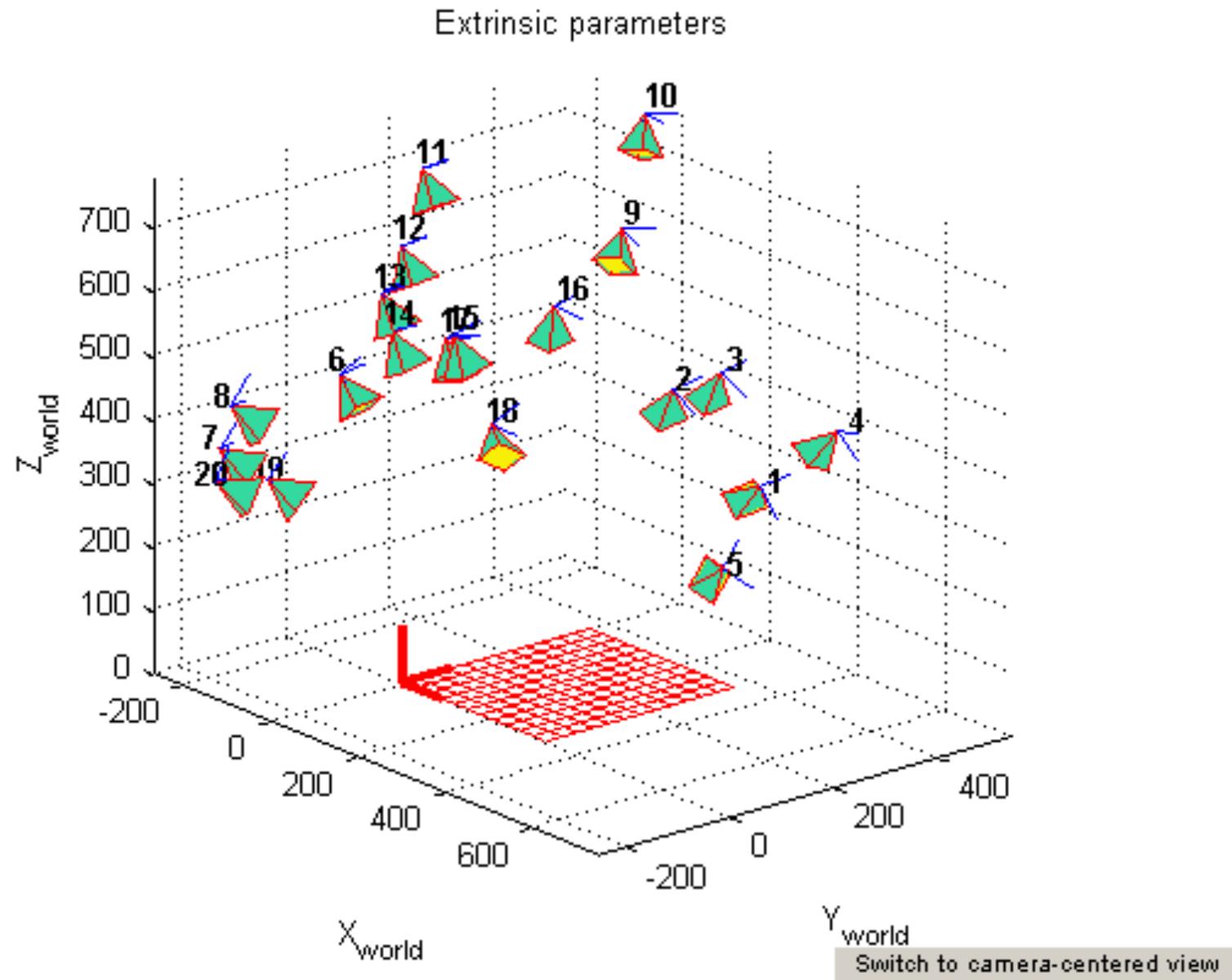


# Calibration Procedure



[Switch to world-centered view](#)

# Calibration Procedure



# Next lecture

- Single view reconstruction