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Mark Scheme (Results)
Summer 2009

GCE

GCE Mathematics (6665/01)

A PEARSON COMPANY

June 2009
6665 Core Mathematics C3
Mark Scheme

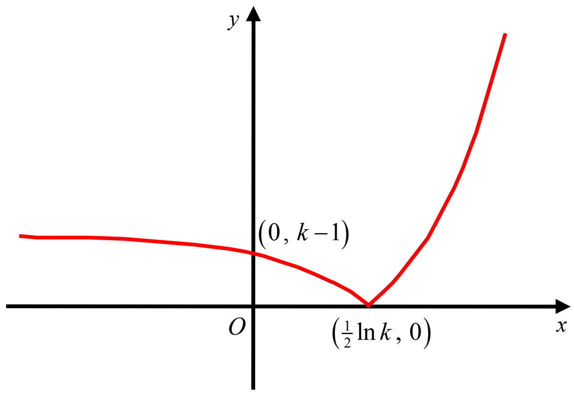
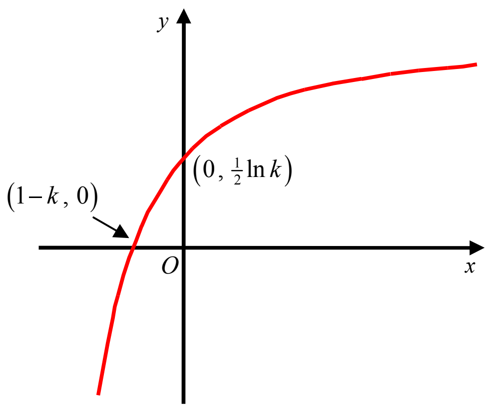
Question Number	Scheme		Marks
Q1 (a)	Iterative formula: $x_{n+1} = \dfrac{2}{(x_n)^2} + 2$, $x_0 = 2.5$		
	$x_1 = \dfrac{2}{(2.5)^2} + 2$	An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320	M1
	$x_1 = 2.32$ $x_2 = 2.371581451...$	Both $x_1 = 2.32(0)$ and $x_2 =$ awrt 2.372	A1
	$x_3 = 2.355593575...$ $x_4 = 2.360436923...$	Both $x_3 =$ awrt 2.356 and $x_4 =$ awrt 2.360 or 2.36	A1 cso
			(3)
(b)	Let $\mathrm{f}(x) = -x^3 + 2x^2 + 2 = 0$		
	$\mathrm{f}(2.3585) = 0.00583577...$	Choose suitable interval for x, e.g. [2.3585, 2.3595] or tighter	M1
	$\mathrm{f}(2.3595) = -0.00142286...$	any one value awrt 1 sf or truncated 1 sf	dM1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	both values correct, sign change and conclusion	A1
		At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states "change of sign, hence root".	(3)
			[6]

Question Number	Scheme		Marks
Q2 (a)	$\cos^2\theta + \sin^2\theta = 1 \quad (\div \cos^2\theta)$		
	$\underline{\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}}$	Dividing $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$ to give underlined equation.	M1
	$1 + \tan^2\theta = \sec^2\theta$		
	$\tan^2\theta = \sec^2\theta - 1$ (as required) **AG**	Complete proof. No errors seen.	A1 cso (2)
(b)	$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2, \quad (\text{eqn } *) \quad 0 \le \theta < 360^\circ$		
	$2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$	Substituting $\tan^2\theta = \sec^2\theta - 1$ into eqn * to get a quadratic in $\sec\theta$ only	M1
	$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$		
	$\underline{3\sec^2\theta + 4\sec\theta - 4} = 0$	Forming a three term "one sided" quadratic expression in $\sec\theta$.	M1
	$(\sec\theta + 2)(3\sec\theta - 2) = 0$	Attempt to factorise or solve a quadratic.	M1
	$\sec\theta = -2 \quad \text{or} \quad \sec\theta = \frac{2}{3}$		
	$\dfrac{1}{\cos\theta} = -2 \quad \text{or} \quad \dfrac{1}{\cos\theta} = \dfrac{2}{3}$		
	$\underline{\cos\theta = -\frac{1}{2}}; \quad \text{or} \quad \cos\theta = \frac{3}{2}$	$\underline{\cos\theta = -\frac{1}{2}}$	A1;
	$\alpha = 120^\circ \quad \text{or} \quad \alpha = \text{no solutions}$		
	$\theta_1 = \underline{120^\circ}$	$\underline{120^\circ}$	A1
	$\theta_2 = 240^\circ$	$\underline{240^\circ}$ or $\theta_2 = 360^\circ - \theta_1$ when solving using $\cos\theta = ...$	B1 $\sqrt{}$
	$\theta = \{120^\circ, 240^\circ\}$	Note the final A1 mark has been changed to a B1 mark.	(6)
			[8]

