Collisions

Supplementary Questions

Part 1

1. Momentum before $= 20 - 12$

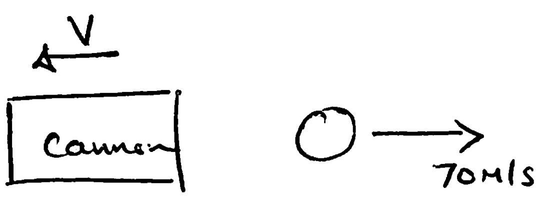
$= 8$ Ns

Momentum After $= Y - 3$

Conservation of Momentum : $8 = Y - 3$

$\therefore$ $Y = 11$ Ns

2.



Momentum before $= 0$

Momentum after $= -1500V + 5 \times 70$

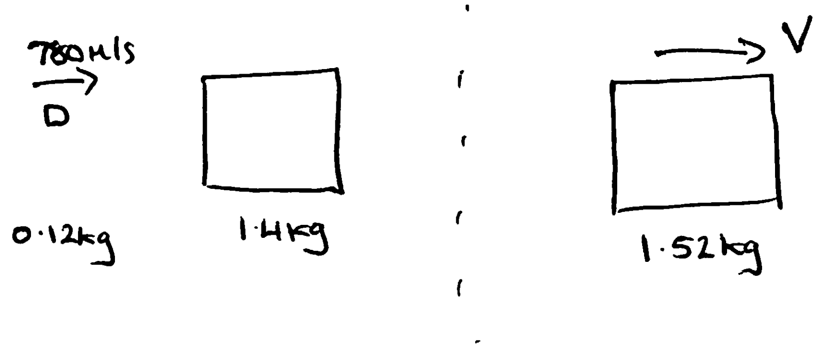
$= 350 - 1500V$

Conservation of Momentum : $350 - 1500V = 0$

$\therefore$ $V = \frac{1500}{350}$

$\therefore$ $V = 4.29$ m/s

3.



Momentum before $= 780 \times 0.12$

$= \underline{\underline{93.6 \text{ kg m/s}}}$

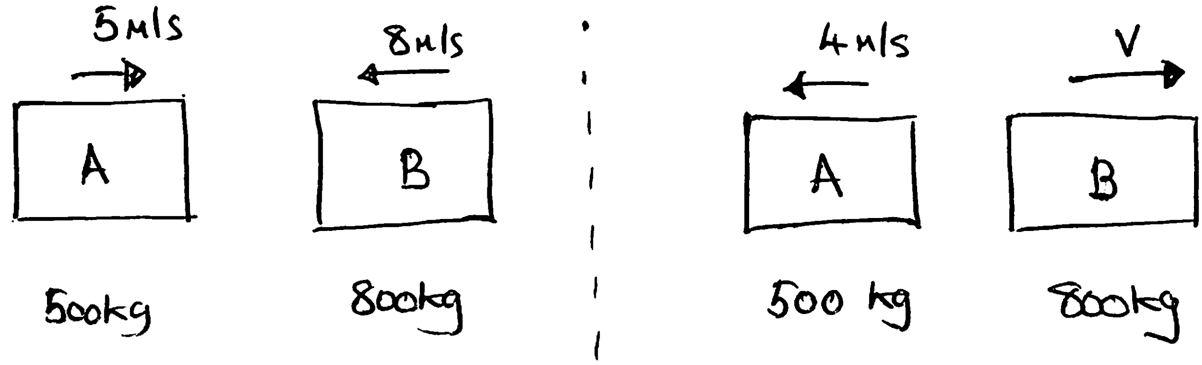
Momentum after $= 1.52V$

Conservation of momentum: $93.6 = 1.52V$

$\therefore V = \dfrac{93.6}{1.52}$

$\therefore \underline{\underline{V = 61.6 \text{ m/s}}}$

4.



Momentum before $= 5 \times 500 - 8 \times 800$

$= -3900$ kgm/s

Momentum after $= -4 \times 500 + 800V$

$= 800V - 2000$

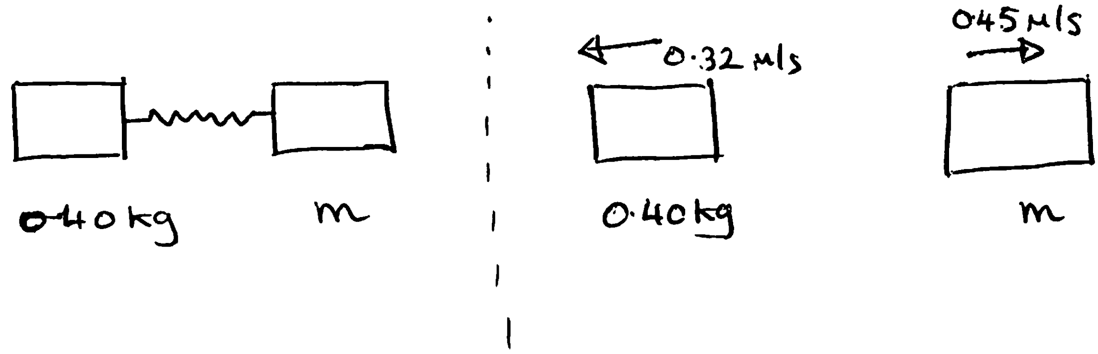
Conservation of Momentum :

