

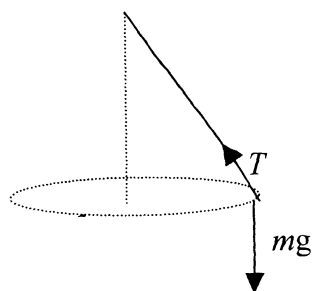
January 2004

Advanced Subsidiary / Advanced Level  
General Certificate of Education

SUBJECT: MECHANICS 6679

PAPER NO: M3

1.



(a)  $(\updownarrow) \quad T \cos 60^\circ = mg \Rightarrow T = 2mg$  \*

(b)  $(\leftrightarrow) \quad T \sin 60^\circ = mr\omega^2$

$$r = L \sin 60^\circ$$

$$\omega = \sqrt{\frac{2g}{L}}$$

(c) Applying Hooke's Law:  $2mg = \frac{\lambda}{(\frac{3}{5}L)} (L - \frac{3}{5}L); \quad \lambda = 3mg$

B1 (1)

M1A1

B1

A1 (4)

M1;A1 (2) [7]

2.

(a) Integrating to find expression for  $v$  [ $v = 2e^{-2t}$  (+c)]

Using initial conditions to find c (-1) or  $v - 1 = [f(t)]'_0$

$$v = 2e^{-2t} - 1 \text{ ms}^{-1}$$

(b) Finding  $t$  when  $v = 0$ ; [ $T = \frac{1}{2} \ln 2, 0.347$ ]

**Integrating**  $v$  w.r.t  $t$ ;  $x = -e^{-2t} - t$  (+c)

Using  $t=0, x=0$  **and** finding value for  $c$  ( $c=1$ )

Substituting  $T$  in equation for  $x$  and finding value for  $x$

[Def. integral:  $x = [-e^{-2t} - t]_0^T$  M1; correct use of limits M1]

$x = \frac{1}{2}(1 - \ln 2) \text{ m}$  (or equiv. **two terms**) or **0.15** or **0.153 m** (awrt)

M1

M1

A1 (3)

M1

M1;A1√

M1

M1

A1 (6) [9]

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3. (a)	$F = \frac{k}{x^2}$ [k may be seen as $Gm_1m_2$ , for example]	M1
	Equating $F$ to $mg$ at $x = R$ ,	M1
	Convincing completion [ $k = mgR^2$ ] to give $F = \frac{mgR^2}{x^2}$ *	A1 (3)
(b)	Equation of motion: $(m)a = (-) \frac{(m)gR^2}{x^2}$ ; using $a = v \frac{dv}{dx}$	B1;M1
	Integrating: $\frac{1}{2} v^2 = \frac{gR^2}{x}$ (+ c) or equivalent	M1A1
	Use of $v^2 = \frac{3gR}{2}$ , $x = R$ to find $c$ [ $c = -\frac{1}{4}gR$ ] or use in def. int.	M1
	Substituting $x = 3R$ and finding $V$ ; $V = \sqrt{\frac{gR}{6}}$	M1;A1 (7)
	<i>Alternative in (b)</i>	[10]
	Work/energy $(-) \int_R^a \frac{mgR^2}{x^2} dx$ ; $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$	B1;M1
	Integrating: $[\frac{mgR^2}{x} - \frac{mgR^2}{R}] = \frac{1}{2}mv^2 - \frac{1}{2}m\frac{3gR}{2}$	M1A1M1
	Final 2 marks as scheme	M1A1

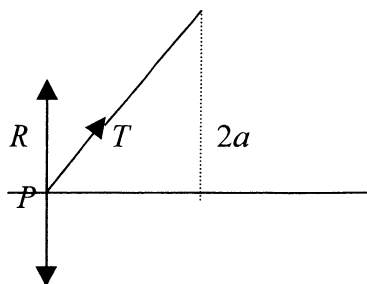
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4.



