

1 Energy in DB

$$\mathcal{H}(t) = \frac{1}{2} \int_{-\infty}^{\infty} hu^2 + gh^2 + \frac{h^3}{3} \left(\frac{\partial u}{\partial x} \right)^2 dx$$

For the Soliton at $t = 0$

$$h(x, 0) = a_0 + a_1 \text{sech}^2(\kappa x)$$

$$u(x, 0) = c \left(1 - \frac{a_0}{a_0 + a_1 \text{sech}^2(\kappa x)} \right)$$

$$\kappa = \frac{\sqrt{3a_1}}{2a_0\sqrt{a_0 + a_1}} \quad (1)$$

and

$$c = \sqrt{g(a_0 + a_1)} \quad (2)$$

For $a_0 = 1.0$, $a_1 = 0.7$, $g = 9.81$

$$\kappa = \frac{\sqrt{2.1}}{2\sqrt{1.7}}$$

$$\kappa = \sqrt{\frac{21}{68}}$$

$$c = \sqrt{9.81(1.7)}$$

$$c = \sqrt{16.677}$$

$$h(x, 0) = 1 + 0.7 \text{sech}^2 \left(\sqrt{\frac{21}{68}} x \right)$$

$$u(x, t) = \sqrt{16.677} \left(1 - \frac{1}{1 + 0.7 \text{sech}^2 \left(\sqrt{\frac{21}{68}} x \right)} \right)$$

$$\frac{\partial}{\partial x} u(x, t) = - \frac{3.17718 \tanh \left(\sqrt{\frac{21}{68}} x \right) \operatorname{sech}^2 \left(\sqrt{\frac{21}{68}} x \right)}{\left(1 + 0.7 \operatorname{sech}^2 \left(\sqrt{\frac{21}{68}} x \right) \right)^2}$$

Using Wolfram

$$\int g h^2 dx = 9.81x + \tanh(0.555719x) (2.88329 \operatorname{sech}^2(0.555719x) + 30.4805)$$

$$\int h u^2 dx = 21.0068 \tanh(0.555719x) - 19.2569 \tanh^{-1}(0.641689 \tanh(0.555719x))$$

$$\begin{aligned} & \int \frac{h^3}{3} \left(\frac{\partial u}{\partial x} \right)^2 dx \\ &= 307.641 \left(\tanh(0.55719x) (0.049539 - 0.00937224 \operatorname{sech}^2(0.555719x)) \right. \\ & \quad \left. - 0.0625954 \tanh^{-1}(0.641689 \tanh(0.555719x)) \right) \quad (3) \end{aligned}$$