

Mark Scheme (Results)

June 2011

GCE Core Mathematics C2 (6664) Paper 1

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June 2011
Publications Code UA027657
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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 75.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
 - M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - B marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod benefit of doubt
- ft follow through
- the symbol will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- · dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark



June 2011 Core Mathematics C2 6664 Mark Scheme

O	Wark Sci	101110		
Question Number	Schem	ne	Mar	ks
1.	$f(x) = 2x^3 - 7x^2 - 5x + 4$			
(a)	Remainder = $f(1) = 2 - 7 - 5 + 4 = -6$	Attempts $f(1)$ or $f(-1)$.	M1	
(a)	= -6	- 6		[2]
		Attempts $f(-1)$.	M1	[4]
(b)	$f(-1) = 2(-1)^3 - 7(-1)^2 - 5(-1) + 4$	f(-1) = 0 with no sign or substitution	IVII	
	and so $(x + 1)$ is a factor.	(-1) = 0 with no sign of substitution errors and for conclusion.	A1	[2]
(c)	$f(x) = \{(x+1)\}(2x^2 - 9x + 4)$	errors and for conclusion.	M1 A	
	= (x+1)(2x-1)(x-4)			
		(a)) for the final three marks in this mant	dM1	
	(Note : Ignore the ePEN notation of (b) (should be	(c)) for the final three marks in this part).		[4] 8
(a)	M1 for <i>attempting</i> either $f(1)$ or $f(-1)$. Can be in	nplied. Only one slip permitted.	ı	
	M1 can also be given for an attempt (at least two "s			
	remainder which is independent of x . A1 can be gi	_	ision	
	working. Award A0 for a candidate who finds -6	but then states that the remainder is 6.		
	Award M1A1 for -6 without any working.			
(b)	M1: attempting only $f(-1)$. A1: must correctly s	show $f(-1) = 0$ and give a conclusion <i>in part</i>	(b) onl	ly.
	Note : Stating "hence factor" or "it is a factor" or a	-		
	Note also that a conclusion can be implied from a property of the conclusion of th	·	"	
	Note: Long division scores no marks in part (b)			
(c)	1 st M1: Attempts long division or other method, to			
	Working need not be seen as this could be done "by	y inspection." $(2x^2 \pm ax \pm b)$ must be seen <i>in</i> .	part (c)
	<i>only</i> . Award 1 st M0 if the quadratic factor is clearly found from dividing $f(x)$ by $(x-1)$. Eg. Some			
	candidates use their $(2x^2 - 5x - 10)$ in part (c) foun	d from applying a long division method in part	(a).	
	1 st A1: For seeing $(2x^2 - 9x + 4)$.			
	2 nd dM1: Factorises a 3 term quadratic. (see rule f previous method mark being awarded. This mark quadratic formula correctly.			
	2^{nd} A1: is cao and needs all three factors on one lin	ne. Ignore following work (such as a solution t	o a	
	quadratic equation.)			
	Note: Some candidates will go from $\{(x+1)\}(2x^2)$	$-9x + 4$) to $\{x = -1\}$, $x = \frac{1}{2}$, 4, and not list a	all three	e
	factors. Award these responses M1A1M1A0.			
	<u>Alternative:</u> 1^{st} M1: For finding either $f(4) = 0$	or $f\left(\frac{1}{2}\right) = 0$.		
	1 st A1: A second correct factor of usually $(x-4)$ of	or $(2x - 1)$ found. Note that any one of the oth	ner corr	rect
	factors found would imply the 1 st M1 mark.			
	2 nd dM1: For using two known factors to find the t			
	2^{nd} A1 for correct answer of $(x+1)(2x-1)(x-4)$.			
	Alternative: (for the first two marks)			
	1 st M1: Expands $(x+1)(2x^2+ax+b)$ {giving $2x^3+(a+2)x^2+(b+a)x+b$ } then compare			
	coefficients to find <u>values</u> for a and b . 1^{st} A1:	a = -9, b = 4		
	Not dealing with a factor of 2: $(x+1)(x-\frac{1}{2})(x-4)$ or $(x+1)(x-\frac{1}{2})(2x-8)$ scores M1A1M1A0.			
	Answer only, with one sign error: eg. $(x+1)(2x+1)(x-4)$ or $(x+1)(2x-1)(x+4)$ scores			
	M1A1M1A0. (c) Award M1A1M1A1 for Listing all three correct factors with no working.			



Question Number	Scheme		Mark	S
2. (a)	$\left\{ (3+bx)^5 \right\} = (3)^5 + \frac{{}^5C_1(3)^4(b\underline{x}) + \frac{{}^5C_2(3)^3(b\underline{x})^2}{2} + \dots}{= 243 + 405bx + 270b^2x^2 + \dots}$	243 as a constant term seen. 405bx (${}^{5}C_{1} \times \times x$) or (${}^{5}C_{2} \times \times x^{2}$) $270b^{2}x^{2}$ or $270(bx)^{2}$	B1 B1 <u>M1</u> A1 [[4]
(b)	${2(\text{coeff } x) = \text{coeff } x^2} \Rightarrow 2(405b) = 270b^2$	Establishes an equation from their coefficients. Condone 2 on the wrong side of the equation.	M1	
	So, $\left\{b = \frac{810}{270} \Rightarrow\right\} b = 3$	b = 3 (Ignore $b = 0$, if seen.)	A1	[2]
(a) (b)	The terms can be "listed" rather than added. Ignore any e 1^{st} B1: A constant term of 243 seen. Just writing (3) ⁵ is B2 2^{nd} B1: Term must be simplified to $405bx$ for B1. The x $405 + bx$ is B0. M1: For either the x term or the x^2 term. Requires correct power of x , but the other part of the coefficient (per wrong or missing). Allow binomial coefficients such as $\binom{5}{2}$, $\binom{5}{2}$, $\binom{5}{1}$, $\binom{5}{1}$. A1: For either $270b^2x^2$ or $270(bx)^2$. (If $270bx^2$ follow Alternative: Note that a factor of 3^5 can be taken out first: $3^5 \left(1 + \frac{bx}{3}\right)$. Ignore subsequent working (isw): Isw if necessary after e.g. $243 + 405bx + 270b^2x^2 + \dots$ leading to $9 + 15bx + 15$	is required for this mark. Note is $\frac{1}{2}$ binomial coefficient in any form $\frac{1}{2}$ branched for $\frac{1}{2}$ coefficient of $\frac{1}{2}$ coefficients from part (a) havinch is A0.	with the b) may b	6
	Note: The mistake $k \left(1 + \frac{bx}{3}\right)^5$, $k \ne 243$ would give a m Note: For $270bx^2$ in part (a), followed by $2(405b) = 270$			\1 .