$$800V - 2000 = -3900$$

$$\therefore \quad 800V = -1900$$

$$\therefore \quad \underline{\underline{V = -2.38 \text{ m/s}}}$$

5.



momentum before $= 0$

momentum after $= 0.45m - 0.32 \times 0.4$

$= 0.45m - 0.128$

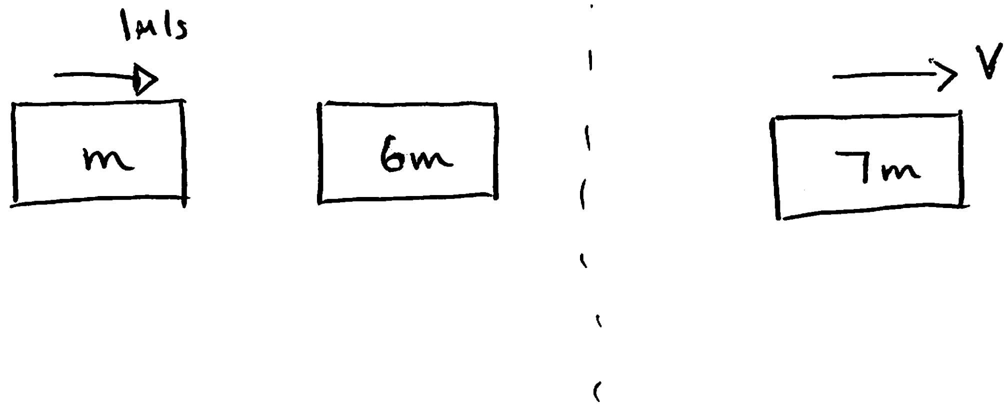
Conservation of momentum :

$0.45m - 0.128 = 0$

$\therefore \; m = \dfrac{0.128}{0.4}$

$\therefore \; m = 0.32 \text{ kg}$

6.



Momentum before $= 1 \times m$

$= m$

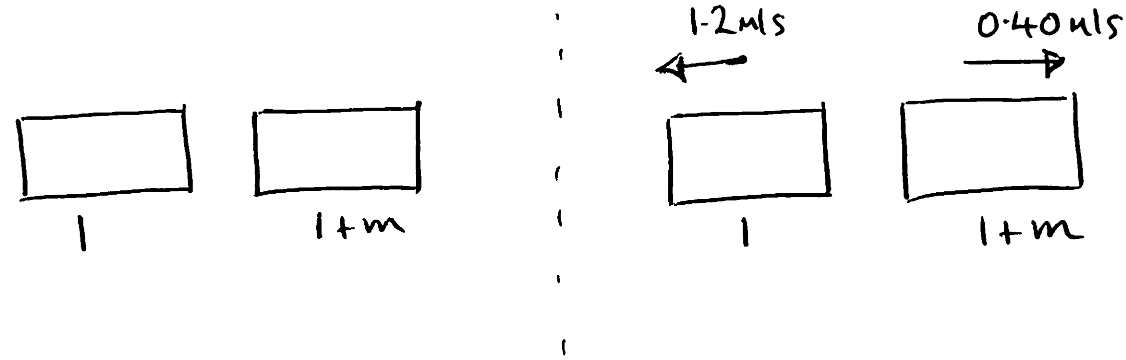
Momentum after $= 7mV$

Conservation of momentum:

$$m = 7mV$$

$$\therefore \; \underline{\underline{V = 1/7 \text{ m/s}}}$$

7.



momentum before $= 0$

momentum after $= 0.4(1+m) - 1.2 \times 1$

$= 0.4m - 0.8$

Conservation of momentum:

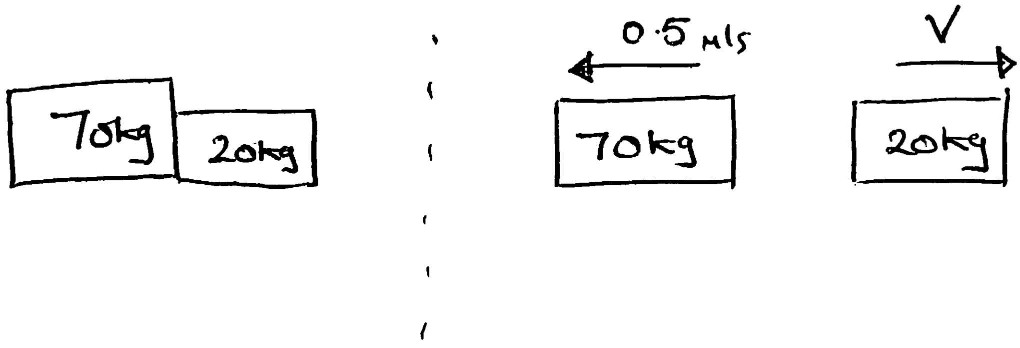
$0 = 0.4m - 0.8$

$\therefore \; m = \dfrac{0.8}{0.4}$

$\therefore \; \underline{\underline{m = 2\,kg}}$

8. Speed astronaut needs $= \frac{15}{30}$

$= \underline{\underline{0.5 \text{ m/s}}}$



Momentum before $= 0$

momentum after $= -70 \times 0.5 + 20V$

$= 20V - 35$

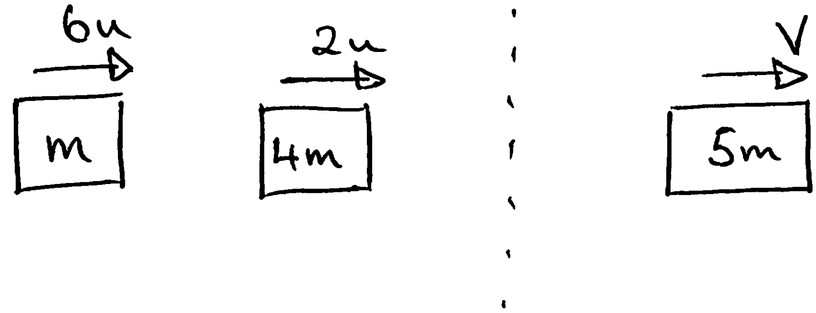
Conservation of momentum :

$20V - 35 = 0$

$V = \frac{35}{20}$

$\therefore \underline{\underline{V = 1.75 \text{ m/s}}}$

9.*



Momentum before $= 6mu + 8mu$

$= 14mu$

Momentum after $= 5mV$

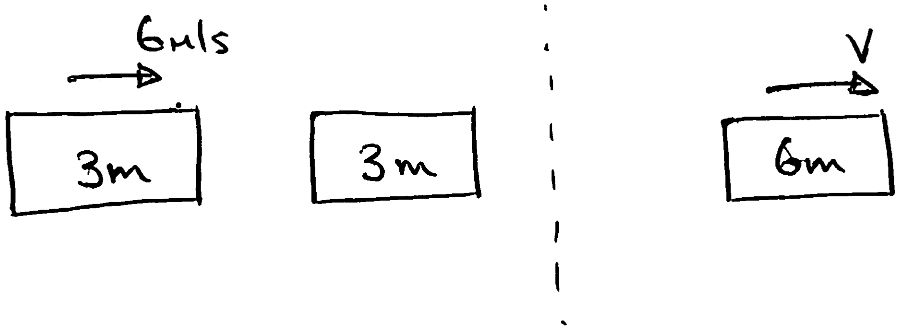
Conservation of momentum :

$14mu = 5mV$

$\therefore \; V = \dfrac{14}{5}u$

$\therefore \; \underline{\underline{V = 2.8u}}$

10.*



Momentum before $= 6 \times 3m$

$= 18m$

Momentum after $= 6mV$

Conservation of momentum:

$$18m = 6mV$$

$\therefore$ $\underline{\underline{V = 3\text{ m/s}}}$

11.i) The boat will move backwards and forwards along the line between the two people. As the boy throws the ball forwards, the boat will move backwards until the girl receives the ball. By catching the ball, the boat will stop. When the girl throws the ball back to the boy, the boat will move in the opposite direction. The boat will then stop when the boy catches the ball.

ii) The boats will move further and further apart, picking up speed as the game progresses. The total momentum of the boats and the ball will always sum to zero.

