Mark Scheme (Results)

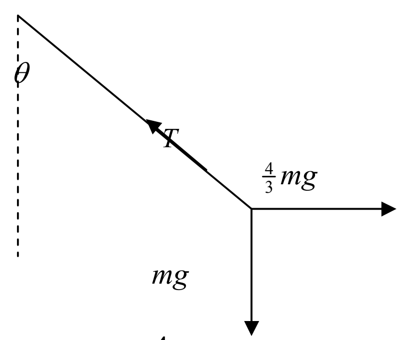
January 2009

GCE

GCE Mathematics (6679/01)

Edexcel Limited. Registered in England and Wales No. 4496750
Registered Office: One90 High Holborn, London WC1V 7BH

January 2009
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks
1	N2L $\quad 3a = -\left(9 + \dfrac{15}{(t+1)^2}\right)$	B1
	$3v = -9t + \dfrac{15}{t+1}\ (+A)$	M1 A1ft
	$v = 0, t = 4 \quad \Rightarrow \quad 0 = -36 + 3 + A \quad \Rightarrow \quad A = 33$	M1 A1
	$v = -3t + \dfrac{5}{t+1} + 11$	
	$t = 0 \quad \Rightarrow \quad v = 16$	M1 A1 (7) **[7]**
2		
(a)	$(\leftarrow) \quad T\sin\theta = \dfrac{4}{3}mg$	M1 A1
	$(\uparrow) \quad T\cos\theta = mg$	A1
	$T^2 = \left(\dfrac{4}{3}mg\right)^2 + (mg)^2$	M1
	Leading to $\quad T = \dfrac{5}{3}mg$	A1 (5)
(b)	HL $\quad T = \dfrac{\lambda x}{a} \quad \Rightarrow \quad \dfrac{5}{3}mg = \dfrac{3mge}{a}$ $\qquad$ ft their T	M1 A1ft
	$e = \dfrac{5}{9}a$	
	$E = \dfrac{\lambda x^2}{2a} = \dfrac{3mg}{2a} \times \left(\dfrac{5}{9}a\right)^2 = \dfrac{25}{54}mga$	M1 A1 (4)
		[9]

Question Number	Scheme	Marks
3	$\omega = \frac{80\times 2\pi}{60}$ rad s^{-1} $\left(=\frac{8\pi}{3}\approx 8.377...\right)$	B1
	Accept $v=\frac{16\pi}{75}\approx 0.67\,\text{ms}^{-1}$ as equivalent	
	$(\uparrow)$ $R = mg$	B1
	For least value of μ $(\leftarrow)$ $\mu mg = mr\omega^2$	M1 A1=A1
	$\mu = \frac{0.08}{9.8}\times\left(\frac{8\pi}{3}\right)^2 \approx 0.57$ accept 0.573	M1 A1 (7)
		[7]
4 (a)	$a = 8$	B1
	$T=\frac{25}{2}=\frac{2\pi}{\omega}$ $\Rightarrow$ $\omega=\frac{4\pi}{25}\,(\approx 0.502\,...)$	M1 A1
	$v^2=\omega^2\left(a^2-x^2\right)$ $\Rightarrow$ $v^2=\left(\frac{4\pi}{25}\right)^2\left(8^2-3^2\right)$ ft their $a,\ \omega$	M1 A1ft
	$v=\frac{4\pi}{25}\sqrt{55}\approx 3.7$ $\left(\text{m h}^{-1}\right)$ awrt 3.7	M1 A1 (7)
(b)	$x=a\cos\omega t$ $\Rightarrow$ $3=8\cos\left(\frac{4\pi}{25}t\right)$ ft their $a,\ \omega$	M1 A1ft
	$t\approx 2.3602\ \ldots$	M1
	time is 12 22	A1 (4)
		[11]

Question Number	Scheme	Marks
5 (a)	Let x be the distance from the initial position of B to C	
	GPE lost = EPE gained	
	$mgx \sin 30° = \dfrac{6mgx^2}{2a}$	M1 A1=A1
	Leading to $x = \dfrac{a}{6}$	M1
	$AC = \dfrac{7a}{6}$	A1 (5)
(b)	The greatest speed is attained when the acceleration of B is zero, that is where the forces on B are equal.	
	$(\nwarrow)$ $T = mg \sin 30° = \dfrac{6mge}{a}$	M1
	$e = \dfrac{a}{12}$	A1
	CE $\dfrac{1}{2}mv^2 + \dfrac{6mg}{2a}\left(\dfrac{a}{12}\right)^2 = mg\dfrac{a}{12}\sin 30°$	M1 A1=A1
	Leading to $v = \sqrt{\left(\dfrac{ga}{24}\right)} = \dfrac{\sqrt{6ga}}{12}$	M1 A1 (7)
		[12]
	Alternative approaches to (b) *are considered on the next page.*	