		1
Question Number	Scheme	Marks
3.	(a) $5^x = 10$ and (b) $\log_3(x - 2) = -1$	
(a)	$x = \frac{\log 10}{\log 5} \text{or} x = \log_5 10$	M1
	x = 1.430676558 = 1.43 (3 sf)	A1 cao [2]
(b)	$(x-2) = 3^{-1}$ $(x-2) = 3^{-1}$ or $\frac{1}{3}$	M1 oe
	$x \left\{ = \frac{1}{3} + 2 \right\} = 2\frac{1}{3}$ $2\frac{1}{3}$ or 2.3 or awrt 2.33	A1
		[2] 4
(a)	M1: for $x = \frac{\log 10}{\log 5}$ or $x = \log_5 10$. Also allow M1 for $x = \frac{1}{\log 5}$	
(b)	1.43 with no working (or any working) scores M1A1 (even if left as $5^{1.43}$). Other answers which round to 1.4 with no working score M1A0. Trial & Improvement Method: M1: For a method of trial and improvement by trialing f (value between 1.4 and 1.43) = Value below 10 and f (value between 1.431 and 1.5) = Value over 10. A1 for 1.43 cao. Note: $x = \log_{10} 5$ by itself is M0; but $x = \log_{10} 5$ followed by $x = 1.430676558$ is M1. M1: Is for correctly eliminating log out of the equation. Eg 1: $\log_3(x-2) = \log_3(\frac{1}{3}) \Rightarrow x-2=\frac{1}{3}$ only gets M1 when the logs are correctly remove Eg 2: $\log_3(x-2) = -\log_3(3) \Rightarrow \log_3(x-2) + \log_3(3) = 0 \Rightarrow \log_3(3(x-2)) = 0 \Rightarrow 3(x-2) = 3^0$ only gets M1 when the logs are correctly removed, but $3(x-2) = 0$ would score M0. Note: $\log_3(x-2) = -1 \Rightarrow \log_3\left(\frac{x}{2}\right) = -1 \Rightarrow \frac{x}{2} = 3^{-1}$ would score M0 for incorrect use $\frac{1}{\log_{10}(x-2)} = 1 \Rightarrow \log_{10}(x-2) = -\log_{10} 3 \Rightarrow \log_{10}(x-2) + \log_{10} 3 = 0 \Rightarrow \log_{10} 3(x-2) = 0 \Rightarrow 3(x-2) = 10^0$. At this point M1 is scored. A correct answer in (b) without any working scores M1A1.	



Question Number	Scheme	Mark	S
4.	$x^2 + y^2 + 4x - 2y - 11 = 0$		
(a)	$\left\{ (x+2)^2 - 4 + (y-1)^2 - 1 - 11 = 0 \right\} $ (±2, ±1), see notes.	M1	
	Centre is $(-2, 1)$. $(-2, 1)$.	A1 cao	[2]
(b)	$(x+2)^2 + (y-1)^2 = 11 + 1 + 4$ $r = \sqrt{11 \pm "1" \pm "4"}$	M1	
	So $r = \sqrt{11 + 1 + 4} \implies r = 4$ 4 or $\sqrt{16}$ (Award A0 for ± 4).	A1 [2]	
(c)	When $x = 0$, $y^2 - 2y - 11 = 0$ Putting $x = 0$ in C or their C . $y^2 - 2y - 11 = 0$ or $(y - 1)^2 = 12$, etc	M1 A1 aef	
	$y = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)} \left\{ = \frac{2 \pm \sqrt{48}}{2} \right\}$ Attempt to use formula or a method of completing the square in order to find $y = \dots$	M1	
	So, $y = 1 \pm 2\sqrt{3}$ $1 \pm 2\sqrt{3}$	A1 cao cs	0
		[4]	8
	Note: Please mark parts (a) and (b) together. Answers only in (a) and/or (b) get full mar	ks.	
(a)	Note in part (a) the marks are now M1A1 and not B1B1 as on ePEN. M1. for $(+2,+1)$. Otherwise M1 for any tensor to the approximate $(+2)^2$. 0	
(a)	M1: for $(\pm 2, \pm 1)$. Otherwise, M1 for an attempt to complete the square eg. $(x \pm 2)^2 \pm \alpha$, α		
(1.)	$(y \pm 1)^2 \pm \beta$, $\beta \neq 0$. M1A1: Correct answer of $(-2, 1)$ stated from any working gets M1A1		
(b)	M1: to find the radius using 11, "1" and "4", ie. $r = \sqrt{11 \pm "1" \pm "4"}$. By applying this meth	od candida	tes
	will usually achieve $\sqrt{16}$, $\sqrt{6}$, $\sqrt{8}$ or $\sqrt{14}$ and not 16, 6, 8 or 14.		
	Note: $(x+2)^2 + (y-1)^2 = -11 - 5 = -16 \Rightarrow r = \sqrt{16} = 4$ should be awarded M0A0.		
	Alternative: M1 in part (a): For comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down		
	$(-g, -f)$ directly. Condone sign errors for this M mark. M1 in part (b): For using $r = \sqrt{g^2}$	$+f^2-c$.	
	Condone sign errors for this method mark. $(x+2)^2 + (y-1)^2 = 16 \Rightarrow r = 8 \text{ scores M0A0, but } r = \sqrt{16} = 8 \text{ scores M1A1 isw.}$		
(c)	1^{st} M1: Putting $x = 0$ in either $x^2 + y^2 + 4x - 2y - 11 = 0$ or their circle equation usually give	ven in nart (a) or
	part (b). 1^{st} A1 for a correct equation in y in any form which can be implied by later working	_	u) 01
	2^{nd} M1: See rules for using the formula. Or completing the square on a 3TQ to give $y = a \pm \sqrt{a}$	-	
	\sqrt{b} is a surd, $b \neq$ their 11 and $b > 0$. This mark should not be given for an attempt to factorise.		
	2^{nd} A1: Need exact pair in simplified surd form of $\{y = \}$ $1 \pm 2\sqrt{3}$. This mark is also cso.		
	Do not need to see $(0, 1 + 2\sqrt{3})$ and $(0, 1 - 2\sqrt{3})$. Allow 2^{nd} A1 for bod $(1 + 2\sqrt{3}, 0)$ and $(1 - 2\sqrt{3})$.	$-2\sqrt{3},0$).	
	Any incorrect working in (c) gets penalised the final accuracy mark. So, beware: incorrect		
	$(x-2)^2 + (y-1)^2 = 16$ leading to $y^2 - 2y - 11 = 0$ and then $y = 1 \pm 2\sqrt{3}$ scores M1A1M1A0.		
	Special Case for setting $y = 0$: Award SC: M0A0M1A0 for an attempt at applying the formula Award SC: M0A0M1A0 for compared to their equation in x which	pleting the	1157
	$x = \frac{-4 \pm \sqrt{(-4)^2 - 4(1)(-11)}}{2(1)} \left\{ = \frac{-4 \pm \sqrt{60}}{2} = -2 \pm \sqrt{15} \right\} $ square to their equation in x which will usually be $x^2 + 4x - 11 = 0$ to give $a \pm \sqrt{b}$, where \sqrt{b} is a surd, $b \neq 0$.		
	Special Case: For a candidate not using \pm but achieving one of the correct answers then awar SC: M1A1 M1A0 for one of either $y = 1 + 2\sqrt{3}$ or $y = 1 - 2\sqrt{3}$ or $y = 1 + \sqrt{12}$ or $y = 1 - \sqrt{3}$		



