A Multiphysics Sea-Surface Simulation Environment for Modelling and Control Of Marine Craft

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***Abstract*—Many modern marine craft rely on control systems for navigation and stability. However, testing of such systems via sea-trials can be a costly and time-consuming exercise. Boat dynamics simulators offer a quicker and cheaper alternative for control system validation. In this paper, we present the Sheffield Wave Environment Model (SWEM), a sea-surface simulation environment which can be used in conjunction with a boat dynamics simulator. SWEM models the effects of ocean swell, gusting local wind, surface current and finite water depth on sea-surfaces. This paper outlines the physical models used in SWEM as well as providing a tutorial on its functionality.**

I. Introduction

Marine craft such as cruise ships, oil rigs and fishing boats, operate in dynamic and potentially hazardous environments. Unlike for air and ground vehicles, marine craft can experience large perturbations about all six degrees of freedom due to the effects of surface waves, currents and local wind. Therefore, designing control systems for sea-going vehicles can be a complex task, especially for smaller vessels which are often underactuated (i.e. they only have two control inputs - thrust and rudder) and are more sensitive to environmental disturbances. For marine craft, control theory is most commonly used for designing guidance, navigation and control (GNC) systems. The control objectives of GNC systems are numerous and include trajectory tracking, setpoint regulation, station-keeping and path-following [5]. Recently, research into developing GNC systems for unmanned surface vehicles (USVs) has intensified [10], [12]. Such vehicles can be used in defence, search and rescue, and for scientific research. However, testing such systems in sea trials is costly, time-consuming and does not allow for any control over the environmental operating conditions. Alternatively, boat dynamics simulators offer a cheaper and quicker method of GNC system validation and verification. One such simulator is the Marine Systems Simulator (MSS) [1] Matlab/Simulink library. In this paper, we will present a novel sea-surface simulation environment which incorporates a number of physical factors and which could be used as an environmental disturbance block in simulators such as the MSS. We have named this the Sheffield Wave Environment Model (SWEM) and have made it available to the research community as a Matlab/Simulink library.

The MSS currently offers a variety of sea-surface models including JONSWAP [6], Pierson-Moskowitz [19] and Torsethaugen [22] spectral wave models (SWMs); SWMs will be discussed further in later sections. The sea-surface environment model outlined in this paper aims to augment these existing models by combining the effects of ocean swell, gusting local winds, surface currents and finite water depth, into one wave model. This is achieved by collating several existing empirical physical models. This efficient multiphysics wave environment simulator forms the main contribution of this paper. In the following, a brief overview of sea-surface simulation is provided in Section II, the physical models used in SWEM will be discussed in Section III, Section IV includes a tutorial for using SWEM, and concluding remarks are given in Section V.

II. Sea-Surface Simulation

The majority of sea-surface simulation models use simplified forms of the Navier-Stokes equations as their basis. The Navier-Stokes equations are statements pertaining to the conservation of mass and momentum in a fluid. When deriving sea-surface models, simplifying approximations to the Navier-Stokes equations can be made under certain assumptions on the depth of the water and amplitude of the waves. When shallow water is assumed, partial differential equation (PDE) type models can be derived, such as the shallow water equations and Boussinesq equations [15]. However, as these are stiff nonlinear sets of PDEs, they can be computationally expensive to simulate. When small amplitude waves are assumed (relative to wavelength), Linear Wave Theory can be used to simulate sea-surfaces for all but very shallow water depths. This theory, also referred to as Airy Wave Theory, is derived from a one-dimensional linearised velocity potential form of the Navier-Stokes equations. In one dimension, Linear Wave Theory assumes sinusoidal wave profiles travelling in the positive x-direction of the form:

$$\eta(x,t) = a\sin(kx - \omega t), \tag{1}$$

where η is the elevation of the surface relative to the mean water depth, t is time, $a := h/2$ is wave amplitude, h is wave height - the vertical distance from crest to trough, $k := 2\pi/\lambda$ is termed the wavenumber, λ is the wavelength - the horizontal distance between two crests/troughs, $\omega := 2\pi/T$ is angular frequency, and T is the wave period - the time taken for two peaks/troughs to pass through a fixed point in x. An important result from Linear Wave Theory is the linear dispersion relation for surface waves which is given as:

$$\omega^2 = gk\tanh(kd), \tag{2}$$

where g denotes acceleration due to gravity and d is water depth. The dispersion relation in (2) relates the temporal properties of a wave to its spatial properties for a given water depth.

