Circular Motion Supplementary Questions

Part 1

1.i) $T = \frac{60}{45} = 1.33$ Seconds

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.33} = \underline{4.71 \text{ rad/s}}$$

ii) $V = r\omega$

$= 0.15 \times 4.71$

$= \underline{0.707 \text{ m/s}}$

2. $V = r\omega \Rightarrow \omega = \frac{V}{r}$; $125 \text{ km/h} = \frac{125 \times 10^3}{60 \times 60} \text{ m/s}$

$= \underline{34.72 \text{ m/s}}$

$$\omega = \frac{34.72}{1.2 \times 10^3} = \underline{0.0289 \text{ rad/s}}$$

3. i) $T = \frac{60}{300} = 0.2$ Seconds

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} = \underline{\underline{31.4 \text{ rad/s}}}$$

ii) $V = r\omega$

$= 0.150 \times 31.4$

$= \underline{\underline{4.71 \text{ m/s}}}$

4. $F = \frac{mv^2}{r} = \frac{70 \times 9.4^2}{38}$

$\therefore \quad \underline{\underline{F = 163 \text{ N}}}$

5. $F = \frac{mv^2}{r} = \frac{100 \times 30^2}{0.60 \times 10^3}$

$\underline{\underline{F = 1500 \text{ N}}}$

6.i) $\frac{360^\circ}{365} = \underline{\underline{0.986^\circ}}$ (per day)

ii) $0.986^\circ = 0.986 \times \frac{2\pi}{360^\circ} = \underline{\underline{0.0172}}$ rad (1 day)

iii) $\omega = \frac{0.0172}{24 \times 60^2} = 1.99 \times 10^{-7}$ rad/s

$V = r\omega = 1.5 \times 10^{11} \times 1.99 \times 10^{-7}$

$\underline{\underline{V = 2.99 \times 10^4 \text{ m/s}}}$

iv) $a = \frac{V^2}{r} = \frac{(2.99 \times 10^4)^2}{1.5 \times 10^{11}}$

$\underline{\underline{a = 5.94 \times 10^{-3} \text{ m/s}^2}}$

v) $F = ma = 6 \times 10^{24} \times 5.94 \times 10^{-3}$

$= \underline{\underline{3.57 \times 10^{22} \text{ N}}}$

7. i) $a = \frac{v^2}{r}$

$= \frac{2^2}{0.5}$ $= \underline{8\ m/s^2}$

ii) $F = ma$

$= 0.3 \times 8$

$= \underline{2.4\ N}$

8. i) $a = \omega^2 r$

$\omega = \sqrt{a/r} = \sqrt{\frac{20 \times 9.81}{17.8}}$

$\underline{\omega = 3.32\ rad/s}$

ii) $a = \frac{v^2}{r} \Rightarrow v = \sqrt{ar}$

$v = \sqrt{20 \times 9.81 \times 17.8}$

$\underline{v = 59.1\ m/s}$

9. $F \leq 6000\ N$

$$\underbrace{\frac{mv^2}{r}}_{F} \leq 6000\ N$$

$$v^2 \leq \frac{6000r}{m}$$

$$v \leq \sqrt{\frac{6000 \times 30}{1100}} = \underline{\underline{12.8\ m/s}}$$

10. $F = \frac{mv^2}{r}$, $W = mg$

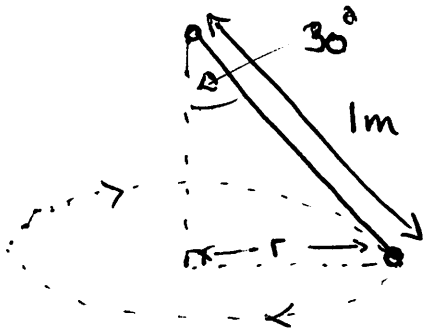
$$\frac{F}{W} = \frac{v^2}{rg} = \frac{650^2}{80 \times 10^3 \times 9.81} = \underline{\underline{0.538}}$$

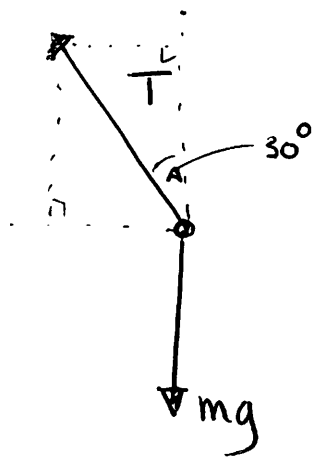
11.* In one turn, the nut travels p.