Question		
Number	Scheme	Marks
5.	$\frac{1}{2}r^2\theta = \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right) = 6\pi \text{ or } 18.85 \text{ or awrt } 18.8 \text{ (cm)}^2$ Using $\frac{1}{2}r^2\theta$ (See notes)	M1
(a)	2^{7} $2^{(6)}$ (3) 6 π or 18.85 or awrt 18.8	A1
		[2]
(b)	$\sin\left(\frac{\pi}{6}\right) = \frac{r}{6-r}$ $\sin\left(\frac{\pi}{6}\right) \text{ or } \sin 30^\circ = \frac{r}{6-r}$	
	$\begin{vmatrix} \frac{1}{2} = \frac{r}{6 - r} \\ 6 - r = 2r \Rightarrow r = 2 \end{vmatrix}$ Replaces sin by numeric value	dM1
	$6 - r = 2r \Rightarrow r = 2$ $r = 2$	A1 cso [3]
(c)	Area = $6\pi - \pi(2)^2 = 2\pi$ or awrt 6.3 (cm) ² their area of sector – πr^2 2π or awrt 6.3	M1 A1 cao [2]
(a)	M1: Needs θ in radians for this formula. Candidate could convert to degrees and use the degrees formula. A1: Does not need units. Answer should be either 6π or 18.85 or awrt 18.8 Correct answer with no working is M1A1. This M1A1 can only be awarded in part (a).	,
(b)	M1: Also allow $\cos\left(\frac{\pi}{3}\right)$ or $\cos 60^\circ = \frac{r}{6-r}$. 1 st M1: Needs correct trigonometry method. Candidates could state $\sin\left(\frac{\pi}{6}\right) = \frac{r}{x}$ and $x + r = \frac{r}{3}$.	= 6 or
(c)	equivalent in their working to gain this method mark. dM1: Replaces sin by numerical value. $0.009 = \frac{r}{6-r}$ from working "incorrectly" in degree here for dM1. A1: For $r = 2$ from correct solution only. Alternative: 1^{st} M1 for $\frac{r}{oC} = \sin 30$ or $\frac{r}{oC} = \cos 60$. 2^{nd} M1 for $OC = 2r$ and then A1 for $r = \frac{\text{Note}}{\text{Note}}$ seeing $OC = 2r$ is M1M1. Special Case: If a candidate states an answer of $r = 2$ (must be in part (b)) as a guess or from incorrect method then award SC: M0M0B1. Such a candidate could then go on to score M1A(c). M1: For "their area of sector – their area of circle", where $r > 0$ is ft from their answer to part Allow the method mark if "their area of sector" < "their area of circle". The candidate must somewhere in their working that they are subtracting the correct way round, even if their answer negative. Some candidates in part (c) will either use their value of r from part (b) or even introduce a value part (c). You can apply the scheme to award either M0A0 or M1A0 or M1A1 to these candidates. Candidates can get M1 by writing "their part (a) answer $-\pi r^2$ ", where the radius of the not substituted. A1: cao – accept exact answer or awrt 6.3 Correct answer only with no working in (c) gets M1A1 Beware: The answer in (c) is the same as the arc length of the pendant	= 2. A1 in part rt (b). how ver is alue of r didates.