In reality, sea-surfaces are not comprised of a single harmonic term but many random waves of different sizes and shapes travelling in numerous directions. However, Linear Wave Theory can be used to reconstruct such seemingly random two-dimensional surface waves. This is achieved by superimposing multiple two-dimensional sinusoidal basis functions, i.e.:

$$\eta(x,y,t) = \sum_{n_x,n_y} a_{k_x,k_y}\sin\left(k_x x + k_y y - \omega_{k_x,k_y}t + \phi_{k_x,k_y}\right), \tag{3}$$

where $k_x := 2\pi n_x/L_x, k_y := 2\pi n_y/L_y$ are wavenumbers in the x and y horizontal directions, $\sum_{n_x,n_y}$ denotes a double summation over indices $n_x \in [-N_x/2+1, N_x/2]$ and $n_y \in [-N_y/2+1, N_y/2]$, N_x and N_y are the even number of grid points in the x and y directions respectively, L_x and L_y are the dimensions of the simulated sea-surface in the x and y directions respectively, a_{k_x,k_y} is the amplitude of each wave component, ω_{k_x,k_y} is calculated for each wave component using (2) where $k := (k_x^2 + k_y^2)^{1/2}$, and $\phi_{k_x,k_y} \in [0, 2\pi]$ is randomly distributed added phase. The primary inputs needed to simulate a sea-surface using (3) are the wave amplitudes a_{k_x,k_y}. These can be provided by spectral wave models (SWMs) such as the JONSWAP and Torsethaugen SWMs currently used in MSS as mentioned in Section I. These empirical models are developed from measurements and observations of sea and ocean waves under various environmental conditions. SWEM uses two SWMs, namely the TMA [2] and EY [3] spectral wave models. These will be discussed further in Section III.

III. SWEM - Physics

The state of the sea-surface depends on several factors such as tidal and ocean currents, water depth, bathymetry, and most prominently, wind. Waves on a sea-surface form through two mechanisms. Firstly, waves can be locally generated by the shear forces imparted by the locally prevailing wind.

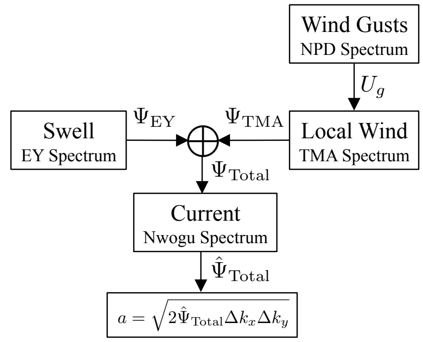

Fig. 1. Schematic of how wave amplitudes are generated in SWEM.

Secondly, waves can be generated by distant weather systems over large distances, meaning that large amplitude waves are possible at a location where there is little or no local wind. This is the mechanism by which ocean waves are formed and is known as ocean swell. SWEM includes the effects of ocean swell, gusting local wind, surface currents and finite water depth, on the surface waves it simulates. Four pre-existing physical models are used to incorporate these effects, namely the NPD wind gust spectrum model [7], the TMA finite water depth SWM [2], the EY unified wind-driven SWM [3], and Nwogu's current influence spectrum [16].

