1 Serre Equations

The Serre Equations read (height/mass)

$$\frac{\partial h}{\partial t} + \frac{\partial uh}{\partial x} = 0$$

Phi

$$\Phi = \frac{\partial b}{\partial x}\left(u\frac{\partial u}{\partial x}\right) + u^2\frac{\partial^2 b}{\partial x^2} + \frac{\partial b}{\partial x}\frac{\partial u}{\partial t}$$

Gamma

$$\Gamma = \left(\frac{\partial u}{\partial x}\right)^2 - u\left(\frac{\partial^2 u}{\partial x^2}\right) - \left(\frac{\partial^2 u}{\partial x \partial t}\right)$$

Pressure

$$p|_{\xi} = p_a + \rho g\xi + \frac{\rho}{2}\xi\left(2h - \xi\right)\Gamma + \rho\xi\Phi$$

Momentum(velocity) x

$$\frac{\partial (uh)}{\partial t} + \frac{\partial}{\partial x}\left(u^2 h + \frac{gh^2}{2} + \frac{h^3}{3}\Gamma + \frac{h^2}{2}\Phi\right) + h\frac{\partial b}{\partial x}\left(g + \frac{h}{2}\Gamma + \Phi\right) = 0$$

z

$$w|_{z} = \frac{z-b}{h}\frac{\partial h}{\partial t} + u\frac{\partial b}{\partial x}$$

For a flat bottom we have

$$\frac{\partial h}{\partial t} + \frac{\partial uh}{\partial x} = 0$$

$$h_t + uh_x + hu_x = 0$$

$$\frac{\partial (uh)}{\partial t} + \frac{\partial}{\partial x}\left(u^2 h + \frac{gh^2}{2} + \frac{h^3}{3}\left[\left(\frac{\partial u}{\partial x}\right)^2 - u\left(\frac{\partial^2 u}{\partial x^2}\right) - \left(\frac{\partial^2 u}{\partial x \partial t}\right)\right]\right) = 0$$

$$u_t h + uh_t + 2uhu_x + u^2 h_x + ghh_x + \frac{\partial}{\partial x}\left(\frac{h^3}{3}\left[\left(\frac{\partial u}{\partial x}\right)^2 - u\left(\frac{\partial^2 u}{\partial x^2}\right) - \left(\frac{\partial^2 u}{\partial x \partial t}\right)\right]\right) = 0$$

$$u_t h + u\left(-u_x h - u h_x\right) + 2uhu_x + u^2 h_x + ghh_x + \frac{\partial}{\partial x}\left(\frac{h^3}{3}\left[\left(\frac{\partial u}{\partial x}\right)^2 - u\left(\frac{\partial^2 u}{\partial x^2}\right) - \left(\frac{\partial^2 u}{\partial x \partial t}\right)\right]\right) = 0$$

$$u_t h + uhu_x + ghh_x + \frac{\partial}{\partial x}\left(\frac{h^3}{3}\left[\left(\frac{\partial u}{\partial x}\right)^2 - u\left(\frac{\partial^2 u}{\partial x^2}\right) - \left(\frac{\partial^2 u}{\partial x \partial t}\right)\right]\right) = 0$$

$$u_t h + uhu_x + ghh_x + h^2 h_x\left[u_x^2 - uu_{xx} - u_{xt}\right] + \frac{h^3}{3}\frac{\partial}{\partial x}\left[u_x^2 - uu_{xx} - u_{xt}\right] = 0$$

$$u_t h + uhu_x + ghh_x + h^2 h_x\left[u_x^2 - uu_{xx} - u_{xt}\right] + \frac{h^3}{3}\left[2u_x u_{xx} - u_x u_{xx} - uu_{xxx} - u_{xtx}\right] = 0$$

Remove factor of h

$$u_t + uu_x + gh_x + hh_x\left[u_x^2 - uu_{xx} - u_{xt}\right] + \frac{h^2}{3}\left[u_x u_{xx} - uu_{xxx} - u_{xtx}\right] = 0$$

Venezian, 1974,1975,1976, (Basco)