edexcel

June 2005
6690 Decision D2
Mark Scheme

Question Number	Scheme	Marks
1) (a)		

	D	E	F
A	20	4	
B		26	6
C			14

m1
A1
(2)

(b) $S_A = 0 \quad S_B = -1 \quad S_C = 7$

$D_D = 21 \quad D_E = 24 \quad D_F = 18$

m1
A1

$I_{13} = I_{AF} = 16 - 0 - 18 = -2$

$I_{21} = I_{BD} = 18 + 1 - 21 = -2$

$I_{31} = I_{CD} = 15 - 7 - 21 = -13$ *

$I_{32} = I_{CE} = 19 - 7 - 24 = -12$

m1
A1✓
A1✓ (5)

(c) eg $CD(+) \rightarrow AD(-) \rightarrow AE(+) \rightarrow BE(-) \rightarrow BF(+) \rightarrow CF(-) \quad \Theta = 14$

m1 A1✓

	D	E	F
A	6	18	
B		12	20
C	14		

cost £1384

A1✓ A1
(4)

(11)

Q1 (a) M1 5 numbers, top LH corner used, a correct solution

A1 c.a.o.

(b) m1 Shadow costs stated - all 6

A1 c.a.o.

m1 4 II's stated

A1√ at least 2 correct

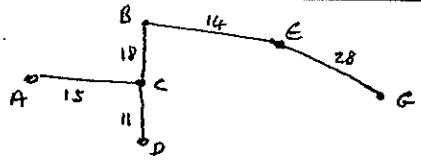
A1√ all 4 c.a.o

(c) M1 Route must√ and be clear. Route has 1 entering + 1 exiting square.

A1√ route correct + Θ correct and clearly given

A1√ new improved solution (5 numbers only)

A1 c.a.o

Question Number	Scheme	Marks
2) (a)	Deleting F leaves r.s.t 	
	r.s.t. length = 86	M1
	So lower bound = 86 + 16 + 19 = 121	A1
	∴ best L.B is 129 by deleting C (✓ from choice)	M1 A1 (4)
		B1✓ (1)
(b)	Add 33 to BF and FB Add 31 to DE and ED	B1 B1 (2)
(c)	Tour, visits each vertex, order correct using table of least distances. e.g. F C D A B E G F (actual route F C D C A B E G F) upper bound of 138 km	M1 A1 A1 A1 (4)
		(11)

@2(a) M1 Finding r.s.t - ie a spanning tree with F removed - needs a "T junction" at C (NN sets no)

A1 86 c.a.o. maybe implicit

M1 Adding 2 least arcs from F - bod

A1 121 c.a.o - if sensible method 121 gets all 4. No method but 121 set sc B2

B1✓ Chooses greatest of 129 and their lower bound from F.

(b) B1 c.a.o

B1 c.a.o

(c) M1 NN each vertex visited once (condone 2 C's if using actual route)

A1 NN stats + finishes at F - stated not drawn

A1 C.a.o

A1 138, if doubled A0 (do not isw)

3)

Let x_{ij} be number of units transported from i to j

where $i \in \{w, x, y\}$ (warehouse) and $j \in \{J, k, L\}$ (supermarket) — B1 (1)

Objective minimise "C" $= 3x_{wJ} + 6x_{wk} + 3x_{wL} + 5x_{xJ} + 8x_{xk} + 4x_{xL} + 2x_{yJ} + 5x_{yk} + 7x_{yL}$ — B1 B1 (2)

Subject to

$x_{wJ} + x_{wk} + x_{wL} = 34$

$x_{xJ} + x_{xk} + x_{xL} = 57$

$x_{yJ} + x_{yk} + x_{yL} = 25$ — m1 A1

$x_{wJ} + x_{xJ} + x_{yJ} = 20$

$x_{wk} + x_{xk} + x_{yk} = 56$

$x_{wL} + x_{xL} + x_{yL} = 40$ — A1 (3)

$x_{ij} \geqslant 0 \quad \forall i \in \{w, x, y\}$ and $j \in \{J, k, L\}$ — B1 (1) [7]

Q3 B1 Introducing decision variables c.a.o (o.e.) need "number" o.e.

