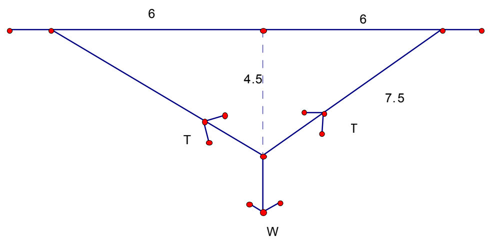
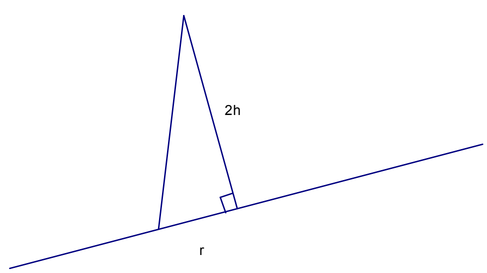
Mark Scheme (Results)

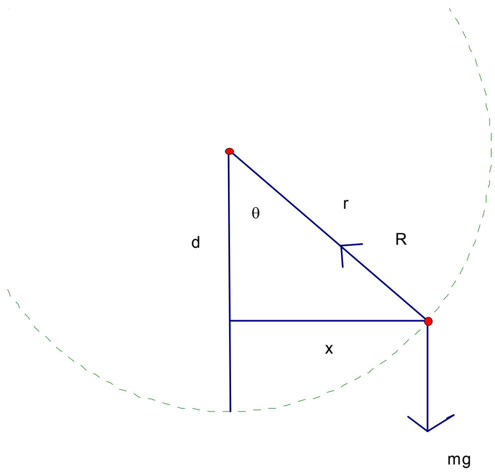
Summer 2009

GCE

GCE Mathematics (6679/01)

June 2009
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	 Resolving vertically: $2T\cos\theta = W$	M1A2,1,0
	Hooke's Law: $T = \dfrac{80\times 3.5}{4}$	M1A1
	$W = 84\text{N}$	A1
(b)	EPE = $2\times\dfrac{80\times 3.5^2}{2\times 4}, = 245$ (or awrt 245)	M1A1ft,A1
	(alternative $\dfrac{80\times 7^2}{16} = 245$)	**[9]**
Q2 (a)	Object / Mass / c of m above base Cone / m / $2h+3h$ Base / $3m$ / h Marker / $4m$ / d	B1(ratio masses) B1(distances)
	m x $5h$ +$3m$ x h = $4m$ x d	M1A1ft
	$d = 2h$	A1
(b)	 $\dfrac{r}{d} = \dfrac{1}{12}$	M1A1ft
	$6r = h$	A1
		[8]

Question Number	Scheme	Marks
Q3 (a)	 $\leftrightarrow \quad R\sin\theta = mx\omega^2$	M1 A1
	$R\times\dfrac{x}{r} = mx\times\dfrac{3g}{2r}$	M1
	$R = \dfrac{3mg}{2}$	A1
(b)	$\updownarrow \quad R\cos\theta = mg$	M1 A1
	$\dfrac{3mg}{2}\times\dfrac{d}{r} = mg$	M1
	$d = \dfrac{2}{3}r$	A1
		[8]
Q4 (a)	Volume $= \int_{\frac{1}{4}}^{1}\pi y^2 dx = \int_{\frac{1}{4}}^{1}\pi\dfrac{1}{x^4}dx$	M1A1
	$= \left[\pi\times\dfrac{-1}{3x^3}\right]_{\frac{1}{4}}^{1}$	A1ft
	$= \pi(\dfrac{-1}{3}+\dfrac{64}{3}) = 21\pi$ *	A1
(b)	$21\pi\rho\bar{x} = \rho\int\pi y^2 x dx = \rho\int\pi\dfrac{1}{x^4}xdx$	M1A1
	$21\pi\bar{x} = \pi\left[\dfrac{-1}{2x^2}\right]_{\frac{1}{4}}^{1}$	A1ft
	$\bar{x} = \dfrac{1}{21}(\dfrac{-1}{2}+\dfrac{16}{2}) = \dfrac{5}{14}$ or awrt 0.36	A1
	$\bar{y} = 0$ by symmetry	B1
		[9]

