Gravitational Fields

Supplementary Questions

Part 1

1. $F = \dfrac{GM_1M_2}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 10 \times 2}{(200 \times 10^{-3})^2}$$

$$= \underline{\underline{3.335 \times 10^{-8}\,N}}$$

2. $F = \dfrac{GM_1M_2}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 15 \times 15}{1.2^2}$$

$$= \underline{\underline{1.04 \times 10^{-8}\,N}}$$

3. $F = \frac{GM_1M_2}{r^2} \Rightarrow r = \sqrt{\frac{GM_1M_2}{F}}$

$$r = \sqrt{\frac{6.67\times10^{-11} \times 1\times10^{12}\times5\times10^{12}}{10}}$$

$$\underline{\underline{r = 5.77\times10^{6}\ m}}$$

4. i) $F = \frac{GM_1M_2}{r^2} = \frac{6.67\times10^{-11} \times 6.0\times10^{24} \times 65}{(6400\times10^{3})^2}$

$$\underline{\underline{F = 635.083\ N}}$$

ii) $F = mg = 65\times9.81$

$= \underline{637.65\ N}$

5. i) $F = \frac{GM_1M_2}{r^2}$

$$= \frac{6.67\times10^{-11} \times 6.0\times10^{24} \times 80}{(8\times10^6)^2}$$

$$= \underline{\underline{500.25\text{ N}}}$$

ii) $F = \frac{GM_1M_2}{r^2}$

$$= \frac{6.67\times10^{-11} \times 6.0\times10^{24} \times 100}{(4.2\times10^7)^2}$$

$$= \underline{\underline{22.69\text{ N}}}$$

iii) $F = \frac{GM_1M_2}{r^2}$

$$= \frac{6.67\times10^{-11} \times 6.0\times10^{24} \times 7.3\times10^{22}}{(3.8\times10^8)^2}$$

$$= \underline{\underline{2.02 \times10^{20}\text{ N}}}$$

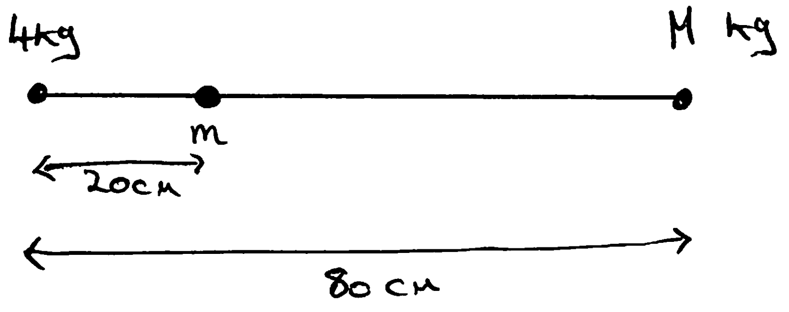
6.

$$F = \frac{GM_1M_2}{r^2}$$

$$= \frac{6.67\times10^{-11} \times 7.3\times10^{22} \times 55}{(1.6\times10^{6})^2}$$

$$= \underline{\underline{104.6N}}$$

7.*



Force on m due to 4kg:

$$F_1 = \frac{G \times 4 \times m}{(20 \times 10^{-2})^2}$$

Force on m due to M kg:

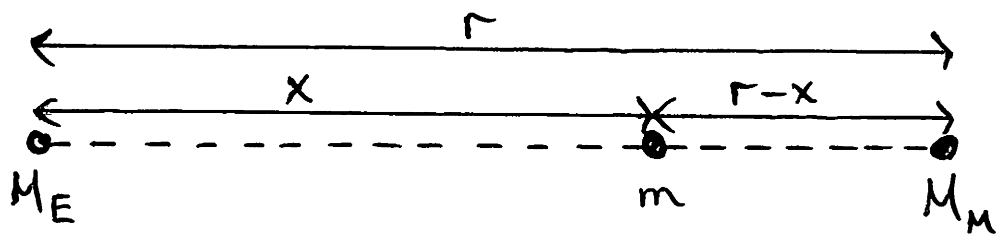
$$F_2 = \frac{G \times M \times m}{(60 \times 10^{-2})^2}$$