For a given set of inputs such as water depth, windspeed etc., each SWM produces a continuous directional wave energy spectrum $\Psi(k_x, k_y)$. In SWEM, EY and TMA spectra are added together to produce a total continuous wave energy spectrum: $\Psi_{\text{Total}} = \Psi_{\text{EY}} + \Psi_{\text{TMA}}$. This is analogous to the Torsethaugen [22] and Ochi-Hubble [17] multi-peak SWMs which have low-frequency (ocean swell) and high frequency (local wind) spectral peaks. However, neither of the aforementioned SWMs include the effects of finite water depth. The total wave energy spectrum is then multiplied by Nwogu's current influence spectrum $C(k_x, k_y)$ to produce the final continuous wave energy spectrum:

$$\hat{\Psi}_{\text{Total}}(k_x, k_y) = C(k_x, k_y)\Psi_{\text{Total}}(k_x, k_y). \tag{4}$$

The discrete wave amplitudes a_{k_x,k_y} are calculated from (4) via:

$$a_{k_x,k_y} = \sqrt{2\hat{\Psi}_{\text{Total}}(k_x, k_y)\Delta k_x \Delta k_y} \tag{5}$$

where $\Delta k_x, \Delta k_y$ are the discrete wavenumber spacings in the x and y directions respectively. Figure 1 illustrates how the physical models are combined to produce the wave amplitudes a_{k_x,k_y}. In the following, the physical models for ocean swell, gusting local wind, and surface currents will be outlined in more detail.

A. *Ocean Swell*

The EY directional SWM [3] is used to model the effects of ocean swell. It was developed by unifying the results from several pre-existing SWMs into one model, and it is widely used for ocean surface reconstruction [11], [13]. The

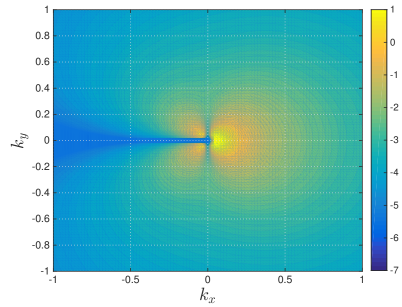

Fig. 2. A contour plot of $\log_{10}\left(\Psi_{\mathrm{EY}}(k_x,k_y)\right)$ for swell windspeed 10m/s, 10^7 km fetch and $d=20m$. The principle wind direction is along the positive x-direction.

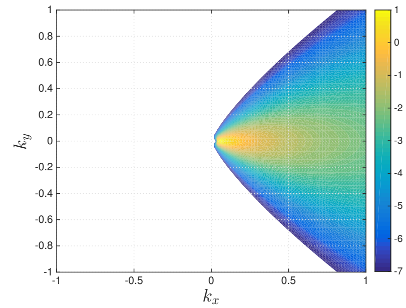

Fig. 3. A contour plot of $\log_{10}\left(\Psi_{\mathrm{TMA}}(k_x,k_y)\right)$ for local windspeed 10m/s, $s=50$ and $d=20m$. The principle wind direction is along the positive x-direction.

mathematical description of the spectrum is beyond the scope of this paper. However, all directional SWMs have the form:

$$\Psi(k_x,k_y)=\Psi(k,\theta)=S(k)D(k,\theta), \tag{6}$$

where $(k_x,k_y) := (k\cos\theta, k\sin\theta)$ relates the cartesian and polar wavenumber coordinates, $S(k)$ is termed the omnidirectional wave spectrum, and $D(k,\theta)$ is called the spreading function. $S(k)$ dictates the amount of energy that is contained in the waves' spatial frequencies, and can be used alone to reconstruct unidirectional surface waves. $D(k,\theta)$ is required to spread the wave energy to waves travelling in directions other than the principle wave direction. For the EY spectrum, $S(k)$ is comprised of a low-frequency wave spectrum, a modified JONSWAP spectrum, and a high-frequency wave spectrum. It incorporates a spreading function which ensures long waves are aligned with the principle wind direction and short waves are more directional; something witnessed from radar observations [3].

