Mark Scheme (Results)

Summer 2009

GCE

GCE Mathematics (6666/01)

A PEARSON COMPANY

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6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
Q1	$\text{f}(x)=\dfrac{1}{\sqrt{(4+x)}}=(4+x)^{-\frac{1}{2}}$	M1
	$=(4)^{-\frac{1}{2}}(1+\ \dots\ )^{\dots}$ $\qquad \dfrac{1}{2}(1+\ \dots\ )^{\dots}$ or $\dfrac{1}{2\sqrt{(1+\dots)}}$	B1
	$=\ \dots\left(1+\left(-\tfrac{1}{2}\right)\left(\dfrac{x}{4}\right)+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(\dfrac{x}{4}\right)^2+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(\dfrac{x}{4}\right)^3+\ \dots\right)$	M1 A1ft
	ft their $\left(\dfrac{x}{4}\right)$	
	$=\dfrac{1}{2}-\dfrac{1}{16}x,+\dfrac{3}{256}x^2-\dfrac{5}{2048}x^3+\ \dots$	A1, A1 (6)
		[6]
	Alternative	
	$\text{f}(x)=\dfrac{1}{\sqrt{(4+x)}}=(4+x)^{-\frac{1}{2}}$	M1
	$=\underline{4^{-\frac{1}{2}}}+\left(-\tfrac{1}{2}\right)4^{-\frac{3}{2}}x+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2}4^{-\frac{5}{2}}x^2+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1.2.3}4^{-\frac{7}{2}}x^3+\ \dots$	<u>B1</u> M1 A1
	$=\dfrac{1}{2}-\dfrac{1}{16}x,+\dfrac{3}{256}x^2-\dfrac{5}{2048}x^3+\ \dots$	A1, A1 (6)

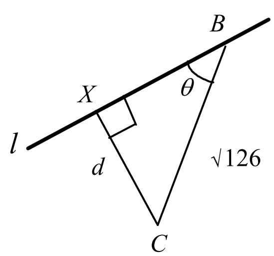
Question Number	Scheme		Marks	
Q2 (a)	1.14805	awrt 1.14805	B1	(1)
(b)	$A \approx \frac{1}{2} \times \frac{3\pi}{8}(\ \ldots\ )$		B1	
	$= \ \ldots\ \left(3 + 2\left(2.77164 + 2.12132 + 1.14805\right) + 0\right)$	0 can be implied	M1	
	$= \ \frac{3\pi}{16}\left(3 + 2\left(2.77164 + 2.12132 + 1.14805\right)\right)$	ft their (a)	A1ft	
	$= \frac{3\pi}{16} \times 15.08202\ \ldots\ \ = 8.884$	cao	A1	(4)
(c)	$\int 3\cos\left(\frac{x}{3}\right)\mathrm{d}x = \frac{3\sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$		M1 A1	
	$= 9\sin\left(\frac{x}{3}\right)$			
	$A = \left[9\sin\left(\frac{x}{3}\right)\right]_0^{\frac{3\pi}{2}} = 9 - 0 = 9$	cao	A1	(3)
				[8]

Question Number	Scheme	Marks
Q3 (a)	$\mathrm{f}(x)=\dfrac{4-2x}{(2x+1)(x+1)(x+3)}=\dfrac{A}{2x+1}+\dfrac{B}{x+1}+\dfrac{C}{x+3}$	
	$4-2x=A(x+1)(x+3)+B(2x+1)(x+3)+C(2x+1)(x+1)$	M1
	A method for evaluating one constant	M1
	$x\to -\frac{1}{2},\quad 5=A\left(\frac{1}{2}\right)\left(\frac{5}{2}\right) \Rightarrow A=4$ any one correct constant	A1
	$x\to -1,\quad 6=B(-1)(2) \Rightarrow B=-3$	
	$x\to -3,\quad 10=C(-5)(-2) \Rightarrow C=1$ all three constants correct	A1 (4)
(b)	(i) $\displaystyle\int\left(\frac{4}{2x+1}-\frac{3}{x+1}+\frac{1}{x+3}\right)\mathrm{d}x$	
	$=\dfrac{4}{2}\ln(2x+1)-3\ln(x+1)+\ln(x+3)+C$ A1 two ln terms correct	M1 A1ft
	All three ln terms correct and "+C" ; ft constants	A1ft (3)
	(ii) $\left[2\ln(2x+1)-3\ln(x+1)+\ln(x+3)\right]_0^2$	
	$=(2\ln 5-3\ln 3+\ln 5)-(2\ln 1-3\ln 1+\ln 3)$	M1
	$=3\ln 5-4\ln 3$	
	$=\ln\left(\dfrac{5^3}{3^4}\right)$	M1
	$=\ln\left(\dfrac{125}{81}\right)$	A1 (3)
		[10]