Zero resultant force : $F_1 = F_2$

$$\frac{G \times 4 \times m}{(20 \times 10^{-2})^2} = \frac{G \times M \times m}{(60 \times 10^{-2})^2}$$

$$M = 4 \times \frac{(60 \times 10^{-2})^2}{(20 \times 10^{-2})^2} = \underline{\underline{36 \text{ kg}}}$$

8.*



$$F_E = \frac{GM_E m}{x^2} \; ; \quad F_M = \frac{GM_M m}{(r-x)^2}$$

$$F_E = F_M \; ; \quad \frac{GM_E m}{x^2} = \frac{GM_M m}{(r-x)^2}$$

$$M_E (r-x)^2 = M_M \cdot x^2$$

$$\left(\frac{r}{x} - 1\right)^2 = \frac{M_M}{M_E}$$

$$x = \frac{r}{1 + \sqrt{\frac{M_M}{M_E}}}$$

$$x = \frac{3.8 \times 10^8}{1 + \sqrt{\frac{7.4 \times 10^{22}}{6.0 \times 10^{24}}}}$$

$$= 3.42 \times 10^8 \text{ m} \quad \text{(away from Earth)}$$

9.*

Force on S due to Earth:

$$F_E = \frac{G m_E m_s}{(9d)^2} = \frac{G m_E m_s}{81 d^2}$$

Force on S due to Moon:

$$F_M = \frac{G m_m m_s}{d^2}$$

$$F_E = F_M \Longrightarrow \frac{G m_E m_s}{81 d^2} = \frac{G m_m m_s}{d^2}$$

$$\therefore \quad \underline{\underline{m_E = 81 m_m}}$$

10.* $\rho = \frac{m}{V} \implies m = \rho V$

* mass of sphere of radius r:

$$m_1 = \rho \cdot \frac{4\pi r^3}{3} = \frac{4\pi \rho r^3}{3}$$

* mass of sphere of radius $2r$:

$$m_2 = \rho \cdot \frac{4\pi (2r)^3}{3} = \frac{32\pi \rho r^3}{3} = \underline{\underline{8m_1}}$$

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* Force between two spheres of radius r, (separated by $2r$):

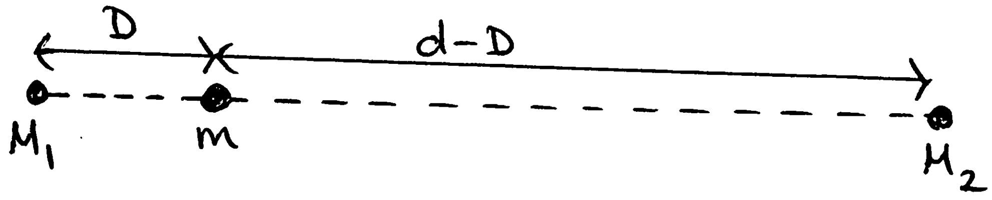
$$F = \frac{G m_1 m_1}{(2r)^2} = \frac{G m_1^2}{4r^2}$$

* Force between two spheres of radius $2r$, (separated by $4r$):

$$F' = \frac{G m_2 m_2}{(4r)^2} = \frac{G m_2^2}{16r^2} = \frac{G(8m_1)^2}{16r^2}$$

$$= \frac{64 G m_1^2}{16 r^2} = 16 \times \underbrace{\frac{G m_1^2}{4r^2}}_{=F} = \underline{\underline{16F}}$$

11.*



$$F_1 = \frac{GM_1 m}{D^2} \quad ; \quad F_2 = \frac{GM_2 m}{(d-D)^2}$$

$$F_1 = F_2 \implies \frac{GM_1 m}{D^2} = \frac{GM_2 m}{(d-D)^2}$$

$$\therefore \quad \frac{(d-D)^2}{D^2} = \frac{M_2}{M_1}$$

$$\therefore \quad \left(\frac{d}{D} - 1\right)^2 = \frac{M_2}{M_1}$$

$$\frac{d}{D} - 1 = \pm\sqrt{\frac{M_2}{M_1}}$$

$$\therefore \boxed{D = \frac{d}{1 \pm \sqrt{\frac{M_2}{M_1}}}}$$

choose +ve root.

Reason: If $M_1 = M_2$, would expect position of neutral point to be equidistant, ie. $D = \frac{d}{2}$.