In order to compute the spectrum, a windspeed at 10 metres above sea level, which we will term the swell windspeed, a principle wind direction and a fetch is required. The fetch is the distance the waves have travelled under the effects of the wind at the given windspeed. In SWEM, this spectrum is calculated once and, unlike other aspects of the model, does not vary with time. An example EY spectrum is plotted in Figure 2. It can be seen that the EY spectrum is highly directional with a significant portion of energy given to waves travelling in directions far from the principle wind direction.

B. Gusting Local Wind

The TMA SWM [2] is used to model local wind effects on the wave amplitudes. This model is an omnidirectional JONSWAP spectrum, $S(k)$, which has been modified to include the effects of finite water depth. For a given set of input parameters, the peak of the TMA spectrum decreases with reducing water depth, i.e. the waves have less energy. For deep water, $d\to\infty$, the TMA spectrum converges to the JONSWAP spectrum. In order to compute the directional wave spectrum as in (6), a spreading function is required. SWEM uses the "Cosine-2s" spreading function [8] with the TMA omnidirectional spectrum to model local wind effects, which is given as:

$$D(k,\theta)=\left(\frac{2^{(2s-1)}}{\pi}\right)\left(\frac{\Gamma^2(s+1)}{\Gamma(2s+1)}\right)\cos^{2s}\left(\frac{\theta-\theta_0}{2}\right), \tag{7}$$

where s is called the spreading parameter, θ_0 is the principle wind direction, and $\Gamma(\cdot)$ denotes the gamma function [20]. Some researchers have proposed a k-dependence on the spreading parameter s [9]. However, in SWEM a user-specified constant value for s is used. The larger the value of s, the more the waves are aligned with the principle wind direction. It makes physical sense that waves generated by local winds will be closely aligned with the wind direction, therefore a high spreading parameter is recommended (e.g. $s\geq 50$). In order to compute the directional local wind spectrum, a windspeed 10 metres above sea level, which we will term the local windspeed, a principle wind direction, and a spreading parameter is required. An example directional TMA spectrum is plotted in Figure 3. When compared to the EY spectrum plotted in Figure 2, the TMA spectrum is shown to be a lot less directional, with the majority of energy allotted to a narrow band of waves closely aligned with the principle wind direction.

In reality, local winds constantly change in both speed and direction. Therefore, SWEM generates a new local wind spectrum for each time step of the simulation, calculated from a time-varying local windspeed and wind direction. The local windspeed U_l is decomposed in the following manner:

$$U_l(t)=\bar{U}+\lambda_g U_g(t), \tag{8}$$

where $\bar{U}$ is the mean windspeed 10 metres above sea level, U_g is the gusting wind component and λ_g is a gust factor. In a similar fashion to surface waves, several spectral models have been developed for wind gusts. The NPD wind gust spectrum [7] is used in SWEM, which has the form:

$$S(f)=320\frac{\left(\frac{\bar{U}}{10}\right)^2}{(1+x^n)^{\frac{5}{3n}}},\quad x=172f\left(\frac{\bar{U}}{10}\right)^{-\frac{3}{4}}, \tag{9}$$

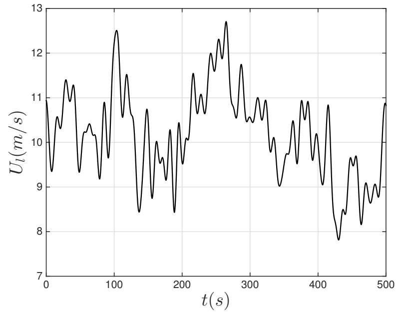

Fig. 4. Local windspeed variation with gusting component calculated from the NPD wind gust spectrum, for $\bar{U} = 10$m/s, $\lambda_g = 1$ and 100 frequency components.