Question Number	Scheme	Marks
Q4 (a)	$\mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x} - 2y\,\mathrm{e}^{-2x} = 2 + 2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$ A1 correct RHS	M1 A1
	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(y\,\mathrm{e}^{-2x}\right) = \mathrm{e}^{-2x}\dfrac{\mathrm{d}y}{\mathrm{d}x} - 2y\,\mathrm{e}^{-2x}$	B1
	$\left(\mathrm{e}^{-2x} - 2y\right)\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2 + 2y\,\mathrm{e}^{-2x}$	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2 + 2y\,\mathrm{e}^{-2x}}{\mathrm{e}^{-2x} - 2y}$	A1 (5)
(b)	At P , $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2 + 2\mathrm{e}^{0}}{\mathrm{e}^{0} - 2} = -4$	M1
	Using $mm' = -1$	
	$m' = \dfrac{1}{4}$	M1
	$y - 1 = \dfrac{1}{4}(x - 0)$	M1
	$x - 4y + 4 = 0$ or any integer multiple	A1 (4)
		[9]
	Alternative for (a) *differentiating implicitly with respect to y.*	
	$\mathrm{e}^{-2x} - 2y\,\mathrm{e}^{-2x}\dfrac{\mathrm{d}x}{\mathrm{d}y} = 2\dfrac{\mathrm{d}x}{\mathrm{d}y} + 2y$ A1 correct RHS	M1 A1
	$\dfrac{\mathrm{d}}{\mathrm{d}y}\left(y\,\mathrm{e}^{-2x}\right) = \mathrm{e}^{-2x} - 2y\,\mathrm{e}^{-2x}\dfrac{\mathrm{d}x}{\mathrm{d}y}$	B1
	$\left(2 + 2y\,\mathrm{e}^{-2x}\right)\dfrac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^{-2x} - 2y$	M1
	$\dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{\mathrm{e}^{-2x} - 2y}{2 + 2y\,\mathrm{e}^{-2x}}$	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2 + 2y\,\mathrm{e}^{-2x}}{\mathrm{e}^{-2x} - 2y}$	A1 (5)

Question Number	Scheme	Marks
Q5 (a)	$\frac{dx}{dt} = -4\sin 2t$, $\frac{dy}{dt} = 6\cos t$	B1, B1
	$\frac{dy}{dx} = -\frac{6\cos t}{4\sin 2t}$ $\left(= -\frac{3}{4\sin t}\right)$	M1
	At $t = \frac{\pi}{3}$, $m = -\frac{3}{4\times\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87	A1 (4)
(b)	Use of $\cos 2t = 1 - 2\sin^2 t$	M1
	$\cos 2t = \frac{x}{2}$, $\sin t = \frac{y}{6}$	
	$\frac{x}{2} = 1 - 2\left(\frac{y}{6}\right)^2$	M1
	Leading to $y = \sqrt{(18-9x)}$ $\left(= 3\sqrt{(2-x)}\right)$ cao	A1
	$-2 \le x \le 2$ $k = 2$	B1 (4)
(c)	$0 \le f(x) \le 6$ either $0 \le f(x)$ or $f(x) \le 6$	B1
	Fully correct. Accept $0 \le y \le 6$, $[0, 6]$	B1 (2)
		[10]
	Alternatives to (a) *where the parameter is eliminated*	
	① $y = (18-9x)^{\frac{1}{2}}$	
	$\frac{dy}{dx} = \frac{1}{2}(18-9x)^{-\frac{1}{2}} \times (-9)$	B1
	At $t = \frac{\pi}{3}$, $x = \cos\frac{2\pi}{3} = -1$	B1
	$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{(27)}} \times -9 = -\frac{\sqrt{3}}{2}$	M1 A1 (4)
	② $y^2 = 18 - 9x$	
	$2y\frac{dy}{dx} = -9$	B1
	At $t = \frac{\pi}{3}$, $y = 6\sin\frac{\pi}{3} = 3\sqrt{3}$	B1
	$\frac{dy}{dx} = -\frac{9}{2\times 3\sqrt{3}} = -\frac{\sqrt{3}}{2}$	M1 A1 (4)