Part 2

1. $g = \frac{GM_E}{r^2}$

$$g = \frac{6.67\times10^{-11} \times 6.0\times10^{24}}{(6.56\times10^{6})^2}$$

$$= \underline{\underline{9.30 \ m/s^2}}$$

2. i) $F = \frac{mv^2}{r} = \frac{GM_M m}{r^2}$

$$v^2 = \frac{GM_m}{r} \Longrightarrow r = \frac{GM_m}{v^2}$$

$$r = \frac{6.67\times10^{-11} \times 7.34\times10^{22}}{(1.65\times10^{3})^2}$$

$$\underline{\underline{r = 1.80\times10^{6} m}}$$

ii) $h = r - R_m$

$$= 1.80\times10^{6} - 1.64\times10^{6}$$

$$= \underline{\underline{1.60\times10^{5} m}}$$

3.i) $$g = \frac{GM_m}{r^2}$$

$$M_m = \frac{gr^2}{G}$$

$$= \frac{1.7 \times (1.7\times10^6)^2}{6.67\times10^{-11}}$$

$$= \underline{\underline{7.37 \times10^{22} \text{ kg}}}$$

ii) $$g = \frac{GM_m}{r^2} \Rightarrow gr^2 = GM_m$$
$$= \text{constant}.$$

$$\therefore g_1 r_1^2 = g_2 r_2^2$$

$$\therefore g_2 = g_1 \times \frac{r_1^2}{r_2^2} = g_1\left(\frac{r_1}{r_2}\right)^2$$

$$g_2 = 1.7 \times \left(\frac{1.7\times10^6}{2.7\times10^6}\right)^2$$

$$= \underline{\underline{0.674 \text{ N/kg}}}$$

4. i) $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times \left(\frac{20\times10^3}{2}\right)^3$

$= \underline{\underline{4.189 \times 10^{12}\ m^3}}$

$\rho = \frac{m}{V} = \frac{2\times10^{30}}{4.189\times10^{12}}$

$= \underline{\underline{4.77\times10^{17}\ kg/m^3}}$

ii) $g = \frac{GM}{r^2}$

$= \frac{6.67\times10^{-11}\times 2\times10^{30}}{\left(\frac{20\times10^3}{2}\right)^2}$

$= \underline{\underline{1.33\times10^{12}\ N/kg}}$

5. $W = mg = \dfrac{GM_E m}{R^2}$

i) $$\frac{mg}{4} = \frac{GM_E m}{4R^2} = \frac{GM_E m}{(2R)^2}$$

For weight to decrease by a factor of 4, need to be at a distance of 2R from the center.

$\therefore$ Need to be at R above the surface.

ii) $$\frac{mg}{2} = \frac{GM_E m}{2R^2} = \frac{GM_E m}{(\sqrt{2}R)^2}$$

Using a similar argument to that above, need to be at $(\sqrt{2} - 1)R$ above surface.

6.*

$$g_x = \frac{GM}{(2000 \times 10^3)^2} \quad ;$$

$$g_y = \frac{GM}{(3000 \times 10^3)^2}$$

$$\frac{g_x}{g_y} = \frac{GM}{(2 \times 10^6)^2} \times \frac{(3 \times 10^6)^2}{GM}$$

$$= \left(\frac{3}{2}\right)^2$$

$$= \frac{9}{4} \quad (= 2.25)$$

7.*

$$M = \rho V \; ; \quad V = \frac{4}{3}\pi R^3$$

$$\therefore \quad M = \frac{4}{3}\pi \rho R^3$$

$$\Longrightarrow \quad g = \frac{GM}{R^2}$$

$$= \frac{G \times \frac{4}{3}\pi\rho R^3}{R^2}$$

$$\underline{\underline{g = \frac{4}{3}\pi G\rho R}}$$

8.* i) If M_P = mass of planet and M_E = mass of Earth.

