Serre Equations with non flat bottom:

$$\frac{\partial G}{\partial t}+\frac{\partial}{\partial x}\left(G\bar{u}+\frac{gh^2}{2}-\frac{2h^3}{3}\left(\frac{\partial \bar{u}}{\partial x}\right)^2\right)=-\frac{\partial}{\partial x}\left(\frac{h^2}{2}\Psi\right)-\left(gh+\frac{h^2}{2}\Phi+h\Psi\right)\frac{\partial b}{\partial x}$$

where:

$$G=\bar{u}h-h^2\frac{\partial h}{\partial x}\frac{\partial \bar{u}}{\partial x}+\frac{h^3}{3}\frac{\partial^2 \bar{u}}{\partial x^2}$$

However I'm going to assume that we do not want temporal derivative in our source term, so we want to modify G so that we can do this. The important thing here to note is that b is independent of time. By simple calculus it is true that:

$$\frac{\partial}{\partial t}\left(\frac{h^2}{2}\frac{\partial b}{\partial x}\frac{\partial \bar{u}}{\partial x}\right)=h\frac{\partial h}{\partial t}\frac{\partial b}{\partial x}\frac{\partial \bar{u}}{\partial x}+\frac{h^2}{2}\frac{\partial^2 b}{\partial t\partial x}\frac{\partial \bar{u}}{\partial x}+\frac{h^2}{2}\frac{\partial b}{\partial x}\frac{\partial^2 \bar{u}}{\partial t\partial x}$$

since b is independent of time, so is its derivative with respect to x and so $\frac{\partial^2 b}{\partial t\partial x}=0$. Thus:

$$\frac{\partial}{\partial t}\left(\frac{h^2}{2}\frac{\partial b}{\partial x}\frac{\partial \bar{u}}{\partial x}\right)=h\frac{\partial h}{\partial t}\frac{\partial b}{\partial x}\frac{\partial \bar{u}}{\partial x}+\frac{h^2}{2}\frac{\partial b}{\partial x}\frac{\partial^2 \bar{u}}{\partial t\partial x}$$

Applying the equation for conservation of mass [] gives:

$$\frac{\partial}{\partial t}\left(\frac{h^2}{2}\frac{\partial b}{\partial x}\frac{\partial \bar{u}}{\partial x}\right)=h\frac{\partial (\bar{u}h)}{\partial x}\frac{\partial b}{\partial x}\frac{\partial \bar{u}}{\partial x}+\frac{h^2}{2}\frac{\partial b}{\partial x}\frac{\partial^2 \bar{u}}{\partial t\partial x}$$

$$\frac{h^2}{2}\frac{\partial b}{\partial x}\frac{\partial^2 \bar{u}}{\partial t\partial x}=h^2\frac{\partial \bar{u}}{\partial x}\frac{\partial b}{\partial x}\frac{\partial \bar{u}}{\partial x}+h\bar{u}\frac{\partial h}{\partial x}\frac{\partial b}{\partial x}\frac{\partial \bar{u}}{\partial x}-\frac{\partial}{\partial t}\left(\frac{h^2}{2}\frac{\partial b}{\partial x}\frac{\partial \bar{u}}{\partial x}\right)$$

So the first equation can be rearranged to give:

$$\frac{\partial G}{\partial t}+\frac{\partial}{\partial x}\left(G\bar{u}+\frac{h^2}{2}\Psi+\frac{gh^2}{2}-\frac{2h^3}{3}\left(\frac{\partial \bar{u}}{\partial x}\right)^2\right)-\frac{h^2}{2}\frac{\partial^2 \bar{u}}{\partial x\partial t}\frac{\partial b}{\partial x}=-\left(gh+\frac{h^2}{2}\left(\frac{\partial \bar{u}}{\partial x}\right)^2-\bar{u}\frac{\partial^2 \bar{u}}{\partial x^2}+h\Psi\right)\frac{\partial b}{\partial x}$$

$$\frac{\partial H}{\partial t}+\frac{\partial}{\partial x}\left(H\bar{u}+\frac{gh^2}{2}-\frac{2h^3}{3}\left(\frac{\partial \bar{u}}{\partial x}\right)^2\right)=-\left(gh-\frac{h^2}{2}\left(\frac{\partial \bar{u}}{\partial x}\right)^2-\bar{u}\frac{\partial^2 \bar{u}}{\partial x^2}+h\Psi-h\bar{u}\frac{\partial h}{\partial x}\frac{\partial \bar{u}}{\partial x}\right)\frac{\partial b}{\partial x}$$

Where:

$$H=\bar{u}h-h^2\frac{\partial h}{\partial x}\frac{\partial \bar{u}}{\partial x}+\frac{h^3}{3}\frac{\partial^2 \bar{u}}{\partial x^2}+\frac{h^2}{2}\frac{\partial b}{\partial x}\frac{\partial \bar{u}}{\partial x}$$