(a) Length of string  $(L) = \frac{10}{3}a$

B1

$$\text{EPE} = \frac{\frac{1}{2}mg}{2a} (L - a)^2$$

M1

$$= \frac{49}{36} mga$$

A1 (3)

$(1.36mga, 13ma \text{ or } 13.3ma)$

(b) Energy equation:  $\frac{1}{2}mv^2 + \frac{\frac{1}{2}mg}{2a} a^2 = (\frac{49}{36} mga)_C$

M1A1✓

$$v = \frac{2}{3} \sqrt{5ga} \text{ or equivalent } [ \text{allow } 1.49\sqrt{ga}, \frac{14}{3}\sqrt{a} ]$$

A1 (3)

(c)  $T_V = T \sin \theta$  [implied by  $R + T \sin \theta = mg$ ]

B1

$$T_V \text{ in terms of } \theta [ \frac{mg}{2a} (\frac{2a}{\sin \theta} - a) \sin \theta ]$$

M1

$$\text{or in terms of } x \text{ or } AP [ \text{or } \frac{mg(AP - a)}{2} \cdot \frac{2a}{AP} \text{ or } \frac{mg}{2} \frac{x}{a(a+x)} ]$$

(i)  $T = \frac{1}{2}mg(2 - \sin \theta)$  or  $R = \frac{1}{2}mg \sin \theta$

A1

Complete method to show  $R > 0$

M1A1

OR

OR

(ii)  $T = \frac{1}{2}mg(2 - \sin \theta); mg(1 - \frac{a}{AP}); \frac{mgx}{a+x}$

A1

Complete method to show  $T_V < mg$  or that  $T_V \geq mg$  not poss

M1A1

OR

OR

(iii)  $T = \frac{1}{2}mg(2 - \sin \theta)$

A1

as  $\theta$  increases  $T_V$  decreases;  $T_V < T_{V \max} = \frac{7}{10}mg < mg$

M1A1 (5)

[In all cases: For A1 all working correct and arg. convincing]

[11]

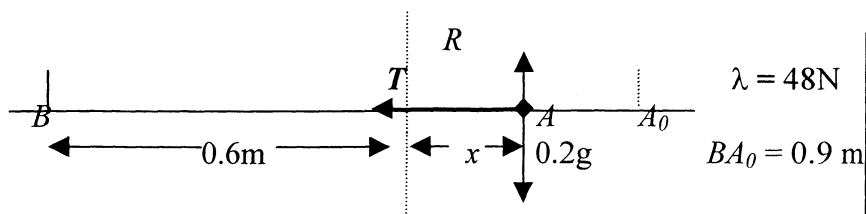
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5. (a)



Applying Hooke's Law in general position [  $T = \frac{48x}{0.6}$  ( $=80x$ ) ] M1

[Note:  $x$  may be other forms e.g. (" $x$ " - 0.6) or "(0.3 - " $x$ ")]

Equation of motion:  $T = (-)0.2 \ddot{x}$  (or  $a$ ) M1

Correct equation: e.g.:  $\frac{48x}{0.6} = -0.2 \ddot{x}$ ,  $0.2 \ddot{x} = \frac{48x}{0.6} (0.3 - "x")$  A1

Equation of the form  $\ddot{x} = -\omega^2 x$ , so SHM M1

Period ( $= \frac{2\pi}{\omega_c}$ ) =  $\frac{2\pi}{20} = \frac{\pi}{10}$  \* (no incorrect working seen) A1 (5)

(b)  $\max v = a\omega$ ;  $= 0.3 \times 20 = 6 \text{ ms}^{-1}$  M1;A1(2)  
[

(c) Using  $x = a \cos 20t$  or  $x = a \sin 20T$  or  $\cos \alpha = \frac{x}{a}$  M1

Using  $x = 0.15$ :  $\cos 20t = \frac{0.15}{a}$  or  $\sin 20T = \frac{0.15}{a}$  or  $\cos \alpha = \frac{0.15}{a}$  M1

Either  $t = \frac{\pi}{60}$ , ( $\frac{5\pi}{60}$ ) or  $T = \frac{\pi}{120}$  or  $\alpha = \frac{\pi}{3}$  A1

Complete method for time:

$t_2 - t_1$ , or  $\frac{\pi}{10} - 2t_1$ , or  $\frac{\pi}{20} + 2T$  or  $\frac{2\pi - 2\alpha}{2\pi} = \frac{t}{(\pi/10)}$  M1