Question Number	Scheme	Marks
Q6 (a)	$\int \sqrt{(5-x)}\,dx = \int (5-x)^{\frac{1}{2}}\,dx = \dfrac{(5-x)^{\frac{3}{2}}}{-\frac{3}{2}} \quad (+C)$	M1 A1 (2)
	$\left(= -\dfrac{2}{3}(5-x)^{\frac{3}{2}} + C\right)$	
(b)	(i) $\int (x-1)\sqrt{(5-x)}\,dx = -\dfrac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \dfrac{2}{3}\int (5-x)^{\frac{3}{2}}\,dx$	M1 A1ft
	$= \quad \dots \quad + \dfrac{2}{3} \times \dfrac{(5-x)^{\frac{5}{2}}}{-\frac{5}{2}} \quad (+C)$	M1
	$= -\dfrac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \dfrac{4}{15}(5-x)^{\frac{5}{2}} \quad (+C)$	A1 (4)
	(ii) $\left[-\dfrac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \dfrac{4}{15}(5-x)^{\frac{5}{2}}\right]_1^5 = (0-0) - \left(0 - \dfrac{4}{15} \times 4^{\frac{5}{2}}\right)$	
	$= \dfrac{128}{15}\left(= 8\dfrac{8}{15} \approx 8.53\right)$ awrt 8.53	M1 A1 (2)
		[8]
	Alternatives for (b) *and* (c)	
	(b) $u^2 = 5 - x \Rightarrow 2u\dfrac{du}{dx} = -1 \quad \left(\Rightarrow \dfrac{dx}{du} = -2u\right)$	
	$\int (x-1)\sqrt{(5-x)}\,dx = \int (4-u^2)u\dfrac{dx}{du}\,du = \int (4-u^2)u(-2u)\,du$	M1 A1
	$= \int (2u^4 - 8u^2)\,du = \dfrac{2}{5}u^5 - \dfrac{8}{3}u^3 \quad (+C)$	M1
	$= \dfrac{2}{5}(5-x)^{\frac{5}{2}} - \dfrac{8}{3}(5-x)^{\frac{3}{2}} \quad (+C)$	A1
	(c) $x = 1 \Rightarrow u = 2, \quad x = 5 \Rightarrow u = 0$	
	$\left[\dfrac{2}{5}u^5 - \dfrac{8}{3}u^3\right]_2^0 = (0-0) - \left(\dfrac{64}{5} - \dfrac{64}{3}\right)$	M1
	$= \dfrac{128}{15}\left(= 8\dfrac{8}{15} \approx 8.53\right)$ awrt 8.53	A1 (2)

Question Number	Scheme	Marks
Q7 (a)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$	M1
	$\mathbf{r} = \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ *or* $\mathbf{r} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ accept equivalents	M1 A1ft (3)
(b)	$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 9 \\ 9 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -10 \end{pmatrix}$ or $\overrightarrow{BC} = \begin{pmatrix} -1 \\ -5 \\ 10 \end{pmatrix}$	
	$CB = \sqrt{\left(1^2 + 5^2 + (-10)^2\right)} = \sqrt{(126)} \quad (= 3\sqrt{14} \approx 11.2)$ awrt 11.2	M1 A1 (2)
(c)	$\overrightarrow{CB}.\overrightarrow{AB} = \left\lvert\overrightarrow{CB}\right\rvert \left\lvert\overrightarrow{AB}\right\rvert \cos\theta$	
	$(\pm)(2+5+20) = \sqrt{126}\sqrt{9}\cos\theta$	M1 A1
	$\cos\theta = \dfrac{3}{\sqrt{14}} \Rightarrow \theta \approx 36.7°$ awrt 36.7°	A1 (3)
(d)		
	$\dfrac{d}{\sqrt{126}} = \sin\theta$	M1 A1ft
	$d = 3\sqrt{5}(\approx 6.7)$ awrt 6.7	A1 (3)
(e)	$BX^2 = BC^2 - d^2 = 126 - 45 = 81$	M1
	$!\ CBX = \dfrac{1}{2} \times BX \times d = \dfrac{1}{2} \times 9 \times 3\sqrt{5} = \dfrac{27\sqrt{5}}{2} (\approx 30.2)$ awrt 30.1 or 30.2	M1 A1 (3)
		[14]
	Alternative for (e)	
	$!\ CBX = \dfrac{1}{2} \times d \times BC \sin\angle XCB$	M1
	$= \dfrac{1}{2} \times 3\sqrt{5} \times \sqrt{126}\sin(90 - 36.7)°$ sine of correct angle	M1
	≈ 30.2 $\quad \dfrac{27\sqrt{5}}{2}$, awrt 30.1 or 30.2	A1 (3)

Question Number	Scheme	Marks
Q8 (a)	$\int \sin^2\theta \, d\theta = \frac{1}{2}\int (1-\cos 2\theta)\, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \quad (+C)$	M1 A1 (2)
(b)	$x = \tan\theta \Rightarrow \frac{dx}{d\theta} = \sec^2\theta$	
	$\pi\int y^2 \, dx = \pi\int y^2 \frac{dx}{d\theta} d\theta = \pi\int (2\sin 2\theta)^2 \sec^2\theta \, d\theta$	M1 A1
	$= \pi\int \frac{(2\times 2\sin\theta\cos\theta)^2}{\cos^2\theta} d\theta$	M1
	$= 16\pi\int \sin^2\theta \, d\theta$ $\qquad k = 16\pi$	A1
	$x = 0 \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0, \quad x = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$	B1 (5)
	$\left(V = 16\pi\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta\right)$	
(c)	$V = 16\pi\left[\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{6}}$	M1
	$= 16\pi\left[\left(\frac{\pi}{12} - \frac{1}{4}\sin\frac{\pi}{3}\right) - (0-0)\right]$ $\qquad$ Use of correct limits	M1
	$= 16\pi\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) = \frac{4}{3}\pi^2 - 2\pi\sqrt{3}$ $\qquad p = \frac{4}{3},\ q = -2$	A1 (3)
		[10]