| Question Number | Scheme | Marks |
|---|---|---|
| **1.** (*a*) | see table below | M1 A1 **(2)** |
| (*b*) | *AE* (12), *EF* (16), *FB* (16), *BC* (10), *CD* (15), *DA* (32), i.e. *AEFBCDA* | M1 A1 |
| | Upper bound = 101 | A1 **(3)** |
| (*c*) | In the original network *AD* is not a direct path.<br>The tour becomes *AEFBCDEA* | B1 **(1)** |
| (*d*) | For example,<br>*B C D E A F B*<br>*C D E A F B C*<br>*D C B F A E D*     length 98<br>*E A F B C D E*<br>*F A E D C B F* | M1 A1 **(2)** |
| | | **(8 marks)** |
| **2.** (*a*) | Row minima: –5, –1, –4, –1  max is –1 | M1 |
| | Column minima: 0, 5, –1, 4  min is –1 | A1 |
| | Play safe is *A* plays II or IV and *B* plays III | A1 A1 **(4)** |
| (*b*) | Since (–1) – (–1) = 0 there is a stable solution | B1 |
| | Saddle point (II, III) and (IV, III) | M1 A1 ft **(3)** |
| (*c*) | Value of game to *B* is –(–1) = 1 | B1 **(1)** |
| | | **(8 marks)** |

Table for 1(*a*):

| | *A* | *B* | *C* | *D* | *E* | *F* |
|---|---|---|---|---|---|---|
| *A* | 0 | 20 | 30 | 32 | 12 | 15 |
| *B* | 20 | 0 | 10 | 25 | 32 | 16 |
| *C* | 30 | 10 | 0 | 15 | 35 | 19 |
| *D* | 32 | 25 | 15 | 0 | 20 | 34 |
| *E* | 12 | 32 | 35 | 20 | 0 | 16 |
| *F* | 15 | 16 | 19 | 34 | 16 | 0 |

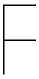
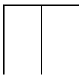
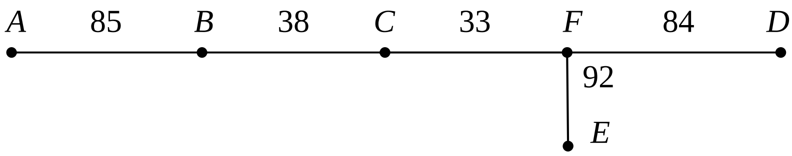
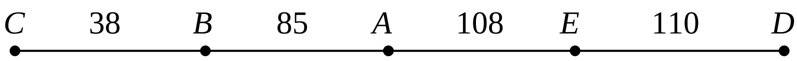
ft = follow-through mark

| Question Number | Scheme | Marks |
|---|---|---|
| **3.** (*a*) | | |

| Stage | Initial state | Action | Destination | Value | Marks |
|---|---|---|---|---|---|
| 1 | $D$ | $DT$ | $T$ | 8 * | |
| | $E$ | $ET$ | $T$ | 10 * | M1 A1 |
| | $F$ | $FT$ | $T$ | 6 * | |
| 2 | $A$ | $AD$ | $D$ | max (7, 8) = 8 * | M1 A1 ft |
| | | $AE$ | $E$ | max (8, 10) = 10 | |
| | $B$ | $BE$ | $E$ | max (9, 10) = 10 | |
| | | $BF$ | $F$ | max (3, 6) = 6 * | A1 ft |
| | $C$ | $CE$ | $E$ | max (6, 10) = 10 | |
| | | $CF$ | $F$ | max (9, 6) = 9 * | A1 ft |
| 3 | $S$ | $SA$ | $A$ | max (9, 8) = 9 | |
| | | $SB$ | $B$ | max (7, 6) = 7 * | M1 A1 ft **(8)** |
| | | $SC$ | $C$ | max (6, 9) = 9 | |

| Question Number | Scheme | Marks |
|---|---|---|
| (*b*) | Minimax route is *SBFT* | M1 |
| | Maximum amount of fuel used is 7 units | A1 **(2)** |
| | | **(10 marks)** |
| **4.** (*a*) | Row 1 dominates row 3<br>Column 1 dominates column 3 | M1 A1 |
| | Thus row 3 and column 3 may be deleted | A1 **(3)** |
| (*b*) | Let $A$ play row 3 with probability $p$ and hence row 3 with probability $(1 – p)$ | |
| | If $B$ plays 1, $A$'s expected gain is $3p + 6(1 – p) = 6 – 3p$<br>If $B$ plays 2, $A$'s expected gain is $5p + 3(1 – p) = 2p + 3$ | M1 A1 |
| | Optimal when $6 – 3p = 2p + 3$ | |
| | $5p = 3$ | |
| | $p = \frac{3}{5}$ | A1 |
| | Hence $A$ should play row 1 with probability $\frac{3}{5}$<br>and row 2 with probability $\frac{2}{5}$ | A1 ft **(4)** |
| | Similarly, let $B$ play column 1 with probability $q$ | |
| | $3q + 5(1 – q) = 6q + 3(1 – q)$ | M1 A1 |
| | $5q = 2$ | |
| | $q = \frac{2}{5}$ | A1 ft |
| | So $B$ should play column 1 with probability $\frac{2}{5}$<br>and column 2 with probability $\frac{3}{5}$ | |
| | Value of game is $4\frac{1}{5}$ to $A$ | A1 **(4)** |
| | | **(11 marks)** |