Question Number	Scheme	Marks
6. (a)	$\{ ar = 192 \text{ and } ar^2 = 144 \}$	
()	$r = \frac{144}{192}$ Attempt to eliminate a. (See notes.)	M1
	$\begin{array}{c c} 192 \\ r = \frac{3}{4} \text{ or } 0.75 \end{array}$	A1
	4 52 57.5	[2]
(b)	a(0.75) = 192	M1
	$a\left\{ = \frac{192}{0.75} \right\} = 256$	A1
(c)	$S_{\infty} = \frac{256}{1 - 0.75}$ Applies $\frac{a}{1 - r}$ correctly using both their a and their $ r < 1$.	[2] M1
	So, $\{S_{\infty} = \} 1024$	A1 cao [2]
(d)	$256(1-(0.75)^n)$ Applies S _n with their a and r and "uses" 1000	
	$\frac{256(1-(0.75)^n)}{1-0.75} > 1000$ at any point in their working. (Allow with = or <).	M1
	$(0.75)^n < 1 - \frac{1000(0.25)}{256} $ $\left\{ = \frac{6}{256} \right\}$ Attempt to isolate $+(r)^n$ from S_n formula.	M1
	(Allow With = or >).	
	$n\log(0.75) < \log\left(\frac{6}{256}\right)$ Uses the power law of logarithms correctly. (Allow with = or >). (See notes.)	M1
	$n > \frac{\log(\frac{6}{256})}{\log(0.75)} = 13.0471042 \Rightarrow n = 14$ See notes and $n = 14$	A1 cso
		[4] 10
(a)	M1: for eliminating a by eg. $192r = 144$ or by either dividing $ar^2 = 144$ by $ar = 192$ or div	iding
	$ar = 192$ by $ar^2 = 144$, to achieve an equation in r or $\frac{1}{r}$ Note that $r^2 - r = \frac{144}{192}$ is M0.	
	Note also that any of $r = \frac{144}{192}$ or $r = \frac{192}{144} \left\{ = \frac{4}{3} \right\}$ or $\frac{1}{r} = \frac{192}{144}$ or $\frac{1}{r} = \frac{144}{192}$ are fine for the aw	vard of
	M1. Note: A candidate just writing $r = \frac{144}{192}$ with no reference to a can also get the method in	mark.
	Note : $ar^2 = 192$ and $ar^3 = 144$ leading to $r = \frac{3}{4}$ scores M1A1. This is because r is the rational between any two consecutive terms. These candidates, however, will usually be penalised in particular to the rational score of the	
(b)	M1 for inserting their r into either of the correct equations of either $ar = 192$ or $\{a = \}$ $\frac{192}{r}$ or	
	$ar^2 = 144$ or $\{a = \}$ $\frac{144}{r^2}$. No slips allowed here for M1.	
	M1: can also be awarded for writing down $144 = a \left(\frac{192}{a}\right)^2$	
	A1 for $a = 256$ only. Note 256 from any working scores M1A1.	
		44:
	Note : Some candidates incorrectly confuse notation to give $r = \frac{4}{3}$ or 1.33 in part (a) (g M1A0). In part (b), they recover to write $a = 192 \times \frac{4}{3}$ for M1 and then 256 for A1.	getting



Question Number	Scheme	Marks	
(c)	M1: for applying $\frac{a}{1-r}$ correctly (no slips allowed!) using both their a and their r, where $ r < 1$.		
(d)	A1: for 1024, cao. In parts (a) or (b) or (c), the correct answer with no working scores full marks. 1^{st} M1: For applying S_n with their a and either "the letter r " or their r and "uses" 1000.		
	2^{nd} M1: For isolating $+(r)^n$ and not $(ar)^n$, (eg. $(192)^n$) as the subject of an equation or inequality.		
	$+(r)^n$ must be derived from the S_n formula.		
	3^{rd} M1: For applying the power law to $\lambda^k = \mu$ to give $k \log \lambda = \log \mu$ oe. where $\lambda, \mu > 1$	0.	
	or 3 rd M1: For solving $\lambda^k = \mu$ to give $k = \log_{\lambda} \mu$, where $\lambda, \mu > 0$.		
	A1: cso If a candidate uses inequalities, a fully correct method with inequalities is require So, an <u>incorrect</u> inequality statement at any stage in a candidate's working for this part lose mark.		
	Note: Some candidates do not realise that the direction of the inequality is reversed in the	final line	
	of their solution. Or A1: cso Note a candidate can achieve full marks here if they do not use inequalities.		
	So, if a candidate uses equations rather than inequalities in their working then they need to state in the		
	final line of their working that $n = 13.04$ (truncated) or $n = \text{awrt } 13.05 \Rightarrow n = 14$ for A1. n = 14 from no working gets SC: M0M0M1A1.		
	n = 14 from no working gets Sc. Molviol/17A1. A method of $T_n > 1000 \Rightarrow 256(0.75)^{n-1} > 1000$ can score M0M0M1A0 for a correct application of		
	the power law of logarithms.	cation of	
	Trial & Improvement Method:		
	For $a = 256$ and $r = 0.75$, apply the following scheme:	N # 1	
	$S_{13} = \frac{256(1 - (0.75)^{13})}{1 - 0.75} = 999.6725616$ Attempt to find either S_{13} or S_{14} . EITHER (1) $S_{13} = \text{awrt } 999.7$ or truncated	M1	
	$1 - 0.75$ EITHER (1) $S_{13} = a \text{wit } 777.7$ of truncated	M1	
	truncated 1005.		
	05.6(1 (0.75)14)	M1	
	BOTH (1) $S_{13} = \text{awrt } 999.7 \text{ or truncated}$		
	999 AND (2) $S_{14} = awrt 1005.8 \text{ or}$	A1	
	So, $n = 14$. truncated 1005 AND $n = 14$.		



