Importance of Dispersion for Shoaling Waves

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Abstract: Flooding has been responsible for some of the deadliest natural disasters in human history because, our densest areas of population occur along rivers and/or close to the coast. The worst of these in living memory was the 2004 Indian Ocean tsunami, which sparked much of the current research we do into inundation at the ANU via the ANUGA software,which was developed in collaboration with Geoscience Australia. The interaction of waves with bathymetry and particularly the shoaling of waves plays a central role in modelling the inundation caused by a tsunami. Current industrial models for tsunami inundation such as ANUGA utilize the non-dispersive shallow water wave equations. Current research into tsunami inundation includes building numerical models based on dispersive equations, such as the Serre equations. Dispersive equations allow waves of different frequencies to travel at different speeds thereby capturing more fluid behaviour. They are theoretically a better choice as the basis for inundation models. However, these methods are computationally expensive and the detailed behaviour is not always significant. When the water depth is large compared to the wave height, the propagation of a wave is not strongly influenced by different frequency speeds associated with dispersion waves. In this case the numerical results for the non-dispersive and dispersive equations are similar. The behaviour of the waves in deep water is well understood, this is not the case for the behaviour of waves in the nearshore. We investigate these waves by numerically modelling experimental data of the propagation of waves over a submerged trapezoidal bar using both the non-dispersive shallow water wave equations and the dispersive Serre equations. Comparing the results from these simulations provides an opportunity to assess the importance of dispersion for the propagation of waves over a submerged bar and shoaling waves on a linear beach.

Keywords: *Tsunamis, shoaling waves, dispersion, models*

1 INTRODUCTION

2 PERIODIC WAVE OVER A SUBMERGED BAR

Beji and A. Battjes (1994) conducted a series of experiments where waves travel over a submerged trapezoidal bar. Their wave tank was 37.7m in length, 0.8m wide and 0.75m high with a constant water stage of 0.4m. The bed had the following profile; $(x, z_b) =$ [(0m,0m), (6m,0m), (12m,0.3m), (14m,0.3m), (17m,0m), (18.95m,0m), (28.95m,0.4m)]. Waves were generated by a wave maker located at $x =$ 0m, and were absorbed downstream by a 3m long gravel beach, with a slope of $1 : 25$. There were seven wave gauges that recorded the depth of water over time at the following locations; WG1: 5.7m, WG2: 10.5m, WG3: 12.5m, WG4: 13.5m, WG5: 14.5m, WG6: 15.7m and WG7: 17.3m.

In this paper we focus on one of these experiments that investigated periodic non-breaking sinusoidal waves with low frequency propagating over the bar. These waves had a period of $T =$ 2s, a wavelength of $\lambda \approx 3.69$m and an amplitude of $A = 0.01$m. These waves travel without deformation towards the bar. The waves then steepen as they travel up the front slope of the bar due to shoaling. On the horizontal top of the bar higher harmonic waves are slowly released. This release of waves is then accelerated as the waves travel down slope. After the bar the waves reach an equilibrium and again begin to travel without deformation. Although bars do occur throughout the worlds oceans, in the context of tsunami modelling we are more interested in the behaviour of waves on the front slope region and the horizontal top of the bar and so we focus on wave gauges 1 through 4.

We have numerically simulated this experiment using a numerical method for the Serre equations by Zoppou et al. (2017) and the numerical modelling software ANUGA for the shallow water wave equations []. The results for the Serre equations are presented in Figure 1 and the results for the shallow water wave equations are presented in Figure [].

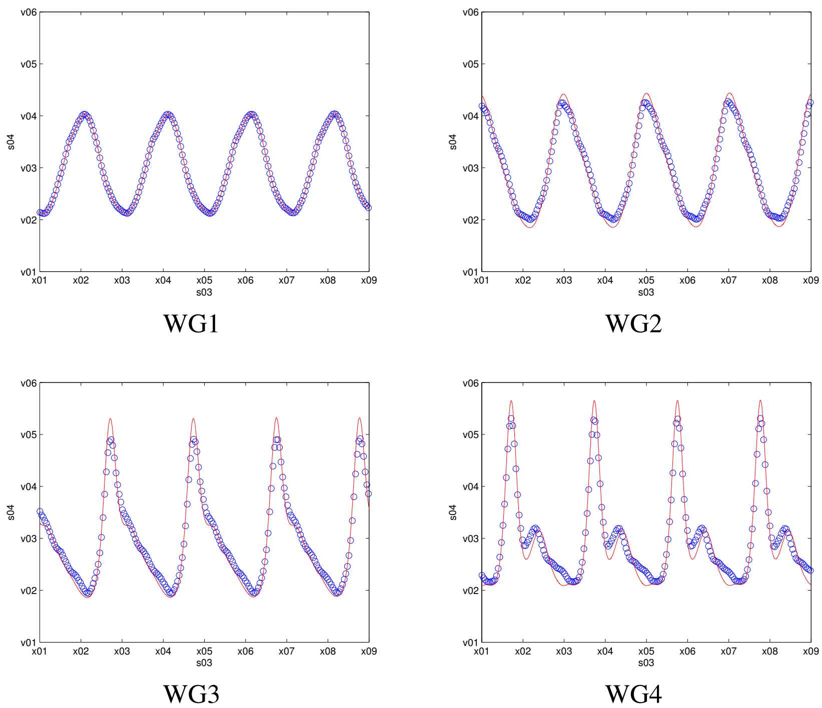

Figure 1. Simulated and observed water surface at several gauges for a sinusoidal wave with period, $T = 2$s traveling over a submerged reef conducted by Beji and A. Battjes (1994).