Question Number	Scheme		Marks	
Q3	$P = 80\,\mathrm{e}^{\frac{t}{5}}$			
(a)	$t = 0 \Rightarrow P = 80\mathrm{e}^{\frac{0}{5}} = 80(1) = \underline{80}$	$\underline{80}$	B1	
				(1)
(b)	$P = 1000 \Rightarrow 1000 = 80\mathrm{e}^{\frac{t}{5}} \Rightarrow \dfrac{1000}{80} = \mathrm{e}^{\frac{t}{5}}$	Substitutes $P = 1000$ and rearranges equation to make $\mathrm{e}^{\frac{t}{5}}$ the subject.	M1	
	$\therefore\ t = 5\ln\left(\dfrac{1000}{80}\right)$			
	$t = 12.6286...$	$\boxed{\text{awrt 12.6 or 13 years}}$	A1	
		Note $t = 12$ or $t = \text{awrt } 12.6 \Rightarrow t = 12$ will score A0		(2)
(c)	$\dfrac{\mathrm{d}P}{\mathrm{d}t} = 16\mathrm{e}^{\frac{t}{5}}$	$k\mathrm{e}^{\frac{1}{5}t}$ and $k \neq 80$.	M1	
		$16\mathrm{e}^{\frac{1}{5}t}$	A1	
				(2)
(d)	$50 = 16\mathrm{e}^{\frac{t}{5}}$			
	$\therefore\ t = 5\ln\left(\dfrac{50}{16}\right)$ $\qquad \{= 5.69717...\}$	Using $50 = \frac{\mathrm{d}P}{\mathrm{d}t}$ and an attempt to solve to find the value of t or $\frac{t}{5}$.	M1	
	$P = 80\,\mathrm{e}^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$ or $P = 80\,\mathrm{e}^{\frac{1}{5}(5.69717...)}$	Substitutes their value of t back into the equation for P.	dM1	
	$P = \dfrac{80(50)}{16} = \underline{250}$	$\underline{250}$ or awrt 250	A1	
				(3)
				[8]

Question Number	Scheme		Marks
Q4 (i)(a)	$y = x^2 \cos 3x$		
	Apply product rule: $\begin{Bmatrix} u = x^2 & v = \cos 3x \\ \frac{du}{dx} = 2x & \frac{dv}{dx} = -3\sin 3x \end{Bmatrix}$		
	$\frac{dy}{dx} = 2x\cos 3x - 3x^2 \sin 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x \cos 3x \pm \beta x^2 \sin 3x$	M1
		Any one term correct	A1
		Both terms correct and no further simplification to terms in $\cos \alpha x^2$ or $\sin \beta x^3$.	A1
			(3)
(b)	$y = \frac{\ln(x^2+1)}{x^2+1}$		
	$u = \ln(x^2+1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2+1}$	$\ln(x^2+1) \rightarrow \frac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \rightarrow \frac{2x}{x^2+1}$	A1
	Apply quotient rule: $\begin{Bmatrix} u = \ln(x^2+1) & v = x^2+1 \\ \frac{du}{dx} = \frac{2x}{x^2+1} & \frac{dv}{dx} = 2x \end{Bmatrix}$		
	$\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2+1}\right)(x^2+1) - 2x\ln(x^2+1)}{(x^2+1)^2}$	Applying $\frac{vu' - uv'}{v^2}$	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)
	$\left\{\frac{dy}{dx} = \frac{2x - 2x\ln(x^2+1)}{(x^2+1)^2}\right\}$	{Ignore subsequent working.}	

