1 Elliptic Equation

$$-u_x = f$$

We begin by multiplying by a test functioin then integrating
for all v

$$-u_x v = fv$$

$$-\int_I u_x v = \int_{I_i} fv$$

Assuming Dirchlet boundary conditions we have by integration by parts

$$\int_I u v_x = \int_I fv$$

Now we also have basis functions ϕ such that

$$u = \sum_{i=1}^{N} u_i \phi_i$$

and it covers the whole space of interest so
for all ϕ_i

$$\int_I u(\phi_x)_i = \int_I f\phi_i$$

also can write this per element as
for all ϕ_i

$$\sum_{\forall e} \int_e u(\phi_x)_i dx = \sum_{\forall e} \int_e f\phi_i dx$$

The sum is simple, what we are interest in is one abstract e
We have

$$\phi_i(x) = \begin{cases} 0 & x < x_{i-1} \\ \frac{x-x_{i-1}}{\Delta x} & x_{i-1} \geq x < x_i \\ 1 - \frac{x-x_{i-1}}{\Delta x} & x_i \geq x < x_{i+1} \\ 0 & x_{i+1} \geq x \end{cases}$$

Thus

$$(\phi_x)_i = \begin{cases} 0 & x < x_{i-1} \\ 1/\Delta x & x_{i-1} \geq x < x_i \\ -1/\Delta x & x_i \geq x < x_{i+1} \\ 0 & x_{i+1} \geq x \end{cases}$$