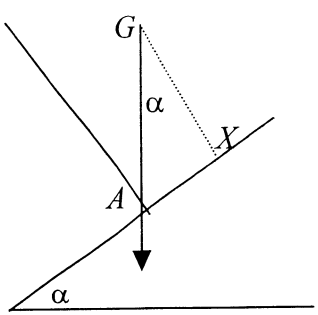
Time =  $\frac{\pi}{15}$  s (must be in terms of  $\pi$ ) A1 (5) [12]

January 2004

Advanced Subsidiary / Advanced Level  
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6. (a)
- |   | Cylinder  | Hemisphere                             | S                                     |        |
|---|---|--|---------------------------------------|--------|
| Masses  | $(\rho)\pi(2a)^2(\frac{3}{2}a)$<br>[6 $\pi a^3$ ]<br>[18] | $(\rho)\frac{2}{3}\pi a^3$<br>[2]      | $(\rho)(\frac{16}{3}\pi a^3)$<br>[16] | M1A1   |
| [M1 for attempt at C, H and S = C - H masses]   |   |  |                                       |        |
| Distances of CM from  |   |  |                                       |        |
| O   | $\frac{3}{4}a$  | $\frac{3}{8}a$                         | $\bar{x}$                             | B1B1   |
| or lower face   | $\frac{3}{4}a$  | $\frac{a}{2} + \frac{5a}{8}$           | $\bar{x}'$                            |        |
| Moments equation: $6\pi a^3(\frac{3}{4}a) - \frac{2}{3}\pi a^3(\frac{3}{8}a) = \frac{16}{3}\pi a^3 \bar{x}$ |   |  |                                       |        |
|   |   | $\bar{x} = \frac{51}{64}a$<br>(0.797a) |                                       | A1 (6) |
- (b)
- 

G above "A" seen or implied  
or  $mg \sin \alpha (GX) = mg \cos \alpha (AX)$

$$\tan \alpha = \frac{AX}{XG} = \frac{2a}{\frac{3}{2}a - \bar{x}}$$
- $[GX = \frac{3}{2}a - \frac{51}{64}a = \frac{45}{64}a, \tan \alpha = \frac{128}{45}] \quad \alpha = 70.6^\circ$
- (c) Finding F and R :  $R = mg \cos \beta, F = mg \sin \beta$
- Using  $F = \mu R$  and finding  $\tan \beta$  [= 0.8]
- $\beta = 38.7^\circ$

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**PAPER NO: M3**

7.	(a) Attempt at conservation of energy: $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga \sin \theta$	M1
	$v^2 = \frac{3}{2}ga + 2ga \sin \theta$	A1 (2)
	(b) Radial equation: $T - mg \sin \theta = m \frac{v^2}{a}$	M1A1
	$T = \frac{3mg}{2}(1 + 2\sin \theta) \text{ any form}$	A1✓ (3)
	(c) Setting $T = 0$ and solving trig. equation; $(\sin \theta = -\frac{1}{2}) \Rightarrow \theta = 210^\circ *$	M1;A1(2)
	(d) Setting $v = 0$ in (a) and (i) solving for $\theta$	M1
	$\sin \theta = -\frac{3}{4}$ so not complete circle	A1 (2)
	or (ii) looking at energy at top ( $mga$ ) and at start ( $\frac{3}{4}mga$ ) so not possible	
	or substituting $\theta = 270^\circ$ in (a); $v^2 < 0$ so not possible to complete	
	(e) No change in PE $\Rightarrow$ no change in KE (Cof E) so $v = u$	B1 (1)
	(f) When string becomes slack, $v^2 = \frac{1}{2}ga$ [ $\sin \theta = -\frac{1}{2}$ in (a)]	B1✓
	Working horizontally : $\sqrt{\frac{ga}{2}} \cos 60^\circ = \sqrt{\frac{3ga}{2}} \cos \phi$	M1A1✓
	or vertically : $\frac{3}{2}ag \sin^2 \phi = \frac{1}{2}ag \cdot \sin^2 60^\circ + ag (= \frac{11ag}{8})$	
	or finding $V_V$ and $V_H$ and using to find $\tan \phi$	
	$\phi = 73^\circ$ or $73.2^\circ$	A1 (4) [14]