12.* Initial speed for a jump of 0.5m high:

$u = ?$
$V = 0$
$A = -9.81$
$S = 0.5$

$$v^2 = u^2 + 2as$$

$$u^2 = v^2 - 2as$$

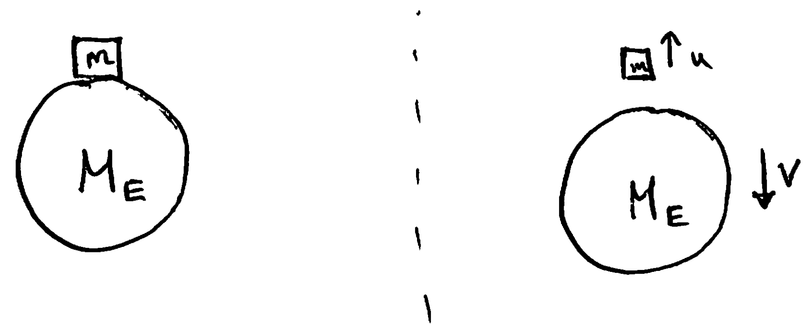
$$= 0^2 - 2\times -9.81 \times 0.5$$

$$= 9.81$$

$$\therefore u = \sqrt{9.81}$$

$$= \underline{\underline{3.13\,m/s}}$$

Total mass of people $= 7\times10^9 \times 60 = 4.2\times10^{11}$ kg



Momentum before $= 0$

Momentum after $= mu - M_E V$

Conservation of momentum:

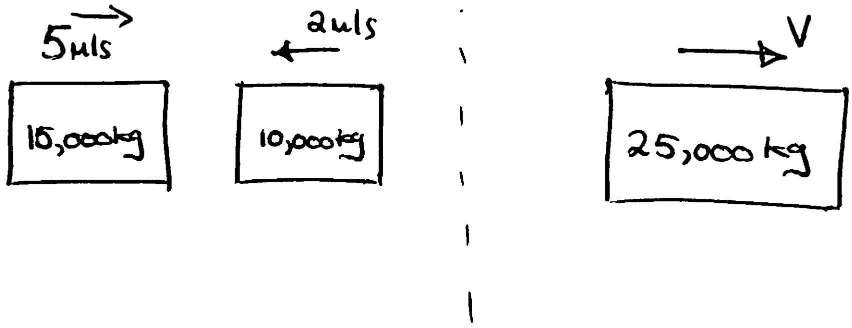
$$mu - M_E V = 0$$

$$V = \frac{mu}{M_E} = \frac{4.2\times10^{11} \times 3.13}{6\times10^{24}}$$

$$\therefore V \sim \underline{\underline{2.2\times10^{-13}\,m/s}}$$

Part 2

1. i)



Momentum before $= 15{,}000 \times 5 - 10{,}000 \times 2$

$= \underline{\underline{55{,}000 \text{ kgm/s}}}$

Momentum after $= 25{,}000\,V$

Conservation of momentum:

$55{,}000 = 25{,}000\,V$

$\underline{\underline{V = 2.2 \text{ m/s}}}$

ii) Kinetic energy before $= 0.5 \times 15{,}000 \times 5^2$
$+ 0.5 \times 10{,}000 \times 2^2$

$= 207{,}500 \text{ J}$

Kinetic energy after $= 0.5 \times 25{,}000 \times 2.2^2$

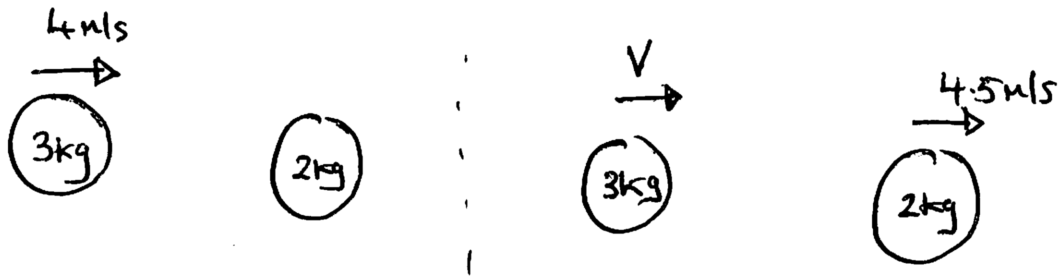
$= 60{,}500 \text{ J}$

energy lost during collision:

$= 207{,}500 - 60{,}500$

$= \underline{\underline{147{,}000 \text{ J}}}$

2. i)