ft = follow-through mark

| Question Number | Scheme | Marks |
|---|---|---|
| **5.** (*a*) | Reducing rows | |
| | $\begin{matrix} 9 & 0 & 3 & 2 \\ 0 & 10 & 4 & 3 \\ 4 & 5 & 0 & 6 \\ 0 & 2 & 4 & 8 \end{matrix} \quad \xrightarrow[\text{columns}]{\text{reducing}} \quad \begin{matrix} 9 & 0 & 3 & 0 \\ 0 & 10 & 4 & 1 \\ 4 & 5 & 0 & 4 \\ 0 & 2 & 4 & 6 \end{matrix}$ | M1 A1 A1 **(3)** |
| (*b*) | Testing for optimality – 3 lines are enough | M1 A1 |
| |  or  Minimum uncovered element is 1 | A1 |
| | $\begin{matrix} 10 & 0 & 3 & 0 \\ 0 & 9 & 3 & 0 \\ 5 & 5 & 0 & 4 \\ 0 & 1 & 3 & 6 \end{matrix} \quad \text{or} \quad \begin{matrix} 10 & 0 & 4 & 0 \\ 0 & 9 & 3 & 0 \\ 4 & 4 & 0 & 3 \\ 0 & 1 & 4 & 5 \end{matrix}$ 4 lines now needed | M1 A1 **(5)** |
| (*c*) | Final matching | |
| | Machine 1 – Job 2 (5) | |
| | Machine 2 – Job 4 (5) | M1 A1 |
| | Machine 3 – Job 3 (3) | |
| | Machine 4 – Job 1 (2) | |
| | Minimum time: 15 hours | A1 **(3)** |
| | | **(11 marks)** |
| **6.** (*a*) | Order of arcs: *AB*, *BC*, *CF*, *FD*, *FG* | M1 A1 A1 |
| |  | A1 **(6)** |
| (*b*) (i) | $2 \times 372 = 744$ | M1 A1 **(2)** |
| (ii) | e.g. *DA* saves 105 giving 639 | |
| | or *AE* saves 180 giving 564 | M1 A1 **(2)** |
| (*c*) | Residual MST | |
| | *AB*, *BC*, *AE*, *ED* | M1 |
| |  | A1 |
| | Lower bound = 341 + 73 + 84 | M1 |
| | = 498 | A1 **(4)** |
| | | **(12 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **7.** (*a*) | $\begin{array}{c\|ccc} & B_1 & B_2 & B_3 \\ \hline F_1 & 20 & 15 & \\ F_2 & & 10 & 15 \\ F_3 & & & 15 \end{array}$ | M1 A1 **(2)** |
| (*b*) | $S(F_1) = 0$ $S(F_2) = 1$ $S(F_3) = 0$<br>$D(B_1) = 10$ $D(B_2) = 4$ $D(B_3) = 7$ | M1 A1 |
| | $I_{13} = 11 - 0 - 7 = 4$<br>$I_{21} = 12 - 1 - 10 = 1$<br>$I_{31} = 9 - 0 - 10 = -1$<br>$I_{33} = 6 - 0 - 4 = 2$ | M1 A1 |
| | Since $I_{31}$ is negative, pattern is not optimal | A1 **(5)** |
| (*c*) | $\begin{array}{c\|ccc} & B_1 & B_2 & B_3 \\ \hline F_1 & 20-\theta & 15+\theta & \\ F_2 & & 10-\theta & 15+\theta \\ F_3 & \theta & & 15-\theta \end{array}$<br>Entering square $F_3\ B_1$<br>Exiting square $F_2\ B_2$<br>$\theta = 10$ | M1 A1 |
| | $\begin{array}{c\|ccc} & B_1 & B_2 & B_3 \\ \hline F_1 & 10 & 25 & \\ F_2 & & & 25 \\ F_3 & 10 & & 5 \end{array}$ | A1 **(3)** |
| (*d*) | $S(F_1) = 0$ $S(F_2) = 0$ $S(F_3) = -1$<br>$D(B_1) = 10$ $D(B_2) = 4$ $D(B_3) = 8$ | M1 A1 |
| | $I_{13} = 11 - 0 - 8 = 3$<br>$I_{21} = 12 - 0 - 10 = 2$<br>$I_{31} = 5 - 0 - 4 = 1$<br>$I_{33} = 6 - (-1) - 4 = 3$ all positive $\therefore$ optimal | A1 |
| | Cost = $(10 \times 10) + (25 \times 4) + (25 \times 8) + (10 \times 9) + (5 \times 7) = 525$ units | M1 A1 **(5)** |
| | | **(15 marks)** |