B1 Minimise

B1 Function accept any letter, but need equation

m1 At least 3 equalities listed, with 3 variables in each (accept $\leq$ or $\geqslant$ here)

A1 3 correct

A1 6 correct - penalise $\leq$ here

B1 non-negativity constraints - all x values dealt with.

penalise bad notation only once per question, by losing first A or B mark earned

4) (a) The route from start to finish in which the arc of minimum length is as large as possible.
e.g. must be practical, involve choice of route, have arc 'cost'.

B 2, 1, 0
B 1 (3)

(b)

Stage	State	Action	Value
1	H	H K	18 *
	I	I K	19 *
	J	J K	21 *
2	F	F H	min(16, 18) = 16
		F I	min(23, 19) = 19 *
		F J	min(17, 21) = 17
	G	G H	min(20, 18) = 18
		G I	min(15, 19) = 15
		G J	min(28, 21) = 21 *
3	B	B G	min(18, 21) = 18 *
	C	C F	min(25, 19) = 19 *
		C G	min(16, 21) = 16
	D	D F	min(22, 19) = 19 *
		D G	min(19, 21) = 19 *
	E	E F	min(14, 19) = 14 *
4	A	A B	min(24, 18) = 18
		A C	min(25, 19) = 19 *
		A D	min(27, 19) = 19 *
		A E	min(23, 14) = 14

M1 A1 (2)

M1 A1 A1 (3)

M1 A1√

A1√ (3)

(c) Routes A C F I K, A D F I K, A D G J K

A1√
A1√A1 (3)
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(a) B2 Good definition: route, arc of min length maximised. Clear
B1 pretty good - all there but confused. 'bad' gets B1
B1 Practical, choice of route, arc 'cost'.

(b) M1 1st stage completed
A1 c.a.o. (+ *)
M1 2nd stage completed bod
A1 F state correct finding mins + then maximum c a o
A1 G state correct .. .. - - - c a o
M1 3rd + 4th stage completed bod
A1√ 3rd stage c.a.o
A1√ 4th stage c.a.o
A1√ 1 correct route

Minimum + maximum stick to scheme - so M1A1, M1A0A0, M1A0A0, A1√, A1√, A0 (max)

only penalise silly statements here and on last A marks.

penalise * with 1st A earned once only (but maybe 2nd lot)

penalise * once only (but maybe 3 correct)

must be able to ✓ on these 3 A marks (unless if only 1 be 1 silly)

(c) A1√ a second correct route
A1 a third correct route - and no more! ie, c.a.o

5) (a) To maximise, subtract all entries from $n \geqslant 30$

e.g. $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 5 & 3 & 6 \\ 0 & 3 & 5 & 9 \end{bmatrix}$ — m1

A2, 1, 0 (3)

minimum uncovered element is 1 : so $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 6 & 3 & 0 \\ 0 & 4 & 2 & 5 \\ 0 & 2 & 4 & 8 \end{bmatrix}$ — m1 A2/1/0 (3)

min. el. = 2

or

min. el = 2 — m1

$\begin{bmatrix} 7 & 0 & 0 & 2 \\ 0 & 4 & 1 & 0 \\ 0 & 2 & 0 & 5 \\ 0 & 0 & 2 & 8 \end{bmatrix}$ $\begin{bmatrix} 7 & 0 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 2 & 6 \end{bmatrix}$ — A2/1/0 (3)

A-2 B-4 C-3 D-1
A-3 B-4 C-1 D-2 — m1 A1/(2)

(b) £1160 000 — B2, 1, 0 (2)

(c) Gives other solution — m1 A1 (2)

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Q5(a) m1 Attempt to subtract all terms from n, $n \geqslant 30$ + Rows + cols if necess.

A2, 1, 0 – 1 e.e. for my table

m1 (Drawing 2 lines) + dble covered + e, uncovered – e, once covered unchanged at least once

A2/1/ ca. 0. – 1 e.e ~~for my table~~

m1 (Drawing 3 lines, 3 elements 1 dble covered, 1 once covered + 1 uncovered treated correctly.