Question Number	Scheme		Marks
(ii)	$y = \sqrt{4x+1},\ \ x > -\frac{1}{4}$		
	At P, $y = \sqrt{4(2)+1} = \underline{\sqrt{9}} = \underline{3}$	At P, $y = \underline{\sqrt{9}}$ or $\underline{3}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \underline{\frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)}$	$\pm k(4x+1)^{-\frac{1}{2}}$ $2(4x+1)^{-\frac{1}{2}}$	M1* A1 aef
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{(4x+1)^{\frac{1}{2}}}$		
	At P, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{(4(2)+1)^{\frac{1}{2}}}$	Substituting $x = 2$ into an equation involving $\frac{\mathrm{d}y}{\mathrm{d}x}$;	M1
	Hence m(**T**) $= \frac{2}{3}$		
	Either **T**: $y - 3 = \frac{2}{3}(x-2)$; or $y = \frac{2}{3}x + c$ and $3 = \frac{2}{3}(2) + c \Rightarrow c = 3 - \frac{4}{3} = \frac{5}{3}$;	$y - y_1 = m(x-2)$ or $y - y_1 = m(x - \text{their stated } x)$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient', their x and their y_1.	dM1*;
	Either **T**: $3y - 9 = 2(x-2)$;		
	T: $3y - 9 = 2x - 4$		
	T: $\underline{2x - 3y + 5 = 0}$	$\underline{2x - 3y + 5 = 0}$ Tangent must be stated in the form $ax + by + c = 0$, where a, b and c are integers.	A1
			(6)
	or **T**: $y = \frac{2}{3}x + \frac{5}{3}$		
	T: $3y = 2x + 5$		
	T: $\underline{2x - 3y + 5 = 0}$		
			[13]

Question Number	Scheme		Marks
Q5 (a)		Curve retains shape when $x > \frac{1}{2}\ln k$	B1
		Curve reflects through the x-axis when $x < \frac{1}{2}\ln k$	B1
		$(0, k-1)$ and $(\frac{1}{2}\ln k, 0)$ marked in the correct positions.	B1
			(3)
(b)		Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)	B1
		$(1-k, 0)$ and $(0, \frac{1}{2}\ln k)$	B1
			(2)
(c)	Range of f: $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k, \infty)}$	Either $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k, \infty)}$ or $\underline{f > -k}$ or $\underline{\text{Range} > -k}$.	B1
			(1)
(d)	$y = e^{2x} - k \Rightarrow y + k = e^{2x}$	Attempt to make x (or swapped y) the subject	M1
	$\Rightarrow \ln(y+k) = 2x$ $\Rightarrow \frac{1}{2}\ln(y+k) = x$	Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x+k)}$	$\underline{\frac{1}{2}\ln(x+k)}$ or $\underline{\ln\sqrt{(x+k)}}$	A1 cao
			(3)
(e)	$f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k, \infty)}$	Either $\underline{x > -k}$ or $\underline{(-k, \infty)}$ or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer	B1 $\sqrt{}$
			(1)
			[10]

Question Number	Scheme		Marks
Q6 (a)	$A = B \Rightarrow \cos(A+A) = \cos 2A = \underline{\cos A\cos A - \sin A\sin A}$	Applies $A = B$ to $\cos(A+B)$ to give the underlined equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$	M1
	$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives		
	$\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required)	<u>Complete proof, with a link between LHS and RHS</u>. No errors seen.	A1 AG
			(2)
(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$	Eliminating y correctly.	M1
	$3\sin 2x = 4\left(\dfrac{1-\cos 2x}{2}\right) - 2\cos 2x$	Using result in part (a) to substitute for $\sin^2 x$ as $\dfrac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\dfrac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles.	M1
	$3\sin 2x = 2(1-\cos 2x) - 2\cos 2x$		
	$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$		
	$3\sin 2x + 4\cos 2x = 2$	Rearranges to give correct result	A1 AG
			(3)
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$		
	$3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$		
	Equate $\sin 2x$: $3 = R\sin\alpha$ Equate $\cos 2x$: $4 = R\cos\alpha$		
	$R = \sqrt{3^2 + 4^2}; = \sqrt{25} = 5$	$R = 5$	B1
	$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...^\circ$	$\tan\alpha = \pm\frac{3}{4}$ or $\tan\alpha = \pm\frac{4}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{4}{\text{their } R}$	M1
		awrt 36.87	A1
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$		
			(3)