Question	Scheme	Marks	
Number	Note: A similar scheme would apply for T&I for candidates using their a and their r . So,.		
	1^{st} M1: For attempting to find one of the correct S_n 's either side (but next to) 1000.		
	2^{nd} M1: For one of these S_n 's correct for their a and their r . (You may need to get your calculators		
	out!)		
	3^{rd} M1: For attempting to find both of the correct S_n 's either side (but next to) 1000. A1: Cannot be gained for wrong a and/or r .		
	Trial & Improvement Cumulative Approach:		
	A similar scheme to T&I will be applied here:	,	
	1^{st} M1: For getting as far as the cumulative sum of 13 terms. 2^{nd} M1: (1) S ₁₃ = awrt 999.7 truncated 999. 3^{rd} M1: For getting as far as the cumulative sum to 14 terms. Also at this s		
	$S_{13} < 1000 \text{ and } S_{14} > 1000$. A1: BOTH (1) $S_{13} = \text{awrt } 999.7$ or truncated 999 AND (2)	iage	
	$S_{14} = \text{awrt } 1005.8 \text{ or truncated } 1005 \text{ AND } n = 14.$		
	Trial & Improvement Method: for $(0.75)^n < \frac{6}{256} = 0.0234375$		
	3^{rd} M1: For evidence of examining both $n = 13$ and $n = 14$.		
	Eg: $(0.75)^{13}$ { = 0.023757 } and $(0.75)^{14}$ { = 0.0178179 }		
	A1: $n = 14$		
	<u>Any misreads</u> , $S_n > 10000$ etc, please escalate up to your Team Leader.		
7.	(a) $3\sin(x+45^\circ) = 2$; $0 \le x < 360^\circ$ (b) $2\sin^2 x + 2 = 7\cos x$; $0 \le x < 2\pi$		
(a)	$\sin(x+45^\circ) = \frac{2}{3}$, so $(x+45^\circ) = 41.8103$ $(\alpha = 41.8103)$ $\sin^{-1}(\frac{2}{3})$ or awrt 41.8	M1	
	or awrt 0.73°		
	So, $x + 45^{\circ} = \{138.1897, 401.8103\}$ $x + 45^{\circ} = \text{either "}180 - \text{their } \alpha \text{" or } \alpha \text{"} = 111$	M1	
	" 360° + their α " (α could be in radians). Either awrt 93.2° or awrt 356.8°	A 1	
	and $x = \{93.1897, 356.8103\}$ Both awrt 93.2° and awrt 356.8°	A1	
	Both awit 93.2 and awit 330.8	A1 [4]	
(b)	$2(1-\cos^2 x) + 2 = 7\cos x$ Applies $\sin^2 x = 1 - \cos^2 x$	M1	
	$2\cos^2 x + 7\cos x - 4 = 0$ Correct 3 term, $2\cos^2 x + 7\cos x - 4 = 0$	A1 oe	
	$(2\cos x - 1)(\cos x + 4) = 0$, $\cos x =$ Valid attempt at solving and $\cos x =$	M1	
		A1 cso	
	$\cos x = \frac{1}{2}, \left\{\cos x = -4\right\}$ $\cos x = \frac{1}{2} \text{(See notes.)}$	AT CSO	
	$\left(\beta = \frac{\pi}{3}\right)$		
	$x = \frac{\pi}{3}$ or 1.04719° Either $\frac{\pi}{3}$ or awrt 1.05°	B1	
	$x = \frac{5\pi}{3}$ or 5.23598° Either $\frac{5\pi}{3}$ or awrt 5.24° or 2π – their β (See notes.)	B1 ft	
		[6] 10	



Question Number	Scheme	Marks
(a)	1 st M1: can also be implied for $x = \text{awrt} - 3.2$	
	2^{nd} M1: for $x + 45^{\circ}$ = either "180 – their α " or "360° + their α ". This can be implied by later	
	working. The candidate's α could also be in radians.	
	Note that this mark is not for $x = \text{either "}180 - \text{their } \alpha \text{" or "}360^\circ + \text{their } \alpha \text{"}.$	
	Note: Imply the first two method marks or award M1M1A1 for either awrt 93.2° or awrt 35	56.8°.
	Note: Candidates who apply the following incorrect working of $3\sin(x + 45^\circ) = 2$	
	\Rightarrow 3(sin x + sin 45) = 2, etc will usually score M0M0A0A0.	
	If there are any EXTRA solutions inside the range $0 \le x < 360$ and the candidate would other	rwise
	score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the que. Also ignore EXTRA solutions outside the range $0 \le x < 360$.	stion).
	Working in Radians: Note the answers in radians are $x = \text{awrt } 1.6$, awrt 6.2	
	If a candidate works in radians then mark part (a) as above awarding the A marks in the same v. If the candidate would then score FULL MARKS then withhold the final aA2 mark (the final n this part of the question.)	
	No working: Award M1M1A1A0 for one of awrt 93.2° or awrt 356.8° seen without any w	orking.
	Award M1M1A1A1 for both awrt 93.2° and awrt 356.8° seen without any working. Allow benefit of the doubt (FULL MARKS) for final answer of	
	$\sin x \{ \text{and not } x \} = \{ \text{awrt } 93.2, \text{ awrt } 356.8 \}$	



