1 Energy in DB

$$\mathcal{H}(t) = \frac{1}{2}\int_{-\infty}^{\infty} hu^2 + gh^2 + \frac{h^3}{3}\left(\frac{\partial u}{\partial x}\right)^2 dx$$

For the Soliton at $t = 0$

$$h(x,0) = a_0 + a_1 sech^2(\kappa x)$$
$$u(x,0) = c\left(1 - \frac{a_0}{a_0 + a_1 sech^2(\kappa x)}\right)$$

$$\kappa = \frac{\sqrt{3a_1}}{2a_0\sqrt{a_0 + a_1}} \tag{1}$$

and

$$c = \sqrt{g(a_0 + a_1)} \tag{2}$$

For $a_0 = 1.0$, $a_1 = 0.7$, $g = 9.81$

$$\kappa = \frac{\sqrt{2.1}}{2\sqrt{1.7}}$$
$$\kappa = \sqrt{\frac{21}{68}}$$

$$c = \sqrt{9.81(1.7)}$$
$$c = \sqrt{16.677}$$

$$h(x,0) = 1 + 0.7 sech^2\left(\sqrt{\frac{21}{68}}x\right)$$

$$u(x,t) = \sqrt{16.677}\left(1 - \frac{1}{1 + 0.7 sech^2\left(\sqrt{\frac{21}{68}}x\right)}\right)$$

$$u(x,t) = \sqrt{16.677}\left(\frac{0.7sech^2\left(\sqrt{\frac{21}{68}}x\right)}{1+0.7sech^2\left(\sqrt{\frac{21}{68}}x\right)}\right)$$

Using Wolfram

$$\int_{-50}^{0} hu^2\,dx = 6.35155$$

$$\int_{0}^{250} hu^2\,dx = 6.35155$$

$$\int_{-50}^{0} gh^2 dx = 520.981$$

$$\int_{0}^{250} gh^2 dx = 2482.98$$

$$\int_{-50}^{0} \frac{h^3}{3}\left(\frac{\partial u}{\partial x}\right)^2 dx = 0.584961$$

$$\int_{0}^{250} \frac{h^3}{3}\left(\frac{\partial u}{\partial x}\right)^2 dx = 0.584961$$

So we have

$$\mathcal{H}(0) = \frac{1}{2}\left[2\times 6.35155 + 2\times 0.584961 + 520.981 + 2482.98\right]$$

$$\mathcal{H}(0) = 6.35155 + 0.584961 + \frac{1}{2}\left[520.981 + 2482.98\right]$$

$$\mathcal{H}(0) = 6.936511 + \frac{1}{2}\left[3003.961\right]$$

$$\mathcal{H}(0) = 6.936511 + 1501.9805$$

$$\mathcal{H}(0) = 1508.917011$$