$M_P = M_E$ but $\rho_P = 2\rho_E$, then radius of planet will be smaller:

$$M = \rho V = \frac{4}{3}\pi\rho R^3$$

$$\therefore \frac{4}{3}\pi\rho_p R_p^3 = \frac{4}{3}\pi\rho_E R_E^3$$

$$\therefore \underbrace{\rho_p}_{=2\rho_E} R_p^3 = \rho_E R_E^3$$

$$\therefore R_p^3 = \frac{1}{2}R_E^3 \implies \underline{\underline{R_p = 2^{-1/3} R_E}}$$

$$g_p = \frac{GM_P}{R_p^2} = \frac{GM_E}{2^{-2/3}R_E^2} = 2^{2/3} \cdot \underbrace{\frac{GM_E}{R_E^2}}_{g_E = 9.8}$$

$$\therefore g_p = 15.556\ldots$$

$$g_p = \underline{\underline{15.6\ m/s^2}}$$

8.* ii) $\rho_p = \rho_E$ but $R_p = 2R_E$.

$$\therefore \quad M_p = \frac{4}{3}\pi \underbrace{\rho_p}_{\rho_E} \underbrace{R_p^3}_{(2R_E)^3}$$

$$= 8 \times \underbrace{\frac{4}{3}\pi \rho_E R_E^3}_{M_E}$$

$$\therefore \quad \underline{\underline{M_p = 8M_E}}$$

$$g_p = \frac{GM_p}{R_p^2} = \frac{G \times 8M_E}{(2R_E)^2}$$

$$= 2 \cdot \underbrace{\frac{GM_E}{R_E^2}}_{g_E = 9.8}$$

$$\therefore \quad g_p = \underline{\underline{19.6 \text{ m/s}^2}}$$

8.* iii) $M_p = M_E$, $R_p = 2R_E$.

$$g_p = \frac{GM_p}{R_p^2}$$

$$= \frac{GM_E}{(2R_E)^2}$$

$$= \frac{1}{4} \times \underbrace{\frac{GM_E}{R_E^2}}_{=9.8}$$

$$\therefore \underline{\underline{g_p = 2.45 \, m/s^2}}$$

Part 3

1. i) $$g = \frac{GM}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{(4.2 \times 10^{7})^2}$$

$$= \underline{\underline{0.227 \text{ m/s}^2}}$$

ii) $$ma = \frac{GMm}{r^2}$$

$$\therefore \quad a = g = \frac{GM}{r^2}$$

Since r is half that of geo synchronous orbit, acceleration will be <u>four</u> times larger:

$$\underline{\underline{a = 0.907 \text{ m/s}^2}}$$

1. iii) $a = \frac{v^2}{r}$

$$V = \sqrt{ar}$$

$$= \sqrt{0.907 \times 2.1\times10^7}$$

$$= \underline{\underline{4365 \text{ m/s}}}$$

iv) Using V from iii) :

$d = 2\pi r$;

$V = 4365$ m/s ;

$$\therefore t = \frac{d}{V} = \frac{2\pi \times 2.1\times10^7}{4365}$$

$$\therefore t = \underline{\underline{30225.3 \text{ seconds}}}$$

(= 8 hrs 23 mins 45 seconds)

Using Kepler's 3rd Law :

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$$\therefore T = \sqrt{\frac{4\pi^2 \times (2.1\times10^7)^3}{6.67\times10^{-11} \times 6.0\times10^{24}}}$$

$$= \underline{\underline{30225.3 \text{ seconds}}}$$

2.i) $r = R_E + h$

$= 6400 \times 10^3 + 350 \times 10^3$

$= \underline{\underline{6.75 \times 10^6 \text{ m}}}$

$$g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{(6.75 \times 10^6)^2}$$

$\therefore \underline{\underline{g = 8.78 \text{ m/s}^2}}$

ii) $g = \frac{v^2}{r} \Rightarrow v = \sqrt{gr}$

$\therefore v = \sqrt{8.78 \times 6.75 \times 10^6}$

$= 7699.9$

$= \underline{\underline{7700 \text{ m/s}}}$

iii) Using Kepler's 3rd Law: $T^2 = \frac{4\pi^2}{GM} r^3$

$$\therefore T = \sqrt{\frac{4\pi^2 \times (6.75 \times 10^6)^3}{6.67 \times 10^{-11} \times 6 \times 10^{24}}} = \underline{\underline{5508 \text{ seconds}}}$$