Momentum before $= 3 \times 4$

$= 12$ kg m/s

Momentum after $= 4.5 \times 2 + 3V$

$= 9 + 3V$

Conservation of momentum : $12 = 9 + 3V$

$\therefore$ $\underline{\underline{V = 1 \text{ m/s}}}$

ii) Kinetic energy before $= 0.5 \times 3 \times 4^2$

$= 24\text{J}$

Kinetic energy after $= 0.5 \times 3 \times 1^2 + 0.5 \times 2 \times 4.5^2$

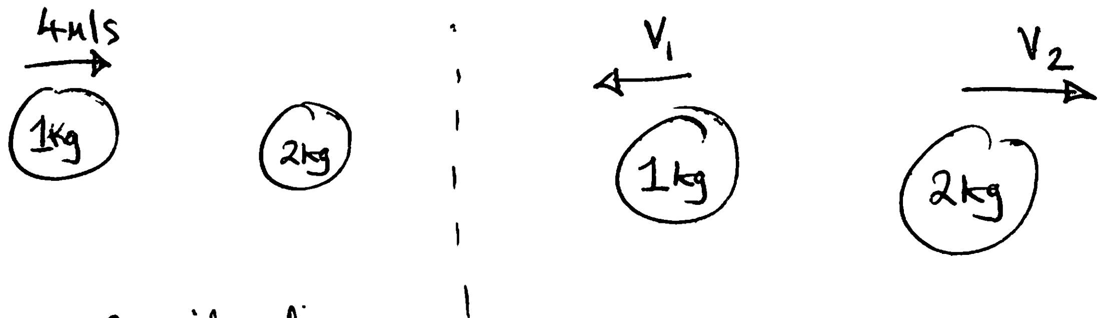
$= 21.75\text{J}$

energy lost $= 24 - 21.75$

$= \underline{\underline{2.25\text{J}}}$

iii) Since some energy is lost, the collision is inelastic.

3.



Momentum consideration

Momentum before $= 4 \times 1$

$= 4 \text{ kg m/s}$

Momentum after $= 2V_2 - V_1$

Conservation of momentum : $\boxed{2V_2 - V_1 = 4.}$

Energy consideration

Kinetic energy before $= 0.5 \times 1 \times 4^2$

$= 8 \text{ J}$

Kinetic energy after $= 0.5 V_1^2 + V_2^2$

Conservation of energy : $\boxed{8 = 0.5 V_1^2 + V_2^2}$

$$V_1^2 + 2V_2^2 = 16 \quad ; \quad V_1 = 2V_2 - 4$$

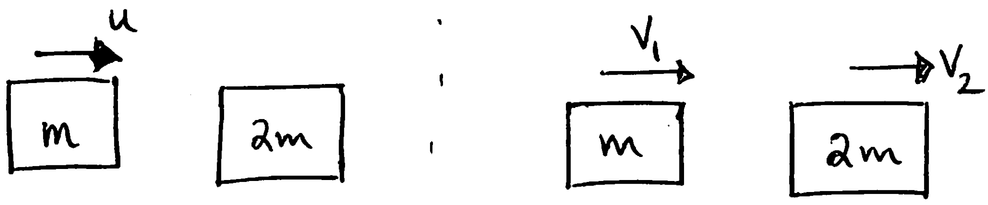
$$(2V_2 - 4)^2 + 2V_2^2 = 16$$

$$6V_2^2 - 16V_2 = 0 \implies 2V_2(3V_2 - 8) = 0$$

1. $V_2 = 0 \implies V_1 = -4$ ← NO collision occurs

2. $\underline{\underline{V_2 = 8/3}} \implies \underline{\underline{V_1 = 4/3}}$

4. i)



Momentum before $= mu$

Momentum after $= mV_1 + 2mV_2$

Conservation of momentum : $mu = mV_1 + 2mV_2$

$\therefore \boxed{u = V_1 + 2V_2}$

Kinetic energy before $= \frac{1}{2} mu^2$

Kinetic energy after $= \frac{1}{2} mV_1^2 + mV_2^2$

Conservation of energy : $\boxed{u^2 = V_1^2 + 2V_2^2}$

$V_1 = u - 2V_2$

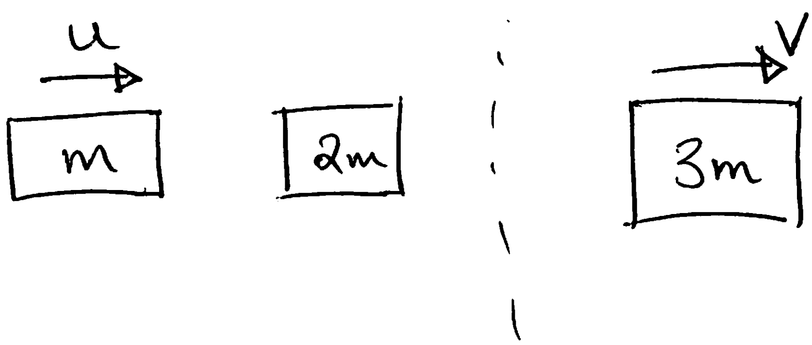
$\therefore \quad u^2 = (u - 2V_2)^2 + 2V_2^2$

$\quad\quad = u^2 - 4uV_2 + 4V_2^2 + 2V_2^2$

$\therefore \; 6V_2^2 - 4uV_2 = 0 \implies 2V_2(3V_2 - 2u) = 0$

$\underline{\underline{V_2 = \frac{2}{3}u}} \implies \underline{\underline{V_1 = -\frac{1}{3}u}}$

4ii)



