

Paper Reference(s)

6690/01

Edexcel GCE

Decision Mathematics D2

Advanced/Advanced Subsidiary

Wednesday 18 January 2006 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination

Nil

Items included with question papers

D1 Answer book

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates must NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

Write your answers for this paper in the D2 answer book provided.

In the boxes on the answer book, write your centre number, candidate number, your surname, initials and signature.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

Full marks may be obtained for answers to ALL questions.

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2)

There are 6 questions in this question paper. The total mark for this question paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Write your answers in the D2 answer booklet for this paper.

1. A theme park has four sites, *A*, *B*, *C* and *D*, on which to put kiosks. Each kiosk will sell a different type of refreshment. The income from each kiosk depends on what it sells and where it is located. The table below shows the expected daily income, in pounds, from each kiosk at each site.

| | Hot dogs and beefburgers (H) | Ice cream (I) | Popcorn, candyfloss and drinks (P) | Snacks and hot drinks (S) |
|---------------|------------------------------|---------------|------------------------------------|---------------------------|
| Site <i>A</i> | 267 | 272 | 276 | 261 |
| Site <i>B</i> | 264 | 271 | 278 | 263 |
| Site <i>C</i> | 267 | 273 | 275 | 263 |
| Site <i>D</i> | 261 | 269 | 274 | 257 |

Reducing rows first, use the Hungarian algorithm to determine a site for each kiosk in order to maximise the total income. State the site for each kiosk and the total expected income. You must make your method clear and show the table after each stage.

(13)

2. An engineering firm makes motors. They can make up to five in any one month, but if they make more than four they have to hire additional premises at a cost of £500 per month. They can store up to two motors for £100 per motor per month. The overhead costs are £200 in any month in which work is done.

Motors are delivered to buyers at the end of each month. There are no motors in stock at the beginning of May and there should be none in stock after the September delivery.

The order book for motors is:

| Month | May | June | July | August | September |
|------------------|-----|------|------|--------|-----------|
| Number of motors | 3 | 3 | 7 | 5 | 4 |

Use dynamic programming to determine the production schedule that minimises the costs, showing your working in the table provided in the answer book.

(12)

3. Three depots, F , G and H , supply petrol to three service stations S , T and U . The table gives the cost, in pounds, of transporting 1000 litres of petrol from each depot to each service station.

| | S | T | U |
|-----|-----|-----|-----|
| F | 23 | 31 | 46 |
| G | 35 | 38 | 51 |
| H | 41 | 50 | 63 |

F , G and H have stocks of 540 000, 789 000 and 673 000 litres respectively.

S , T and U require 257 000, 348 000 and 412 000 litres respectively.

The total cost of transporting the petrol is to be minimised.

Formulate this problem as a linear programming problem. Make clear your decision variables, objective function and constraints.

(8)

4. The following minimising transportation problem is to be solved.

| | <i>J</i> | <i>K</i> | Supply |
|----------|----------|----------|--------|
| <i>A</i> | 12 | 15 | 9 |
| <i>B</i> | 8 | 17 | 13 |
| <i>C</i> | 4 | 9 | 12 |
| Demand | 9 | 11 | |

- (a) Complete the first table in the answer book. (2)
- (b) Explain why an extra column was added to the table in the answer book. (2)

A possible north-west solution is:

| | <i>J</i> | <i>K</i> | Supply |
|----------|----------|----------|--------|
| <i>A</i> | 9 | 0 | |
| <i>B</i> | | 11 | 2 |
| <i>C</i> | | | 12 |

- (c) Explain why it was necessary to place a zero in the first row of the second column. (1)

After three iterations of the stepping-stone method the table becomes:

| | <i>J</i> | <i>K</i> | Supply |
|----------|----------|----------|--------|
| <i>A</i> | | 8 | 1 |
| <i>B</i> | | | 13 |
| <i>C</i> | 9 | 3 | |