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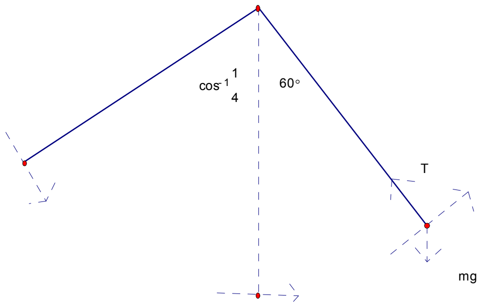
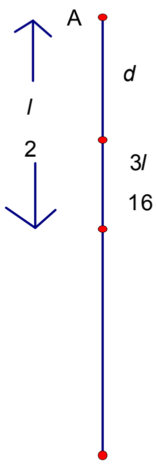
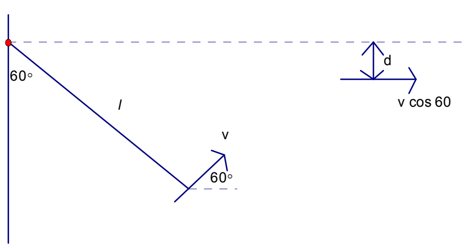
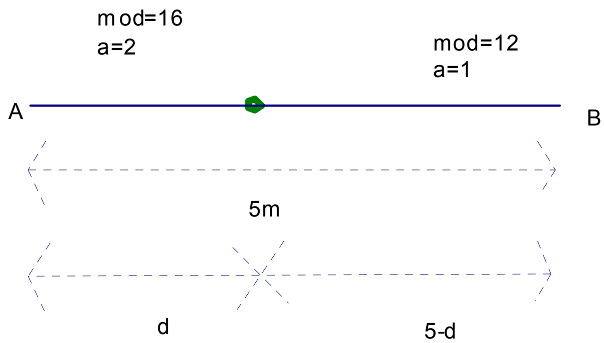
Q5 (a)		
	Energy:	
	$(\frac{1}{2}mu^2+)mgl(\cos\theta-\frac{1}{4})=\frac{1}{2}mv^2$	M1A1
	Resolving:	
	$T-mg\cos\theta=\frac{mv^2}{l}$	M1A1
	Eliminate v^2:	
	$T=mg\cos\theta+\frac{1}{l}(2mgl(\cos\theta-\frac{1}{4}))$	M1
	$T=3mg\cos\theta-\frac{mg}{2}$ *	A1
(b)		
	$\theta=60°\Rightarrow mv^2=2mgl(\frac{1}{2}-\frac{1}{4})$	M1
	$\Rightarrow v^2=\frac{gl}{2}$	
	vertical motion under gravity:	
	$\updownarrow\ 0=(v\cos 30°)^2-2gs$	M1
	$0=\frac{gl}{2}\times\frac{3}{4}-2gs\Rightarrow s=\frac{3l}{16}$	A1
	Distance below A $=\frac{l}{2}-\frac{3l}{16}=\frac{5l}{16}$	M1A1
		[11]
Alternative for end of (b) using energy		
	$\frac{1}{2}mv^2-mgl\cos 60=\frac{1}{2}m(v\cos 60)^2-mgd$	M1A1
	$\frac{gl}{4}-\frac{gl}{2}=\frac{gl}{4}\times\frac{1}{4}-gd$	M1
	$d=\frac{1-4+8}{16}l=\frac{5l}{16}$	A1

Question Number	Scheme	Marks
Q6 (a)	At max v, driving force = resistance	
	Driving force = $\frac{80}{v}$	B1
	$\Rightarrow \frac{80}{20} = k \times 20^2 \Rightarrow k = \frac{1}{100}$	M1A1
	F = ma $\Rightarrow 100a = \frac{80}{v} - kv^2 \quad (= \frac{8000 - v^3}{100v})$	M1
	* $\Rightarrow v\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{8000 - v^3}{10000v}$ *	A1
(b)	$\int_4^8 \frac{10000v^2}{8000 - v^3}\mathrm{d}v = \int_0^D 1\mathrm{d}x$	M1A1
	$D = \left[-\frac{10000}{3}\ln\lvert 8000 - v^3 \rvert \right]_4^8$	A1
	$= (-\frac{10000}{3}\ln\frac{7488}{7936}) = 193.7..... \approx 194\,\mathrm{m}$ (accept 190)	M1 A1
(c)	$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{8000 - v^3}{10000v} \Rightarrow \int_0^T 1\mathrm{d}t = \int_4^8 \frac{10000v}{8000 - v^3}\mathrm{d}v$	M1A1
	$\Rightarrow T \approx \frac{1}{2} \times 2 \times 10000 \times \left\{ \frac{4}{7936} + \frac{2 \times 6}{7784} + \frac{8}{7488} \right\}$	M1
	$\Rightarrow T (= 31.1409....) \approx 31$	A1
		[14]

Question Number	Scheme	Marks
Q7 (a)		
	Hooke's law: Equilibrium $\Rightarrow \dfrac{16(d-2)}{2} = \dfrac{12(4-d)}{1}$	M1A1A1
	$\Rightarrow d = 3.2$	A1
	so extensions are 1.2m and 0.8m.	A1
(b)	If the particle is displaced distance x towards $\boldsymbol{B}$ then	
	$-m\ddot{x} = \dfrac{16(1.2+x)}{2} - \dfrac{12(0.8-x)}{1} (= 20x)$	M1A1ft A1ft
	$\Rightarrow \ddot{x} = -40x$ or $\ddot{x} = -\dfrac{20}{m} (\Rightarrow \text{SHM})$	A1
(c)	$T = \dfrac{2\pi}{\sqrt{40}}$	B1ft
	$a = \dfrac{\sqrt{10}}{\text{their } \omega}$	B1ft
	$x = a\sin\omega t$ their a, their ω	M1
	$\dfrac{1}{4} = \dfrac{1}{2}\sin\sqrt{40}t$	A1
	$\sqrt{40}t = \dfrac{\pi}{6} (\Rightarrow t = \dfrac{\pi}{6\sqrt{40}})$	M1
	Proportion $\dfrac{4t}{T} = \dfrac{4\pi}{6\sqrt{40}} \times \dfrac{\sqrt{40}}{2\pi} = \dfrac{1}{3}$	M1A1
		[16]