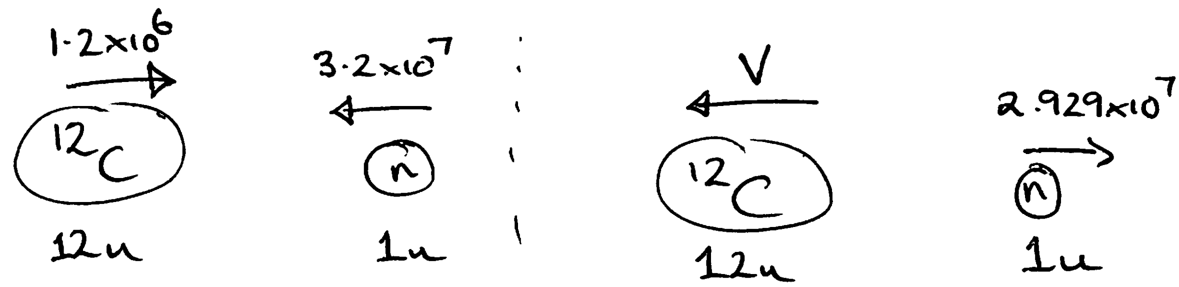
momentum before = mu

momentum after = $3mV$

Conservation of momentum : $mu = 3mV$

$\therefore \; V = \frac{1}{3}u$

5. a) i)



Momentum before $= 12u \times 1.2\times10^6 - 3.2\times10^7 \times 1u$

$= -1.76\times10^7\,u$

Momentum after $= 2.929\times10^7 \times 1u - 12uV$

Conservation of momentum:

$$-1.76\times10^7\,u = 2.929\times10^7\,u - 12uV$$

$$\therefore\ V = \frac{2.929\times10^7 + 1.76\times10^7}{12}$$

$$\therefore V = \underline{\underline{3.91\times10^6\ \text{m/s}}} \qquad (3.9075\times10^6)$$

ii) Kinetic energy before $= 0.5\times12u\times(1.2\times10^6)^2$

$+ 0.5\times1u\times(3.2\times10^7)^2$

$= \underline{\underline{5.2064\times10^{14}}}\ u$

Kinetic energy after $= 0.5\times12u\times(3.9075\times10^6)^2$

$+ 0.5\times1u\times(2.929\times10^7)^2$

$= \underline{\underline{5.2056\times10^{14}\,u}}$

Kinetic energy before $\simeq$ Kinetic energy after,

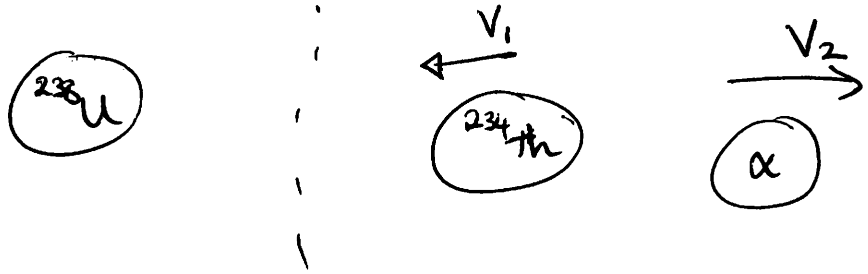
$\therefore$ collision is elastic.

iii) mass after $= 13u$, Velocity after $= V$.

$13uV = -1.76\times10^7 u \implies V = \underline{\underline{-1.35\times10^6\ \text{m/s}}}$

(1.35×10⁶ m/s to the left)

6.



Conservation of momentum: $234uV_1 = 4uV_2$

$\therefore \; \underline{\underline{V_2 = 58.5\,V_1}}$

Kinetic energy of $\alpha = \frac{1}{2} \times 4u \times V_2^2$

$= 2uV_2^2$

Kinetic energy of $^{234}Th = \frac{1}{2} \times 234u \times V_1^2$

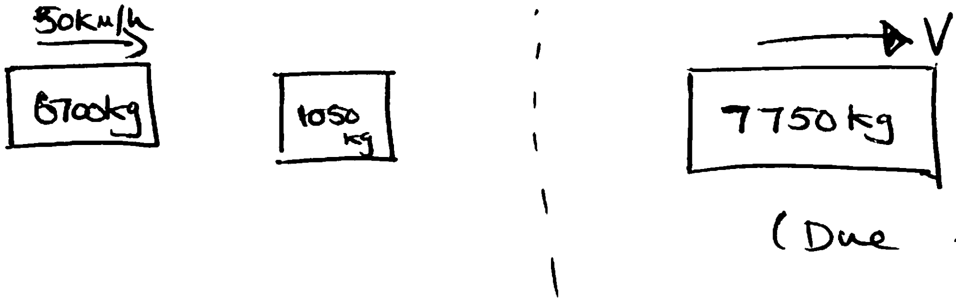
$= 117\,u\,V_1^2$

$$\frac{KE_\alpha}{KE_{234Th}} = \frac{2uV_2^2}{117uV_1^2} = \frac{2 \times (58.5\,V_1)^2}{117\,V_1^2}$$

$= \underline{\underline{58.5}}$

α particle has 58.5 times more kinetic energy than 234Th nucleus.

