

June 2006
6684 Statistics S2
Mark Scheme

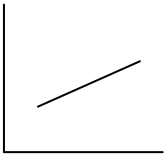
Question Number	Scheme	Marks
1.	<p>(a) Saves time / cheaper / easier any one or <u>A census/asking all members</u> takes a long time or is expensive or difficult to carry out</p> <p>(b) <u>List, register or database of all club members/golfers</u> or <u>Full membership list</u></p> <p>(c) Club <u>member(s)</u></p>	<p>B1 (1)</p> <p>B1 (1)</p> <p>B1 (1)</p> <p>Total 3 marks</p>
2.	<p>(a) $P(L < -2.6) = 1.4 \times \frac{1}{8} = \frac{7}{40}$ or 0.175 or equivalent</p> <p>(b) $P(L < -3.0 \text{ or } L > 3.0) = 2 \times \left(1 \times \frac{1}{8}\right) = \frac{1}{4}$ M1 for 1/8 seen</p> <p>(c) $P(\text{within 3mm}) = 1 - \frac{1}{4} = 0.75$ B(20,0.75) recognises binomial Using B(20,p) Let X represent number of rods within 3mm $P(X \leq 9 / p = 0.25) \text{ or } 1 - P(X \leq 10 / p = 0.75)$ $= 0.9861$ awrt 0.9861</p>	<p>B1 (1)</p> <p>M1;A1 (2)</p> <p>B1 M1 M1 A1 (4) Total 7 marks</p>

Question Number	Scheme	Marks
3.	<p>(a) Let X represent the number of properties sold in a week</p> <p>$\therefore X \sim P_0(7)$ must be in part a</p> <p>Sales occur independently/randomly, singly, at a constant rate context needed once</p> <p>(b) $P(X = 5) = P(X \leq 5) - P(X \leq 4)$ or $\frac{7^5 e^{-7}}{5!}$</p> <p>$= 0.3007 - 0.1730$</p> <p>$= 0.1277$ awrt 0.128</p> <p>(c) $P(X > 181) \approx P(Y \geq 181.5)$ where $Y \sim N(168, 168)$ $N(168, 168)$</p> <p>$= P\left(z \geq \frac{181.5 - 168}{\sqrt{168}}\right)$ ± 0.5 stand with μ and σ</p> <p>Give A1 for 1.04 or correct expression</p> <p>$= P(z \geq 1.04)$</p> <p>$= 1 - 0.8508$ attempt correct area $1-p$ where $p > 0.5$</p> <p>$= 0.1492$ awrt 0.149</p>	<p>B1</p> <p>B1 B1 (3)</p> <p>M1</p> <p>A1 (2)</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (6)</p> <p>Total 11 marks</p>

Question Number	Scheme	Marks
4.	<p>(a) Let X represent the number of breakdowns in a week.</p> <p>$X \sim P_o(1.25)$ implied</p> <p>$P(X < 3) = P(0) + P(1) + P(2)$ or $P(X \leq 2)$</p> $= e^{-1.25} \left(1 + 1.25 + \frac{(1.25)^2}{2!} \right)$ <p>$= 0.868467 \dots \dots$ awrt 0.868 or 0.8685</p> <p>(b) $H_0 : \lambda = 1.25 ; H_1 : \lambda \neq 1.25$ (or $H_0 : \lambda = 5 ; H_1 : \lambda \neq 5$) λ or μ</p> <p>Let Y represent the number of breakdowns in 4 weeks</p> <p>Under $H_0, Y \sim P_o(5)$ may be implied</p> <p>$P(Y \geq 11) = 1 - P(Y \leq 10)$ or $P(X \geq 11) = 0.0137$</p> <p>One needed for M</p> <p>$P(X \geq 10) = 0.0318$</p> <p>$= 0.0137$ CR $X \geq 11$</p> <p>$0.0137 < 0.025, 0.0274 < 0.05, 0.9863 > 0.975, 0.9726 > 0.95$ or $11 \geq 11$ any</p> <p>.allow %</p> <p>$\sqrt{\quad}$ from H_1</p> <p>Evidence that the rate of breakdowns has changed /decreased context</p> <p>From their p</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1 (4)</p> <p>B1 B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>B1√ (7)</p> <p>Total 11 marks</p>

Question Number	Scheme	Marks
5. (a)	Binomial Let X represent the number of green mugs in a sample	B1 (1)
(b)	$X \sim B(10, 0.06)$ may be implied or seen in part a $P(X=3) = {}^{10}C_3(0.06)^3(0.94)^7$ ${}^{10}C_3(p)^3(1-p)^7$ $= 0.016808\dots$ awrt 0.0168	B1 M1 A1 (3)
(c) (i)	Let X represent number of green mugs in a sample of size 125 $X \sim P_0(125 \times 0.06 = 7.5)$ may be implied $P(10 \leq X \leq 13) = P(X \leq 13) - P(X \leq 9)$ $= 0.9784 - 0.7764$ $= 0.2020$ awrt 0.202	B1 M1 A1 (3)
(ii)	$P(10 \leq X \leq 13) \approx P(9.5 \leq Y \leq 13.5)$ where $Y \sim N(7.5, 7.05)$ 7.05 9.5, 13.5 $= P\left(\frac{9.5-7.5}{\sqrt{7.05}} \leq z \leq \frac{13.5-7.5}{\sqrt{7.05}}\right)$ ± 0.5 stand. both values or both correct expressions. $= P(0.75 \leq z \leq 2.26)$ awrt 0.75 and 2.26 $= 0.2147$ awrt 0.214 or 0.215	B1 B1 M1 M1 A1 A1 (6) Total 13 marks