Question Number	Scheme	Marks
5	*Alternative approach to* (b) *using calculus with energy.*	
	Let distance moved by B be x	
	CE $\quad \frac{1}{2}mv^2 + \frac{6mg}{2a}x^2 = mgx\sin 30°$	M1 A1=A1
	$v^2 = gx - \frac{6g}{a}x^2$	
	For maximum v $\quad \frac{\mathrm{d}}{\mathrm{d}x}\left(v^2\right) = 2v\frac{\mathrm{d}v}{\mathrm{d}x} = g - \frac{12g}{a}x = 0$	M1 A1
	$x = \frac{a}{12}$	
	$v^2 = g\left(\frac{a}{12}\right) - \frac{6g}{a}\left(\frac{a}{12}\right)^2 = \frac{ga}{24}$	M1
	$v = \sqrt{\left(\frac{ga}{24}\right)}$	A1 (7)
	Alternative approach to (b) *using calculus with Newton's second law.*	
	As before, the centre of the oscillation is when extension is $\frac{a}{12}$	M1 A1
	N2L $\quad mg\sin 30° - T = m\ddot{x}$	
	$\frac{1}{2}mg - \frac{6mg\left(\frac{a}{12}+x\right)}{a} = m\ddot{x}$	M1 A1
	$\ddot{x} = -\frac{6g}{a}x \quad \Rightarrow \quad \omega^2 = \frac{6g}{a}$	A1
	$v_{\max} = \omega a = \sqrt{\left(\frac{6g}{a}\right)} \times \frac{a}{12} = \sqrt{\left(\frac{ga}{24}\right)}$	M1 A1 (7)

Question Number	Scheme	Marks
6 (a)	$\int y^2\,dx = \int (4-x^2)^2\,dx = \int (16-8x^2+x^4)\,dx$	
	$= 16x - \frac{8x^3}{3} + \frac{x^5}{5}$	M1 A1
	$\left[16x - \frac{8x^3}{3} + \frac{x^5}{5}\right]_0^2 = \frac{256}{15}$	M1 A1
	$\int xy^2\,dx = \int x(4-x^2)^2\,dx = \int (16x-8x^3+x^5)\,dx$	
	$= 8x^2 - 2x^4 + \frac{x^6}{6}$	M1 A1
	$\left[8x^2 - 2x^4 + \frac{x^6}{6}\right]_0^2 = \frac{32}{3}$	M1A1
	$\bar{x} = \frac{\int xy^2\,dx}{\int y^2\,dx} = \frac{32}{3} \times \frac{15}{216} = \frac{5}{8}$ *	M1 A1 (10)
(b)	$A \times \bar{x} = (\pi r^2 l) \times \frac{l}{2}$	M1
	$\frac{256}{15}\pi \times \frac{5}{8} = \pi \times 16l \times \frac{l}{2}$	A1 ft
	Leading to $l = \frac{2\sqrt{3}}{3}$ accept exact equivalents or awrt 1.15	M1 A1 (4)
		[14]

Question Number	Scheme	Marks
7 (a)	Let speed at C be u	
	CE $\quad \frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mga(1-\cos\theta)$	M1 A1
	$u^2 = \frac{9ga}{4} - 2ga\cos\theta$	
	$mg\cos\theta \ (+R) = \frac{mu^2}{a}$	M1 A1
	$mg\cos\theta = \frac{9mg}{4} - 2mg\cos\theta$ eliminating u	M1
	Leading to $\cos\theta = \frac{3}{4}$ *	M1 A1 (7)
(b)	At C $\quad u^2 = \frac{9ga}{4} - 2ga \times \frac{3}{4} = \frac{3}{4}ga$	B1
	$(\rightarrow)$ $\quad u_x = u\cos\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{3}{4} = \sqrt{\left(\frac{27ga}{64}\right)} = 2.033\sqrt{a}$	M1 A1ft
	$(\downarrow)$ $\quad u_y = u\sin\theta = \sqrt{\left(\frac{3ga}{4}\right)} \times \frac{\sqrt{7}}{4} = \sqrt{\left(\frac{21ga}{64}\right)} = 1.792\sqrt{a}$	M1
	$v_y^2 = u_y^2 + 2gh \ \Rightarrow \ v_y^2 = \frac{21}{64}ga + 2g \times \frac{7}{4}a = \frac{245}{64}ga$	M1 A1
	$\tan\psi = \frac{v_y}{u_x} = \sqrt{\left(\frac{245}{27}\right)} \approx 3.012 \ldots$	M1
	$\psi \approx 72°$ awrt $72°$	A1 (8)
	Or 1.3^c (1.2502^c) awrt 1.3^c	**[15]**
	Alternative for the last five marks	
	Let speed at P be v.	
	CE $\quad \frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{ag}{4}\right) = mg \times 2a$ or equivalent	M1
	$v^2 = \frac{17mga}{4}$	M1 A1
	$\cos\psi = \frac{u_x}{v} = \sqrt{\left(\frac{27}{64} \times \frac{4}{17}\right)} = \sqrt{\left(\frac{27}{272}\right)} \approx 0.315$	M1
	$\psi \approx 72°$ awrt $72°$	A1
	Note: The time of flight from C to P is $\frac{\sqrt{235} - \sqrt{21}}{8}\sqrt{\left(\frac{a}{g}\right)} \approx 1.38373\sqrt{\left(\frac{a}{g}\right)}$	