Oscillations Supplementary Questions

Part 1

1. $X = A\cos(\omega t)$

i) $A = 0.2$, $T = 2 \Rightarrow \omega = \frac{2\pi}{T} = \underline{\underline{\pi}}$

$\therefore X = 0.2\cos(\pi t)$

at $t = 0$, $\underline{\underline{X = 0.2\,m}}$

ii) $t = 0.25$,

$X = 0.2\cos(0.25\pi)$

$= \underline{\underline{0.141\,m}}$

iii) $t = 0.50$

$X = 0.2\cos(0.5\pi)$

$= \underline{\underline{0\,m}}$

iv) $t = 1.00$

$X = 0.2\cos(\pi)$

$= \underline{-0.2m}$

2. $A = 40 \times 10^{-3}\ m, \quad f = 4\ Hz$

$\therefore \quad \omega = 2\pi f$

$= 8\pi$

$a = -\omega^2 x$

$\boxed{a = -64\pi^2 x}$

i) at $x = \pm 40 \times 10^{-3}$

$a = -64\pi^2 \times (40 \times 10^{-3})$

$= -25.3\ m/s^2$

$25.3\ m/s^2$ towards equilibrium position

at $x = -40 \times 10^{-3}$

$a = -64\pi^2 \times (-40 \times 10^{-3})$

$= 25.3\ m/s^2$

ii) at equilibrium position, $x = 0$

$\therefore \quad a = 0$

iii) at $x = 20 \times 10^{-3}\ m$,

$a = -64\pi^2 \times (20 \times 10^{-3})$

$= -12.6\ m/s^2$

3. $X = 1.8 \cos(4\pi t)$

i) $\underline{A = 1.8\,m}$

ii) $4\pi = \omega = 2\pi f$

$\therefore \underline{f = 2\,Hz}$

iii) $T = \frac{1}{f} = \frac{1}{2}$

$\therefore \underline{T = 0.5 \text{ seconds}}$

iv) $V_{max} = \omega A$

$= 4\pi \times 1.8$

$= \underline{22.6\,m/s}$

v) $a_{max} = \omega^2 A$

$= 16\pi^2 \times 1.8$

$= \underline{284\,m/s^2}$

4 i) $A = 0.025\ m, \quad T = 0.25\ s.$

$$f = \frac{1}{T} = \frac{1}{0.25}$$

$$\underline{f = 4\ Hz}$$

ii) $$a_{max} = \omega^2 A$$

$$\omega = 2\pi f = 8\pi$$

$$\therefore a_{max} = (8\pi)^2 \times 0.025$$

$$= \underline{15.8\ m/s^2}$$

iii) $$V_{max} = \omega A$$

$$= 8\pi \times 0.025$$

$$= \underline{0.628\ m/s}$$

5. $f = 40\text{Hz}$; $A = 0.5 \times 10^{-3}\ \text{m}$

$\omega = 2\pi f$

$\underline{\omega = 80\pi}$

i) $V_{Max} = \omega A$

$= 80\pi \times 0.5 \times 10^{-3}$

$= \underline{0.126\ \text{m/s}}$

ii) $a_{max} = \omega^2 A = (80\pi)^2 \times 0.5 \times 10^{-3}$

$= \underline{31.6\ \text{m/s}^2}$

6. i) 1 revolution = 1 complete cycle of piston's SHM.

20,000 per minute = 333.3 Hz

ii) $V_{max} = \omega A$

$\omega = 2\pi f = 2094.4$ rad/s

$\therefore V_{max} = 2094.4 \times 0.0125$

$= 26.2$ m/s

iii) $a_{max} = \omega^2 A$

$= (2094.4)^2 \times 0.0125$

$= 54,831.4$

$= 54,800$ m/s^2 (= 5589 g)

iv) $F = ma$

$= 0.2 \times 54,800$

$= 10966.3$

$= 11000$ N

7.

$$a = -300x$$

$$\therefore \quad 300 = \omega^2$$

$$\therefore \quad \omega = \sqrt{300} = 10\sqrt{3}$$

$$= 2\pi f$$

$$\therefore \quad f = \frac{5\sqrt{3}}{\pi}$$

$$= \underline{2.76\ \text{Hz}}$$

8. ignore

$$a_{max} = -\omega^2 x > 9.81$$

$$\uparrow$$

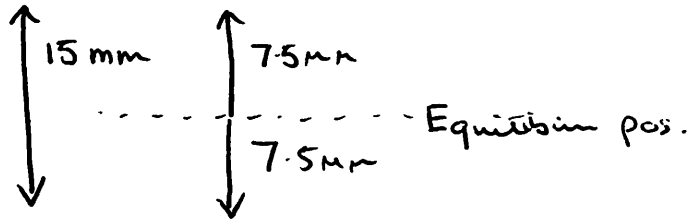
$$A$$

$$\therefore$$

$$\therefore \quad \omega^2 > \frac{9.81}{A} \iff f > \frac{1}{2\pi}\sqrt{\frac{9.81}{A}}$$

$$\therefore \quad f \geq \frac{1}{2\pi}\sqrt{\frac{9.81}{1.6\times10^{-6}}} = \underline{394\ \text{Hz}}$$

9.*



$\therefore$ $\underline{A = 7.5 \text{ mm}}$

25 Stitches per 6 seconds $= 4.1\dot{6}$ Stitches per Second.

One Stitch = one cycle.

$\therefore$ $\underline{f = 4.1\dot{6} \text{ Hz}}$

$$V_{Max} = \omega A$$

$$= 2\pi \times 4.1\dot{6} \times 7.5 \times 10^{-3}$$

$$= \underline{0.196 \text{ m/s}}$$

10.* $A = 0.1 \times 10^{-2}\ m$

$= \underline{1 \times 10^{-3}\ m}$

$a_{max} > g$

$\therefore\ \omega^2 A > g$

$\therefore\ \omega > \sqrt{\frac{g}{A}}$

$\therefore\ f \geq \frac{1}{2\pi}\sqrt{\frac{g}{A}}$

$f > \frac{1}{2\pi}\sqrt{\frac{9.81}{1 \times 10^{-3}}} = \underline{15.8\ Hz}$

11.* Assume tide height is SHM.

½ cycle = 6 hours 15 mins.

$\therefore$ T = 12 hours 30 mins

= 750 mins

"Equilibrium position" : X = 20

Amplitude = 10m.

$\therefore$ $X = 20 + 10\cos(\omega t + \epsilon)$ ← to correct for tide being at lowest point when $t=0$.

$\Rightarrow 10\cos\epsilon = -10$

$\therefore \underline{\epsilon = \pi}$

~~$\omega = \frac{2\pi}{750}$~~ if t has units of mins.

$\therefore$ $X = 20 + 10\cos\left(\frac{2\pi t}{750} + \pi\right)$

$\therefore$ $17.5 = 20 + 10\cos\left(\frac{2\pi t}{750} + \pi\right)$

$\therefore$ $\cos\left(\frac{2\pi t}{750} + \pi\right) = -0.25$

$\therefore$ $\frac{2\pi t}{750} + \pi = 1.823$ or 4.4597

$$\therefore \quad \frac{2\pi t}{750} = 1.318$$

$$t = \frac{1.318 \times 750}{2\pi}$$

$$= 157.3 \text{ mins after 12pm.}$$

∴ The ship can first enter the dock at <u>2:37 pm</u>