1 Energy in DB

$$\mathcal{H}(t) = \frac{1}{2}\int_{-\infty}^{\infty} hu^2 + gh^2 + \frac{h^3}{3}\left(\frac{\partial u}{\partial x}\right)^2 dx$$

For the Soliton at $t = 0$

$$h(x,0) = a_0 + a_1 sech^2\left(\kappa x\right)$$

$$u(x,0) = c\left(1 - \frac{a_0}{a_0 + a_1 sech^2\left(\kappa x\right)}\right)$$

$$\kappa = \frac{\sqrt{3a_1}}{2a_0\sqrt{a_0 + a_1}} \tag{1}$$

and

$$c = \sqrt{g\left(a_0 + a_1\right)} \tag{2}$$

For $a_0 = 1.0$, $a_1 = 0.7$, $g = 9.81$

$$\kappa = \frac{\sqrt{2.1}}{2\sqrt{1.7}}$$

$$\kappa = \sqrt{\frac{21}{68}}$$

$$c = \sqrt{9.81\left(1.7\right)}$$

$$c = \sqrt{16.677}$$

$$h(x,0) = 1 + 0.7 sech^2\left(\sqrt{\frac{21}{68}}x\right)$$

$$u(x,t) = \sqrt{16.677}\left(1 - \frac{1}{1 + 0.7 sech^2\left(\sqrt{\frac{21}{68}}x\right)}\right)$$

$$\frac{\partial}{\partial x}u(x,t) = -\frac{3.17718\tanh\left(\sqrt{\frac{21}{68}}x\right)\operatorname{sech}^2\left(\sqrt{\frac{21}{68}}x\right)}{\left(1+0.7sech^2\left(\sqrt{\frac{21}{68}}x\right)\right)^2}$$

Using Wolfram

$$\int gh^2dx = 9.81x + \tanh(0.555719x)\left(2.88329\operatorname{sech}^2(0.555719x) + 30.4805\right)$$

$$\int hu^2\,dx = 21.0068\tanh(0.555719x) - 19.2569\tanh^{-1}(0.641689\tanh(0.555719x))$$

$$\begin{aligned}\int \frac{h^3}{3}\left(\frac{\partial u}{\partial x}\right)^2\,dx \\ = 307.641\Bigg(&\tanh(0.55719x)\left(0.049539 - 0.00937224\operatorname{sech}^2(0.555719x)\right) \\ &- 0.0625954\tanh^{-1}(0.641689\tanh(0.555719x))\Bigg) \quad (3)\end{aligned}$$