1 Momentum in Soliton

$$Momentum(t) = \int_{-\infty}^{\infty} uh\,dx$$

For the Soliton at $t = 0$

$$h(x,0) = a_0 + a_1 sech^2\left(\kappa x\right)$$

$$u(x,0) = c\left(1 - \frac{a_0}{a_0 + a_1 sech^2\left(\kappa x\right)}\right)$$

$$\kappa = \frac{\sqrt{3a_1}}{2a_0\sqrt{a_0 + a_1}} \tag{1}$$

and

$$c = \sqrt{g\left(a_0 + a_1\right)} \tag{2}$$

Integrate from x_0 to x_1

$$Momentum(0) = \int_{x_0}^{x_1}\left(c\left(1 - \frac{a_0}{a_0 + a_1 sech^2\left(\kappa x\right)}\right)\left(a_0 + a_1 sech^2\left(\kappa x\right)\right)\right) dx$$

$$Momentum(0) = \int_{x_0}^{x_1} c\left(a_0 + a_1 sech^2\left(\kappa x\right) - a_0\right) dx$$

$$Momentum(0) = \int_{x_0}^{x_1} c\left(a_1 sech^2\left(\kappa x\right)\right) dx$$

$$Momentum(0) = ca_1\left[\frac{\tanh\left(\kappa x\right)}{\kappa}\right]_{x_0}^{x_1}$$