Question			
Number	Scheme	Marks	
(b)	1 st M1: for a correct method to use $\sin^2 x = 1 - \cos^2 x$ on the given equation.		
	Give bod if the candidate omits the bracket when substituting for $\sin^2 x$, but		
	$2 - \cos^2 x + 2 = 7\cos x$, without supporting working, (eg. seeing " $\sin^2 x = 1 - \cos^2 x$ ") would score		
	1 st M0.		
	Note that applying $\sin^2 x = \cos^2 x - 1$, scores M0.		
	1 st A1: for obtaining either $2\cos^2 x + 7\cos x - 4$ or $-2\cos^2 x - 7\cos x + 4$.		
	1 st A1: can also awarded for a correct three term equation eg. $2\cos^2 x + 7\cos x = 4$ or		
	$2\cos^2 x = 4 - 7\cos x$ etc. 2^{nd} M1: for a valid attempt at factorisation of a quadratic (either 2TQ or 3TQ) in cos, can use	anv	
	variable here, c , y , x or $\cos x$, and an attempt to find at least one of the solutions. See introd		
	the Mark Scheme. Alternatively, using a correct formula for solving the quadratic. Either the		
	formula must be stated correctly or the correct form must be implied by the substitution.		
	2^{nd} A1: for $\cos x = \frac{1}{2}$, BY A CORRECT SOLUTION ONLY UP TO THIS POINT. Ignore 6	extra	
	answer of $\cos x = -4$, but penalise if candidate states an incorrect result e.g. $\cos x = 4$. If the	y have	
	used a substitution, a correct value of their c or their y or their x .		
	Note: 2^{nd} A1 for $\cos x = \frac{1}{2}$ can be implied by later working.		
	1 st B1: for either $\frac{\pi}{3}$ or awrt 1.05°		
	2^{nd} B1: for either $\frac{5\pi}{3}$ or awrt 5.24° or can be ft from 2π – their β or 360° – their β where		
	$\beta = \cos^{-1}(k)$, such that $0 < k < 1$ or $-1 < k < 0$, but $k \ne 0$, $k \ne 1$ or $k \ne -1$.		
	If there are any EXTRA solutions inside the range $0 \le x < 2\pi$ and the candidate would otherwise		
	score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the que Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$.	stion).	
	Working in Degrees: Note the answers in degrees are $x = 60$, 300		
	If a candidate works in degrees then mark part (b) as above awarding the B marks in the same If the candidate would then score FULL MARKS then withhold the final bB2 mark (the final this part of the question.) Answers from no working:	-	
	$x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$ scores M0A0M0A0B1B1,		
	x = 60 and $x = 300$ scores M0A0M0A0B1B0,		
	$x = \frac{\pi}{3}$ ONLY or $x = 60$ ONLY scores M0A0M0A0B1B0,		
	$x = \frac{5\pi}{3}$ ONLY or $x = 120$ ONLY scores M0A0M0A0B0B1.		
	No working: You cannot apply the ft in the B1ft if the answers are given with NO working.		
	Eg: $x = \frac{\pi}{5}$ and $x = \frac{9\pi}{3}$ FROM NO WORKING scores M0A0M0A0B0B0.		
	For candidates using trial & improvement, please forward these to your Team Leader.		



Question Number	Scheme	Marks
8. (a)	$\{V = \} \ 2x^2y = 81$ $2x^2y = 81$	B1 oe
(a)	$\{L = 2(2x + x + 2x + x) + 4y \Rightarrow L = 12x + 4y\}$	
	$y = \frac{81}{2x^2} \Rightarrow L = 12x + 4\left(\frac{81}{2x^2}\right)$ Making y the subject of their expression and substitute this into the correct L formula.	M1
	So, $L = 12x + \frac{162}{x^2}$ AG Correct solution only. AG.	A1 cso
		[3]
(b)	$\frac{dL}{dx} = 12 - \frac{324}{x^3} \left\{ = 12 - 324x^{-3} \right\}$ Either $12x \to 12$ or $\frac{162}{x^2} \to \frac{\pm \lambda}{x^3}$	M1
	Correct differentiation (need not be simplified).	A1 aef
	$\begin{cases} \frac{dL}{dr} = \begin{cases} 12 - \frac{324}{r^3} = 0 \implies x^3 = \frac{324}{12}; = 27 \implies x = 3 \end{cases}$ $L' = 0 \text{ and "their } x^3 = \pm \text{ value"}$ or "their $x^{-3} = \pm \text{ value"}$	M1;
	$x = \sqrt[3]{27} \text{ or } x = 3$	A1 cso
	$\{x = 3,\}$ $L = 12(3) + \frac{162}{3^2} = 54$ (cm) Substitute candidate's value of $x \neq 0$ into a formula for L .	ddM1
	$\frac{3^2}{54}$	A1 cao [6]
	Correct ft L'' and considering sign.	M1
(c)	$\{\text{For } x = 3\}, \ \frac{\mathrm{d}^2 L}{\mathrm{d}x^2} = \frac{972}{x^4} > 0 \implies \text{Minimum}$	A1 [2]
		11
(a)	B1: For any correct form of $2x^2y = 81$. (may be unsimplified). Note that $2x^3 = 81$ is B0. O candidates can use any symbol or letter in place of y.	tnerwise,
(b)	M1: Making y the subject of their formula and substituting this into a correct expression for hal: Correct solution only. Note that the answer is given. Note you can mark parts (b) and (c) together.	L.
	2^{nd} M1: Setting their $\frac{dL}{dt} = 0$ and "candidate's ft correct power of $x = a$ value". The power of	of x must
	be consistent with their differentiation. If inequalities are used this mark cannot be gained un candidate states value of x or L from their x without inequalities. $L' = 0 \text{ can be implied by } 12 = \frac{324}{x^3}.$	
	$2^{\text{nd}} \text{ A1: } x^3 = 27 \implies x = \pm 3 \text{ scores A0.}$	
	2^{nd} A1: can be given for no value of x given but followed through by correct working leading $L = 54$.	g to
(c)	3 rd M1: Note that this method mark is dependent upon the two previous method marks being M1: for attempting correct ft second derivative and <u>considering its sign</u> .	awarded.
	A1: Correct second derivative of $\frac{972}{r^4}$ (need not be simplified) and a valid reason (e.g. > 0),	<u>and</u>
	conclusion. Need to conclude minimum (allow x and not L is a minimum) or indicate by a tic a minimum. The actual value of the second derivative, if found, can be ignored, although subtheir L and not x into L'' is A0. Note: 2 marks can be scored from a wrong value of x , no value found or from not substituting in the value of their x into L'' . Gradient test or testing values either side of their x scores M0A0 in part (c).	ck that it is estituting
	Throughout this question allow confused notation such as $\frac{dy}{dx}$ for $\frac{dL}{dx}$.	