A2/, 1/ ca o – 1 ee

m1 All 4 people allocated to all 4 companies – a list

A1 ca o

(b) B2 £1160 000

B1 116 or £116 etc (multiple of 10 art)

(c) m1 All 4 people allocated to all 4 companies – a list

A1 ca o

6) (a) A zero-sum game is one in which the sum of the gains for all players is zero. (o.e.) — B1 (1)

(b)

	I	II	III	
I	5	2	3	min 2
II	3	5	4	min 3 ← max
	max 5	5	4 ↑ min	

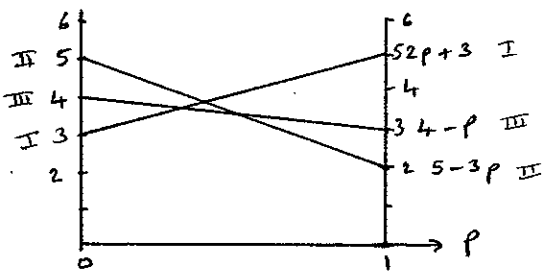
m1 A1

Since $3 \neq 4$ not stable — A1 (3)

(c) Let A play I with probability p
.. .. play II .. $(1-p)$

If B play I A's gains are $5p + 3(1-p) = 2p + 3$
.. .. .. II .. .. $2p + 5(1-p) = 5 - 3p$
.. .. .. III .. .. $3p + 4(1-p) = 4 - p$

m1 A1 (2)



A2,1,0 (2)

Intersection of $2p+3$ and $4-p$ $\Rightarrow p = \frac{1}{3}$ — m1 A1√ (2)

$\therefore$ A should play I $\frac{1}{3}$ of time and II $\frac{2}{3}$ of time; value (to A) $= 3\frac{2}{3}$ — A1√ A1√ (2)

(d) Let B play I with probability q_1, II with probability q_2 and III with probability q_3 — B1

e.g. Let $x_1 = \frac{q_1}{V}$ $x_2 = \frac{q_2}{V}$ $x_3 = \frac{q_3}{V}$ (If reduced x_2 vanishes) dep √ if vars correctly deleted — m1

maximise $P = x_1 + x_2 + x_3$ — A1

subject to $5x_1 + 2x_2 + 3x_3 \leq 1$
$3x_1 + 5x_2 + 4x_3 \leq 1$
$x_1, x_2, x_3 \geqslant 0$

A2,1,0 (5)

Alt: eg. $\begin{bmatrix} -5 & -3 \\ -2 & -5 \\ -3 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 3 & 2 \end{bmatrix}$ (If reduced $\begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix} + q_2$ vanishes.) dep √ if vars correctly deleted

maximise $P = V$
subject to $V - q_1 - 4q_2 - 3q_3 \leq 0$
$V - 3q_1 - q_2 - 2q_3 \leq 0$
$V, q_1, q_2, q_3 \geqslant 0$
$q_1 + q_2 + q_3 \leq 1$

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Q6(a) B1 c a o (o.e) Condone assumption it's a 2 player game

(b) M1 Finding row maximin and column minimax. All 5 values listed enough

A1 row maximin = 3, col minimax = 4. Identified in some way

A1 row maximin $3 \neq 4$ col minimax stated + statement (not stable) a clear link

(c) M1 Set up 3 probability equations (implicit definition of p)

A1 all correct - may be unsimplified.

A2,1,0 3 lines correctly drawn + domain $0 \leq p \leq 1$ + scale clear + lines labelled −1 e.e.

M1 Using correct eqns to find max - but ✓ from their eqns.

A1✓ $p = \frac{1}{3}$ c a o but ✓ their eqns.

A1✓ strategy clear - both row + value ✓ their eqns

A1✓ value clear must ✓ their eqns.

(d) B1 Set up of B probabilities - all 3 (or reducing)

M1 Setting up to formulate as LP. Defining x's, adapting matrix etc. Still dealing with 3 (or 2 if reduced)

A1 Objective fn + maximise, 3 variables. (or 2 if reduced)

A2,1,0 constraints (incl non negativity) −1 each constraint error (non-neg count as 1 error)
−1 for equations

Alt 2 maximise $P = V$ If reduced

subject to $V + 2q_1 - q_2 \leq 3$

$V - q_1 + q_2 \leq 2$ $\quad q_1, q_2, V \geq 0$

Alt 3 If adapt matrix main scheme becomes. If reduced "x_2" varies

minimise $P = x_1 + x_2 + x_3$

subject to $x_1 + 4x_2 + 3x_3 \leq 1$

$3x_1 + x_2 + 2x_3 \leq 1$ $\quad x_1, x_2, x_3 \geq 0$