7.*



(Due to inelastic collision)

$$50\ \text{km/h} = \frac{50\times10^3}{60^2}\ \text{m/s} = 13.\dot{8}\ \text{m/s}.$$

momentum before $= 13.\dot{8} \times 6700$

$= 93055.\dot{5}$ kg m/s

Momentum after $= 7750V$

conservation of momentum: $93055.\dot{5} = 7750V$

$$\underline{\underline{V = 12.007\ \text{m/s}}}$$

Kinetic energy of wreckage

$= 0.5 \times 7750 \times (12.007)^2$

$= \underline{\underline{558666.87\ \text{J}}}$

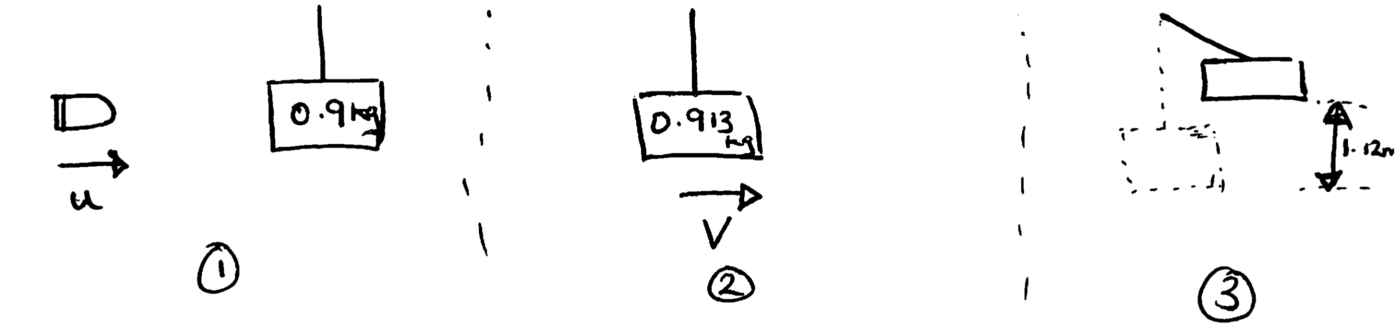
G.P.E. of wreckage when stopped $= 7750 \times 9.81 \times h = 76027.5h$

$$\therefore\ h = \frac{558666.87}{76027.5} = 7.35\text{m vertically}$$

Distance along road $= 12.5 \times 7.35$

$= \underline{\underline{91.85\text{m}}}$

8.*



① momentum before $= \underline{1.3 \times 10^{-2}}\, u$

② momentum after $= \underline{0.913}\, V$

Conservation of momentum: $\boxed{1.3 \times 10^{-2}\, u = 0.913\, V}$

② Kinetic energy after $= 0.5 \times 0.913\, V^2$

$= \underline{0.4565\, V^2}$

③ GPE at end of motion $= 0.913 \times 9.81 \times 1.12$

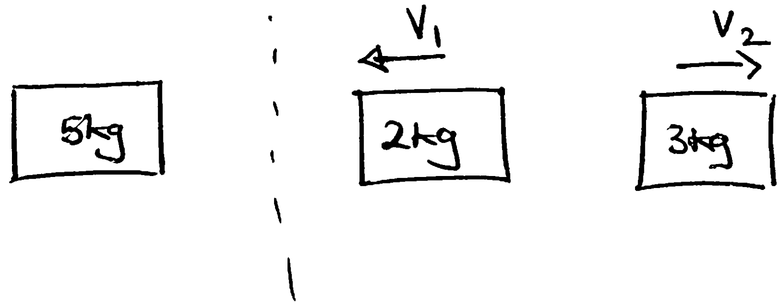
$= \underline{10.0313\text{ J}}$

Conservation of energy: $\boxed{0.4565\, V^2 = 10.0313}$

$\therefore \quad \underline{V = 4.688\text{ m/s}}$

$$\therefore \quad u = \frac{0.913 \times 4.688}{1.3 \times 10^{-2}} = \underline{329.2\text{ m/s}}$$

