Evolution equations:

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(f(q)\right) = 0 \tag{1}$$

For forcing we get

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(f(q)\right) = H \tag{2}$$

Integrate from $x_{j-1/2}$ to $x_{j+1/2}$

$$\int_{x_{j-1/2}}^{x_{j+1/2}} \frac{\partial q}{\partial t}\ dx + \int_{x_{j-1/2}}^{x_{j+1/2}} \frac{\partial}{\partial x}\left(f(q)\right)\ dx = \int_{x_{j-1/2}}^{x_{j+1/2}} H\ dx \tag{3}$$

Defining

$$\bar{q}^n = \frac{1}{\Delta x}\int_{x_{j-1/2}}^{x_{j+1/2}} q^n\ dx$$

$$\Delta x\left(\frac{\partial \bar{q}}{\partial t}\right) + \big(f(q_{j+1/2}) - f(q_{j-1/2})\big) = \int_{x_{j-1/2}}^{x_{j+1/2}} H\ dx \tag{4}$$

Integrate from t^{n+1} to t^n

$$\Delta x \int_{t^n}^{t^{n+1}} \left(\frac{\partial \bar{q}}{\partial t}\right)\ dt + \int_{t^n}^{t^{n+1}} \big(f(q_{j+1/2}) - f(q_{j-1/2})\big)\ dx = \int_{t^n}^{t^{n+1}}\int_{x_{j-1/2}}^{x_{j+1/2}} H\ dx\ dt \tag{5}$$

$$\Delta x\left(\bar{q}^{n+1} - \bar{q}^n\right) + \int_{t^n}^{t^{n+1}} \big(f(q_{j+1/2}) - f(q_{j-1/2})\big)\ dt = \int_{t^n}^{t^{n+1}}\int_{x_{j-1/2}}^{x_{j+1/2}} H\ dx\ dt \tag{6}$$

$$\Delta x\left(\bar{q}^{n+1} - \bar{q}^n\right) + \int_{t^n}^{t^{n+1}} f(q_{j+1/2})\ dt - \int_{t^n}^{t^{n+1}} f(q_{j-1/2})\ dt = \int_{t^n}^{t^{n+1}}\int_{x_{j-1/2}}^{x_{j+1/2}} H\ dx\ dt \tag{7}$$

Defining

$$F_{i-1/2} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} f(q_{j-1/2})\ dx$$

$$F_{i+1/2} = \frac{1}{\Delta t} \int_{t^n}^{t^{n+1}} f(q_{j+1/2})\ dx$$

$$\Delta x \left(\bar{q}^{n+1} - \bar{q}^{n}\right) + \Delta t \left(F_{i+1/2} - F_{i-1/2}\right) = \int_{t^n}^{t^{n+1}} \int_{x_{j-1/2}}^{x_{j+1/2}} H\ dx\ dt \tag{8}$$

$$\bar{q}^{n+1} = \bar{q}^{n} - \frac{\Delta t}{\Delta x} \left(F_{i+1/2} - F_{i-1/2}\right) + \frac{1}{\Delta x} \int_{t^n}^{t^{n+1}} \int_{x_{j-1/2}}^{x_{j+1/2}} H\ dx\ dt \tag{9}$$

We can break H up further as
$H = \frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(f(q)\right)$
So that

$$Force = \bar{q}^{n+1} - \bar{q}^{n} + \frac{\Delta t}{\Delta x} \left(F_{i+1/2} - F_{i-1/2}\right)$$

where $Force$ uses the analytic values of these quantities rather than the approximate values.