Question Number	Scheme	Marks
9.	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$	
(a)	{Curve = Line} \Rightarrow - x^2 + 2 x + 24 = x + 4 Eliminating y correctly.	B1
	$x^2 - x - 20 = 0$ $\Rightarrow (x - 5)(x + 4) = 0$ $\Rightarrow x =$ Attempt to solve a resulting quadratic to give $x =$ their values.	M1
	So, $x = 5, -4$ Both $x = 5$ and $x = -4$.	A1
	So corresponding y-values are $y = 9$ and $y = 0$. See notes below.	B1ft [4]
(b)	$\left\{ \int (-x^2 + 2x + 24) dx \right\} = -\frac{x^3}{3} + \frac{2x^2}{2} + 24x \left\{ + \epsilon \right\}$ $1^{\text{st}} \text{ A1 at least two out of three terms.}$ $2^{\text{nd}} \text{ A1 for } \underline{\text{correct answer.}}$	M1A1A1
	$\left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x \right]_{-4}^{5} = () - ()$ Substitutes 5 and -4 (or their limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{ \left(-\frac{125}{3} + 25 + 120 \right) - \left(\frac{64}{3} + 16 - 96 \right) = \left(103\frac{1}{3} \right) - \left(-58\frac{2}{3} \right) = 162 \right\}$	
	Area of $\Delta = \frac{1}{2}(9)(9) = 40.5$ Uses correct method for finding area of triangle.	M1
	So area of R is $162 - 40.5 = 121.5$ Area under curve – Area of triangle.	M1
	121.5	Al oe cao
		[7] 11



Question Number	Scheme	Marks	
(a)	1 st B1: For correctly eliminating either x or y . Candidates will usually write $-x^2 + 2x + 24 = x + 4$. This mark can be implied by the resulting quadratic. M1: For solving their quadratic (which must be different to $-x^2 + 2x + 24$) to give $x =$ See introduction for Method mark for solving a 3TQ. It must result from some attempt to eliminate one of the variables. A1: For both $x = 5$ and $x = -4$. 2 nd B1ft: For correctly substituting their values of x in equation of line or parabola to give both correct ft y -values. (You may have to get your calculators out if they substitute their x into $y = -x^2 + 2x + 24$). Note: For $x = 5, -4 \Rightarrow y = 9$ and $y = 0 \Rightarrow eg. (-4, 9)$ and $(5, 0)$, award B1 isw. If the candidate gives additional answers to $(-4, 0)$ and $(5, 9)$, then withhold the final B1 mark. Special Case: Award SC: B0M0A0B1 for $\{A\}(-4, 0)$. You may see this point marked on the diagram. Note: SC: B0M0A0B1 for solving $0 = -x^2 + 2x + 24$ to give $\{A\}(-4, 0)$ and/or $(6, 10)$.		
(b)	Note: Do not give marks for working in part (b) which would be creditable in part (a). 1^{st} M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $24 \to 24x$ is sufficient for M1. 1^{st} A1 at least two out of three terms correctly integrated. 2^{nd} A1 for correct integration only and no follow through. Ignore the use of a '+ c'. 2^{nd} M1: Note that this method mark is dependent upon the award of the first M1 mark in pa Substitutes 5 and -4 (and not 4 if the candidate has stated $x = -4$ in part (a).) (or the limit candidate has found from part(a)) into an "integrated function" and subtracts, either way rou one slip! 3^{rd} M1: Area of triangle $= \frac{1}{2}$ (their x_2 – their x_1)(their x_2) or Area of triangle $= \int_{x_1}^{x_2} x + 4 \left\{ c \right\}$ Where x_1 = their -4 , x_2 = their 5 and x_2 = their x_2 usually found in part (a). x_2 and Area under curve x_3 A1: A1: x_4 A2: x_4 A2: x_4 A3: x_4 A3: x_4 A4: x_4 A4: x_4 A6: x_4 A6: x_4 A1: x_4 A1: x_4 A1: x_4 A2: x_4 A3: x_4 A4: x_4 A6: x_4 A6: x_4 A7: x_4 A7: x_4 A8: x_4 A8: x_4 A8: x_4 A8: x_4 A9: x_4 A9: x_4 A1:		