Question Number	Scheme	Marks
6	<p>(a) $\int_1^4 \frac{1+x}{k} dx = 1$</p> <p>$\therefore \left[\frac{x}{k} + \frac{x^2}{2k} \right]_1^4 = 1$</p> <p>$k = \frac{21}{2} *$</p> <p>(b) $P(X \leq x_0) = \int_1^{x_0} \frac{2}{21}(1+x)$</p> <p>$= \left[\frac{2x}{21} + \frac{x^2}{21} \right]_1^{x_0}$</p> <p>$= \frac{2x_0 + x_0^2 - 3}{21} \text{ or } \frac{(3+x)(x-1)}{21}$</p> <p>$F(x) = \begin{cases} 0, & x < 1 \\ \frac{x^2 + 2x - 3}{21} & 1 \leq x < 4 \\ 1 & x \geq 4 \end{cases}$</p> <p>(c) $E(X) = \int_1^4 \frac{2x}{21}(1+x)dx$</p> <p>$= \left[\frac{x^2}{21} + \frac{2x^3}{63} \right]_1^4$</p> <p>$= \frac{171}{63} = 2\frac{5}{7} = \frac{19}{7} = 2.7142....$</p>	<p>$\int f(x) = 1$ Area = 1</p> <p>correct integral/correct expression</p> <p>cs0</p> <p>$\int f(x)$ variable limit or +C</p> <p>correct integral + limit of 1</p> <p>May have k in</p> <p>middle; ends</p> <p>valid attempt $\int xf(x)$</p> <p>x^2 and x^3</p> <p>correct integration</p> <p>awrt 2.71</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>B1✓; B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(3)</p> <p>(5)</p> <p>(3)</p>

Question Number	Scheme	Marks
(d)	$F(m) = 0.5 \Rightarrow \frac{x^2 + 2x - 3}{21} = \frac{1}{2}$ <p>putting their $F(x) = 0.5$</p> $\therefore 2x^2 + 4x - 27 = 0 \quad \text{or equiv}$ $\therefore x = \frac{-4 \pm \sqrt{16 - 4 \cdot 2(-27)}}{4}$ <p>attempt their 3 term quadratic</p> $\therefore x = -1 \pm 3.8078\dots$ <p>i.e. $x = 2.8078\dots$ awrt 2.81</p>	<p>M1</p> <p>M1</p> <p>A1 (3)</p>
(e)	Mode = 4	B1 (1)
(f)	<p><u>Mean < median < mode</u> (\Rightarrow negative skew)</p> <p>Or</p> <p><u>Mean < median</u></p> <p>allow numbers in place of words</p>  <p>w diagram but line must not cross y axis</p>	<p>B1 (1)</p> <p>Total 16 marks</p>

Question Number	Scheme	Marks
7. (a)	<p>Let X represent the number of bowls with minor defects.</p> <p>$\therefore X \sim B; (25, 0.20)$ may be implied</p> <p>$P(X \leq 1) = 0.0274$ or $P(X = 0) = 0.0038$ need to see at least one. prob for $X \leq$ no For M1</p> <p>$P(X \leq 9) = 0.9827; \Rightarrow P(X \geq 10) = 0.0173$ either</p> <p>$\therefore \text{CR is } \{X \leq 1 \cup X \geq 10\}$</p>	<p>B1; B1</p> <p>M1A1</p> <p>A1</p> <p>A1 (6)</p>
b)	<p>Significance level = $0.0274 + 0.0173$</p> <p>$= 0.0447$ or 4.477% awrt 0.0447</p>	<p>B1 (1)</p>
c)	<p>$H_0 : p = 0.20; H_1 : p < 0.20;$</p> <p>Let Y represent number of bowls with minor defects</p> <p>Under $H_0 Y \sim B(20, 0.20)$ may be implied</p> <p>$P(Y \leq 2)$ or $P(Y \leq 2) = 0.2061$ either</p> <p>$= 0.2061$ $P(Y \leq 1) = 0.0692$</p> <p>CR $Y \leq 1$</p> <p>$0.2061 > 0.10$ or $0.7939 < 0.9$ or $2 > 1$ their p</p> <p>Insufficient evidence to suggest that the proportion of defective bowls has decreased.</p>	<p>B1 B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>B1✓ (7)</p>