Spring 2020

**Homework: Data-Driven Methods for Complex Dynamical Systems**
DUE: Wednesday, June 10, 2020

Consider the following historical (and classic) data set concerning Canadian lynx and snowshoe hare populations from 1845 to 1903.

| Year | Snowshoe Hare Pelts (thousands) | Canada Lynx Pelts (thousands) |
|---|---|---|
| 1845 | 20 | 32 |
| 1847 | 20 | 50 |
| 1849 | 52 | 12 |
| 1851 | 83 | 10 |
| 1853 | 64 | 13 |
| 1855 | 68 | 36 |
| 1857 | 83 | 15 |
| 1859 | 12 | 12 |
| 1861 | 36 | 6 |
| 1863 | 150 | 6 |
| 1865 | 110 | 65 |
| 1867 | 60 | 70 |
| 1869 | 7 | 40 |
| 1871 | 10 | 9 |
| 1873 | 70 | 20 |

| Year | Snowshoe Hare Pelts (thousands) | Canada Lynx Pelts (thousands) |
|---|---|---|
| 1875 | 100 | 34 |
| 1877 | 92 | 45 |
| 1879 | 70 | 40 |
| 1881 | 10 | 15 |
| 1883 | 11 | 15 |
| 1885 | 137 | 60 |
| 1887 | 137 | 80 |
| 1889 | 18 | 26 |
| 1891 | 22 | 18 |
| 1893 | 52 | 37 |
| 1895 | 83 | 50 |
| 1897 | 18 | 35 |
| 1899 | 10 | 12 |
| 1901 | 9 | 12 |
| 1903 | 65 | 25 |

Figure 1: Population data.

1. Develop a DMD model to forecast the future population states
2. Do a time-delay DMD model to produce a forecast and compare with regular DMD. Determine if it is likely that there are latent variables.
3. Empirical Predator-Prey models such as Lotka-Volterra are commonly used to models such phenomenon. Consider the model $\dot{x} = (b - py)x$ and $\dot{y} = (rx - d)y$. Use the data to fit values of $b, p, r$ and $d$.
4. Find the best fit nonlinear, dynamical systems model to the data using sparse regression.

Download the accompanying ZIP file which includes MATLAB code for solving (i) A reaction-diffusion system of equations, and (ii) The Kuramoto-Sivashinsky (KS) equation.

1. Train a NN that can advance the solution from $t$ to $t + \Delta t$ for the KS equation
2. Compare your evolution trajectories for your NN against using the ODE time-stepper provided with different initial conditions
3. For the reaction-diffusion system, first project to a low-dimensional subspace via the SVD and see how forecasting works in the low-rank variables.

For the Lorenz equations, consider the following.

1. Train a NN to advance the solution from $t$ to $t + \Delta t$ for $\rho = 10, 28$ and $40$. Now see how well your NN works for future state prediction for $\rho = 17$ and $\rho = 35$.
2. See if you can train your NN to identify (for $\rho = 28$) when a transition from one lobe to another is imminent. Determine how far in advance you can make this prediction. (NOTE: you will have to label the transitions in a test set in order to do this task)

Download the data set BZ.mat (which is a snipet from a Belousov-Zhabotinsky chemical oscillator movie – check them out on youtube).

1. get the data: download from the course website (It is next to HW 2)
2. See what you can do with the data (i.e. repeat the first two steps above)

The following code may be helpful for view the data.

```
[m,n,k]=size(BZ_tensor); % x vs y vs time data

for j=1:k
   A=BZ_tensor(:,:,j);
   pcolor(A), shading interp, pause(0.2)
end
```

This is an exploratory homework. So play around with the data and make sure to make lots of plots. Good luck, and have fun.

# Grading and Homework Write Ups and GitHub

This homework should be written as if it were an article/tutorial being prepared for submission, parts of which will be part of your GitHub page. The following is the expected format for homework submission in addition to porting the write-up to your GitHub:

**MAXIMUM NUMBER OF PAGES**: 10 (All your code will be on GitHub)

Title/author/abstract Title, author/address lines, and short (100 words or less) abstract.
Sec. I. Introduction and Overview
Sec. II. Theoretical Background
Sec. III. Algorithm Implementation and Development
Sec. IV. Computational Results
Sec. V. Summary and Conclusions

I will grade based upon how completely you solved the homework as well as neatness and little things like: did you label your graphs and include figure captions.

NOTE 1: The report does not have to be long. But it does have to be complete.

NOTE 2: This report is not for me, it is for you! Specifically, for the future you. So write a nice report so that you could reproduce the results if you need the methods addressed here in another year or more.

A few things should be kept in mind when generating your reports:

1. Use a professional grade word processor (Latex or MSword, for example)
2. For equations: Latex already does a nice job, but in Word, use Microsoft Equation Editor
3. Label your graphs. Include brief figure captions. Reference the figure in the text.
4. Figures should be set flush with the top or bottom of a page.
5. Label all equations.
6. Provide references where appropriate.
7. All coding should be shuffled to Appendix A and B. Reference it when necessary.
8. Always remember: this report is being written for YOU! So be clear and concise.
9. Spellcheck.