Question Number	Scheme	Marks
Number	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$ 3^{rd} M1: Uses integral of $(x + 4)$ with	
Aliter 9.(b) Way 2	Area of $R = \int_{-4}^{5} (-x^2 + 2x + 24) - (x + 4) dx$ Correct ft limits. 4 th M1: Uses "curve" - "line" function with correct ft limits.	
	$x^3 x^2$ M: $x^n o x^{n+1}$ for any one term.	M1
	$= -\frac{x^3}{3} + \frac{x^2}{2} + 20x \{+c\}$ A1 at least two out of three terms Correct answer (Ignore + c).	A1ft A1
	$\left[-\frac{x^3}{3} + \frac{x^2}{2} + 20x \right]_{-4}^{5} = () - ()$ Substitutes 5 and -4 (or <i>their limits</i> from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{ \left(-\frac{125}{3} + \frac{25}{2} + 100 \right) - \left(\frac{64}{3} + 8 - 80 \right) = \left(70\frac{5}{6} \right) - \left(-50\frac{2}{3} \right) \right\}$	
	See above working to decide to award 3 rd M1 mark here:	M1
	See above working to decide to award 4 th M1 mark here:	M1
	So area of R is = 121.5	A1 oe cao
		[7] 11
(b)	1^{st} M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms.	
	Note that $20 \rightarrow 20x$ is sufficient for M1.	
	1^{st} A1 at least two out of three terms correctly ft. Note this accuracy mark is ft in Way 2. 2^{nd} A1 for correct integration only and no follow through. Ignore the use of a '+ c'.	
	Allow 2 nd A1 also for $-\frac{x^3}{3} + \frac{2x^2}{2} + 24x - \left(\frac{x^2}{2} + 4x\right)$. Note that $\frac{2x^2}{2} - \frac{x^2}{2}$ or $24x - 4x$	only counts
	as one integrated term for the 1 st A1 mark. Do not allow any extra terms for the 2 nd A1 mark 2 nd M1: Note that this method mark is dependent upon the award of the first M1 mark in pa Substitutes 5 and -4 (and not 4 if the candidate has stated $x = -4$ in part (a).) (or the limit candidate has found from part(a)) into an "integrated function" and subtracts, either way rou one slip!	rt (b). s the and. Allow
	3 rd M1: Uses the integral of $(x + 4)$ with correct ft limits of their x_1 and their x_2 (usually for	-
	(a)) {where $(x_1, y_1) = (-4, 0)$ and $(x_2, y_2) = (5, 9)$.} This mark is usually found in the first candidate's working in part (b). 4 th M1: Uses "curve" – "line" function with correct ft (usually found in part (a)) limits. Sub be correct way round. This mark is usually found in the first line of the candidate's working	traction must
	Allow $\int_{-4}^{5} (-x^2 + 2x + 24) - x + 4 \{dx\}$ for this method mark.	
	3 rd A1: 121.5 oe cao. Note: SPECIAL CASE for this alternative method	
	Area of $R = \int_{-4}^{5} (x^2 - x - 20) dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - 20x \right]_{-4}^{5} = \left(\frac{125}{3} - \frac{25}{2} - 100 \right) - \left(-\frac{64}{3} - 8 + 80 \right)$	0
	The working so far would score SPEICAL CASE M1A1A1M1M1M0A0.	
	The candidate may then go on to state that $=\left(-70\frac{5}{6}\right) - \left(50\frac{2}{3}\right) = -\frac{243}{2}$	
	If the candidate then multiplies their answer by -1 then they would gain the 4 th M1 and 121.5 the final A1 mark.	5 would gain



Question Number	Scheme	Marks
Aliter	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$	
9. (a)	{Curve = Line} $\Rightarrow y = -(y-4)^2 + 2(y-4) + 24$ Eliminating x correctly.	B1
Way 2	$y^2 - 9y = 0$ $\Rightarrow y(y - 9) = 0$ $\Rightarrow y = 0$ Attempt to solve a resulting quadratic to give $y = 0$ their values.	M1
	So, $y = 0, 9$ Both $y = 0$ and $y = 9$.	A1
	So corresponding y-values are $x = -4$ and $x = 5$. See notes below.	B1ft [4]
	2^{nd} B1ft: For correctly substituting their values of y in equation of line or parabola to give b ox-values.	oth correct ft
9. (b)	Alternative Methods for obtaining the M1 mark for use of limits: There are two alternative methods can candidates can apply for finding "162". Alternative 1: $ \int_{-4}^{0} (-x^2 + 2x + 24) dx + \int_{0}^{5} (-x^2 + 2x + 24) dx \\ = \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x \right]_{-4}^{0} + \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x \right]_{0}^{5} \\ = (0) - \left(\frac{64}{3} + 16 - 96 \right) + \left(-\frac{125}{3} + 25 + 120 \right) - (0) \\ = \left(103\frac{1}{3} \right) - \left(-58\frac{2}{3} \right) = 162 $ Alternative 2: $ \int_{-4}^{6} (-x^2 + 2x + 24) dx - \int_{5}^{6} (-x^2 + 2x + 24) dx \\ = \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x \right]_{-4}^{6} - \left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x \right]_{5}^{6} \\ = \left\{ \left(-\frac{216}{3} + 36 + 144 \right) - \left(\frac{64}{3} + 16 - 96 \right) \right\} - \left\{ \left(-\frac{216}{3} + 36 + 144 \right) - \left(-\frac{125}{3} + 25 + 120 \right) - \left(-125$	25 + 120)}



Marks

Appendix

List of Abbreviations

- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ft or $\sqrt{}$ denotes "follow through"
- cao denotes "correct answer only"
- aef denotes "any equivalent form"
- cso denotes "correct solution only"
- AG or * denotes "answer given" (in the question paper.)
- awrt denotes "anything that rounds to"
- aliter denotes "alternative methods"

Extra Solutions

Question

If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Scheme

Number	Scheme	With			
	$(x+2)^2 + (y-1)^2 = 16$, centre $(x_1, y_1) = (-2, 1)$ and radius $r = 4$.				
Aliter	$d_1 = \sqrt{4^2 - 2^2} = \sqrt{12}$ Applying $\sqrt{\text{their } r^2 - \left \text{their } x_1 \right ^2}$	M1			
4. (c)	$\sqrt{12}$	A1 aef			
Way 2	Hence, $y = 1 \pm \sqrt{12}$ Applies $y = \text{their } d$	M1			
	Hence, $y = 1 \pm \sqrt{12}$ Applies $y = \text{their } y_1 \pm \text{ their } d$ So, $y = 1 \pm 2\sqrt{3}$ $1 \pm 2\sqrt{3}$	A1 cao			
	1=2,0	cso			
		[4]			
	Special Case: Award Final SC: M1A1 M1A0 if candidate achieves any one of either				
	$y = 1 + 2\sqrt{3}$ or $y = 1 - 2\sqrt{3}$ or $y = 1 + \sqrt{12}$ or $y = 1 - \sqrt{12}$.				
Aliter 8. (a)	$2x^2 \left(\frac{L-12x}{4}\right) = 81$ $2x^2 \left(\frac{L-12x}{4}\right) = 81$	B1 oe			
Way 2	Rearranges their equation to make y the subject.	M1			
	$\Rightarrow x^2(L-12x) = 162 \Rightarrow L = 12x + \frac{162}{x^2}$ Rearranges their equation to make y the subject. Correct solution only. AG.	A1 cso			
		[3]			

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Order Code UA027654 June 2011

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