$$\frac{p}{V} = T = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi V}{p}.$$

12.*





$r = 1 \times \sin 30°$

$r = 0.5\text{m}$

Tension

$$T\sin 30° = \frac{mV^2}{r} \quad ; \quad T\cos 30° = mg$$

i) $$\frac{T\sin 30°}{T\cos 30°} = \frac{mV^2}{r} \times \frac{1}{mg} = \frac{V^2}{rg}$$

$\frac{T\sin 30°}{T\cos 30°}$ = $\tan 30°$

$$\therefore \quad V = \sqrt{rg\tan 30°} = \underline{1.68\,\text{m/s}} \quad ;$$

time period

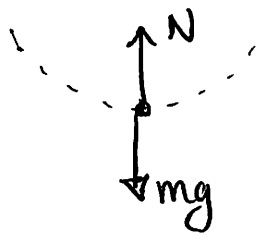
$$T = \frac{2\pi r}{V} = \frac{2\pi \times 0.5}{1.68}$$

$\underline{T = 1.87 \text{ seconds}}$

ii) $$T = \frac{mg}{\cos 30°} = \frac{0.5 \times 9.81}{\cos 30°} = \underline{5.66\text{N}}$$

Part 2

1. i)



$$F = N - mg = \frac{mV^2}{r} = \frac{30 \times 6^2}{3.2}$$

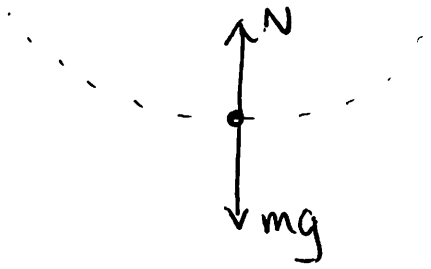
$$\underline{F = 337.5\ N}$$

ii) $N - mg = 337.5\ N$

$N = 337.5 + 30 \times 9.81$

$= \underline{631.8\ N}$

2. i)



$a \leq 5g$

$\frac{v^2}{r} \leq 5g \Rightarrow r \geq \frac{v^2}{5g} = \frac{170^2}{5 \times 9.81}$

$r \geq 589m$

min. radius : $r = 589m$

ii) $N - mg = F = ma$

$N = mg + ma$

$= 85(g + 5g)$

$= 85 \times 6 \times 9.81$

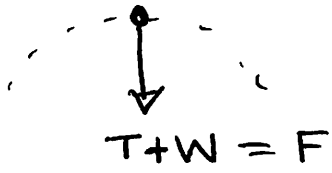
$N = 5003.1N$

3. $F = \frac{mv^2}{r} = \frac{4 \times 5^2}{2} = \underline{50N}$

$W = mg = 4 \times 9.81$

$= \underline{39.24N}$

* Top of circle :

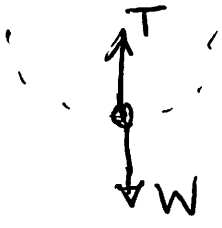


$T = F - W$

$= 50 - 39.24$

$= \underline{10.76N}$ ← Minimum tension

* Bottom of circle :



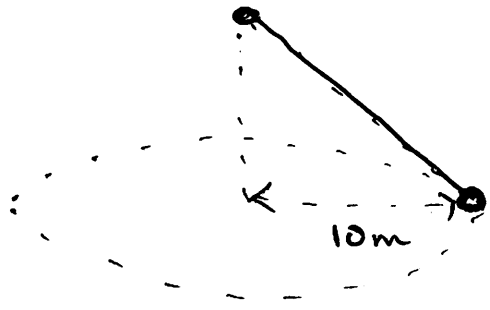
$T - W = F$

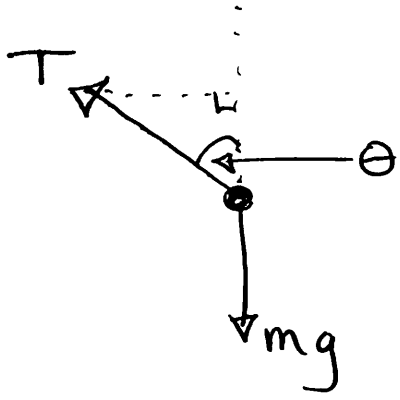
$\therefore T = F + W$

$= 50 + 39.24$

$T = \underline{89.24N}$

4.