3 Propagation of a Soliton over a Slope.

The propagation of periodic waves over a submerged bar found significant differences between the numerical solution of the Serre equations and the shallow water wave equations. These differences are pronounced due to the fact that the experimental set-up had a rather large ratio of wave amplitude to water depth, making the shallow water wave equations an inappropriate model for the experiment. To rectify this another numerical experiment was performed simulating the propagation of a small amplitude wave over a linear slope.

In this simulation the initial conditions are a soliton of the Serre equations [] with an amplitude $0.01m$ on top of quiescent water $1m$ deep. The stage is kept at a fixed $1m$ throughout the scenario but the bed profile changes and can be defined in the following way; (x, z_b) = [(-100m,0m), (100m,0m), (149.5m,0.99m), (250m,0.99m)]. The initial conditions and the numerical solutions for both the Serre equations and the shallow water wave equations are plotted in Figure 2.

It can be seen that the behaviour of the numerical solutions of the Serre equations and the shallow water wave equations are very similar when the ratio of wave amplitude to water depth is small, with the soliton travelling over a large distance compared to the water depth with very little deformation for both equations after $t = 20s$. For the Serre equations the soliton travels without deformation due to a balance between non-linearity and dispersion, for the shallow water wave equations it travels with little deformation because the effects of non-linearity are small as the wave amplitude is small. As the soliton progresses up the slope there is also very little difference between the numerical solutions throughout the shoaling process as can be seen at $t = 40s$. It is only when the bed becomes flat after the slope that differences begin to arise although both numerical solutions are quite similar at $t = 60s$ and $100s$. The main difference between the Serre equations and the shallow water wave equations is the emergence of undulations on top of the bore that has formed with these being physical oscillations that can be seen in well validated numerical solutions of the Serre equations[] and even the Euler equations []. These undulations have the effect of increasing the amplitude of the incoming bore while keeping the mean amplitude of the bore the same as the shallow water wave equations. We have also found that undular bores of the Serre equations travel slightly quicker than the bores of the shallow water wave equations []. This suggests for innundation problems that the Serre equations will produce slightly faster and higher incoming tsunami waves than the shallow water wave equations.

4 Conclusions

References

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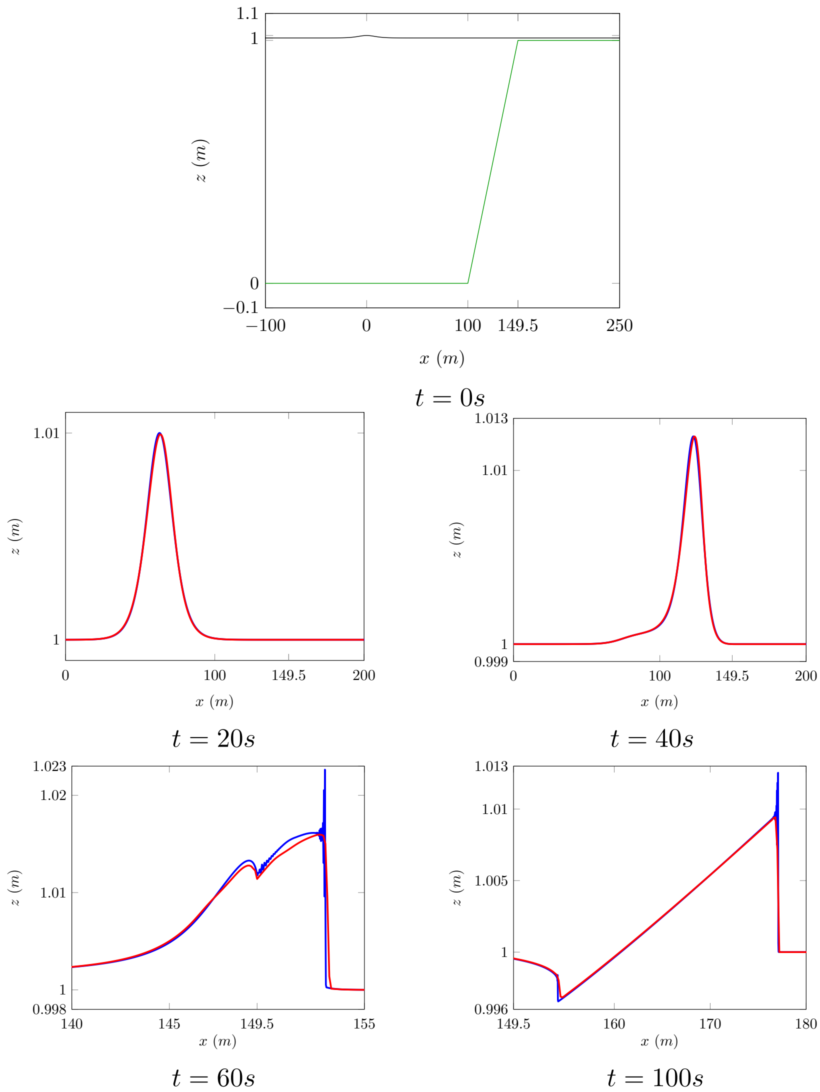

Figure 2. The stage from the numerical solution for the Serre equations (—) and the shallow water wave equations (—) for the soliton travelling over a slope, at different times. The initial conditions for both numerical models are also presented in terms of stage (—) and bed profile (—).