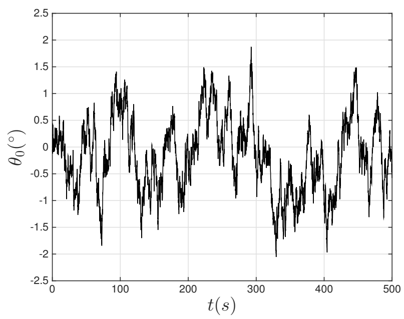

Fig. 5. Local wind angle variation, with fluctuations to the mean calculated using a 1st-order Gauss-Markov process, for $\mu = 0.1$ and $\bar{\theta}_0 = 0$.

where n=0.468 and f is frequency in Hz. The time-varying wind gust component is then calculated from:

$$U_g(t) = \sum_{i=1}^{N_f} \left[\sqrt{2S(f_i)\Delta f} \cos\left(2\pi f_i t + \phi_i\right) \right], \qquad (10)$$

where f_i is the i^{th} discrete sampled frequency component, Δf is the sample frequency interval, and $\phi \in [0, 2\pi]$ is random added phase. The wind gust factor λ_g in (8) can be varied to intensify or attenuate the strength of the gust component. An example plot of the gusting local wind variation is shown in Figure 4.

The local wind direction angle, θ_0, also varies with time. Like for the local windspeed, the wind angle can also be decomposed into a mean component and varying component, i.e. $\theta_0 = \bar{\theta}_0 + \theta_0'$. The varying component is modelled using a 1st-order Gauss-Markov process [4]:

$$\dot{\theta}_0' + \mu\theta_0' = w, \qquad (11)$$

where $\mu > 0$ is a constant fluctuation coefficient and w is white noise. Figure 5 shows an example plot of the wind angle variation.

C. Surface Current

Surface currents can be produced by various environmental forces such as tide, wind and the Coriolis effect. Currents of sufficient speed can have large effects on surface waves. When waves travel against a current, they increase in amplitude and may eventually break. Waves travelling with a current will reduce in amplitude. Nwogu [16] devised a method for altering a wave energy spectrum to include the effects of a surface current. This is achieved by multiplying a wave spectrum with no current, Ψ, with a current influence spectrum C, such that $\hat{\Psi}(k, \theta) = C\Psi(k_0, \theta_0)$. The current influence spectrum is defined as:

$$C(k, k_0, \theta) := \frac{kC_{g0}}{k_0(C_g + U_c\cos\theta)} \left[1 - \frac{U_c k\cos\theta}{\omega}\right], \qquad (12)$$

where $C_g := \partial\omega/\partial k$ is the group velocity of the waves relative to the current, and U_c is the surface current speed; variables with subscript $_0$ are values without the effects of current. The main limitation with the above model is that it produces unrealistically large values for the directional energy spectrum as $C_g + U_c\cos\theta$ approaches zero. Waves can no longer penetrate the current when the limit $C_g = -U_c\cos\theta$ is reached and in reality, would break before reaching that point. SWEM neither models nor simulates breaking waves. Therefore, Nwogu's current influence spectrum is modified in two ways. Firstly, the amplitude of "breaking waves" is set to zero, i.e. $C = 0$ if $C_g \leq -U_c\cos\theta$. Secondly, saturation limits are applied to the higher wavenumbers in the spectrum to prevent unphysical amplitudes of high spatial frequency waves. This is achieved by multiplying the current influence spectrum by k_0^{-5} for wavenumbers $k_0 > 1$. Figure 6 shows the effects of current on the EY spectrum shown in Figure 2. The current is travelling in the opposite direction to the principle swell wind direction. This has the effect of reducing the directionality of the spectrum and increasing the energy of the lower spatial frequency wave components. The current direction angle, θ_c, varies with time in the same manner as the local wind direction angle.

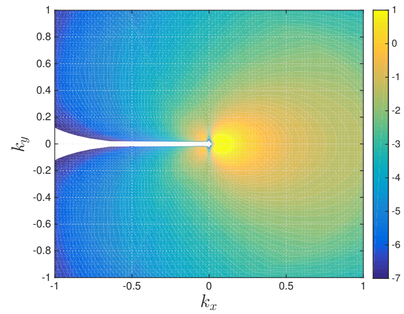

Fig. 6. A contour plot of $\log_{10}(\hat{\Psi}_{\text{EY}}(k_x, k_y))$ for surface current speed $U_c = 2$m/s, swell windspeed 10m/s, 10^7 km fetch and $d = 20m$. The principle swell wind direction is along the positive x-direction and the principle current direction is along the negative x-direction.