i) $a = \frac{v^2}{r} = \frac{8^2}{10} = \underline{6.4\ m/s^2}$

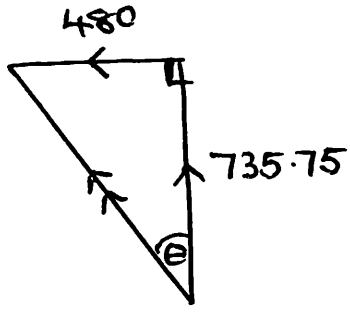
ii) $F = ma = 75 \times 6.4$

$= \underline{480\ N}$

iii) $W = mg = 75 \times 9.81$

$= \underline{735.75\ N}$

iv)



$\tan\theta = \frac{480}{735.75}$

$\theta = \arctan\left(\frac{480}{735.75}\right)$

$= \underline{33.1^\circ}$

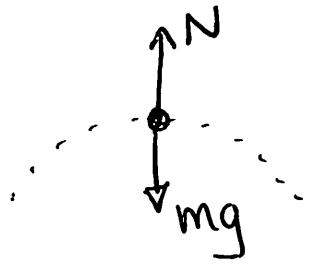
5. $a = 19.6\ m/s^2$, $r = 7.0\ m$

$$a = \frac{v^2}{r} \quad ; \quad v = \sqrt{ar}$$

$$v = \sqrt{19.6 \times 7}$$

$$v = 11.7\ m/s$$

6. i)



$$F = mg - N$$

$$N = mg - F = mg - \frac{mv^2}{r}$$

$$N = 950\left(9.81 - \frac{17^2}{50}\right) = 3828.5\ N$$

ii) For N to equal Zero (i.e. no force of contact), we require $mg = \frac{mv^2}{r}$

$$v = \sqrt{gr} = \sqrt{9.81 \times 50}$$

$$= 22.1\ m/s$$

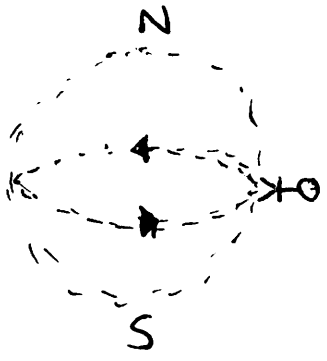
7.* i) At North pole, explorer is not travelling around in a circle due to motion of the Earth.

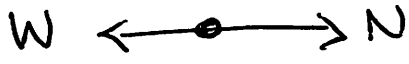
$$\therefore \quad N = W = mg$$

$$= 75 \times 9.81$$

$$N = \underline{735.75N}$$

ii) On equator the explorer is travelling around in a circle:





$$F = \frac{MV^2}{r} = W - N \quad ; \quad V = \frac{2\pi r}{T} = \frac{2\pi \times 6400 \times 10^3}{24 \times 60^2}$$

$$V = 465.42 \text{ m/s}$$

$$N = W - \frac{MV^2}{r} = 75 \times 9.81 - \frac{75 \times 465.42^2}{6400 \times 10^3}$$

$$\underline{N = 733.2N}$$

8.* i)

$r = 0.29 \sin 30°$

$= \underline{0.145 \text{ m}}$

ii) $2\pi r = 2\pi \times 0.145$

$= 0.911 \text{ m}$

$\underline{V = 0.911 \text{ m/s}}$

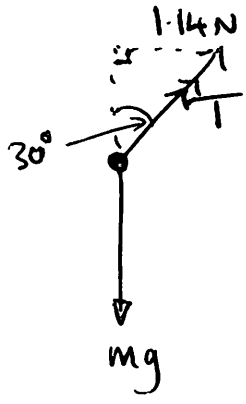
iii) $a = \frac{V^2}{r} = \frac{0.911^2}{0.145}$

$= \underline{5.72 \text{ m/s}^2}$

iv) $F = ma \quad = 0.2 \times 5.72$

$= \underline{1.14 \text{ N}}$

v)



$T = \sqrt{1.14^2 + (0.2 \times 9.81)^2}$

$= \underline{2.27 \text{ N}}$

9.* At top of circle, $F = \frac{mv^2}{r} = mg$ at minimum speed.

$mg\Delta h = \frac{1}{2}mv^2$ and $v^2 = gr$

$\therefore \quad mg\Delta h = \frac{1}{2} m g r$

$$\boxed{\Delta h = \frac{r}{2}}$$