

1. A curve C is described by the equation

Find an equation of the tangent to C at the point $(1, -2)$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

Question 1 continued

Leave
blank

Handwriting practice area with 30 horizontal lines.

(Total 7 marks)

Q1

Small rectangular box for marking.



Leave
blank

2. (a) Given that $y = \sec x$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}, \frac{\pi}{8}$ and $\frac{\pi}{4}$.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	1			1.20269	

(2)

- (b) Use the trapezium rule, with all the values for y in the completed table, to obtain an estimate for $\int_0^{\frac{\pi}{4}} \sec x \, dx$. Show all the steps of your working, and give your answer to 4 decimal places.

(3)

The exact value of $\int_0^{\frac{\pi}{4}} \sec x \, dx$ is $\ln(1 + \sqrt{2})$.

- (c) Calculate the % error in using the estimate you obtained in part (b).

(2)



Question 2 continued

Lined area for writing the answer to Question 2.

Leave blank

Q2

Mark box for Question 2.

(Total 7 marks)



Leave
blank

3. Using the substitution $u^2 = 2x - 1$, or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{2x-1}} dx.$$

(8)



Question 3 continued

Leave
blank

Handwriting practice area with 30 horizontal lines.

(Total 8 marks)

Q3

Marking box for Q3



Leave
blank

4.

Figure 1

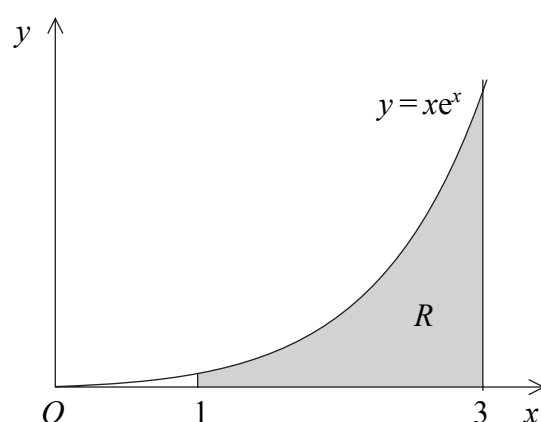


Figure 1 shows the finite shaded region, R , which is bounded by the curve $y = xe^x$, the line $x = 1$, the line $x = 3$ and the x -axis.

The region R is rotated through 360 degrees about the x -axis.

Use integration by parts to find an exact value for the **volume** of the solid generated.

(8)



Question 4 continued

Lined area for writing the answer to Question 4.

Leave
blank

Q4

Box for marking the answer to Question 4.

(Total 8 marks)



5.

(a) Find the values of A and C and show that $B = 0$.

(4)

(7)

Question 5 continued

Blank lined area for writing the answer to Question 5.

Leave
blank

Q5

(Total 11 marks)



- 6.** The line l_1 has vector equation

$$\mathbf{r} = 8\mathbf{i} + 12\mathbf{j} + 14\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where λ is a parameter.

The point A has coordinates $(4, 8, a)$, where a is a constant. The point B has coordinates $(b, 13, 13)$, where b is a constant. Points A and B lie on the line l_1 .

- (a) Find the values of a and b .

(3)

Given that the point O is the origin, and that the point P lies on l_1 such that OP is perpendicular to l_1 ,

- (b) find the coordinates of P .

(5)

- (c) Hence find the distance OP , giving your answer as a simplified surd.

(2)



Leave
blank

Question 6 continued

(Total 10 marks)

Q6



7. The volume of a spherical balloon of radius r cm is V cm³, where $V = \frac{4}{3}\pi r^3$.

(a) Find $\frac{dV}{dr}$.

The volume of the balloon increases with time t seconds according to the formula

$$\frac{dV}{dt} = \frac{1000}{(2t+1)^2}, \quad t \geq 0.$$

(b) Using the chain rule, or otherwise, find an expression in terms of r and t for $\frac{dr}{dt}$. (2)

(c) Given that $V = 0$ when $t = 0$, solve the differential equation $\frac{dV}{dt} = \frac{1000}{(2t+1)^2}$, to obtain V in terms of t .

(4)

(d) Hence, at time $t = 5$,

(i) find the radius of the balloon, giving your answer to 3 significant figures, (3)

(ii) show that the rate of increase of the radius of the balloon is approximately $2.90 \times 10^{-2} \text{ cm s}^{-1}$. (2)



Question 7 continued

Leave
blank

Lined area for writing the answer to Question 7.



Question 7 continued

Leave
blank

Handwriting practice area with 30 horizontal lines.



Leave
blank

Question 7 continued

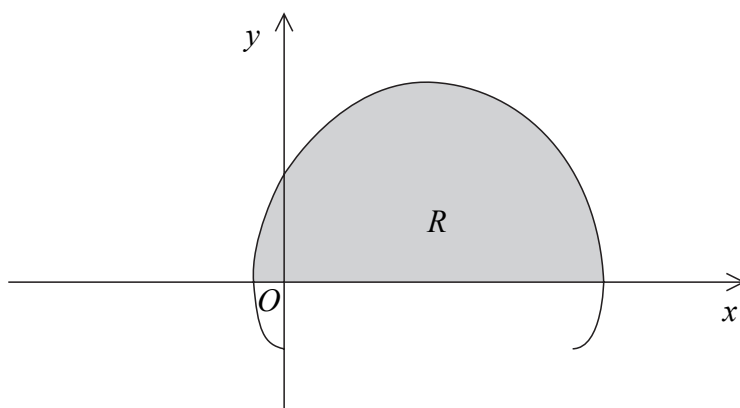
(Total 12 marks)

Q7



8.

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = t - 2 \sin t, \quad y = 1 - 2 \cos t, \quad 0 \leq t \leq 2\pi.$$

- (a) Show that the curve crosses the x -axis where $t = \frac{\pi}{3}$ and $t = \frac{5\pi}{3}$.

(2)

The finite region R is enclosed by the curve and the x -axis, as shown shaded in Figure 2.

- (b) Show that the area of R is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2 \cos t)^2 dt.$$

(3)

- (c) Use this integral to find the exact value of the shaded area.

(7)



Question 8 continued

Leave
blank

Lined area for writing the answer to Question 8.



Leave
blank

Question 8 continued

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