IV. SWEM - Tutorial

SWEM requires Matlab R2014b or later and an up to date C-compiler to run. The latest version of SWEM can be downloaded as a zip file from

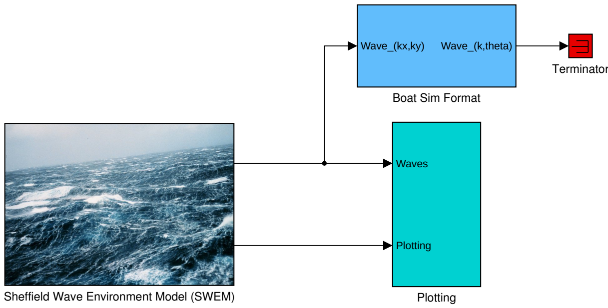

Fig. 7. The SWEM block set viewed in Simulink.

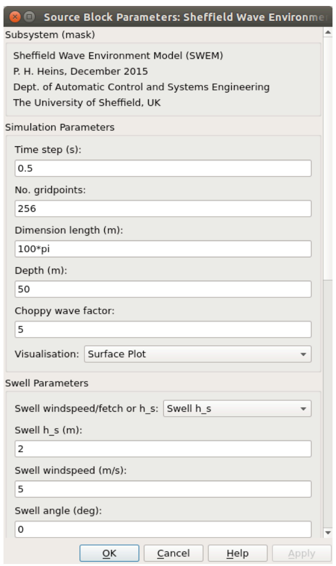

Fig. 8. The SWEM parameters menu.

`github.com/P-Heins/SWEM/releases`. Once downloaded, open Matlab and move `SWEM-x.x.zip` to the working directory. Unzipping this file will generate a folder containing the Simulink model `SWEM.mdl` and a subfolder titled `SWEM_Functions`. Right-click on `SWEM_Functions` and select `Add to Path`. Double-clicking on `SWEM.mdl` will open the model in Simulink and should show the four blocks in Figure 7. Before using SWEM, first configure Simulink by going to `Simulation/Model Configuration Parameters`. In the `Solver` tab, choose `Fixed-step` as the solver type from the drop-down menu and enter a `Fixed-step size`. In the `Hardware Implementation` tab, ensure that the number of bits assigned to `native` is equal to the number assigned to `long`; changing `native` if required. Once finished, close this menu.

Double-clicking on the `SWEM` block will open the SWEM parameters menu shown in Figure 8. The simulated sea-surface is uniquely determined by specifying these parameters. The menu is spit into four sections. The first section, `Simulation Parameters`, consists of general parameters required to run and visualise the simulation. All SWEM simulations require a fixed time step which can be any positive real number. The time step inputted into the SWEM parameters menu must match the previously set `Fixed-step size`. SWEM visualises simulated sea-surfaces on square grids. Therefore, the `No. of gridpoints`: $N = N_x = N_y$. Similarly, the `Dimension length`: $L = L_x = L_y$. `Depth` should be a positive real number.

A "choppy wave" model (CWM) [14] is used in SWEM for visualisation purposes only. The CWM transforms linear two-dimensional surface waves generated from (3) into weakly nonlinear three-dimensional surface waves. The sea-surfaces generated using the CWM appear more physically realistic hence its use for visualisation. Boat dynamics simulators tend to only model the effects of linear waves and therefore the CWM is not incorporated into the `Boat Sim Format` block. The `Choppy wave factor` is used to tune the "choppiness" of the waves [21]. The higher the factor, the sharper the waves' peaks appear to be. SWEM currently offers two visualisation options: a contour plot or a surface plot of wave elevations $\eta(x, y, t)$. Both also indicate the directions of the swell, local wind and current, and state the simulation time, significant wave height h_s (defined as the mean of the one-third largest waves), and the sea state. The sea state is estimated using the Beaufort wind scale and Douglas sea scale [18] and is based solely on h_s.