- (d) Taking the most negative improvement index as the entering square for the stepping stone method, solve the transportation problem. You must make your shadow costs and improvement indices clear and demonstrate that your solution is optimal. (11)

5. A two-person zero-sum game is represented by the following pay-off matrix for player A .

| | B plays 1 | B plays 2 | B plays 3 | B plays 4 |
|-------------|-------------|-------------|-------------|-------------|
| A plays 1 | -2 | 1 | 3 | -1 |
| A plays 2 | -1 | 3 | 2 | 1 |
| A plays 3 | -4 | 2 | 0 | -1 |
| A plays 4 | 1 | -2 | -1 | 3 |

- (a) Verify that there is no stable solution to this game.

(3)

- (b) Explain why the 4×4 game above may be reduced to the following 3×3 game.

| | | |
|----|----|----|
| -2 | 1 | 3 |
| -1 | 3 | 2 |
| 1 | -2 | -1 |

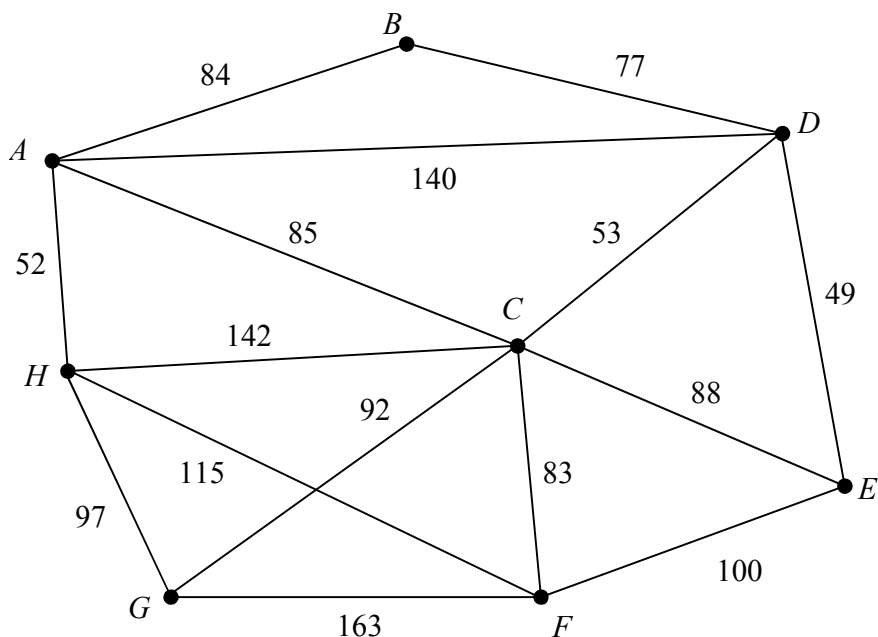
(2)

- (c) Formulate the 3×3 game as a linear programming problem for player A . Write the constraints as inequalities. Define your variables clearly.

(8)

6.

Figure 1



The network in Figure 1 shows the distances in km, along the roads between eight towns A , B , C , D , E , F , G and H . Keith has a shop in each town and needs to visit each one. He wishes to travel a minimum distance and his route should start and finish at A .

By deleting D , a lower bound for the length of the route was found to be 586 km.

By deleting F , a lower bound for the length of the route was found to be 590 km.

(a) By deleting C , find another lower bound for the length of the route. State which is the best lower bound of the three, giving a reason for your answer.

(5)

(b) By inspection, complete the table of least distances.

(4)

The table can now be taken to represent a complete network.

The nearest neighbour algorithm was used to obtain upper bounds for the length of the route:

Starting at D , an upper bound for the length of the route was found to be 838 km.

Starting at F , an upper bound for the length of the route was found to be 707 km.

(c) Starting at C , use the nearest neighbour algorithm to obtain another upper bound for the length of the route. State which is the best upper bound of the three, giving a reason for your answer.

(4)

TOTAL FOR PAPER: 75 MARKS

END