3. Io : $r = 4.2$, $T = 1.8$ days.

Ganymede : $r = 10.7$, $T = ?$

$$\frac{r_1^3}{r_2^3} = \frac{T_1^2}{T_2^2}$$

$$\therefore \quad T_2^2 = T_1^2 \times \frac{r_2^3}{r_1^3}$$

$$\therefore \quad T_2 = \sqrt{1.8^2 \times \frac{10.7^3}{4.2^3}}$$

$$\therefore \quad T_2 = 7.32 \text{ days}$$

4. Earth : $r = 1$ Au, $T = 1$ year.

Small planet : $r = 14$ Au, $T = ?$

$$\frac{r_1^3}{r_2^3} = \frac{T_1^2}{T_2^2}$$

$$T_2^2 = T_1^2 \times \frac{r_2^3}{r_1^3}$$

$$\therefore \quad T_2 = \sqrt{1^2 \times \frac{14^3}{1^3}}$$

$$\underline{\underline{T_2 = 52.4 \text{ years}}}$$

5. Using methods of Q's 3 + 4 :

$$T_2 = \sqrt{1^2 \times \frac{2.6^3}{1^3}}$$

$$= \underline{\underline{4.19 \text{ years}}}$$

6. If $r = 2R_E$ then use Kepler's 3rd Law:

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$$\therefore T = \sqrt{\frac{4\pi^2 \times (2 \times 6400 \times 10^3)^3}{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}}$$

$$\therefore \underline{\underline{T = 14383 \text{ seconds}}}$$

7. Orbiting same body : use ratios :

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

Deimos : $T_1 =$ 30 hours 18 mins $=$ 1818 mins

Phobos : $T_2 =$ 7 hours 39 mins $=$ 459 mins.

$$r_2^3 = r_1^3 \times \frac{T_2^2}{T_1^2}$$

$$\therefore r_2 = \sqrt[3]{(2.3 \times 10^4 \times 10^3)^3 \times \frac{459^2}{1818^2}}$$

$$= \underline{\underline{9.19 \times 10^3 \text{ km}}} \qquad (\text{or } 9.19 \times 10^6 \text{ m})$$

8.*

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$2.3 \times 10^7 m$

From Q7: $$M = \frac{4\pi^2}{G} \cdot \frac{r^3}{T^2}$$

$30 \times 60^2 + 18 \times 60 = 109080$ seconds

$$= \frac{4\pi^2}{6.67 \times 10^{-11}} \times \frac{(2.3 \times 10^7)^3}{(109,080)^2}$$

$$\therefore \quad \underline{\underline{M = 6.05 \times 10^{23} \text{ kg}}}$$

At 100 km above surface, let $r = R + h$

$$\therefore \quad T^2 = \frac{4\pi^2}{GM} (R+h)^3$$

and $$g = \frac{GM}{(R+h)^2} \Longrightarrow (R+h)^3 = \left(\frac{GM}{g}\right)^{3/2}$$

$$\therefore \quad T^2 = \frac{4\pi^2}{GM} \times \left(\frac{GM}{g}\right)^{3/2} = \frac{4\pi^2}{g}\sqrt{\frac{GM}{g}}$$

$$\therefore \quad T = \left[\frac{4\pi^2}{3.32}\sqrt{\frac{6.67 \times 10^{-11} \times 6.05 \times 10^{23}}{3.32}}\right]^{1/2}$$

$$= \underline{\underline{6439 \text{ seconds}}} = 1 \text{ hour } 47 \text{ mins } 19 \text{ seconds.}$$

9.* $R = 1.6\times10^{6}$ m

$M = 7.3\times10^{22}$ kg

$T = 27.3$ days

$= \underline{\underline{2358720}}$ Seconds

$$T^2 = \frac{4\pi^2}{GM} \cdot r^3 \implies r^3 = \frac{GM}{4\pi^2} \cdot T^2$$

$$\therefore r = \sqrt[3]{\frac{6.67\times10^{-11} \times 7.3\times10^{22}}{4\pi^2} \times (2358720)^2}$$

$$\therefore \underline{\underline{r = 8.82 \times10^{7} \text{ m}}}$$

Height above Surface $= 8.82\times10^{7} - 1.6\times10^{6}$

$= \underline{\underline{8.66\times10^{7}\text{m}}}$