The swell component of the sea-surface is specified in the `Swell Parameters` section of the menu. The magnitude of the swell can be determined either by choosing a swell significant wave height `Swell h_s`, or by choosing a `Swell windspeed` and `Swell fetch`. This must be specified in the drop-down menu. `Swell angle` selects the direction of the swell. All direction angles, in degrees, are defined as increasing clock-wise from zero with respect to the x-axis. The `Local Wind Parameters` specify the magnitude and direction of the time-varying gusting local wind. The `Mean local windspeed` and `Gust factor` are defined in (8). The `Gust freq. spacing`, Δf, and the `No. of gusting freq. components`, N_f, are used to discretise frequency to form the NPD wind gust spectrum in (10), such that $f_i = n_i \Delta f$ for $n_i = 1, 2 \cdots N_f$. `Mean local wind angle` and `Wind angle fluctuation coeff.` are defined in (11). The `Current speed` and `Mean current angle` are set in the `Current Parameters` section of the menu. The `Current angle fluctuation coeff.` is analogous to the `Wind angle fluctuation coeff.` as defined in (11).

Once the desired parameters have been inserted into the menu, click on `Apply` and then `OK`. Then, to run the simulation, select `Simulation/Run`. Example visualisations from SWEM simulations are shown in Figures 9 and 10. The

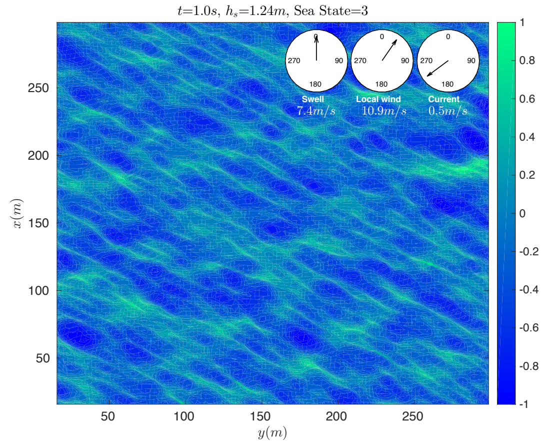

Fig. 9. A contour plot of wave elevations η from a SWEM simulation.

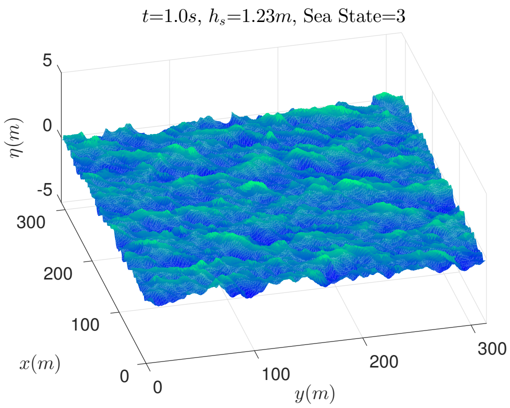

Fig. 10. A surface plot from a SWEM simulation.

contour plot in Figure 9 includes three dials in the top-right corner. These indicate the directions that the swell, local wind and current are moving towards. The numerical values below these dials indicate, from left to right, the offshore windspeed used to generate the swell, the local windspeed, and the surface current speed.

V. Conclusion

In this paper, a new multiphysics sea-surface simulation environment has been presented called the Sheffield Wave Environment Model (SWEM). SWEM has been developed to be used as an environmental disturbance block in boat dynamics simulators such as MSS [1], and is capable of modelling the effects of ocean swell, gusting local wind, surface current and finite water depth on the sea-surfaces it simulates. The four preexisting physical models incorporated into SWEM have been briefly discussed and a tutorial on the functionality of SWEM has been included. In future work, SWEM shall be used as a tool for the design and testing of USV control systems.

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