$$\mathcal{C}_2 = \frac{2\cos(k\Delta x) - 2}{\Delta x^2}$$

$$\mathcal{C}_4 = \frac{-2\cos(2k\Delta x) + 32\cos(k\Delta x) - 30}{12\Delta x^2}$$

We define:

$$\mathcal{G} = \left[H - \frac{H^3}{3}\mathcal{C}\right]$$

$$G_j = \mathcal{G}u_j$$

$$\mathcal{M}_3 = \frac{26 - 2\cos(k\Delta x)}{24}$$

We again will suppress order subscripts further on, but we also have $\mathcal{M}_1 = \mathcal{M}_2 = 1$.

0.0.1 Reconstruction

So we define $\mathcal{R}_1^+ = e^{ik\Delta x}$ and $\mathcal{R}_1^- = 1$.

So we have that

$$\mathcal{R}_2^- = 1 + \frac{i\sin(k\Delta x)}{2}$$

$$\mathcal{R}_2^+ = e^{ik\Delta x}\left(1 - \frac{i\sin(k\Delta x)}{2}\right)$$

$$R_3^- = \frac{\mathcal{M}_3}{6}\left[5 + -e^{-ik\Delta x} + 2e^{ik\Delta x}\right]$$

$$R_3^+ = \frac{\mathcal{M}_3 e^{ik\Delta x}}{6}\left[5 + 2e^{-ik\Delta x} - e^{ik\Delta x}\right]$$

$$R_2^u = \frac{e^{ik\Delta x} - 1}{2}$$

$$R_3^u = \frac{-3e^{2ik\Delta x} + 27e^{ik\Delta x} + 27 - 3e^{-ik\Delta x}}{48}$$

$$\mathcal{F}^{h,u} = H\mathcal{R}^u$$

$$\mathcal{F}^{h,h} = -\frac{\sqrt{gH}}{2}\left[\mathcal{R}^{+} - \mathcal{R}^{-}\right]$$

$$\mathcal{F}^{u,u} = -\frac{\sqrt{gH}}{2}\mathcal{G}\left[\mathcal{R}^{+} \ - \mathcal{R}^{-}\right]$$

$$\mathcal{F}^{u,h} = \frac{gH\mathcal{R}^{-} + gH\mathcal{R}^{+}}{2}$$

Defining $\mathcal{D} = 1 - e^{-ik\Delta x}$