Question Number	Scheme		Marks
(d)	$3\sin 2x + 4\cos 2x = 2$		
	$5\cos(2x - 36.87) = 2$		
	$\cos(2x - 36.87) = \frac{2}{5}$	$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$	M1
	$(2x - 36.87) = 66.42182...^\circ$	awrt 66	A1
	$(2x - 36.87) = 360 - 66.42182...^\circ$		
	Hence, $x = 51.64591...^\circ,\ 165.22409...^\circ$	One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3	A1
		Both awrt 51.6 AND awrt 165.2	A1
			(4)
		If there are any EXTRA solutions inside the range $0 \le x < 180^\circ$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \le x < 180^\circ$.	
			[12]

Question Number	Scheme		Marks
Q7	$\mathrm{f}(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}$ $x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$		
(a)	$\mathrm{f}(x) = \frac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$	An attempt to combine to one fraction Correct result of combining all three fractions	M1 A1
	$= \frac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$		
	$= \frac{x^2 + x - 12}{\left[(x+4)(x-2)\right]}$	Simplifies to give the correct numerator. Ignore omission of denominator	A1
	$= \frac{(x+4)(x-3)}{\left[(x+4)(x-2)\right]}$	An attempt to factorise the numerator.	dM1
	$= \frac{(x-3)}{(x-2)}$	Correct result	A1 cso **AG**
			(5)
(b)	$\mathrm{g}(x) = \frac{\mathrm{e}^x - 3}{\mathrm{e}^x - 2} \quad x \in \mathbb{R},\ x \neq \ln 2.$		
	Apply quotient rule: $\begin{Bmatrix} u = \mathrm{e}^x - 3 & v = \mathrm{e}^x - 2 \\ \frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x & \frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^x \end{Bmatrix}$		
	$\mathrm{g}'(x) = \frac{\mathrm{e}^x(\mathrm{e}^x - 2) - \mathrm{e}^x(\mathrm{e}^x - 3)}{(\mathrm{e}^x - 2)^2}$	Applying $\frac{vu' - uv'}{v^2}$ Correct differentiation	M1 A1
	$= \frac{\mathrm{e}^{2x} - 2\mathrm{e}^x - \mathrm{e}^{2x} + 3\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$		
	$= \frac{\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$	Correct result	A1 **AG** cso
			(3)

Question Number	Scheme		Marks
(c)	$g'(x) = 1 \Rightarrow \dfrac{e^x}{(e^x - 2)^2} = 1$		
	$e^x = (e^x - 2)^2$	Puts their differentiated numerator equal to their denominator.	M1
	$e^x = e^{2x} - 2e^x - 2e^x + 4$		
	$\underline{e^{2x} - 5e^x + 4} = 0$	$\underline{e^{2x} - 5e^x + 4}$	A1
	$(e^x - 4)(e^x - 1) = 0$	Attempt to factorise or solve quadratic in e^x	M1
	$e^x = 4$ or $e^x = 1$		
	$x = \ln 4$ or $x = 0$	both $x = 0, \ \ln 4$	A1
			(4)
			[12]

Question Number	Scheme		Marks
Q8 (a)	$\sin 2x = \underline{2\sin x\cos x}$	$\underline{2\sin x\cos x}$	B1 aef (1)
(b)	$\operatorname{cosec} x - 8\cos x = 0\,,\quad 0 < x < \pi$		
	$\frac{1}{\sin x} - 8\cos x = 0$	Using $\operatorname{cosec} x = \frac{1}{\sin x}$	M1
	$\frac{1}{\sin x} = 8\cos x$		
	$1 = 8\sin x\cos x$		
	$1 = 4(2\sin x\cos x)$		
	$1 = 4\sin 2x$		
	$\underline{\sin 2x = \frac{1}{4}}$	$\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$	M1
		$\underline{\sin 2x = \frac{1}{4}}$	<u>A1</u>
	Radians $2x = \{0.25268..., 2.88891...\}$ Degrees $2x = \{14.4775..., 165.5225...\}$		
	Radians $x = \{0.12634..., 1.44445...\}$ Degrees $x = \{7.23875..., 82.76124...\}$	Either arwt 7.24 or 82.76 or 0.13 or 1.44 or 1.45 or awrt 0.04π or awrt 0.46π.	A1
		Both <u>0.13</u> and <u>1.44</u>	A1 cao
		Solutions for the final two A marks must be given in x only. If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 < x < \pi$.	(5) **[6]**