9.*



Conservation of momentum : $\boxed{2V_1 = 3V_2}$

Kinetic energy of 2kg block $= 0.5 \times 2 \times V_1^2 = V_1^2$

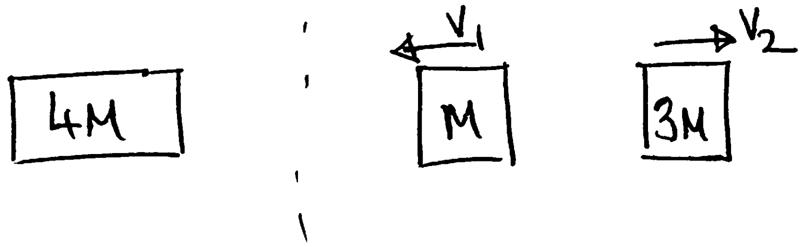
Kinetic energy of 3kg block $= 0.5 \times 3 \times V_2^2 = 1.5 V_2^2$

Ratio of energies : $\dfrac{KE_{2kg}}{KE_{3kg}} = \dfrac{V_1^2}{1.5 V_2^2}$

$$= \frac{1}{1.5}\left(\frac{V_1}{V_2}\right)^2 = \frac{1}{1.5}\left(\frac{3}{2}\right)^2$$

$$= \underline{\underline{1.5}}$$

10.*



Conservation of momentum : $\boxed{MV_1 = 3MV_2}$

Kinetic energy of $M = \frac{1}{2}MV_1^2$

Kinetic energy of $3M = \frac{3}{2}MV_2^2$

$$\therefore \quad E = \frac{1}{2}MV_1^2 + \frac{3}{2}MV_2^2$$

$$V_2 = \frac{1}{3}V_1 \;;\; \therefore E = \frac{1}{2}MV_1^2 + \frac{3}{2}M\left(\frac{V_1}{3}\right)^2$$

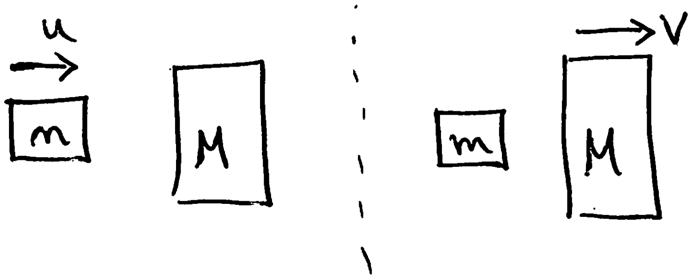
$$= \frac{1}{2}MV_1^2 + \frac{1}{6}MV_1^2$$

$$\boxed{E = \frac{2}{3}MV_1^2}$$

Kinetic energy of $M = \frac{1}{2}MV_1^2 = \frac{3}{4}\left(\frac{2}{3}MV_1^2\right)$

$$= \underline{\underline{\frac{3}{4}E}}$$

11.**



Conservation of momentum : $mu = MV$

initial kinetic energy $= \frac{1}{2} mu^2$

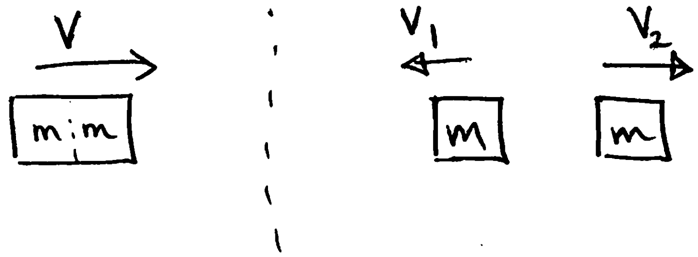
final kinetic energy $= \frac{1}{2} MV^2$

$$\frac{\text{Final KE}}{\text{Initial KE}} = \frac{\frac{1}{2} MV^2}{\frac{1}{2} mu^2} = \frac{MV^2}{mu^2}$$

$$= \frac{mMV^2}{m^2u^2} = \frac{mMV^2}{M^2V^2}$$

$$= \underline{\underline{\frac{m}{M}}}$$

12.**



Conservation of Momentum : $\boxed{2mV = m(V_2 - V_1)}$

Kinetic energy before $= \frac{1}{2} \times 2mV^2 = mV^2$

Kinetic energy after $= \frac{1}{2}MV_1^2 + \frac{1}{2}MV_2^2$

KE doubles : $\boxed{\frac{1}{2}MV_1^2 + \frac{1}{2}MV_2^2 = 2MV^2}$

$$V_1^2 + V_2^2 = 4V^2 \quad ; \quad 2V = V_2 - V_1$$

$$V_1^2 + (2V + V_1)^2 = 4V^2$$

$$2V_1^2 + 4VV_1 + 4V^2 = 4V^2$$

$$\therefore \quad 2V_1(V_1 + 2V) = 0$$

$$\left.\begin{array}{l} V_1 = 0 \implies V_2 = 2V \\ \\ V_1 = -2V \implies V_2 = 0 \end{array}\right\} \text{Symmetric answers}$$

One particle is stationary, and the other travels in initial direction with velocity $= 2V$.